BUKIT VIEW SECONDARY SCHOOL
MID YEAR EXAMINATION 2015
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS **4038/1**

11 May 2015

2 hours

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Answer **all** the questions

1. Show that the equation $2x^2 + x(3x+h) = -3k$ has two real and distinct roots for all negative values of k. [3]

2. Simplify $\sqrt{32} - \sqrt{8} + \sqrt{\frac{1}{8}}$, giving your answer in the form $k\sqrt{2}$. [3]

3. The equation $(k+2)x^2 + 4k = (4k+2)x$ has no real roots.
   Find the range of values of k. [4]

4. The line $y = kx - 3$ is a tangent to the curve $y = 2x^2 + 7$.
   Find the possible values of k. [4]

5. Find the value of $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$. [4]

6. (i) Simplify $2\log_4(x+2y) - \log_4 x - \log_4 y$ into a single logarithmic function. [2]

   (ii) Given that $x^2 + 4y^2 = 12xy$, hence, show that

   $$2\log_4(x+2y) - \log_4 x - \log_4 y = 2.$$ [2]

7. (i) Show that $\frac{1}{\sqrt{p+1}+\sqrt{p}} = \sqrt{p+1} - \sqrt{p}$. [3]

   (ii) Hence, find the value of

   $$\frac{1}{\sqrt{16}+\sqrt{15}} + \frac{1}{\sqrt{15}+\sqrt{14}} + \frac{1}{\sqrt{14}+\sqrt{13}} + \dots\dots\dots + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{1}}.$$ [2]

8. (i) Solve the simultaneous equations.

$x = 1 + 2y$

$x = 6xy - 2y - 3$ [5]

(ii) Explain the geometrical meaning of the solution. [1]

9. By using the substitution $u = 2^x$ or otherwise,

(a) prove that $2^{x+3} - 2^x$, is exactly divisible by 7. [2]

(b) find the values of x such that $2^{2x+1} - 7(2^x) + 3 = 0$. [4]

10. Find the range of values of x which satisfy the inequality $x^2 \leq x^2 - 2x < 4x^2 - 1$.

Represent the solution on a number line. [6]

11. Express $y = 3x^2 - 6x + 11$ in the form $y = a(x+h)^2 + k$, where a, h and k are constants.

(i) State the minimum value of y and the corresponding value of x. [4]

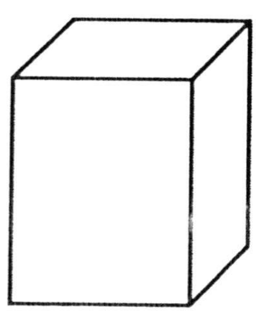
(ii) Sketch the graph $y = 3x^2 - 6x + 11$ showing clearly the coordinates of the minimum point of the curve and y-intercept. [3]

12. A cuboid, as shown below, of volume $\left(18 + 11\sqrt{2}\right)cm^3$ stands on its square base.

Given that the area of the base is $\left(3 + 2\sqrt{2}\right)cm^2$, without using a calculator,

(i) show that the length of its base is $\left(1 + \sqrt{2}\right)cm$, [4]

(ii) find the height of the cuboid in the form $\left(p + q\sqrt{2}\right)cm$, where p and q are integers. [3]



18

13. (a) On the same axes, sketch the graphs $y = 3\left(2^{-0.2x}\right)$ and $y = 3\left(2^{0.2x}\right)$. [2]

(b) The amount, y milligrams, of a radioactive substance present after t years, is given by $y = Ae^{-0.2t}$, where A is a constant. At the beginning of 2000, there was an estimated amount of 500 mg. Find

(i) the value of A, [1]

(ii) the amount present in the year 2005, [2]

(iii) the year in which the amount of substance would be expected to be halved. [3]

(iv) the amount that has decayed after one decade. [3]

14. Solve each of the following equations.

(a) $e^{3x+2} = 43.7$ [2]

(b) $\dfrac{2}{\log_7 2} = \log_2(7 - x)$ [4]

(c) $4^x = 32^{2x+3}$ [4]

THE END

A Maths MYE 2015

1) [3m]

$2x^2 + x(3x+h) = -3k$

$2x^2 + 3x^2 + xh + 3k = 0$

$5x^2 + (h)x + 3k = 0$ [m1]

$\therefore b^2 - 4ac = h^2 - 4(5)(3k)$ [m1]

$= h^2 - 60k$

For All negative values of k, $-60k > 0$

$\therefore h^2 - 60k > 0$ # shown. [A1]

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2) [3m]

$\sqrt{32} - \sqrt{8} + \sqrt{\frac{1}{8}}$

$= \sqrt{16\times 2} - \sqrt{4\times 2} + \frac{1}{\sqrt{4\times 2}}$

$= 4\sqrt{2} - 2\sqrt{2} + \frac{1}{2\sqrt{2}}$ [m1]

$= 2\sqrt{2} + \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ [m1]

$= 2\sqrt{2} + \frac{\sqrt{2}}{4}$

$= \frac{8\sqrt{2}+\sqrt{2}}{4}$

$= \frac{9\sqrt{2}}{4}$ or $\frac{9}{4}\sqrt{2}$ # [A1]

OR $4\sqrt{2} - 2\sqrt{2} + \sqrt{\frac{1}{16}\times 2}$

$= 2\sqrt{2} + \frac{\sqrt{2}}{4}$

$= 2\frac{1}{4}\sqrt{2}$

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3) [4m]

$(k+2)x^2 + 4k = (4k+2)x$

$(k+2)x^2 - (4k+2)x + 4k = 0$ [m1]

$\therefore b^2 - 4ac < 0$

$[-(4k+2)]^2 - 4(k+2)(4k) < 0$ [m1]

$16k^2 + 16k + 4 - 16k(k+2) < 0$

$16k^2 + 16k + 4 - 16k^2 - 32k < 0$

$-16k + 4 < 0$ [m1]

$-16k < -4$

$k > \frac{1}{4}$ # [A1]

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4) [4m]

$kx - 3 = 2x^2 + 7$

$2x^2 - kx + 7 + 3 = 0$

$2x^2 - kx + 10 = 0$ [m1]

$\therefore b^2 - 4ac = 0$

$(-k)^2 - 4(2)(10) = 0$ [m1]

$k^2 = 80$

$k = \pm\sqrt{80}$ or ± 8.94 # [A2]

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5) [4m]

$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \frac{\log_a a}{\log_a abc} + \frac{\log_b b}{\log_b abc} + \frac{\log_c c}{\log_c abc}$ [m1]

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$ [m1]

$= \log_{abc} abc$ [m1]

$= 1$ # [A1]

OR $\log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$

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6) [4m]

(i) [2m]

$2\log_4(x+2y) - \log_4 x - \log_4 y$

$= \log_4 (x+2y)^2 - \log_4 x - \log_4 y$ [m1]

$= \log_4\left(\frac{x^2+4xy+4y^2}{xy}\right)$ # [A1]

(ii) [2m]

$2\log_4(x+2y) - \log_4 x - \log_4 y$

$= \log_4\left(\frac{12xy+4xy}{xy}\right)$

$= \log_4\left(\frac{16xy}{xy}\right)$ [m1]

$= \log_4 16$

$= \log_4 4^2$ [m1]

$= 2\log_4 4 = 2$ # shown.

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7) [5m]

(i) [3m]

$\frac{1}{\sqrt{P+1}+\sqrt{P}} \times \frac{\sqrt{P+1}-\sqrt{P}}{\sqrt{P+1}-\sqrt{P}}$ [m1]

$= \frac{\sqrt{P+1}-\sqrt{P}}{(\sqrt{P+1})^2 - (\sqrt{P})^2}$ [m1]

$= \frac{\sqrt{P+1}-\sqrt{P}}{P+1-P}$ [m1]

$= \sqrt{P+1} - \sqrt{P}$ # shown.

(ii) [2m]

$\frac{1}{\sqrt{16}+\sqrt{15}} + \frac{1}{\sqrt{15}+\sqrt{14}} + \frac{1}{\sqrt{14}+\sqrt{13}} + \cdots + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{1}}$

$= \sqrt{16} - \sqrt{15} + \sqrt{15} - \sqrt{14} + \sqrt{14} - \sqrt{13} + \cdots + \sqrt{3} - \sqrt{2} + \sqrt{2} - \sqrt{1}$ [m1]

$= \sqrt{16} - \sqrt{1}$

$= 4 - 1$

$= 3$ # [A1]

9

[6m]

8) (i) $x = 1 + 2y \quad —(1)$

[2m] $x = 6xy - 2y - 3 \quad —(2)$

Sub (1) into (2):

$1 + 2y = 6y(1+2y) - 2y - 3$ [m1]

$1 + 2y = 6y + 12y^2 - 2y - 3$

$12y^2 + 6y - 2y - 2y - 3 - 1 = 0$

$12y^2 + 2y - 4 = 0$

$2(6y^2 + y - 2) = 0$ } [m1]

$(3y+2)(2y-1) = 0$ [m1]

$y = -\frac{2}{3}$ or $y = \frac{1}{2}$

$\therefore x = 1 + 2(-\frac{2}{3})$ $\quad x = 1 + 2(\frac{1}{2})$

$= -\frac{1}{3}$ $\quad = 2$

$\therefore x = -\frac{1}{3},\ y = -\frac{2}{3}$ # [A1]

$x = 2,\ y = \frac{1}{2}$ # [A1]

(ii) [1m] The line intersects the curve at the points $(2, \frac{1}{2})$ and $(-\frac{1}{3}, -\frac{2}{3})$

OR

The line intersects the curve at 2 points. [B1]

[6m]

9) (a) [2m] $2^{x+3} - 2^x = 2^x \times 2^3 - 2^x$ [m1]

$= 8(2^x) - 2^x$

$= 7(2^x)$ [A1]

$\therefore 2^{x+3} - 2^x$ is exactly divisible by 7. #

(b) [4m] $2^{2x+1} - 7(2^x) + 3 = 0$

$2^{2x} \times 2 - 7(2^x) + 3 = 0$ [m1]

$2u^2 - 7u + 3 = 0$

$(2u-1)(u-3) = 0$ [m1]

$u = \frac{1}{2}$ or $u = 3$

$2^x = 2^{-1}$ $\quad 2^x = 3$

$\therefore x = -1$ # [A1] $\quad x = \log_2 3$

$= 1.58$ # [A1]

[6m]

10) $x^2 \le x^2 - 2x < 4x^2 - 1$

$x^2 \le x^2 - 2x$

$x^2 - x^2 + 2x \le 0$

$2x \le 0$

$x \le 0$ [A1]

$x^2 - 2x < 4x^2 - 1$

$x^2 - 4x^2 - 2x + 1 < 0$

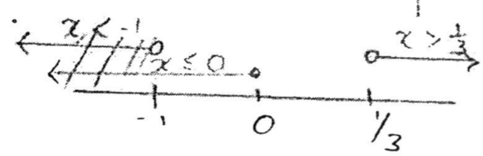
$-3x^2 - 2x + 1 < 0$

$3x^2 + 2x - 1 > 0$ [m1]

$(3x-1)(x+1) > 0$ [m1]

$x < -1$ or $x > \frac{1}{3}$

[A1]



[A1]

$\therefore x < -1$ # [A1]

[7m]

11) $y = 3x^2 - 6x + 11$

[2m] $= 3(x^2 - 2x + \frac{11}{3})$ [m1]

$= 3[(x-1)^2 - 1 + \frac{11}{3}]$

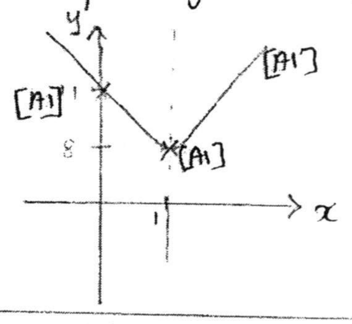
$= 3[(x-1)^2 + 2\frac{2}{3}]$

$= 3(x-1)^2 + 8$ # [A1]

(i) [2m] minimum value of $y = 8$ # [A1]

corresponding value of $x = 1$ # [A1]

(ii) [3m]



when $x = 0$, $y = 11$

[7m]

12) (i) [4m] let $\sqrt{3 + 2\sqrt{2}} = a + b\sqrt{2}$

$3 + 2\sqrt{2} = (a + b\sqrt{2})^2$ [m1]

$= a^2 + 2ab\sqrt{2} + 2b^2$

$= a^2 + 2b^2 + 2ab\sqrt{2}$

$\therefore 2ab = 2 \Rightarrow ab = 1$

$a = \frac{1}{b} \quad —(1)$

$a^2 + 2b^2 = 3 \quad —(2)$ } [m1]

20

Sub (1) into (2):

$\left(\frac{1}{b}\right)^2 + 2b^2 = 3$

$\frac{1}{b^2} + 2b^2 = 3$

$1 + 2b^4 = 3b^2$

$2b^4 - 3b^2 + 1 = 0$

$(2b^2-1)(b^2-1) = 0$ [m1]

$b^2 = \frac{1}{2}$ or $b^2 = 1$

$b = \pm\frac{1}{\sqrt{2}}$ or $b = \pm 1$

$\therefore a = \pm\sqrt{2}$ $\qquad a = \pm 1$

Reject negative values.

$\therefore b = \frac{1}{\sqrt{2}}, \ a = \sqrt{2}$; $b = 1, \ a = 1$ } [m1]

$\therefore$ length $= 1 + \sqrt{2}$ # Shown.

OR $\sqrt{3+2\sqrt{2}}$ [m1]

$= \sqrt{1+2+2\sqrt{2}}$ [m1]

$= \sqrt{1+2\sqrt{2}+2}$

$= \sqrt{1^2 + 2(1)\sqrt{2} + (\sqrt{2})^2}$ [m1]

$= \sqrt{(1+\sqrt{2})^2}$ [m1]

$= 1+\sqrt{2}$ # shown.

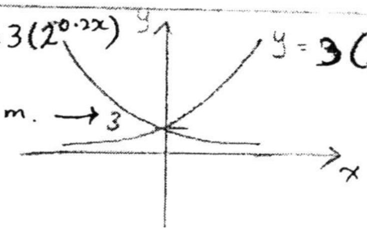
(ii) [3m] $\frac{18+11\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$ [m2]

$= \frac{54 - 36\sqrt{2} + 33\sqrt{2} - 44}{9-8}$

$= 10 - 3\sqrt{2}$ # [A1]

[11m]

13) (a) [2m]



[A2]

(b)(i) [9m] [1m] $500 = Ae^{-0.2(0)}$

$\therefore A = 500$ # [B1]

(ii) [2m] When $t=5$, $y = 500e^{-0.2(5)}$ [m1]

$= 183.9$

≈ 184 # [A1]

(iii) [3m] $\frac{500}{2} = 500e^{-0.2t}$ [m1]

$\frac{1}{2} = e^{-0.2t}$

$-0.2t = \ln\left(\frac{1}{2}\right)$ [m1]

$t = 3.4657$

$\therefore$ The year is $2000 + 3 = 2003$ # [A1]

(iv) [3m] When $t = 10$, $y = 500e^{-0.2(10)}$ [m1]

$= 67.6676$

$\therefore$ Amount decayed

$= 500 - 67.6676$ [m1]

≈ 432 # [A1]

[10m]

14) (a) [2m] $e^{3x+2} = 43.7$

$3x + 2 = \ln(43.7)$ [m1]

$x = 0.592$ # [A1]

(b) [4m] $\frac{2}{\log_7 2} = \log_2(7-x)$

$2\log_2 7 = \log_2(7-x)$ [m1]

$\log_2 7^2 = \log_2(7-x)$ [m1]

$\therefore 49 = 7 - x$ [m1]

$x = 7 - 49$

$= -42$ # [A1]

(c) [4m] $4^x = 32^{2x+3}$

$2^{2x} = 2^{5(2x+3)}$ [m2]

$\therefore 2x = 10x + 15$ [m1]

$-8x = 15$

$x = -1\frac{7}{8}$ # [A1]

or -1.875 #

(b) $\frac{2}{\log_7 2} = \log_2(7-x)$

$2 = \log_2(7-x) \times \log_7 2$

$= \frac{\log_2(7-x)}{\log_2 7}$ [M1]

$2 = \log_7 7-x$. [m1]

$7^2 = 7 - x$

$49 = 7 - x$ } [M1]

$x = -42$ [A1]

21