BEATTY SECONDARY SCHOOL
MID-YEAR EXAMINATION 2015

SUBJECT : Additional Mathematics

LEVEL : Sec 3 Express

PAPER : 4047 / 1

DURATION : 2 hours

SETTER : Mdm Ong Hai Lee

DATE : 14 May 2015

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This paper consists of 5 printed pages (including this cover page)

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the coordinates of the points of intersection of the line $x + 2y = 5$ and the curve $x^2 + 4y^2 + 12x - 29 = 0$. [5]

2 The expression $(2a - 2)x^3 - (b + 6)x^2 + 13x + b$, where a and b are constants, has a factor of $2x - 1$ and leaves a reminder of -45 when divided by $x + 2$.

(i) Find the value of a and of b. [4]

(ii) Using the value of a and of b found in part **(i)**, show that the equation $(2a - 2)x^3 - (b + 6)x^2 + 13x + b = 0$ has only one real root. [2]

3 Given that $x^5 + ax^3 + bx^2 - 3 = (x^2 - 1)Q(x) - x - 2$, where $Q(x)$ is a polynomial.

(i) State the degree of $Q(x)$. [1]

(ii) Find the value of a and of b. [4]

(iii) Using the value of a and of b found in part **(ii)**, find the remainder when $x^5 + ax^3 + bx^2 - 3$ is divided by $x + 2$. [1]

4 **(a)** Factorise completely $27x^3 - (x - 1)^3$. [2]

(b) Express $\dfrac{2x^3 + 5x^2 - 8x + 4}{(2x - 1)(x^2 + 1)}$ in partial fractions. [6]

5 **(a)** Given that $y = ax^b + 2$ and that $y = 9$ when $x = 2$, and $y = 30$ when $x = 4$, find the value of a and of b. [4]

(b) Given that $\dfrac{p^{13-y}q^{x+5}}{\sqrt[4]{q^{8y}}} = (p^{2x+3})^2$, find the value of x and of y. [5]

[Turn over

6 **(a)** Solve the equation $3^{2x+2} - 10(3^x) + 1 = 0$. [4]

(b) Without using a calculator, evaluate 15^x given that
$3^{2x-1} \times 5^{4-x} = 3^{x+3} \times 5^{3-2x}$. [3]

7 The quadratic equation $2x^2 + x - 4 = 0$ has roots α and β.

(i) Find the value of $\alpha^3 + \beta^3$. [3]

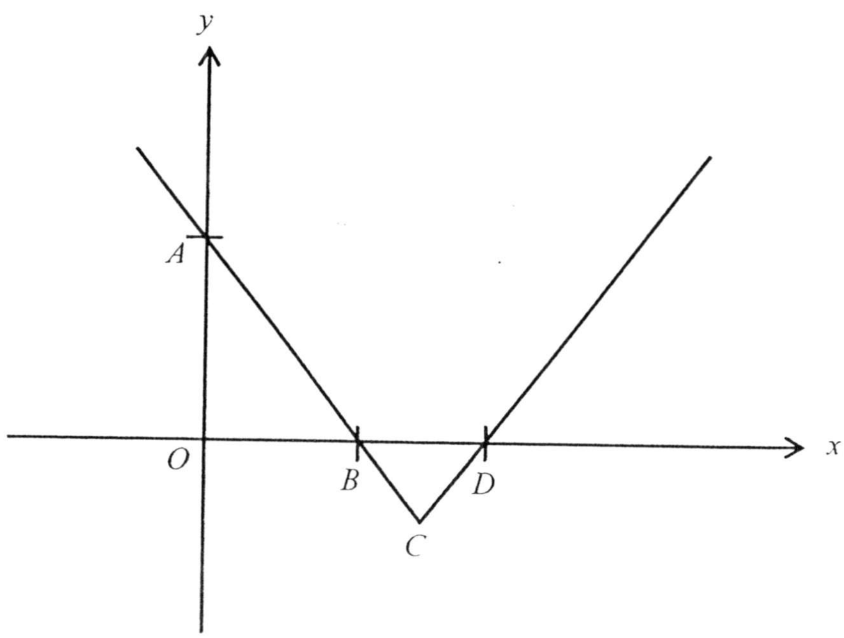
(ii) Hence find the quadratic equation whose roots are $(\alpha^3 + 1)$ and $(\beta^3 + 1)$. [3]

8 **(a)** A curve has the equation $y = kx^2 + (2k-4)x + 3k - 2$, where $k > 0$.
Find the set of values of k for which the curve lies completely above the x-axis. [3]

(b) Show that the equation $3x^2 + mx + m - 5 = 0$ has real and distinct roots for all real values of m. [3]

9 **(a)** Find the set of values of x for which $2x(x+2) - 1 \leq (x+1)(x+3)$. [3]

(b) The diagram shows part of the graph of $y = |4x - 3| - 1$. Find the coordinates of the points A, B, C and D. [4]



10 A curve has the equation $y = -2x^2 + 6x - 5$.

(i) Express $y = -2x^2 + 6x - 5$ in the form of $y = a(x-b)^2 + h$, where a, b and h are real numbers. [2]

(ii) State the turning point of the curve. [1]

(iii) Sketch the graph of $y = -2x^2 + 6x - 5$. [2]

(iv) Using your graph, state the number of solution(s) to each of the following equations.

(a) $|-2x^2 + 6x - 5| = 2$ [1]

(b) $|-2x^2 + 6x - 5| = -3$ [1]

11 (i) Obtain the first four terms of the binomial expansion of $(1+p)^7$, in ascending powers of p. [1]

(ii) Hence,

(a) find the value of $(1.01)^7$ correct to 3 decimal places. [2]

(b) by taking $p = x - x^2$, obtain the expansion of $(1 + x - x^2)^7$ as far as the term in x^3. [2]

12 (a) Find the coefficient of x^3 in the binomial expansion of $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$. [3]

(b) The coefficients of x and x^2 in the binomial expansion of $(3+2x)^n$ are equal.

(i) Find the value of n. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\left(2x - \frac{1}{x}\right)(3+2x)^n$. [2]

~~~ **End of Paper** ~~~
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Answer Keys:

1 $(1, 2)$ and $\left(-2, \frac{7}{2}\right)$

2(i) $a = 2$ and $b = -7$

3(i) 3 **(ii)** $a = -2$ and $b = 1$ **(iii)** -15

4(a) $(2x+1)(13x^2 - 5x + 1)$ **(b)** $1 + \frac{6}{5(2x-1)} + \frac{12x-19}{5(x^2+1)}$

5(a) $a = \frac{7}{4}$ and $b = 2$ **(b)** $x = 1$ and $y = 3$

6(a) $x = -2$ or 0 **(b)** $15^x = \frac{81}{5}$

7(i) $-\frac{25}{8}$ **(ii)** $8x^2 + 9x - 65 = 0$

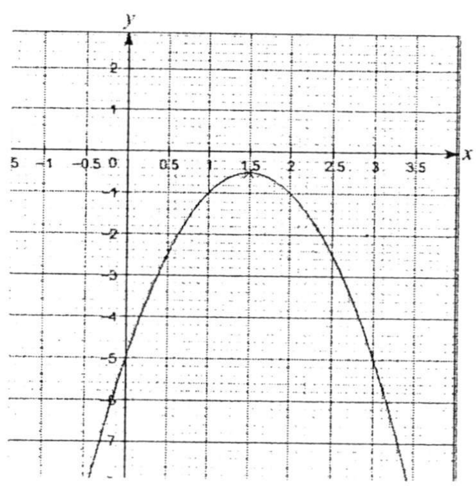
8(a) $k < -2$ (rejected) or $k > 1$

9(a) $-2 \le x \le 2$ **(b)** $A(0, 2)$, $B(0, \frac{1}{2})$, $C(\frac{3}{4}, -1)$ and $D(0, 1)$

10(i) $y = -2\left(x - \frac{3}{2}\right)^2 - \frac{1}{2}$

(ii) $\left(\frac{3}{2}, -\frac{1}{2}\right)$

(iii)



(iv)(a) 2 **(b)** 0

11(i) $1 + 7p + 21p^2 + 35p^3 + \ldots$

(ii)(a) 1.072

(ii)(b) $1 + 7x + 14x^2 - 7x^3 + \ldots$

12(a) $-24\frac{3}{4}$

(b) $n = 4$, 432

2015 Mid-Year Examination Secondary 3 Express Additional Mathematics Paper One

1 Find the coordinates of the points of intersection of the line $x + 2y = 5$ and the curve $x^2 + 4y^2 + 12x - 29 = 0$. [5]

Solution:

$x + 2y = 5$ ... (1)

$x^2 + 4y^2 + 12x - 29 = 0$ ... (2)

From (1), $x = 5 - 2y$ ... (3)

Sub. (3) into (2),

$(5 - 2y)^2 + 4y^2 + 12(5 - 2y) - 29 = 0$ **... [M1, substitute]**

$25 - 20y + 4y^2 + 4y^2 + 60 - 24y - 29 = 0$

$8y^2 - 44y + 56 = 0$

$2y^2 - 11y + 14 = 0$ **... [A1, expand and simplify]**

$(2y - 7)(y - 2) = 0$ **... [√M1, factorise]**

$y = \frac{7}{2}$ or $y = 2$

When $y = 2$, $x = 5 - 2(2) = 1$

When $y = \frac{7}{2}$, $x = 5 - 2\left(\frac{7}{2}\right) = -2$

Therefore the coordinates of the points of intersection are (1 , 2) and $\left(-2, \frac{7}{2}\right)$

... [A1, A1]

[minus 1m if answers not in coordinates form]

4

2 The expression $(2a-2)x^3-(b+6)x^2+13x+b$, where a and b are constants, has a factor of $2x-1$ and leaves a reminder of -45 when divided by $x+2$.

(i) Find the value of a and of b. [4]

(ii) Using the value of a and of b found in part **(i)**, show that the equation $(2a-2)x^3-(b+6)x^2+13x+b=0$ has only one real root. [2]

Solution:

(i) Let $f(x)=(2a-2)x^3-(b+6)x^2+13x+b$

Since $(2x-1)$ is a factor of $f(x)$, $f\left(\frac{1}{2}\right)=0$

$(2a-2)\left(\frac{1}{2}\right)^3-(b+6)\left(\frac{1}{2}\right)^2+13\left(\frac{1}{2}\right)+b=0$ **... [M1, correct substitution]**

$\frac{1}{4}a-\frac{1}{4}-\frac{1}{4}b-\frac{3}{2}+\frac{13}{2}+b=0$

$\frac{1}{4}a+\frac{3}{4}b=-\frac{19}{4}$

$a+3b=-19$ ... (1)

When divided by $(x+2)$, $f(-2)=-45$

$(2a-2)(-2)^3-(b+6)(-2)^2+13(-2)+b=-45$

... [M1, correct substitution]

$-16a+16-4b-24-26+b=-45$

$16a+3b=11$ ... (2)

(2) – (1), $15a=30$,

$a=2$ **... [A1]**

Sub $a=2$ into (1), $2+3b=-19$

$b=-7$ **... [A1]**

(ii) Sub. $a=2$ and $b=-7$ into $(2a-2)x^3-(b+6)x^2+13x+b=0$,

$2x^3+x^2+13x-7=0$

$(2x-1)(x^2+x+7)=0$ **... [A1]**

For $x^2+x+7=0$, Discriminant $=(1)^2-4(1)(7)<0$

So $x^2+x+7=0$ so no solution

Hence $2x^3+x^2+13x-7=0$ has only 1 solution. **... [√B1, or solve with quadratic formula]**

3 Given that $x^5 + ax^3 + bx^2 - 3 = (x^2 - 1)Q(x) - x - 2$, where $Q(x)$ is a polynomial.

(i) State the degree of $Q(x)$. [1]

(ii) Find the value of a and of b. [4]

(iii) Using the value of a and of b found in part **(ii)**, find the remainder when $x^5 + ax^3 + bx^2 - 3$ is divided by $x + 2$. [1]

Solution:

(i) degree of $Q(x) = 3$ ... **[B1]**

(ii) Let $x = 1$,

$(1)^5 + a(1)^3 + b(1)^2 - 3 = 0 - 1 - 2$ ... **[M1, correct substitution]**

$a + b = -1$ ... (1)

Let $x = -1$,

$(-1)^5 + a(-1)^3 + b(-1)^2 - 3 = 0 - (-1) - 2$ ... **[M1, correct substitution]**

$-a + b = 3$ ... (3)

(1) + (2), $2b = 2$

$b = 1$ ... **[A1]**

Sub $b = 1$ into (1),

$a = -2$ ... **[A1]**

(iii) Hence $x^5 + ax^3 + bx^2 - 3 = x^5 - 2x^3 + x^2 - 3$

When divided be $x + 2$,

remainder $= (-2)^5 - 2(-2)^3 + (-2)^2 - 3$

$= -15$ ... **[√B1]**

5

4 **(a)** Factorise completely $27x^3-(x-1)^3$. [2]

(b) Express $\dfrac{2x^3+5x^2-8x+4}{(2x-1)(x^2+1)}$ in partial fractions. [6]

Solution:

(a)

$$27x^3-(x-1)^3 = (3x)^3-(x-1)^3$$

$$= [(3x-(x-1)][(3x)^2+(3x)(x-1)+(x-1)^2] \quad \text{... [M1]}$$

$$= (2x+1)(9x^2+3x^2-3x+x^2-2x+1)$$

$$= (2x+1)(13x^2-5x+1) \quad \text{... [A1]}$$

(b)

$$\frac{2x^3+5x^2-8x+4}{(2x-1)(x^2+1)} = \frac{2x^3+5x^2-8x+4}{2x^3-x^2+2x-1}$$

$$= 1+\frac{6x^2-10x+5}{(2x-1)(x^2+1)} \quad \text{... [M1]}$$

$$\frac{6x^2-10x+5}{(2x-1)(x^2+1)} = \frac{A}{2x-1}+\frac{Bx+C}{x^2+1}$$

$$6x^2-10x+5 = A(x^2+1)+(Bx+C)(2x-1) \quad \text{... [M1]}$$

Let $x=\frac{1}{2}$, $A=\frac{6}{5}$ ... [A1]

Let $x=0$, $C=-\frac{19}{5}$ ... [A1]

Let $x=1$, $B=\frac{12}{5}$ ... [A1]

$$\frac{2x^3+5x^2-8x+4}{(2x-1)(x^2+1)} = 1+\frac{6}{5(2x-1)}+\frac{12x-19}{5(x^2+1)} \quad \text{... [A1]}$$

5 **(a)** Given that $y = ax^b + 2$ and that $y = 9$ when $x = 2$, and $y = 30$ when $x = 4$, find the value of a and of b. [4]

(b) Given that $\dfrac{p^{13-y}q^{x+5}}{\sqrt[4]{q^{8y}}} = \left(p^{2x+3}\right)^2$, find the value of x and of y. [5]

Solution:

(a) When $x = 2$ and $y = 9$, $9 = a(2)^b + 2$

$a(2)^b = 7$ ...(1) ... **[M1, substitution]**

When $x = 4$ and $y = 30$, $30 = a(4)^b + 2$

$a(4)^b = 28$ ... (2) ... **[M1, substitution]**

$\dfrac{(2)}{(1)}$, $\dfrac{a(4)^b}{a(2)^b} = \dfrac{28}{7}$

$(2)^b = 4$

$b = 2$ ... **[A1]**

Sub. $b = 2$ into (1), $a(2)^2 = 7$

$a = \dfrac{7}{4}$ ... **[A1]**

(b) $\dfrac{p^{13-y}q^{x+5}}{\sqrt[4]{q^{8y}}} = \left(p^{2x+3}\right)^2$

$\dfrac{p^{13-y}q^{x+5}}{q^{2y}} = p^{4x+6}$ ... **[M1, power law]**

... **[M1, fractional indices]**

$p^{13-y}q^{x+5} = p^{4x+6}q^{2y}$

Hence, $13 - y = 4x + 6$

$y = 7 - 4x$ ... (1)

And $x + 5 = 2y$ ... (2)

(equations (1) and (2)) ... **[M1, compare indices]**

Sub. (1) into (2), $x + 5 = 2(7 - 4x)$

$x = 1$ ... **[A1]**

Sub $x = 1$ into (1), $y = 3$ ... **[A1]**

6

6 (a) Solve the equation $3^{2x+2} - 10(3^x) + 1 = 0$. [4]

(b) Without using a calculator, evaluate 15^x given that

$$3^{2x-1} \times 5^{4-x} = 3^{x+3} \times 5^{3-2x}.$$ [3]

Solution:

(a) $3^{2x+2} - 10(3^x) + 1 = 0$

Let $u = 3^x$, $\quad 9u^2 - 10u + 1 = 0$ ... **[M1, substitution]**

$(9u-1)(u-1) = 0$

$u = \frac{1}{9}$ or $u = 1$ ... **[M1]**

Hence, $3^x = \frac{1}{9}$ or $3^x = 1$

$3^x = 3^{-2}$ or $3^x = 3^0$

$x = -2$ or $x = 0$ ... **[A1, A1]**

(b) $3^{2x-1} \times 5^{4-x} = 3^{x+3} \times 5^{3-2x}$

$\frac{3^{2x-1}}{3^{x+3}} = \frac{5^{3-2x}}{5^{4-x}}$

$3^{x-4} = 5^{-x-1}$ ... **[M1, law of indices]**

$\frac{3^x}{3^4} = \frac{1}{5^x \times 5}$

$3^x \times 5^x = \frac{3^4}{5}$ ... **[M1]**

$15^x = \frac{81}{5}$ ... **[A1]**

7 The quadratic equation $2x^2 + x - 4 = 0$ has roots α and β .

(i) Find the value of $\alpha^3 + \beta^3$. [3]

(ii) Hence find the quadratic equation whose roots are $(\alpha^3 + 1)$ and $(\beta^3 + 1)$. [3]

Solution:

(i) $2x^2 + x - 4 = 0$ has roots α and β

Sum of roots $= \alpha + \beta = -\frac{1}{2}$ ... **[B1]**

Product of roots $= \alpha\beta = \frac{-4}{2} = -2$ ... **[B1]**

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$$

$$= (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$$

$$= \left(-\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right)^2 - 3(-2)\right) = -\frac{25}{8}$$ ... **[B1]**

(ii) $(\alpha^3 + 1) + (\beta^3 + 1) = \alpha^3 + \beta^3 + 2$

$$= -\frac{25}{8} + 2 = -\frac{9}{8}$$ ... **[√M1]**

$(\alpha^3 + 1)(\beta^3 + 1) = \alpha^3\beta^3 + \alpha^3 + \beta^3 + 1$

$$= (-2)^3 + \left(-\frac{25}{8}\right) + 1$$

$$= -\frac{81}{8}$$ ... **[√M1]**

Equation: $x^2 + \frac{9}{8}x - \frac{81}{8} = 0$

$8x^2 + 9x - 81 = 0$ ... **[A1]**

7

8 **(a)** A curve has the equation $y = kx^2 + (2k-4)x + 3k - 2$, where $k > 0$.
Find the set of values of k for which the curve lies completely above the x-axis. [3]

(b) Show that the equation $3x^2 + mx + m - 5 = 0$ has real and distinct roots for all real values of m. [3]

Solution:

(a) $y = kx^2 + (2k-4)x + 3k - 2$ where $k > 0$

Since curve lies completely above $y = 0$, $b^2 - 4ac < 0$

$(2k-4)^2 - 4(k)(3k-2) < 0$ ... **[M1]**

$4k^2 - 16k + 16 - 12k^2 + 8k < 0$

$-8k^2 - 8k + 16 < 0$ ... **[√M1]**

$k^2 + k - 2 > 0$

$(k+2)(k-1) > 0$

$k < -2$ (rejected) or $k > 1$ ... **[A1, must reject $k < -2$]**

(b) $3x^2 + mx + m - 5 = 0$

Discriminant, $b^2 - 4ac = m^2 - 4(3)(m-5)$ ... **[M1]**

$= m^2 - 12m + 60$

$= m^2 - 12m + 6^2 - 6^2 + 60$

$= (m-6)^2 + 24$ ... **[A1]**

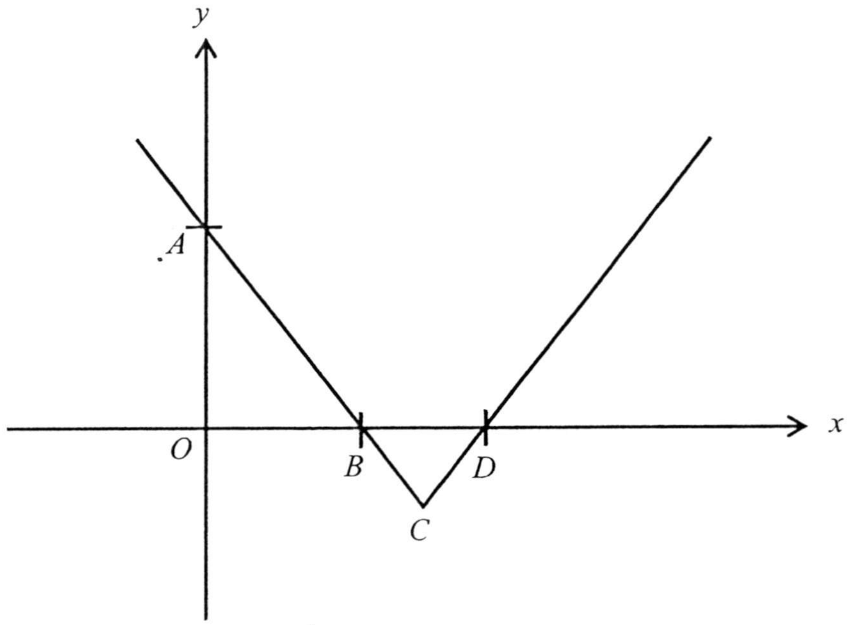
Since $(m-6)^2 \geq 0$, $(m-6)^2 + 24 > 0$

So discriminant > 0, hence $3x^2 + mx + m - 5 = 0$

has real and distinct roots for all real values of m. ... **[B1]**

9 **(a)** Find the set of values of x for which $2x(x+2)-1 \le (x+1)(x+3)$. [3]

(b) The diagram shows part of the graph of $y = |4x-3|-1$. Find the coordinates of the points A, B, C and D. [4]



Solution:

(a)
$$2x(x+2)-1 \le (x+1)(x+3)$$
$$2x^2+4x-1 \le x^2+4x+3$$
$$x^2-4 \le 0$$ ... **[M1, expand and simplify]**
$$(x+2)(x-2) \le 0$$ ... **[M1, factorise]**
$$-2 \le x \le 2$$ ... **[A1]**

(b) When $x = 0$, $y = |-3|-1 = 2$, hence $A(0, 2)$ ... **[B1]**

When $y = 0$, $|4x-3|-1 = 0$

$4x-3 = \pm 1$

$x = \frac{1}{2}$ or 1, hence $B(\frac{1}{2}, 0)$ and $D(1, 0)$

... **[B1, B1]**

$$x = \frac{\frac{1}{2}+1}{2} = \frac{3}{4} \text{ and } y = \left|4\left(\frac{3}{4}\right)-3\right|-1 = -1$$

Hence $C(\frac{3}{4}, -1)$ ... **[B1]**

8

10 A curve has the equation $y = -2x^2 + 6x - 5$.

(i) Express $y = -2x^2 + 6x - 5$ in the form of $y = a(x-b)^2 + h$, where a, b and h are real numbers. [2]

(ii) State the turning point of the curve. [1]

(iii) Sketch the graph of $y = -2x^2 + 6x - 5$. [2]

(iv) Using your graph, state the number of solution(s) to each of the following equations.

(a) $|-2x^2 + 6x - 5| = 2$ [1]

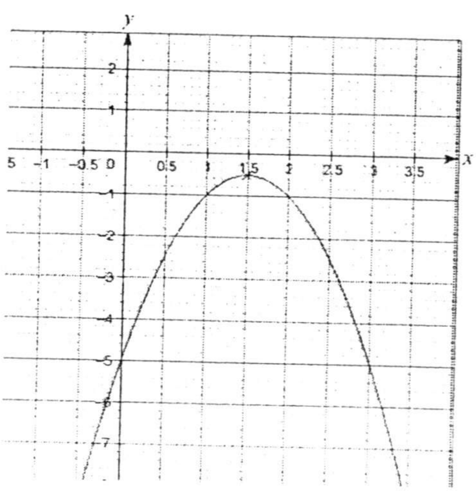
(b) $|-2x^2 + 6x - 5| = -3$ [1]

Solution:

(i)
$$y = -2x^2 + 6x - 5$$
$$y = -2\left(x^2 - 3x + \frac{5}{2}\right)$$
$$y = -2\left(x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{2}\right) \quad \text{... [M1]}$$
$$y = -2\left(\left(x - \frac{3}{2}\right)^2 + \frac{1}{4}\right)$$
$$y = -2\left(x - \frac{3}{2}\right)^2 - \frac{1}{2} \quad \text{... [A1]}$$

(ii) turning point is $\left(\frac{3}{2}, -\frac{1}{2}\right)$ ... [√B1]

(iii) [G1] – shape and correct y-intercept
]G1] – correct turning point



(iv)(a) 2 solutions ... [B1]

(iv)(b) 0 solution ... [B1]

11 **(i)** Obtain the first four terms of the binomial expansion of $(1+p)^7$, in ascending powers of p. [1]

(ii) Hence,

(a) find the value of $(1.01)^7$ correct to 3 decimal places. [2]

(b) by taking $p = x - x^2$, obtain the expansion of $\left(1 + x - x^2\right)^7$ as far as the term in x^3. [2]

Solution:

(i) $(1+p)^7 = 1 + 7p + 21p^2 + 35p^3 + \ldots$ **... [B1]**

(ii)

(a) Taking $x = 0.01$,

$(1.01)^7 = 1 + 7(0.01) + 21(0.01)^2 + 35(0.01)^3 + \ldots$ **... [M1]**

$= 1.072135$

$= 1.072$ (3 d p) **... [A1]**

(b) $(1 + x - x^2)^7 = 1 + 7(x - x^2) + 21(x - x^2)^2 + 35(x - x^2)^3 + \ldots$ **... [M1]**

$= 1 + 7x - 7x^2 + 21(x^2 - 2x^3 + x^4) + 35x^3 + \ldots$

$= 1 + 7x - 7x^2 + 21x^2 - 42x^3 + 21x^4 + 35x^3 + \ldots$

$= 1 + 7x + 14x^2 - 7x^3 + \ldots$ **... [A1]**

12 **(a)** Find the coefficient of x^3 in the binomial expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$. [3]

(b) The coefficients of x and x^2 in the binomial expansion of $(3+2x)^n$ are equal.

(i) Find the value of n. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\left(2x-\frac{1}{x}\right)(3+2x)^n$. [2]

Solution:

(a) $T_{r+1} = \binom{12}{r}\left(\frac{1}{x}\right)^{12-r}\left(-\frac{x^2}{2}\right)^r$ ... **[M1]**

$= \binom{12}{r}\left(-\frac{1}{2}\right)^r (x)^{3r-12}$

Let $3r-12=3$, $r=5$ ... **[M1]**

Hence coefficient of $x^3 = \binom{12}{5}\left(-\frac{1}{2}\right)^5 = -24\frac{3}{4}$ ... **[A1]**

(b)(i) $(3+2x)^n = (3)^n + \binom{n}{1}(3)^{n-1}(2x)^1 + \binom{n}{2}(3)^{n-2}(2x)^2 + \ldots$ ... **[M1]**

Coefficient of x = Coefficient of x^2

$\binom{n}{1}(3)^{n-1}(2) = \binom{n}{2}(3)^{n-2}(2)^2$

$n(3)^{n-1}(2) = \frac{n(n-1)}{2}(3)^{n-2}(4)$ ... **[M1]**

$1 = \frac{(n-1)}{3}$

$n = 4$ ... **[A1]**

(b)(i) Hence $\left(2x-\frac{1}{x}\right)(3+2x)^n$

$= \left(2x-\frac{1}{x}\right)(81+216x+216x^2+96x^3+\ldots)$ ... **[M1]**

$= 432x^2 - 96x^2 + \ldots$

$= 336x^2$

Therefore, coefficient of $x^2 = 336$ ... **[A1]**

~ **End of Paper** ~

Class	Register No	Name

Bukit Merah Secondary School

Mid-Year Examination 2015

E

Secondary 3 Express

ADDITIONAL MATHEMATICS

4047

Additional Material : Writing Paper (6 sheets)

Cover Page

12 May 2015

2 hours

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of the marks for this paper is **80**.

This document consists of 4 printed pages.

[Turn Over

10

1. Find the coordinates of the points of intersection of the line $2x+3y=4$ and the curve $(2x+1)^2+5(y-2)^2=45$. [4]

2. The roots of the quadratic equation $2x^2-8x+5=0$ are α and β.

(i) State the value of $\alpha+\beta$ and of $\alpha\beta$. [2]

(ii) Find the quadratic equation in x whose roots are $\frac{2}{\beta^3}$ and $\frac{2}{\alpha^3}$. [6]

3. Given that $2\sqrt{5}x-6=3x+\sqrt{5}$, find x in the form $p+q\sqrt{5}$, where p and q are rational numbers. [4]

4. (a) Given that $3^{2x+3}\times 5^{2x}=27^x\times 125^{x+1}$, evaluate 15^x. [3]

(b) Without using a calculator, find the value of p such that $\left(\frac{2}{\sqrt{75}}-\frac{\sqrt{147}}{3}\right)\div\sqrt{2}=p\sqrt{6}$. [4]

5. (a) Using an appropriate substitution, solve $3^{2x}-81^{\frac{x+1}{4}}=4$. [4]

(b) Solve the equation $\sqrt{10+x^2-\sqrt{3x+4}}-3=x$, giving your answers correct to 2 decimal places. [5]

6. Given that $f(x)=3x^5-11x^3+30x^2+36=(x-2)(x+3)Q(x)+ax+b$ for all values of x and that $Q(x)$ is a polynomial,

(i) find the values of a and of b. [4]

(ii) Hence, find the remainder when $f(x)+2$ is divided by (x^2+x-6). [2]

7. Express $\dfrac{8x^3 - 7x^2 - 11x + 16}{x(2x+1)^2}$ in partial fractions. [6]

8. The function $f(x) = 2x^3 + 7x^2 + ax + b$, where a and b are constants, has a factor of $x + 4$ and leaves a remainder of 15 when divided by $x + 3$.

(i) Prove that the value of a is -10 and of b is -24. [3]

(ii) Hence, solve the equations

(a) $f(x) = 0$, [3]

(b) $2y^6 + 7y^4 + ay^2 + b = 0$. [2]

9. **(a)(i)** Write down and simplify the first three terms of the expansion $(2 - x)^6$ in ascending powers of x. [2]

(ii) Given that the coefficients of x and x^2 in the expansion of $(1 + ax)^2(2 - x)^6$ are -128 and b respectively, calculate the value of a and of b. [5]

(b) Find the term independent of x in the expansion of $\left(2x^3 + \dfrac{1}{3x^2}\right)^{10}$. [3]

10. Solve

(a) $2\log_2 6 - \log_2(x - 1) = 3 + \log_2(4x - 7)$, [4]

(b) $\log_5 x + 1 = \log_{\sqrt{5}} x + 2$, [4]

(c) $\log_2 x + 3\log_x 2 = 4$. [4]

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11. Solve the simultaneous equations, giving the answers in exact values.

$$\left(9^x\right)\left(27^y\right)=1,$$

$$\frac{e^{2x}}{\left(e^{y+2}\right)^3}=\frac{1}{e^2}.$$

[6]

~~~~~~~ End of Paper ~~~~~~~
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MARKING SCHEME

1. Find the coordinates of the points of intersection of the line $2x+3y=4$ and the curve $(2x+1)^2+5(y-2)^2=45$. [4]

$2x+3y=4$ ---- (1)
$(2x+1)^2+5(y-2)^2=45$ ----(2)
From (1): $2x=4-3y$ ----- (3)
Sub (3) into (2):
$(5-3y)^2+5(y^2-4y+4)-45=0$ [M1]
$25-30y+9y^2+5y^2-20y+20-45=0$
$14y^2-50y=0$ [M1]
$2y(7y-25)=0$
$y=0$ or $\frac{25}{7}$
When $y=0$, $x=2$, $(2, 0)$ [A1]
When $y=\frac{25}{7}$, $x=-\frac{47}{14}$, $(-\frac{47}{14},\frac{25}{7})$ [A1]

2. The roots of the quadratic equation $2x^2-8x+5=0$ are α and β.

(i) State the value of $\alpha+\beta$ and of $\alpha\beta$. [2]

(ii) Find the quadratic equation in x whose roots are $\frac{2}{\beta^3}$ and $\frac{2}{\alpha^3}$. [6]

(i)	$\alpha+\beta=4$ [B1] $\alpha\beta=\frac{5}{2}$ [B1]
(ii)	Roots: $\frac{2}{\beta^3}$ and $\frac{2}{\alpha^3}$ Sum of roots $=\frac{2}{\beta^3}+\frac{2}{\alpha^3}$ $=\frac{2(\alpha^3+\beta^3)}{(\alpha\beta)^3}$

	$= \dfrac{2\left[(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)\right]}{(\alpha\beta)^3}$ [M1] $= \dfrac{2(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{(\alpha\beta)^3}$ [M1] $= \dfrac{2(4)\left[(4)^2-3\left(\frac{5}{2}\right)\right]}{\left(\frac{5}{2}\right)^3}$ $= \dfrac{544}{125}$ [A1] Product of root $= \left(\dfrac{2}{\beta^3}\right)\left(\dfrac{2}{\alpha^3}\right)$ $= \dfrac{4}{(\alpha\beta)^3}$ [M1] $= \dfrac{32}{125}$ [A1] Equation: $125x^2 - 544x + 32 = 0 \,/\, x^2 - \dfrac{544}{125}x + \dfrac{32}{125} = 0$ [B1/FT1]

3. Given that $2\sqrt{5}x - 6 = 3x + \sqrt{5}$, find x in the form $p + q\sqrt{5}$, where p and q are rational numbers.

[4]

$$2\sqrt{5}x - 3x = 6 + \sqrt{5} \Rightarrow (2\sqrt{5}-3)x = 6+\sqrt{5}$$ [M1]

$$x = \frac{6+\sqrt{5}}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3}$$ [M1]

$$= \frac{12\sqrt{5}+3\sqrt{5}+18+2(5)}{4(5)-3(3)}$$ [M1]

$$= \frac{15\sqrt{5}+28}{11}$$

$$= \frac{28}{11} + \frac{15}{11}\sqrt{5}$$ [A1]

4. (a) Given that $3^{2x+3} \times 5^{2x} = 27^x \times 125^{x+1}$, evaluate 15^x. [3]

$3^{2x+3} \times 5^{2x} = 27^x \times 125^{x+1}$

$3^{2x+3} \times 5^{2x} = 3^{3x} \times 5^{3(x+1)}$ [M1]

$1 = \frac{3^{3x} \times 5^{3(x+1)}}{3^{2x+3} \times 5^{2x}}$

$3^{3x-(2x+3)} \times 5^{3(x+1)-2x} = 1$

$3^{x-3} \times 5^{x+3} = 1$

$3^x \times 3^{-3} \times 5^x \times 5^3 = 1$ [M1]

$3^x \times 5^x \times \frac{125}{27} = 1$

$15^x = \frac{27}{125}$ [A1]

(b) Without using a calculator, find the value of p such that $\left(\frac{2}{\sqrt{75}} - \frac{\sqrt{147}}{3}\right) \div \sqrt{2} = p\sqrt{6}$. [4]

$\left(\frac{2}{\sqrt{75}} - \frac{\sqrt{147}}{3}\right) \div \sqrt{2} = \left(\frac{2}{5\sqrt{3}} - \frac{7\sqrt{3}}{3}\right) \times \frac{1}{\sqrt{2}}$ [M1]

$= \frac{6-35(3)}{15\sqrt{3}} \times \frac{1}{\sqrt{2}}$ [M1]

$= \frac{-99}{15\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ [M1]

$= \frac{-11}{10}\sqrt{6}$

$p = \frac{-11}{10}$ [A1]

5. (a) Using an appropriate substitution, solve $3^{2x} - 81^{\frac{x+1}{4}} = 4$. [4]

$3^{2x} - 81^{\frac{x+1}{4}} = 4$

$3^{2x} - 3^{x+1} - 4 = 0$ [M1]

Let $3^x = u$

$u^2 - 3u - 4 = 0$ [M1]

$(u-4)(u+1) = 0$

$3^x = 4$ or $3^x = -1$ (No solution) [A1 for both correct answers]

$x = \frac{lg4}{lg3} = 1.26$ [A1]

(b) Solve the equation $\sqrt{10+x^2-\sqrt{3x+4}}-3=x$, giving your answers correct to 2 decimal places. [5]

(b)	$\sqrt{10+x^2-\sqrt{3x+4}}=3+x$ $10+x^2-\sqrt{3x+4}=9+6x+x^2$ $(1-6x)^2=3x+4$ $36x^2-12x+1=3x+4$ $36x^2-15x-3=0$ $12x^2-5x-1=0$ $x=\dfrac{5\pm\sqrt{25+48}}{24}$ $=-0.15$ or 0.56 (rejected)	[M1] [M1] [M1] [M1] [A1 for both answers]

6. Given that $f(x)=3x^5-11x^3+30x^2+36=(x-2)(x+3)Q(x)+ax+b$ for all values of x and that $Q(x)$ is a polynomial,

(i) find the values of a and of b. [4]

(ii) Hence, find the remainder when $f(x)+2$ is divided by (x^2+x-6). [2]

(i)	$3x^5-11x^3+30x^2+36=(x-2)(x+3)Q(x)+ax+b$ Sub $x=2$, $164=2a+b$ --- (1) Sub $x=-3$, $-126=-3a+b$ --- (2) Sub (1) into (2), $-126=-3a+(164-2a)$ $-126-164=-5a$ $a=58$ $b=48$	 [M1] [M1] [M1] – for using substitution or elimination method *(ecf allowed)* [A1 for both answers]
(ii)	Since $(x-2)(x+3)=x^2+x-6$, Remainder $=58x+48+2$ $=58x+50$	M1 A1 or B2 *(ecf for values of a and b allowed for both marks)*

7. Express $\dfrac{8x^3-7x^2-11x+16}{x(2x+1)^2}$ in partial fractions. [6]

$x(2x+1)^2 = 4x^3+4x^2+x$

$\dfrac{2(4x^3+4x^2+x)-15x^2-13x+16}{4x^3+4x^2+x} = 2+\dfrac{16-15x^2-13x}{4x^3+4x^2+x}$ [M1 for any method to get quotient and remainder]

$\dfrac{16-15x^2-13x}{x(2x+1)^2} = \dfrac{A}{x}+\dfrac{B}{(2x+1)}+\dfrac{C}{(2x+1)^2}$ [M1]

$16-15x^2-13x = A(2x+1)^2 + Bx(2x+1)+Cx$ [M1]

Let $x = 0$: $A = 16$

$x = -0.5$: $-0.5C = 18.75$, $C = -\dfrac{75}{2}$ (-37.5)

$x = 1$: $3B = -12 - 9(16) + 37.5$, $B = -\dfrac{79}{2}$ [A2,1,0]

$\dfrac{8x^3-7x^2-11x+16}{x(2x+1)^2} = 2+\dfrac{16}{x}-\dfrac{79}{2(2x+1)}-\dfrac{75}{2(2x+1)^2}$ [A1]

Check:

$$2+\frac{16}{x}-\frac{237}{2(2x+1)}-\frac{75}{2(2x+1)^2} = \frac{4x(2x+1)^2+2(16)(2x+1)^2-79x(2x+1)-75x}{2x(2x+1)^2}$$

$$= \frac{4x(4x^2+4x+1)+32(4x^2+4x+1)-158x^2-79x-75x}{2x(2x+1)^2}$$

$$= \frac{16x^3-14x^2-22x+32}{2x(2x+1)^2}$$

$$= \frac{8x^3-7x^2-11x+16}{x(2x+1)^2}$$

8 The function $f(x)=2x^3+7x^2+ax+b$, where a and b are constants, has a factor of $x+4$ and leaves a remainder of 15 when divided by $x+3$.

(a) Prove that the value of a is -10 and of b is -24. [3]

(b) Hence, solve the equation $f(x) = 0$. [3]

(c) Hence solve $2y^6+7y^4+ay^2+b=0$. [2]

(a)

$f(x)=2x^3+7x^2+ax+b$

$f(-4)=0$

$2(-4)^3+7(-4)^2+a(-4)+b=0$ [M1]

$-4a+b=16$ ----(1)

$f(-3)=15$

$2(-3)^3+7(-3)^2+a(-3)+b=15$ [M1]

$-3a+b=6$ ----(2)

(2) – (1): a = -10

$b = 6 + 3(-10) = -24$ [A1 for both answers]

(b) $f(x) = (x+4)(2x^2+bx-6)$

Compare coeff. of x: $-10 = 4b - 6$

$b = -1$

$f(x) = (x+4)(2x^2-x-6)$ [M1] or equivalent method

$f(x) = (x+4)(2x+3)(x-2) = 0$

$x = -4, 2, -\frac{3}{2}$ [A2, 1, 0]

(c) Using the substitution $x = y^2$,

$y^2 = 2, -4, -\frac{3}{2}$ [M1, mark awarded if not rejected]

(NA), (NA)

$y = \pm\sqrt{2}$ [A1]

9. (a)(i) Write down and simplify the first three terms of the expansion $(2-x)^6$ in ascending powers of x. [2]

$$(2-x)^6 = 2^6 - \binom{6}{1}2^5 x + \binom{6}{2}2^4 x^2 + \ldots\ldots$$

$$= 64 - 192x + 240x^2 + \ldots\ldots \qquad \text{[B2, 1]}$$

(ii) Given that the coefficients of x and x^2 in the expansion of $(1+ax)^2(2-x)^6$ are -128 and b respectively, calculate the value of a and of b. [5]

$$(1+ax)^2(2-x)^6 = (1+2ax+a^2x^2)(64-192x+240x^2+\ldots.) \qquad \text{[M1]}$$

$$= 64 + (128a-192)x + (240-384a+64a^2)x^2 + \ldots..$$

Equating the coefficient of x, $128a - 192 = -128 \Rightarrow 128a = -128 + 192$ [M1]

$$a = \frac{64}{128} = \frac{1}{2} \qquad \text{[A1]}$$

Equating the coefficient of x^2, $b = 240 - 384a + 64a^2$ [M1]

$$= 240 + 384\left(\frac{1}{2}\right) + 64\left(\frac{1}{2}\right)^2$$

$$= 64 \qquad \text{[A1]}$$

(b) Find the term independent of x in the expansion of $\left(2x^3+\frac{1}{3x^2}\right)^{10}$. [3]

(b)	$\left(2x^3+\frac{1}{3x^2}\right)^{10}$		
	$T_{r+1}=\binom{10}{r}(2x^3)^{10-r}\left(\frac{1}{3x^2}\right)^r$	[M1]	
	$T_{r+1}=\binom{10}{r}(2)^{10-r}\left(\frac{1}{3}\right)^r x^{30-5r}$		
	For independent term, $30-5r=0$, $r=6$	[M1] OR	
	Coefficient of independent term $=\binom{10}{6}(2)^{10-6}\left(\frac{1}{3}\right)^6$		[M2]
	$=4\frac{148}{243}$	[A1]	[A1]

10. Solve

(i) $2\log_2 6-\log_2(x-1)=3+\log_2(4x-7)$, [4]

(ii) $\log_5 x+1=\log_{\sqrt{5}} x+2$, [4]

(iii) $\log_2 x+3\log_x 2=4$. [4]

(i)	$2\log_2 6-\log_2(x-1)=3+\log_2(4x-7)$,	
	$2\log_2 6-\log_2(x-1)-\log_2(4x-7)=3$	
	$\log_2\frac{6^2}{(x-1)(4x-7)}=3$	[M1]
	$\log_2\frac{36}{4x^2-11x+7}=3$	
	$\frac{36}{4x^2-11x+7}=2^3$ *OR* $36=8(4x^2-11x+7)$	[M1]
	$8x^2-22x+14-9=0\Rightarrow 8x^2-22x+5=0$	
	$(4x-1)(2x-5)=0$	[M1]
	$x=\frac{1}{4}$ (rejected), $x=\frac{5}{2}$	[A1 for both answers]

(ii) $\log_5 x + 1 = \log_{\sqrt{5}} x + 2$ $\log_5 x + 1 = \dfrac{\log_5 x}{\log_5 \sqrt{5}} + 2$ [M1] $\log_5 x + 1 = \dfrac{\log_5 x}{\frac{1}{2}} + 2$ [M1] $\log_5 x + 1 = 2\log_5 x + 2$ $\log_5 x + 1 = 2\log_5 x + 2$ $\log_5 x = -1$ [M1] $x = 5^{-1} = \dfrac{1}{5}$ or 0.2 [A1]

(iii) $\log_2 x + 3\log_x 2 = 4$ $\log_2 x + \dfrac{3}{\log_2 x} = 4$ [M1]   Let $u = \log_2 x$ $u + \frac{3}{u} = 4$ $u^2 - 4u + 3 = 0$ [M1] $(u - 3)(u - 1) = 0$ $u = 3$ or $u = 1$ $\log_2 x = 3 \Rightarrow x = 2^3 = 8$ [A1]   or $\log_2 x = 1 \Rightarrow x = 2^1 = 2$ [A1]

11. Solve the simultaneous equations, giving the answers in exact values.

$$(9^x)(27^y)=1,$$
$$\frac{e^{2x}}{(e^{y+2})^3}=\frac{1}{e^2}.$$

[6]

$3^{2x}3^{3y}=3^0$	
$3^{2x+3y}=3^0$	
Comparing Indices	
$2x+3y=0$.........(1)	[M1]
$\frac{e^{2x}}{e^{3y+6}}=e^{-2}$	
$e^{2x-3y-6}=e^{-2}$	[M1]
Comparing Indices	
$2x-3y-6=-2$	[M1]
$2x-3y=4$.........(2)	[M1] - substitution or elimination
(1) + (2)	
$4x=4$	
$x=1$	[A1]
$y=-\frac{2}{3}$	[A1] (only accept exact value)