聖 嬰 中 學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class Index Number

MID-YEAR EXAMINATION 2015
SECONDARY 3 EXPRESS
ADDITIONAL MATHEMATICS

4047/01

Date: **12 May 2015**

Duration: **2 h**

Additional Materials: *Writing Paper (7 sheets)*

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is **80**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Set by: Mrs Chang Poh Joo

Vetted by: Mrs Nathan & Mdm Hayati

This document consists of 5 printed pages (including cover page).

22

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer all the questions.

1 Solve the simultaneous equations

$$3x+2y=1,$$
$$3x^2-y^2=5x+3y.$$ [4]

2 Solve the equation $8+|x^2-3x|=3x$. [4]

3 **(i)** Find the range of values of p for which the line $y=x-2p$ does not intersect the curve $x^2=3y-1$. [4]

(ii) Hence state the value of p for which the line $y=x-2p$ is a tangent to the curve $x^2=3y-1$. [1]

4 The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula

$$m=28e^{-0.00072t}$$

(i) Find the value of m when $t=20$. [1]

(ii) Find the value of t when the mass is half of its value at $t=0$. [3]

(iii) State the value which m approaches as t becomes very large. [1]

(iv) Sketch the graph of m against t. [1]

5 **(a)** The equation $qx^2+k=qx-q$ where $q\neq 0$, has two equal roots. Express k in terms of q. [3]

(b) **(i)** By completing the square, show that $m^2-8m+20=(m-4)^2+4$. [1]

(ii) Prove that the solutions of the equation $(x-m)(x-3)=m-3x+5$ are real for all real values of m. [3]

03

6 Using a separate diagram for each part, represent on the number line the solution set of

(i) $x^2 - 4x \geq 0$, [2]

(ii) $\frac{3-2x}{5} + \frac{7}{x} > 0$. [4]

State the set of values of x which satisfies both of these inequalities. [1]

7 The roots of the quadratic equation $2x^2 = 10x + 3$ are α and β.

Find the quadratic equation whose roots are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$. [7]

8 (i) Sketch the curve $y = |(x-2)(x+3)|$, giving the coordinates of the maximum point and of the points where the curve meets the axes. [3]

(ii) By drawing a suitable straight line on the same axes, state the number of solutions to the equation $|x^2 + x - 6| = 7$. [2]

(iii) Find the value of k for which the equation $|x^2 + x - 6| = k$ has 3 distinct real roots. [1]

9 (a) A rectangle has an area of $(8\sqrt{2} + 7\sqrt{5})$ cm^2 and a length of $(3\sqrt{2} + \sqrt{5})$ cm.

Express in the form $a + b\sqrt{10}$, where a and b are integers,

(i) the breadth of the rectangle, [3]

(ii) the value of D^2, where D cm is the length of the diagonal of the rectangle. [3]

(b) Solve the equation $\sqrt{13 - \sqrt{2x^2 + 7x + 12}} = 3$. [3]

10 **(a)** Without using a calculator, solve, for p and q, the simultaneous equations.

$$\frac{2^p}{4^q}=\frac{1}{8},$$

$$\sqrt{5^q \times 125^p}=25.$$ [5]

(b) Use an appropriate substitution, or otherwise, solve $7^{2x+1}+20\left(7^x\right)=3$. [4]

(c) Show that $2(9^{n+1})+3^{2n+3}-9^n$ is a multiple of 11 for all positive integer values of n. [3]

11 **(a)** Without using a calculator, evaluate $\log_5 8-\log_5(125^{\frac{2}{3}})+\frac{1}{2}\log_5\frac{25}{64}+1$. [4]

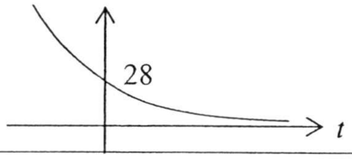
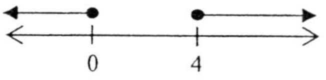
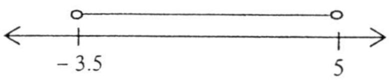
(b) Given that $2\lg(\sqrt{y+1})=3+4\lg x$, express y in terms of x. [4]

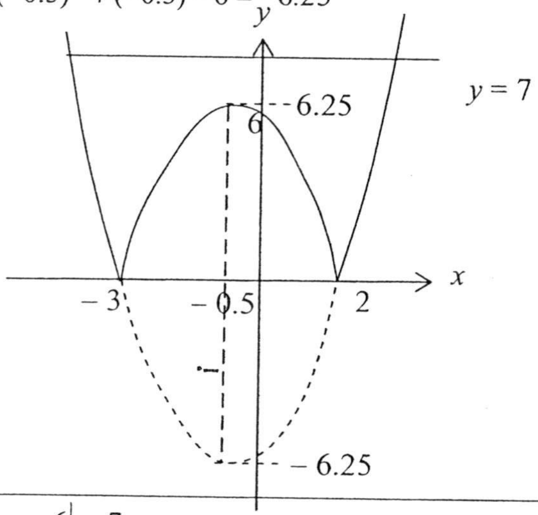
(c) Solve the equation $\log_2 x-4=5\log_x 2$. [5]

End of Paper

24

One-page answers to HIHS Sec 3 Additional Math Mid-Year Examination 2015

No.	Answer	No.	Answer
1	$x=-2\frac{1}{3},\ y=4$ or $x=1,\ y=-1$	9(ai)	$1+\sqrt{10}$
2	$x=4$ or $\sqrt{8}$ [reject $x=2$ and $-\sqrt{8}$]	(aii)	$D^2=34+8\sqrt{10}$
3(i)	$p>\frac{5}{24}$ 3 (ii) $p=\frac{5}{24}$	9(b)	$x=\frac{1}{2}$ or $x=-4$ [deduct 1 m if the answer $x=-4$ is rejected]
4(i)	27.6	10(a)	$p-2q=-3$ ……. equation (1) $3p+q=4$ ……. equation (2) $p=\frac{5}{7},\ q=1\frac{6}{7}$
(ii)	963		
(iii)	m approaches 0 grams [$m=0$ is not accepted]		
(iv)		10(b)	$x=-1$ or $7^x=-3$ (reject as there is no solution)
		10(c)	See the next page for proof
5(a)	$k=-\frac{3}{4}q$	11(a)	Answer is 0 Note : Without using calculator, you must show clear working to get 5^2 from $125^{\frac{2}{3}}$: $125^{\frac{2}{3}}=(5^3)^{\frac{2}{3}}=5^2$ or $125^{\frac{2}{3}}=(\sqrt[3]{125})^2=5^2$
5(bi)	$=m^2-8m+(-4)^2+20-(-4)^2$ $=(m-4)^2+20-16$ $=(m-4)^2+4$		
5(bii)	See the next page for proof.		
6(i)	$x\le 0$ or $x\ge 4$ 	11(b)	$y=1000x^4-1$
(ii)	$-3.5<x<5$ 	11(c)	$x=32$ or $x=\frac{1}{2}$
(iii)	$-3.5<x\le 0$ and $4\le x<5$		
7	$\alpha+\beta=5,\quad \alpha\beta=\frac{-3}{2}$ Sum of new roots $=\frac{-112}{3}$ Product of new roots = 4 Equation is $3x^2+112x+12=0$ or $x^2+\frac{112}{3}x+4=0$		
8	See the next page for answers		

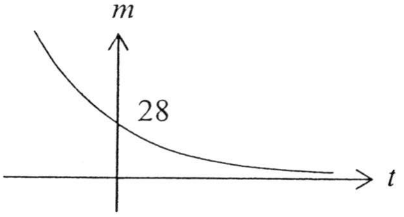
5bii	$(x-m)(x-3)=m-3x+5$ $x^2-3x-mx+3m=m-3x+5$ $x^2-mx+(2m-5)=0$ Discriminant, $b^2-4ac=(-m)^2-4(1)(2m-5)$ $=m^2-8m+20$ $=(m-4)^2+4$ from part (i) Since $(m-4)^2 \geq 0$, for all real values of m hence $(m-4)^2+4>0$ Thus, $b^2-4ac>0$. (Shown). Hence, the line meets the curve and the roots are real.	M1 for forming eqn M1 for use of b^2-4ac correctly to get $m^2-8m+20$ A1 for clear explanation using $(m-4)^2+4>0$
8i	$y=\lvert(x-2)(x+3)\rvert$ Use the value of x at line of symmetry to find max y value. $x=\frac{-3+2}{2}=-0.5$ $y=(-0.5)^2+(-0.5)-6=-6.25$ 	B1 mark for correct turning point (− 0.5, − 6.25) and y-intercept = 6 B1 mark for correct parabola shape and correct modulus function B1 mark for correct x-intercepts and labelled axes [no half mark is allowed]
8ii	$\lvert x^2+x-6\rvert=7$ $\lvert(x-2)(x+3)\rvert=7$ Add the line $y=7$ on the same axes The number of solutions is 2.	M1 for adding $y=7$ A1 for number of solutions
8iii	For the equation to have 3 distinct real roots, $k=$ 6.25	B1
10c	$2(9^{n+1})+3^{2n+3}-9^n$ $=2(9^n\cdot 9^1)+(3^2)^n\cdot 3^3-9^n$ $=18(9^n)+27(9^n)-1(9^n)$ $=44(9^n)$ $=11(4)(9^n)$ Thus, it is a multiple of 11 Alternative method : $2(9^{n+1})+3^{2n+3}-9^n$ $=2(3^{2n+2})+3^{2n+3}-3^{2n}$ $=3^{2n}(2\times 3^2)+3^{2n}(3^3)-3^{2n}(1)$ $=3^{2n}[18+27-1]$ $=3^{2n}(44)$ $=11(4)(3^{2n})$ Thus, it is a multiple of 11.	M1 separate index M1 for $44(9^n)$ A1 for clear conclusion $11(4)(9^n)$

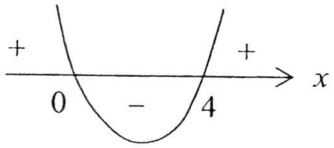
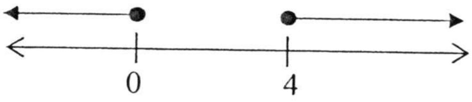
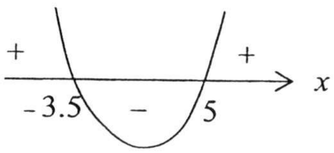
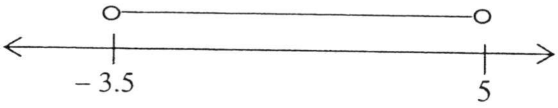
25

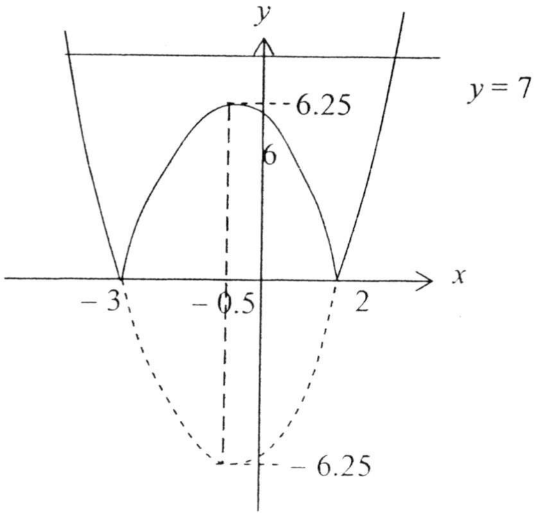
Marking Scheme for Sec 3 Additional Math Mid-Year Examination 2015

No	Solution	Marks / Comments
1	$3x+2y=1$ ....(1) $3x^2-y^2=5x+3y$ .... (2) From eqn (1) : $x=\dfrac{1-2y}{3}$ .... (3) Sub (3) into (2) : $3\left(\dfrac{1-2y}{3}\right)^2-y^2=5\left(\dfrac{1-2y}{3}\right)+3y$	M1 for complete elimination of x or y
	$3\left(\dfrac{1-4y+4y^2}{9}\right)-y^2=\dfrac{5-10y}{3}+3y$ $\dfrac{1-4y+4y^2}{3}-\dfrac{3y^2}{3}=\dfrac{5-10y}{3}+\dfrac{9y}{3}$ Multiply by 3 throughout: $1-4y+4y^2-3y^2=5-10y+9y$	
	$y^2-3y-4=0$ $(y-4)(y+1)=0$ $y=4$ or $y=-1$	M1 for $y^2-3y-4=0$ or $3x^2+4x-7=0$
	Sub y values into eqn (3) to find x values: $x=\dfrac{1-2(4)}{3}$ or $x=\dfrac{1-2(-1)}{3}$	A1 for x values A1 for y values
	$x=-2\frac{1}{3}$ or $x=1$	or
	$\therefore\ x=-2\frac{1}{3},\ y=4$ or $x=1,\ y=-1$	A2 for 2 correct pairs of x and y
2	$8+\lvert x^2-3x\rvert=3x$ $\lvert x^2-3x\rvert=3x-8$	
	$x^2-3x=3x-8$ or $x^2-3x=-(3x-8)$ $x^2-6x+8=0$ or $x^2=8$ $(x-4)(x-2)=0$ or $x=\pm\sqrt{8}$ $x=4$ or $x=2$	M1 for removing modulus sign correctly
	Check validity of answers: $\lvert x^2-3x\rvert=3x-8$ When $x=4$, $\lvert 4\rvert=4$ When $x=2$, $\lvert -2\rvert=-2$ Not valid When $x=\sqrt{8}$, $\lvert -0.49\rvert=0.49$ When $x=-\sqrt{8}$, $\lvert 16.4\rvert=-16.4$ Not valid	M1 for $x=4$, 2 M1 for $x=\pm\sqrt{8}$
	Thus, $x=4$ or $\sqrt{8}$ [accept 2.83 (3 s.f)]	A1 for final 2 answers

3i	$x^2 = 3(x-2p)-1$ $x^2 = 3x-6p-1$ $x^2 - 3x + (6p+1) = 0$ Since line does not meet curve, $b^2 - 4ac < 0$ $(-3)^2 - 4(1)(6p+1) < 0$ $9 - 24p - 4 < 0$ $5 - 24p < 0$ $-24p < -5$ $24p > 5$ $p > \frac{5}{24}$	$(y+2p)^2 = 3y-1$ $y^2 + 4py + 4p^2 = 3y - 1$ $y^2 + (4p-3)y + (4p^2+1) = 0$ $b^2 - 4ac < 0$ $(4p-3)^2 - 4(1)(4p^2+1) < 0$ $16p^2 - 24p + 9 - 16p^2 - 4 < 0$ $-24p + 5 < 0$ $24p > 5$ $p > \frac{5}{24}$	M1 for complete elimination of y M1 for correct quadratic equation M1 for applying $b^2 - 4ac < 0$ A1 [accept $p > 0.208$]
3ii	$p = \frac{5}{24}$		B1
4(i)	$m = 28e^{-0.00072t}$ When $t = 20$, $m = 28e^{-0.00072(20)}$ $= 28e^{-0.0144}$ $= 27.6$	[unit is not required]	B1
4(ii)	When $t = 0$, $m = 28e^0$ $= 28$ grams Half of its mass = 14 grams $m = 28e^{-0.00072t}$ $14 = 28e^{-0.00072t}$ $0.5 = e^{-0.00072t}$ Take *ln* on both sides $\ln 0.5 = \ln e^{-0.00072t}$ $\ln 0.5 = -0.00072t$ $t = \frac{\ln 0.5}{-0.00072}$ $t = 962.7$ Check : When $t = 962$ days, $m = 28e^{-0.00072t} = 14.007$ grams (mass is not halved yet) When $t = 963$ days, $m = 28e^{-0.00072t} = 13.997$ grams $\therefore t =$ **963** days <u>before</u> the mass is halved[unit is not required]	$14 = 28e^{-0.00072t}$ $\ln 14 = \ln(28e^{-0.00072t})$ $\ln 14 = \ln 28 + \ln e^{-0.00072t}$ $\ln 14 = \ln 28 - 0.00072t(\ln e)$ $0.00072t = \ln 28 - \ln 14$ $t = \frac{\ln 28 - \ln 14}{0.00072} = 962.7$	M1 for $0.5 = e^{-0.0072t}$ M1 for applying natural logarithm to both sides OR $t = \frac{\ln 0.5}{-0.00072}$ A1 for 963 days instead of 962.7 days
4iii	m approaches 0 grams	[unit is not required] [Do not accept $m = 0$]	B1

4iv	Sketch of graph 	B1 [axes must be labelled, curve must be very near to t-axis, m-intercept at 28]
5(a)	$qx^2+k=qx-q$ $qx^2-qx+(k+q)=0$ Given equation has equal roots, $b^2-4ac=0$ $(-q)^2-4(q)(k+q)=0$ $q^2-4kq-4q^2=0$ $-3q^2-4kq=0$ $3q^2+4kq=0$ $q(3q+4k)=0$ $q=0$ or $3q+4k=0$ (rej.) $3q=-4k$ $k=-\frac{3}{4}q$ Alternative method : $3q^2+4kq=0$ $4kq=-3q^2$ $k=\frac{-3q^2}{4q}$ $k=\frac{-3q}{4}$	M1 for $(-q)^2-4(q)(k+q)=0$ M1 for forming eqn $-3q^2-4kq=0$ A1
5bi	$m^2-8m+20$ $=m^2-8m+(-4)^2+20-(-4)^2$ $=(m-4)^2+20-16$ $=(m-4)^2+4$ (Shown)	B1 for working
5bii	$(x-m)(x-3)=m-3x+5$ $x^2-3x-mx+3m=m-3x+5$ $x^2-mx+(2m-5)=0$ a = 1, b = $-m$, c =$(2m-5)$ Discriminant, $b^2-4ac=(-m)^2-4(1)(2m-5)$ $=m^2-8m+20$ $=(m-4)^2+4$ from part (i) Since $(m-4)^2\geq 0$, for all real values of m hence $(m-4)^2+4>0$ Thus, $b^2-4ac>0$. (Shown). Hence, the line meets the curve and the roots are real.	M1 for forming eqn M1 for use of b^2-4ac correctly to get $m^2-8m+20$ A1 for clear explanation using $(m-4)^2+4>0$

6i	$x^2-4x\geq 0$ $x(x-4)\geq 0$ $x\leq 0$ or $x\geq 4$   	B1 for answer B1 for number line
6ii	$\frac{3-2x}{5}+\frac{7}{x}>0$ $\frac{(3-2x)x+7(5)}{5x}>0$ $3x-2x^2+35>0$ $2x^2-3x-35<0$ $(2x+7)(x-5)<0$  Solution is $-3.5<x<5$  Deduct 1 mark from overall presentation if number lines drawn without ruler / arrows on both sides.	M1 for adding fraction $\frac{(3-2x)x+7(5)}{5x}>0$ or $x(3-2x)+7(5)>0$ M1 for or $2x+7)(x-5)<0$ A1 for $-3.5<x<5$ B1 for number line
	Range of values of x is $-3.5<x\leq 0$ and $4\leq x<5$	B1
7	$2x^2=10x+3$ $2x^2-10x-3=0$ a = 2, b = –10, c = – 3 Sum of roots, $\alpha+\beta=-\frac{b}{a}=-\frac{(-10)}{2}=5$ Product of roots, $\alpha\beta=\frac{c}{a}=\frac{-3}{2}$ Given new roots are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$, Product of new roots $=\left(\frac{2\alpha}{\beta}\right)\left(\frac{2\beta}{\alpha}\right)$ $=\frac{4\alpha\beta}{\alpha\beta}=4$ Sum of new roots $=\frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}$ $=\frac{2\alpha^2}{\alpha\beta}+\frac{2\beta^2}{\alpha\beta}$	B1 for sum of roots B1 for product of roots M1 for product of new roots = 4 M1 for combining fraction

7	$= \frac{2(\alpha^2+\beta^2)}{\alpha\beta}$ $= \frac{2[(\alpha+\beta)^2-2\alpha\beta]}{\alpha\beta}$ $= \frac{2\left[5^2-2\left(\frac{-3}{2}\right)\right]}{\left(\frac{-3}{2}\right)}$ $= \frac{2(28)}{\left(\frac{-3}{2}\right)}$ $= 56 \div \left(\frac{-3}{2}\right)$ $= \frac{-112}{3}$ [accept $-37\frac{1}{3}$] Equation with new roots is x^2 – (sum of new roots) x + (product of new roots) = 0 $x^2 - \left(\frac{-112}{3}\right)x + 4 = 0$ $x^2 + \frac{112}{3}x + 4 = 0$ or $3x^2 + 112x + 12 = 0$	M1 for substituting correct values into $\frac{2[(\alpha+\beta)^2-2\alpha\beta]}{\alpha\beta}$ A1 for sum of new roots = $\frac{-112}{3}$ A1 for equation
8i	$y = \lvert (x-2)(x+3) \rvert$ Use the value of x at line of symmetry to find max y value. $x = \frac{-3+2}{2} = -0.5$ $y = (-0.5)^2 + (-0.5) - 6 = -6.25$ 	B1 mark for correct turning point (– 0.5, – 6.25) and y-intercept = 6 B1 mark for correct parabola shape and correct modulus function B1 mark for correct x-intercepts and labelled axes [no half mark is allowed]

8ii	$\lvert x^2+x-6\rvert = 7$ $\lvert (x-2)(x+3)\rvert = 7$ Add the line $y = 7$ on the same axes The number of solutions is 2.	M1 for adding $y = 7$ A1 for number of solutions
8iii	$\lvert x^2+x-6\rvert = k$ $\lvert (x-2)(x+3)\rvert = k$ For the equation to have 3 distinct real roots, $k = \lvert -6.25\rvert$ $k = 6.25$	B1
9ai	$\frac{8\sqrt{2}+7\sqrt{5}}{3\sqrt{2}+\sqrt{5}} = \frac{8\sqrt{2}+7\sqrt{5}}{3\sqrt{2}+\sqrt{5}} \times \frac{3\sqrt{2}-\sqrt{5}}{3\sqrt{2}-\sqrt{5}}$ $= \frac{8\sqrt{2}\left(3\sqrt{2}-\sqrt{5}\right)+7\sqrt{5}\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2}$ $= \frac{24(2)-8\sqrt{10}+21\sqrt{10}-7(5)}{9(2)-(5)}$ $= \frac{13+13\sqrt{10}}{13}$ $= 1+\sqrt{10}$ (breadth)	M1 for correct expansion in numerator M1 for correct use of a^2-b^2 in denominator A1
9aii	Using Pythagoras' theorem, $(\text{Diagonal})^2 = (\text{length})^2 + (\text{breadth})^2$ $D^2 = \left(3\sqrt{2}+\sqrt{5}\right)^2 + \left(1+\sqrt{10}\right)^2$ $D^2 = \left(3\sqrt{2}\right)^2 + 2\left(3\sqrt{2}\right)\left(\sqrt{5}\right) + \left(\sqrt{5}\right)^2 + (1)^2 + 2(1)\left(\sqrt{10}\right) + \left(\sqrt{10}\right)^2$ $D^2 = 9(2)+6\sqrt{10}+5+1+2\sqrt{10}+10$ $D^2 = 34+8\sqrt{10}$ [deduct 1 m if final answer is in decimal]	M1 for correct substitution into Pythagoras' theorem [allow ecf from i] M1 for correct expansion A1
9b	$\sqrt{13-\sqrt{2x^2+7x+12}} = 3$ Square both sides, $13-\sqrt{2x^2+7x+12} = 9$ $\sqrt{2x^2+7x+12} = 4$ Square both sides again $2x^2+7x+12 = 16$ $2x^2+7x-4 = 0$ $(2x-1)(x+4) = 0$ $x = \frac{1}{2}$ or $x = -4$ [deduct 1 m if the answer $x = -4$ is rejected]	M1 M1 A1

10a	$\frac{2^p}{4^q} = \frac{1}{8}$ $\frac{2^p}{2^{2q}} = 2^{-3}$ $2^{p-2q} = 2^{-3}$ $p - 2q = -3$ ----- equation (1)	M1 for $p - 2q = -3$ [or equivalent, o.e]
	$\sqrt{5^q \times 125^p} = 25$ $\sqrt{5^q \times 5^{3p}} = 5^2$ $\left(5^{3p+q}\right)^{\frac{1}{2}} = 5^2$ $5^{\frac{3p+q}{2}} = 5^2$ $\boxed{\begin{array}{l} \sqrt{5^q \times 5^{3p}} = 5^2 \\ 5^q \times 5^{3p} = 5^4 \\ 5^{q+3p} = 5^4 \end{array}}$	M1 $\left(5^{3p+q}\right)^{\frac{1}{2}} = 5^{\frac{3p+q}{2}}$
	$\frac{3p+q}{2} = 2$ $3p + q = 4$ ----- equation (2)	M1 for $3p + q = 4$ [or equivalent, o.e]
	Sub $p = 2q - 3$ into equation (2): $3(2q - 3) + q = 4$ $6q - 9 + q = 4$ $7q = 13$ $q = 1\frac{6}{7}$	A1
	$p = 2q - 3 = \frac{5}{7}$	A1
10b	$7^{2x+1} + 20(7^x) = 3$ $7^{2x} \cdot 7^1 + 20(7^x) - 3 = 0$ $7(7^x)^2 + 20(7^x) - 3 = 0$	M1
	Let u be 7^x $7u^2 + 20u - 3 = 0$ $(7u - 1)(u + 3) = 0$	M1
	$u = \frac{1}{7}$ or $u = -3$ $7^x = \frac{1}{7}$ or $7^x = -3$ (no solution, reject) $7^x = 7^{-1}$ $x = -1$	A1 for $x = -1$ A1 for rejecting -3

29

10c	Method 1 : $2(9^{n+1})+3^{2n+3}-9^n$ $=2(9^n\cdot 9^1)+(3^2)^n\cdot 3^3-9^n$ $=18(9^n)+27(9^n)-1(9^n)$ $=44(9^n)$ $=11(4)(9^n)$ Thus, it is a multiple of 11	M1 separate index M1 for $44(9^n)$ A1 for clear conclusion $11(4)(9^n)$
	Method 2 : $2(9^{n+1})+3^{2n+3}-9^n$ $=2(3^{2n+2})+3^{2n+3}-3^{2n}$ $=3^{2n}(2\times 3^2)+3^{2n}(3^3)-3^{2n}(1)$ $=3^{2n}[18+27-1]$ $=3^{2n}(44)$ $=11(4)(3^{2n})$ Thus, it is a multiple of 11.	M1 separate index M1 for $44(3^{2n})$ A1 for clear conclusion $11(4)(3^{2n})$
11a	$\log_5 8-\log_5\left(125^{\frac{2}{3}}\right)+\frac{1}{2}\log_5\left(\frac{25}{64}\right)+1$ $=\log_5 8-\log_5 25+\log_5\left(\frac{25}{64}\right)^{\frac{1}{2}}+\log_5 5$ $=\log_5 8-\log_5 25+\log_5\left(\frac{5}{8}\right)+\log_5 5$ $=\log_5\left(\frac{8}{25}\times\frac{5}{8}\times 5\right)$ $=\log_5 1$ $=0$	M1 for showing clearly $125^{\frac{2}{3}}=(5^3)^{\frac{2}{3}}=5^2$ or $125^{\frac{2}{3}}=(\sqrt[3]{125})^2=5^2$ without calculator M1 for power law $\frac{1}{2}\log_5\frac{25}{64}=\log_5\frac{5}{8}$ M1 for product and quotient law A1 for $\log_5 1=0$

11b	<u>Method 1 :</u> $2\lg(\sqrt{y+1}) = 3 + 4\lg x$ $\lg(\sqrt{y+1})^2 - \lg x^4 = 3$ $\lg(y+1) - \lg x^4 = 3$ $\lg\frac{(y+1)}{x^4} = 3$ Convert logarithmic form to index form, $\frac{y+1}{x^4} = 10^3$ $y+1 = 1000x^4$ $y = 1000x^4 - 1$	M1 for power law $2\lg(\sqrt{y+1}) = \lg(\sqrt{y+1})^2$ and $4\lg x = \lg x^4$ M1 for quotient law M1 for converting to index form A1
11b	<u>Method 2 :</u> $2\lg(\sqrt{y+1}) = 3 + 4\lg x$ $\lg(\sqrt{y+1})^2 = 3\lg 10 + 4\lg x$ $\lg(y+1) = \lg 10^3 + \lg x^4$ $\lg(y+1) = \lg(1000x^4)$ Comparing lg on both sides, $y+1 = 1000x^4$ $y = 1000x^4 - 1$	M1 for power law $2\lg(\sqrt{y+1}) = \lg(\sqrt{y+1})^2$ and $4\lg x = \lg x^4$ M1 for $3 = \lg 10^3$ M1 for product law A1
11c	<u>Method 1 :</u> $\log_2 x - 4 = 5\log_x 2$ $\log_2 x - 4 = \frac{5}{\log_2 x}$ Let p be $\log_2 x$ $p - 4 = \frac{5}{p}$ $p^2 - 4p - 5 = 0$ $(p-5)(p+1) = 0$ $p = 5$ or $p = -1$ $\log_2 x = 5$ or $\log_2 x = -1$ $x = 2^5$ or $x = 2^{-1}$ $x = 32$ or $x = \frac{1}{2}$	M1 for change of base M1 for $p^2 - 4p - 5 = 0$ M1 for solutions to p [or other suitable variables used] A2

	Method 2 :	
	$\log_2 x - 4 = 5\log_x 2$	M1 for change of base
	$\frac{1}{\log_x 2} - 4 = 5\log_x 2$	
	Let m be $\log_x 2$	
	$\frac{1}{m} - 4 = 5m$	M1 for $5m^2 + 4m - 1 = 0$
	$5m^2 + 4m - 1 = 0$	
	$(5m-1)(m+1) = 0$	
	$m = \frac{1}{5}$ or $m = -1$	M1 for solutions to m [or other suitable variables used]
	$\log_x 2 = \frac{1}{5}$ or $\log_x 2 = -1$	
	$x^{\frac{1}{5}} = 2$ or $x^{-1} = 2$	
	$x = 2^5$ or $\frac{1}{x} = 2$	
	$x = 32$ or $x = \frac{1}{2}$	A2