SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
SECOND SEMESTRAL EXAMINATION

Name: ______________________________( ) Class: Sec 3E______

ADDITIONAL MATHEMATICS

Paper 1

4047/01

Tuesday 6 October 2015

2 hours

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of **5** printed pages.

Setter: Ms Liew Jia Meng
Vetter: Ms Zoe Pow

[Turn over

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*We Nurture Students to **Think**, **Care** and **Lead** with **P.R.I.D.E.***

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

Answer **all** questions.

1 Solve $7^x = \frac{10}{3} - 7^{-x}$. [3]

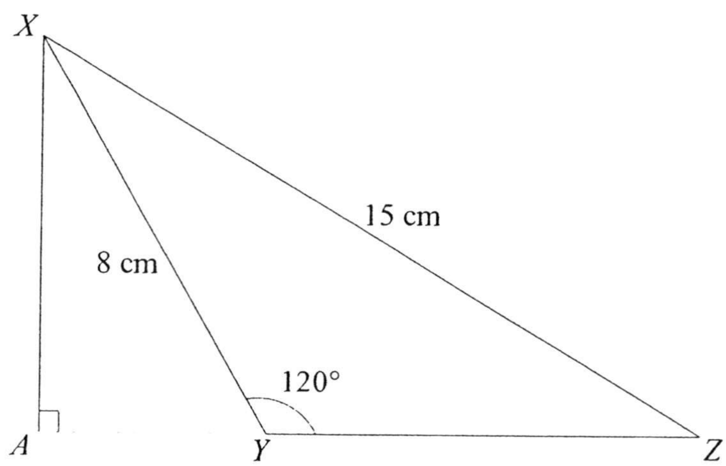
2 Solve the equation $\frac{1}{8}e^x\left(e^x - 3\right) = \frac{1}{2}$. [3]

3 Find the value of $\frac{9^{x+2} + 12\left(9^x\right)}{27^{x+2}\,3^{-x-2}}$. [3]

4 The line $x = y - 2$ intersects the curve $\frac{x^2}{3} + \frac{y^2}{4} = \frac{13}{4}$ at the points A and B.

Find the coordinates of the midpoint of A and B. [5]

5 The diagram shows a triangle XYZ in which $XY = 8$ cm, $XZ = 15$ cm and angle $XYZ = 120°$. The line ZY is extended to the point A where angle $XAY = 90°$.



(i) Find the exact length of AY. [2]

(ii) Show that the angle $XZY = \sin^{-1}\left(\frac{4\sqrt{3}}{15}\right)$. [3]

6 Given that $\tan A = \frac{8}{15}$ and that $\tan A$ and $\sin A$ have different signs, find the exact value of

(i) $\operatorname{cosec}(-A)$, [1]

(ii) $\sin\left(90° - A\right)$, [1]

(iii) $\cos A \cot A$. [2]

7 **(i)** Sketch the curve of $y = e^{2x+1} - 3$. [2]

(ii) In order to solve the equation $\ln(x+6) - 1 = 2x$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{2x+1} - 3$.

(a) Find the equation of this straight line. [2]

(b) By sketching this line on the same axes as (i), determine the number of solutions to the equation $\ln(x+6) - 1 = 2x$. [2]

8 **(i)** Prove the identity $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$. [3]

(ii) Hence, find all angles between 0° and 360° which satisfy the equation

$$(\sec x - \tan x) = \frac{5}{6(\operatorname{cosec} x + 1)}.$$ [3]

9 **(a)** Without using a calculator, express $\left(\frac{\sqrt{48}}{6} + \frac{2}{\sqrt{12}} + \frac{36}{\sqrt{75}}\right) \times \frac{6}{\sqrt{2}}$ in the form of $p\sqrt{6}$. [3]

(b) Given that $\frac{7\sqrt{2}}{3-\sqrt{2}} - \frac{5}{1+\sqrt{2}} = a + b\sqrt{2}$ such that a and b are integers, find the value of a and of b. [4]

10 **(a)** **(i)** Sketch, on the same axes, the graphs of $y^2 = -2x$ and $y = -\frac{5}{x^2}$. [2]

(ii) State the value of k for which the x-coordinate of the points of intersection satisfies the equation $x^5 = k$. [2]

(b) Given that $\log_m 2 = x$ and $\log_m 3 = y$, express $\log_m\left(\frac{\sqrt{2}}{9m}\right)$ in terms of x and y. [3]

11 **(a)** Find all the angles between 0° and 360° which satisfy the equation

$$8\sin(2x - 40^\circ) = -5.$$ [3]

(b) Given that $0 \le x \le 6$, find the values of x for which $3\sin^2 x + \cos x = \cos^2 x$. [4]

12 A certain radioactive substance is known to decay with time such that the amount of substance left after t days is given by $N = 180e^{-kt}$, where k is a constant.

It is found that the amount of substance is halved after 3.5 days.

Find

(i) the initial amount of substance, [1]

(ii) the value of k, [2]

(iii) the number of days it takes before the amount of substance is reduced to 6 grams, [2]

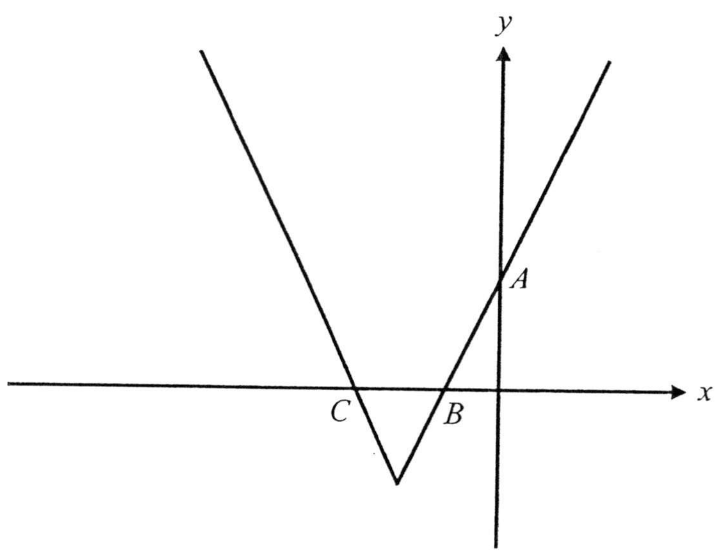
(iv) the amount of substance remaining after 1 week. [2]

13 Express the following in partial fractions.

(a) $\dfrac{3x-21}{(x-3)(x^2+1)}$ [4]

(b) $\dfrac{x^2-x+1}{x^2-5x-6}$ [4]

14 The diagram shows part of the graph of $y = |3x+5| - 2$.



(i) Find the coordinates of the points A, B and C. [3]

(ii) Solve the equation $|3x+5| - 2 = x + 4$. [2]

(iii) Find the number of solutions of the equation $|3x+5| - 2 = mx + 4$ when

(a) $m = -1$, [2]

(b) $m = 3$. [2]

END OF PAPER

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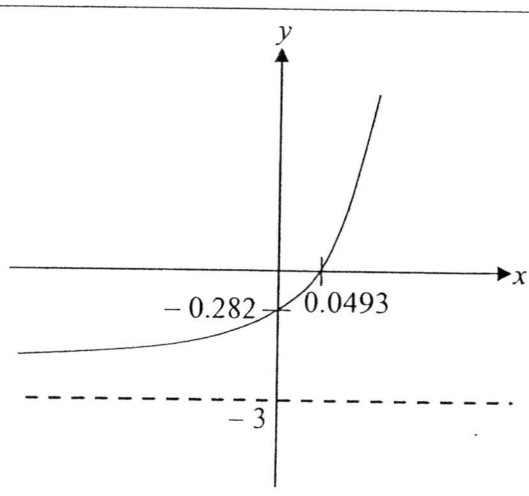
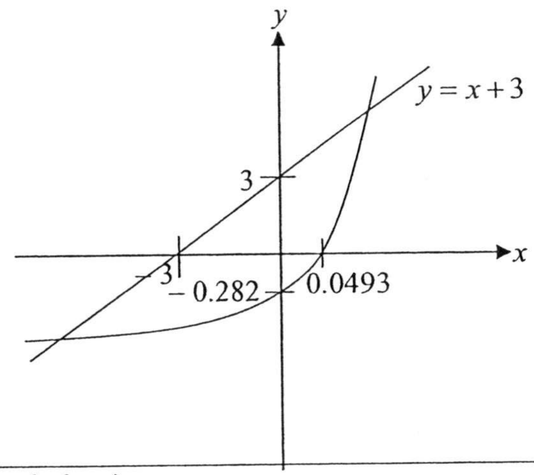
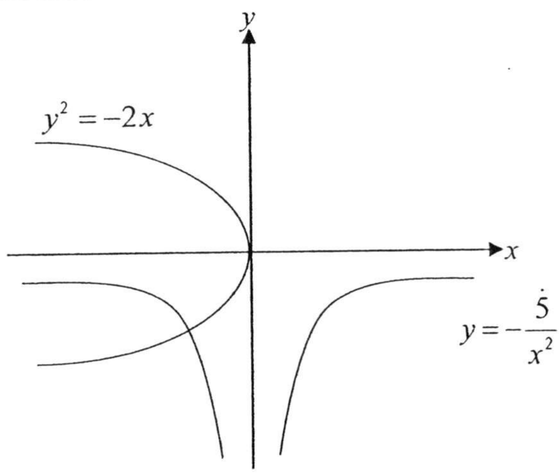
SECOND SEMESTRAL EXAMINATION 2015
Sec 3E AM P1 Solution

Paper 1 (80 marks)

Qn.	Answer Key
1.	$x = 0.565$ (3 sig. fig.) or $x = -0.565$ (3 sig. fig.)
2.	$x = 1.39$ (3 sig. fig.)
3.	$\frac{31}{27}$
4.	Coordinates of A and B $\left(1\frac{2}{7}, 3\frac{2}{7}\right)$ and $(-3,-1)$. Midpoint of AB = $\left(-\frac{6}{7}, 1\frac{1}{7}\right)$
5i.	$AY = 4$ units
6i.	$2\frac{1}{8}$
6ii.	$-\frac{15}{17}$
6iii.	$-1\frac{89}{136}$
7i.	
7iia.	$y = x + 3$

7iib.	Draw the line $y = x + 3$. 2 solutions 
8ii.	$x = 50.2^\circ, 230.2^\circ$ (to 1 decimal place)
9i.	$10\frac{1}{5}\sqrt{6}$
9ii.	$a = 7, b = -2$
10ai.	
10aii.	$k = -12\frac{1}{2}$
10b.	$\frac{1}{2}x - 2y - 1$
11i.	$x = 0.7^\circ, 129.3^\circ, 180.7^\circ, 309.3^\circ$
11ii.	$x = 0$ rad, 2.42 rad, 3.86 rad
12i.	$N = 180$

12ii.	$k = 0.198$ (3 sig. fig.)
12iii.	$t = 18$ days (round up)
12iv.	$N = 45.0$ (3 sig. fig.)
13i.	$\frac{3x-21}{(x-3)(x^2+1)} = -\frac{6}{5(x-3)} + \frac{6x+33}{5(x^2+1)}$
13ii.	$\frac{x^2-x+1}{x^2-5x-6} = 1 + \frac{31}{7(x-6)} - \frac{3}{7(x+1)}$
14i.	Coordinates are $A(0,3)$, $B(-1,0)$ and $C\left(-2\frac{1}{3},0\right)$.
14ii.	$x = \frac{1}{2}$ or $x = -2\frac{3}{4}$
14iiia	$\lvert 3x+5\rvert - 2 = -x+4$ Gradient of line is gentler than gradient of left arm. Line cuts both left and right arms at two points respectively. 2 solutions
14iiib	$\lvert 3x+5\rvert - 2 = 3x+4$ Gradient of line is parallel to right arm. Line cuts left arm at only 1 point. 1 solution

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Paper 1 (80 marks)

Qn.	Solution	Marker's Remarks
1.	$7^x = \frac{10}{3} - 7^{-x}$ $7^x = \frac{10}{3} - \frac{1}{7^x}$ Let $y = 7^x$ $y = \frac{10}{3} - \frac{1}{y}$ $y^2 - \frac{10}{3}y + 1 = 0$ $3y^2 - 10y + 3 = 0$ $(y-3)(3y-1) = 0$ $y = 3$ or $y = \frac{1}{3}$ $7^x = 3$ or $7^x = \frac{1}{3}$ $x = \frac{\ln 3}{\ln 7}$ or $x = \frac{\ln\frac{1}{3}}{\ln 7}$ $x = 0.565$ (3 sig. fig.) or $x = -0.565$ (3 sig. fig.)	M1 - factorise M1 – taking ln A1
2.	$\frac{1}{8}e^x(e^x - 3) = \frac{1}{2}$ Let $y = e^x$ $\frac{1}{8}y(y-3) = \frac{1}{2}$ $y^2 - 3y - 4 = 0$ $(y-4)(y+1) = 0$ $y = 4$ or $y = -1$ $e^x = 4$ or $e^x = -1$ (NA) $x = \ln 4$ x = 1.39 (3 sig. fig.)	M1 – factorise M1 – take 'ln' A1

3.	$\dfrac{9^{x+2}+12(9^x)}{27^{x+2}\ 3^{-x-2}}$	
	$=\dfrac{3^{2(x+2)}+12(3^{2x})}{3^{3(x+2)}\ 3^{-x-2}}$	M1 – change to base 3
	$=\dfrac{3^{2x}(3^4)+12(3^{2x})}{3^{2x+4}}$	M1 – factorise 3^{2x} correctly
	$=\dfrac{3^{2x}(81+12)}{3^{2x}(81)}$	
	$=\dfrac{31}{27}$	A1

<table>
<tr><td>4.</td><td colspan="2">$x = y - 2$ ----- (1)
$\frac{x^2}{3} + \frac{y^2}{4} = \frac{13}{4}$ ----- (2)
Subst (1) into (2):
$\frac{(y-2)^2}{3} + \frac{y^2}{4} = \frac{13}{4}$
$4(y^2 - 4y + 4) + 3y^2 = 39$
$7y^2 - 16y - 23 = 0$
$(7y - 23)(y + 1) = 0$
$y = \frac{23}{7}$ or $y = -1$
Subst $y = \frac{23}{7}$ and $y = -1$ into (1):
$x = \frac{9}{7}$ or $x = -3$
Coordinates of A and B $\left(1\frac{2}{7}, 3\frac{2}{7}\right)$ and $(-3, -1)$.
Midpoint of AB = $\left(\frac{1\frac{2}{7} + (-3)}{2}, \frac{3\frac{2}{7} + (-1)}{2}\right)$
$= \left(-\frac{6}{7}, 1\frac{1}{7}\right)$</td><td>M1 – substitution
M1 – Factorise
M1
M1 – Midpoint formula
A1</td></tr>
<tr><td>5i.</td><td>$\angle XYA = 60°$
$\cos 60° = \frac{AY}{8}$
$AY = 8\cos 60°$
$AY = 4$ units</td><td>$\angle AXY = 30°$
$\frac{AY}{\sin 30°} = \frac{8}{\sin 90°}$
$AY = \frac{8\sin 30°}{\sin 90°} = 8\left(\frac{1}{2}\right) = 4$ units</td><td>M1
A1</td></tr>
</table>

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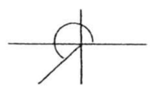
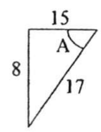
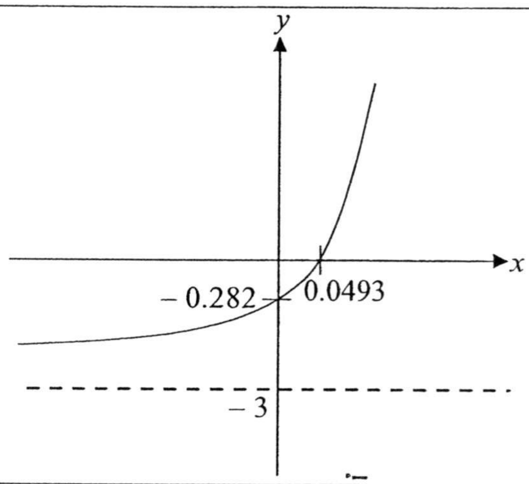
5ii.	$\sin 60° = \frac{AX}{8}$ $AX = 8\sin 60°$ $AX = 8\left(\frac{\sqrt{3}}{2}\right)$ $AX = 4\sqrt{3}$ units M1 $\sin \angle XZY = \frac{4\sqrt{3}}{15}$ M1 $\angle XZY = \sin^{-1}\left(\frac{4\sqrt{3}}{15}\right)$ A1	$\frac{\sin \angle XZY}{8} = \frac{\sin 120°}{15}$ M1 $\sin \angle XZY = \frac{8}{15}\sin 60°$ $= \frac{8}{15}\left(\frac{\sqrt{3}}{2}\right)$ M1 $= \frac{4\sqrt{3}}{15}$ $\angle XZY = \sin^{-1}\left(\frac{4\sqrt{3}}{15}\right)$ A1
6i.	cosec $(-A)$ $= \frac{1}{\sin(-A)}$ $= \frac{1}{-\sin A}$ $= \frac{1}{-\left(-\frac{8}{17}\right)}$ $= 2\frac{1}{8}$ 	B1
6ii	sin $(90° - A)$ $= \cos A$ $= -\frac{15}{17}$	B1

6iii	$\cos A \cot A$ $= \left(-\frac{15}{17}\right)\left(\frac{\cos A}{\sin A}\right)$ $= \left(-\frac{15}{17}\right)\left(\frac{-\frac{15}{17}}{-\frac{8}{17}}\right)$ $= -1\frac{89}{136}$	M1 A1
7i	At $x = 0$, $y = e - 3$ $y = -0.282$ (3 sig. fig.) At $y = 0$, $e^{2x+1} = 3$ $2x + 1 = \ln 3$ $x = \frac{\ln 3 - 1}{2}$ $x = 0.0493$ (3 sig. fig.)  	B1 – shape B1 – label intercepts and indicate asymptote
7iia.	$\ln(x + 6) - 1 = 2x$ $\ln(x + 6) = 2x + 1$ $x + 6 = e^{2x+1}$ $x + 3 = e^{2x+1} - 3$ Equation of line: $y = x + 3$	M1 A1

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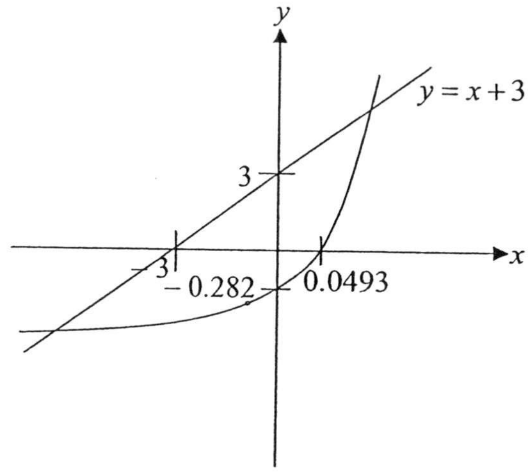
7iib	Draw the line $y = x + 3$. 2 solutions  	B1 – Drawing of line $y = x + 3$ B1 – No of solns
8i	$(\sec x - \tan x)(\operatorname{cosec} x + 1)$ $= \sec x \operatorname{cosec} x + \sec x - \tan x \operatorname{cosec} x - \tan x$ $= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right) + \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right) - \frac{\sin x}{\cos x}$ $= \frac{1}{\sin x \cos x} + \frac{1}{\cos x} - \frac{1}{\cos x} - \frac{\sin x}{\cos x}$ $= \frac{1}{\sin x \cos x} - \frac{\sin x}{\cos x}$ $= \frac{1 - \sin^2 x}{\sin x \cos x}$ $= \frac{\cos^2 x}{\sin x \cos x}$ $= \frac{\cos x}{\sin x}$ $= \cot x$ (shown)	M1 M1 A1

8ii.	$(\sec x - \tan x) = \dfrac{5}{6(\operatorname{cosec} x + 1)}$ $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \dfrac{5}{6}$ $\cot x = \dfrac{5}{6}$ $\tan x = \dfrac{6}{5}$ Basic angle = $\tan^{-1}\dfrac{6}{5} = 50.19442891^\circ$ x lies in the 1st and 3rd quadrants. $x = 50.19442891^\circ,\ 180^\circ + 50.19442891^\circ$ $x = 50.2^\circ,\ 230.2^\circ$ (to 1 decimal place)	M1 – tan x M1 – basic angle A1
9i.	$\left(\dfrac{\sqrt{48}}{6} + \dfrac{2}{\sqrt{12}} + \dfrac{36}{\sqrt{75}}\right) \times \dfrac{6}{\sqrt{2}}$ $= \dfrac{\sqrt{48}}{\sqrt{2}} + \dfrac{12}{(\sqrt{12})(\sqrt{2})} + \dfrac{36(6)}{(\sqrt{75})(\sqrt{2})}$ $= \sqrt{24} + \sqrt{6} + \dfrac{216}{(\sqrt{25 \times 3})(\sqrt{2})}$ $= 2\sqrt{6} + \sqrt{6} + \dfrac{216}{5\sqrt{6}}$ $= 2\sqrt{6} + \sqrt{6} + \dfrac{216}{5\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}}$ $= 2\sqrt{6} + \sqrt{6} + \dfrac{216\sqrt{6}}{30}$ $= 10\dfrac{1}{5}\sqrt{6}$	M1 – simplify to square root 6 M1 – rationalize correctly A1

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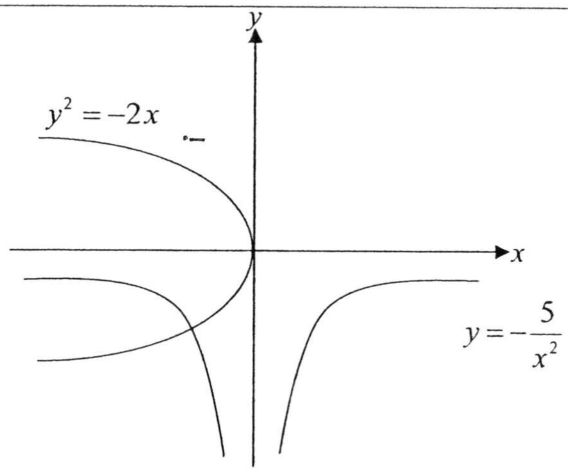
9ii.	$\frac{7\sqrt{2}}{3-\sqrt{2}}-\frac{5}{1+\sqrt{2}}=a+b\sqrt{2}$ $\frac{7\sqrt{2}(1+\sqrt{2})-5(3-\sqrt{2})}{(3-\sqrt{2})(1+\sqrt{2})}=a+b\sqrt{2}$ $\frac{7\sqrt{2}+14-15+5\sqrt{2}}{3+2\sqrt{2}-2}=a+b\sqrt{2}$ $\frac{12\sqrt{2}-1}{1+2\sqrt{2}}\times\frac{1-2\sqrt{2}}{1-2\sqrt{2}}=a+b\sqrt{2}$ $\frac{12\sqrt{2}-24(2)-1+2\sqrt{2}}{-7}=a+b\sqrt{2}$ $\frac{14\sqrt{2}-49}{-7}=a+b\sqrt{2}$ $7-2\sqrt{2}=a+b\sqrt{2}$ $a=7, b=-2$	M1 – single M1 M1 A1
10ai	 	B1 each – Shape
10aii	$-2x=\left(-\frac{5}{x^2}\right)^2$ $-2x=\frac{25}{x^4}$ $x^5=-12\frac{1}{2}$ $k=-12\frac{1}{2}$	M1 A1

10b	Given: $\log_m 2 = x$ and $\log_m 3 = y$ $\log_m\left(\frac{\sqrt{2}}{9m}\right)$ $= \log_m \sqrt{2} - \log_m(9m)$ $= \log_m \sqrt{2} - (\log_m 9 + \log_m m)$ $= \log_m \sqrt{2} - \log_m 9 - 1$ $= \frac{1}{2}\log_m 2 - 2\log_m 3 - 1$ $= \frac{1}{2}x - 2y - 1$	M1 – pdt rule M1 – power rule A1
11i	$8\sin(2x - 40°) = -5$ $\quad 0° \le x \le 360°$ $\sin(2x - 40°) = -\frac{5}{8}$ $\quad 0° \le 2x \le 720°$ $-40° \le 2x - 40° \le 680°$ Basic angle = $\sin^{-1}\left(\frac{5}{8}\right) = 38.68218745°$ $(2x - 40°)$ lies in the 3[rd] and 4[th] quadrants. $(2x - 40°) = -38.68218745°, 180°+38.68218745°, 360° - 38.68218745°, 540°+38.68218745°$ $(2x - 40°) = -38.68218745°, 218.6821874°, 321.3178126°, 578.6821874°, 681.3178126°$ $x = 0.7°, 129.3°, 180.7°, 309.3°$	M1 – basic angle M1 A1

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11ii.	$3\sin^2 x+\cos x=\cos^2 x$ $3\sin^2 x-\cos^2 x+\cos x=0$ $3(1-\cos^2 x)-\cos^2 x+\cos x=0$ $4\cos^2 x-\cos x+3=0$ $(\cos x-1)(4\cos x+3)=0$ $\cos x=1$ or $\cos x=-\frac{3}{4}$ $x=0\text{ rad}$ x lies in 2nd or 3rd quadrant. Basic angle = $\cos^{-1}\frac{3}{4}$ = 0.722734247 rad $x = \pi - 0.722734247$ rad, $\pi + 0.722734247$ rad x = 2.42 rad, 3.86 rad x = 0 rad, 2.42 rad, 3.86 rad	M1 – change to $(1-\cos^2 x)$ M1 – Factorise M1 – basic angle A1
12i	$N=180e^{-kt}$ When t = 0, N = 180 (original amount of substance)	B1
12ii	When t = 3.5, N = 90. $90=180e^{-k(3.5)}$ $\ln\frac{1}{2}=-3.5k$ $k=0.198042051$ $k=0.198$ (3 sig. fig.)	M1 A1
12iii	$N=180e^{-0.198042051t}$ $6=180e^{-0.198042051t}$ $\ln\frac{6}{180}=-0.198042051t$ $t=17.17411714$ days $t=18$ days (round up)	M1 – take ln A1
12iv	1 week = 7 days $N=180e^{-0.198042051(7)}$ N = 45.0 (3 sig. fig.)	M1 A1

13i.	$\frac{3x-21}{(x-3)(x^2+1)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+1}$	
	$3x-21 = A(x^2+1) + (Bx+C)(x-3)$	M1
	Subst $x = 3$, $9-21 = 10A$ $10A = -12$ $A = -\frac{6}{5}$	
	Subst $x = 1$, $-18 = 2(-1\frac{1}{5}) - 2B - 2C$ $B + C = 7\frac{4}{5}$ $B = \frac{39}{5} - C$ Subst $x = 0$, $-21 = -1\frac{1}{5} - 3C$ $C = \frac{33}{5}$	M2
	$B = \frac{6}{5}$	
	$\frac{3x-21}{(x-3)(x^2+1)} = -\frac{6}{5(x-3)} + \frac{6x+33}{5(x^2+1)}$	A1

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13ii.	$\frac{x^2-x+1}{x^2-5x-6}=1+\frac{4x+7}{(x-6)(x+1)}$ Long division: $x^2-5x-6 \overline{)\,x^2-x+1\,}$, quotient 1; $-(x^2-5x-6)$; remainder $4x+7$ $\frac{4x+7}{(x-6)(x+1)}=\frac{A}{x-6}+\frac{B}{x+1}$ $4x+7=A(x+1)+B(x-6)$ Subst $x=-1$, $-4+7=-7B$ $B=-\frac{3}{7}$ Subst $x=6$, $24+7=7A$ $A=\frac{31}{7}$ $\frac{4x+7}{(x-6)(x+1)}=\frac{31}{7(x-6)}-\frac{3}{7(x+1)}$ $\frac{x^2-x+1}{x^2-5x-6}=1+\frac{31}{7(x-6)}-\frac{3}{7(x+1)}$	M1 – Long Division M1 M1 – for both A and B A1
14i	$y=\lvert 3x+5\rvert-2$ At $x=0$, $y=\lvert 3(0)+5\rvert-2$ $y=3$ At $y=0$, $0=\lvert 3x+5\rvert-2$ $3x+5=2$ or $-(3x+5)=2$ $x=-1$ or $x=-2\frac{1}{3}$ Coordinates are $A(0,3)$, $B(-1,0)$ and $C\left(-2\frac{1}{3},0\right)$.	B1 each – for each coordinate

14ii.	$\lvert 3x+5\rvert - 2 = x+4$ $\lvert 3x+5\rvert = x+6$ $3x+5 = x+6$ or $-(3x+5) = x+6$ $2x = 1$ or $4x = -11$ $x = \frac{1}{2}$ or $x = -2\frac{3}{4}$	M1 A1
14iiia	$\lvert 3x+5\rvert - 2 = -x+4$ Gradient of line is gentler than gradient of left arm. Line cuts both left and right arms at two points respectively. 2 solutions	B1 B1
14iiib	$\lvert 3x+5\rvert - 2 = 3x+4$ Gradient of line is parallel to right arm. Line cuts left arm at only 1 point. 1 solution	B1 B1

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SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
SECOND SEMESTRAL EXAMINATIONS

Name: ______________________________ (   ) Class: Sec _______

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Thursday 8 October 2015

2 hours

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Submit Sections A and B separately

This question paper consists of **5** printed pages.

Setter: Mrs Chen Yen Wah
Vetter: Ms Zoe Pow

[Turn over

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*We Nurture Students to **Think**, **Care** and **Lead** with **P.R.I.D.E.***

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots.. + \binom{n}{r}a^{n-r}b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

Section A (42 marks)

1 Find the range of values of x for which $(2x-3)^2 > x$. [3]

2 Find the term independent of x in the expansion of $\left(2x-\frac{1}{x^2}\right)^9$. [4]

3 Solve the equation $\log_3 x^2 - 1 = 3\log_x 3$. [4]

4 Without using a calculator, solve, for x and y, the simultaneous equations

$$3^{x+1} = 27\left(3^{y-1}\right),$$
$$\log_2 6 + 1 = \log_2(6x+3y).$$ [5]

5 Given that the expansion of $(k+x)(1-3x)^n$ in ascending powers of x is $3-44x+px^2+\ldots$ find the values of the constants k, n and p. [6]

6 **(i)** Show that $\frac{\tan A}{\sec A+1}+\frac{\tan A}{\sec A-1} = 2\operatorname{cosec} A$. [4]

(ii) Hence, find all the angles between 0 and 2π which satisfy the equation

$$\frac{\tan A}{\sec A+1}+\frac{\tan A}{\sec A-1} = 5 .$$ [3]

7 **(a)** Find the value of k for which the line $y = 2x+k$ is a tangent to the curve $y = x^2-3x+4$. [3]

(b) Find the range of values of k for which $x^2+12x+9$ is always greater than $4x+k$. [3]

8 The roots of the quadratic equation $2x^2-3x+1=0$ are α and β.

(i) Show that $\alpha^2-\alpha\beta+\beta^2 = (\alpha+\beta)^2-3\alpha\beta$. [1]

(ii) Find the quadratic equation whose roots are α^3 and b^3. [6]

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Begin Section B on a fresh sheet of paper

Section B (38 marks)

9 The function f is defined by $f(x) = 3\cos 2x$ for $0 \le x \le 2\pi$.

(i) State the amplitude and period of f. [2]

(ii) Find the x-coordinate(s) of the points whereby $f(x) = 3\cos 2x$ meets the x-axis. [2]

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le 2\pi$. [3]

(iv) On the diagram drawn in part **(iii)**, sketch the graph of $y = \frac{2x}{3\pi}$ for $0 \le x \le 2\pi$. [1]

(v) State the number of solutions, for $0 \le x \le 2\pi$, of the equation $9\pi \cos 2x = 2x$. [2]

10 A circle passes through the points $A(4, 11)$ and $B(6, 9)$.
Its centre lies on the line $y = 2x$. Find

(i) the equation of the perpendicular bisector of AB, [3]

(ii) the coordinates of the centre of the circle, [2]

(iii) the equation of the circle, [2]

(iv) the equation of the circle reflected about the line $y = x$. [2]

11 **(a)** Solve the equation $2x^3 - 9x^2 + 3x + 4 = 0$. [5]

(b) The expression $6x^3 + px^2 + qx + 10$, where p and q are constants, has a factor of $2x - 1$ and leaves a remainder of -20 when divided by $x + 2$. Find the value of p and of q. [4]

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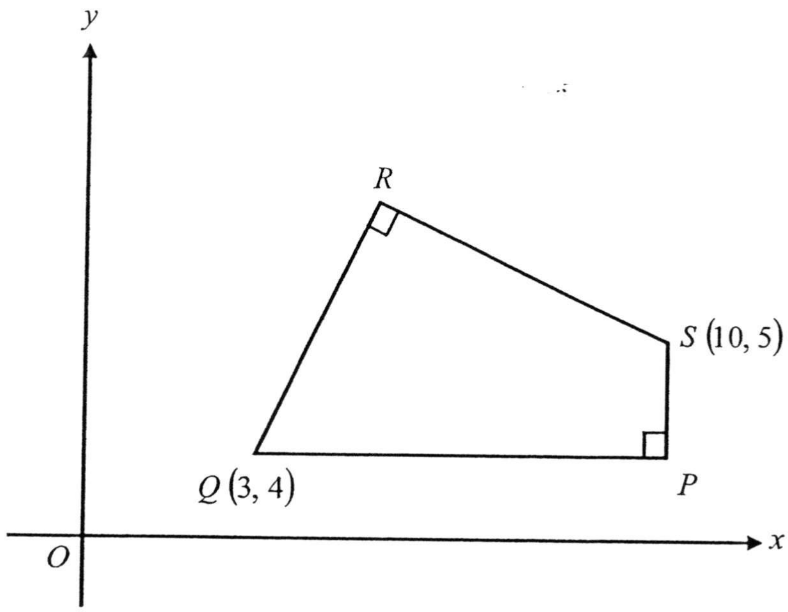

The diagram shows a quadrilateral $PQRS$ in which SR is perpendicular to RQ and QP is perpendicular to PS. The point Q is $(3, 4)$ and the point S is $(10, 5)$.

Given that QR is parallel to the line $6x - 2y = 13$, find

(i) the equation of QR, [2]

(ii) the coordinates of R, [4]

(iii) the area of the quadrilateral $PQRS$. [2]

T is a point on the line SR such that the area of ΔQTR : area of $\Delta QTS = 3 : 2$.

(iv) Find the coordinates of the point T. [2]

End of Paper

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SECOND SEMESTRAL EXAMINATIONS 2015

Sec 3E AM P2 Answer Key

1. $x > \frac{9}{4} = 2\frac{1}{4}$ or $x < 1$
2. -5376
3. $x = 3\sqrt{3} = 5.20$
4. $x = 1\frac{2}{3}$ and $y = \frac{2}{3}$
5. $k = 3$, $n = 5$ and $p = 255$
6. $A = 0.412$ rad (3sf) or $A = 2.73$ rad (3sf)
7. a) $k = -\frac{9}{4} = -2\frac{1}{4}$, b) $k < -7$
8. ii) $8x^2 - 9x + 1 = 0$
9. i) amplitude = 3, period = π, ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, v) 4
10. i) $y = x + 5$, ii) $(5, 10)$, iii) $(x-5)^2 + (y-10)^2 = 2$, iv) $(x-10)^2 + (y-5)^2 = 2$
11. a) $x = 1$, $x = -\frac{1}{2}$ or $x = 4$, b) $q = -19$, $p = -5$
12. i) $y = 3x - 5$, ii) $R(4, 7)$, iii) 13.5 units^2, iv) $T = \left(7\frac{3}{5}, 5\frac{4}{5}\right)$

Paper 1 (80 marks)

Qn. #	Solution	Mark Allocation	Markers' Report
1	$(2x-3)^2 > x$ $4x^2-12x+9 > x$ $4x^2-13x+9>0$ $(4x-9)(x-1)>0$ $x > \frac{9}{4} = 2\frac{1}{4}$ or $x<1$	M1 – Simplification M1 – Factorisation A1	
2	Method 1 $T_{r+1} = \binom{9}{r}(2x)^{9-r}\left(-\frac{1}{x^2}\right)^r$ $= \binom{9}{r}(2)^{9-r}(x)^{9-r}(-1)^r(x^{-2})^r$ $= \binom{9}{r}(2)^{9-r}(-1)^r(x)^{9-r}(x)^{-2r}$ $= \binom{9}{r}(2)^{9-r}(-1)^r(x)^{9-r-2r}$ $(x)^{9-3r} = x^0$ $9-3r=0$ $r=3$ $T_4 = \binom{9}{3}(2)^{9-3}(-1)^3$ $= -5376$ Method 2 $\left(2x-\frac{1}{x^2}\right)^9$ $= (2x)^9 + \binom{9}{1}(2x)^8\left(-\frac{1}{x^2}\right) + \binom{9}{2}(2x)^7\left(-\frac{1}{x^2}\right)^2$ $+\binom{9}{3}(2x)^6\left(-\frac{1}{x^2}\right)^3 + \ldots$ By inspection, the 4[th] term is the term independent of x.	M1 – Simplification M1 – Find r M1 A1 M1 – Pattern clear M1 – Stating term M1 A1	11/2

<table>
<tr><th>Qn. #</th><th>Solution</th><th>Mark Allocation</th><th>Markers' Report</th></tr>
<tr><td></td><td>$T_4 = \binom{9}{3}(2)^6(-1)^3$
$= -5376$</td><td></td><td></td></tr>
<tr><td>3</td><td>$\log_3 x^2 - 1 = 3\log_x 3$
$2\log_3 x - 1 = \frac{3}{\log_3 x}$
Let $u = \log_3 x$.
$2u - 1 = \frac{3}{u}$
$2u^2 - u - 3 = 0$
$(2u-3)(u+1) = 0$
$u = \frac{3}{2}$ or $u = -1$
$\log_3 x = \frac{3}{2}$ or $\log_3 x = -1$
$x = 3^{\frac{3}{2}}$ or $x = 3^{-1}$
$x = \sqrt{3^3}$
$x = \sqrt{27}$ or $x = \frac{1}{3}$
$x = 3\sqrt{3} = 5.20$ (3sf)</td><td>M1 – Convert base

M1 – Factorisation

M1 – Log form to index form

A1 – Both correct</td><td></td></tr>
<tr><td>4</td><td>$3^{x+1} = 27(3^{y-1})$ - Eq (1)
$\log_2 6 + 1 = \log_2(6x+3y)$ - Eq (2)

From (1),
$3^{x+1} = 3^3(3^{y-1})$
$x + 1 = 3 + y - 1$
$x - y = 1$ - Eq (3)

From (2),
$\log_2 6 + \log_2 2 = \log_2(6x+3y)$
$\log_2 6 \times 2 = \log_2(6x+3y)$
$\log_2 12 = \log_2(6x+3y)$
$6x + 3y = 12$
$2x + y = 4$ - Eq (4)</td><td>M1 – Change of base and laws of indices

M1 – Logarithmic laws

M1 – substitution or elimination</td><td></td></tr>
</table>

Qn. #	Solution	Mark Allocation	Markers' Report
	(3) + (4), $3x = 5$ $x = 1\frac{2}{3}$ Sub $x = 1\frac{2}{3}$ into (3), $y = \frac{2}{3}$ Therefore, $x = 1\frac{2}{3}$ and $y = \frac{2}{3}$.	A1/A1	
5	$(k+x)(1-3x)^n = 3 - 44x + px^2 + \ldots$ $(k+x)(1-3x)^n$ $= (k+x)\left[1 + (n)(-3x) + \frac{n(n-1)}{2\times 1}(-3x)^2 +\right.$ $(k+x)(1-3x)^n$ $= (k+x)\left(1 - 3nx + \frac{n(n-1)}{2}(9x^2)\right)$ $(k+x)(1-3x)^n$ $= (k+x)\left(1 - 3nx + \frac{9n(n-1)}{2}x^2\right)$ Therefore, $(k+x)\left(1 - 3nx + \frac{9n(n-1)}{2}x^2\right)$ $= 3 - 44x + px^2 + \ldots$ Comparing constant, $k = 3$. Comparing coefficient of x, $-3kn + 1 = -44$ $-3(3)n = -45$ $n = 5$. Comparing coefficient of x^2, $\frac{9kn(n-1)}{2} - 3n = p$ $\frac{9(3)(5)(5-1)}{2} - 3(5) = p$ $p = 255$. Therefore, $k = 3$, $n = 5$ and $p = 255$.	M1 – Binomial expansion A1 M1 A1 M1 – Simplification A1	113

Qn. #	Solution	Mark Allocation	Markers' Report
6i	LHS $= \frac{\tan A}{\sec A+1}+\frac{\tan A}{\sec A-1}$ $=\frac{\tan A(\sec A-1)+\tan A(\sec A+1)}{(\sec A+1)(\sec A-1)}$ $=\frac{\tan A[(\sec A-1)+(\sec A+1)]}{(\sec A+1)(\sec A-1)}$ $=\frac{\tan A(\sec A-1+\sec A+1)}{\sec^2 A-1^2}$ $=\frac{\tan A(2\sec A)}{\sec^2 A-1^2}$ $=\frac{\tan A(2\sec A)}{\tan^2 A}$ $=\frac{(2\sec A)}{\tan A}$ $=\frac{2}{\cos A}\div\frac{\sin A}{\cos A}$ $=\frac{2}{\cos A}\times\frac{\cos A}{\sin A}$ $=\frac{2}{\sin A}$ $=2\operatorname{cosec}A$ = RHS (proven)	M1 – Trigo Identity $(\sec A+1)(\sec A-1)$ $=\sec^2 A-1$ M1 – Trigo Identity ($\sec^2 A-1=\tan^2 A$) M1 - $\sec A=\frac{1}{\cos A}$ A1 – Simplification	
6ii	$\frac{\tan A}{\sec A+1}+\frac{\tan A}{\sec A-1}=5$ $2\operatorname{cosec}A=5$ $\frac{2}{\sin A}=5$ $\sin A=\frac{2}{5}$ Basic angle, $\alpha=\sin\frac{2}{5}=0.412$ rad $A=0.412$ rad (3sf) or $A=2.73$ rad (3sf)	M1 – Simplification M1 – Basic angle A1	
7a)	$y=2x+k$ - Eq 1 $y=x^2-3x+4$ - Eq 2 Sub (1) into (2), $x^2-3x+4=2x+k$ $x^2-5x+4-k=0$	M1 – Simplifying	

<table>
<tr><th>Qn. #</th><th>Solution</th><th>Mark Allocation</th><th>Markers' Report</th></tr>
<tr><td></td><td>For line to be tangent to curve,
$b^2 - 4ac = 0$.
$(-5)^2 - 4(1)(4-k) = 0$
$25 - 16 + 4k = 0$
$k = -\frac{9}{4} = -2\frac{1}{4}$</td><td>M1 – Discriminant = 0

A1</td><td></td></tr>
<tr><td>7b)</td><td>$x^2 + 12x + 9 > 4x + k$
$x^2 + 8x + 9 - k > 0$

$b^2 - 4ac < 0$
$8^2 - 4(1)(9-k) < 0$
$64 - 36 + 4k < 0$
$4k < -28$
$k < -7$</td><td>M1 – Simplifying

M1 – Discriminant < 0

A1</td><td></td></tr>
<tr><td>8i)</td><td>$\alpha^2 - \alpha\beta + \beta^2$
$= \alpha^2 + 2\alpha\beta + \beta^2 - 3\alpha\beta$
$= (\alpha + \beta)^2 - 3\alpha\beta$
(shown)</td><td>B1</td><td></td></tr>
<tr><td>8ii)</td><td>$2x^2 - 3x + 1 = 0$
$\alpha + \beta = \frac{3}{2}$
$\alpha\beta = \frac{1}{2}$
$\alpha^3 + \beta^3$
$= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$
$= \left(\frac{3}{2}\right)\left[\left(\frac{3}{2}\right)^2 - 3\left(\frac{1}{2}\right)\right]$
$= 1\frac{1}{8}$
$\alpha^3\beta^3 = (\alpha\beta)^3$
$= \left(\frac{1}{2}\right)^3$
$= \frac{1}{8}$</td><td>M1
M1
M1 – factorise using sum of cube $\alpha^3 + \beta^3$

M1

M1

A1</td><td>114</td></tr>
</table>

Qn. #	Solution	Mark Allocation	Markers' Report
	New equation is $x^2 - \frac{9}{8}x + \frac{1}{8} = 0$ $8x^2 - 9x + 1 = 0$		
9i)	amplitude = 3 period = π	B1 B1	
9ii)	When $y = 0$, $3\cos 2x = 0$ $\cos 2x = 0$ $2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	A1/A1 (1 mark for every 2 correct answers)	
9iii) 9iv)		Shape – B1 Period – B1 Coordinates – B1 $y = \frac{2x}{3\pi}$ = B1 $x = \pi$, $y = \frac{2}{3}$ $x = 2\pi$, $y = \frac{4}{3}$	
9v)	$9\pi \cos 2x = 2x$ $3\cos 2x = \frac{2x}{3\pi}$ No. of solutions = 4	 M1 A1	
10i)	Let equation of perpendicular bisector of AB be $y = mx + c$. $m_{AB} = \frac{11-9}{4-6} = -1$ Since $m_{AB} = -\frac{1}{m}$, $m = 1$. Therefore, $y = x + c$	 M1 – gradient of m M1 – midpoint of AB	

Qn. #	Solution	Mark Allocation	Markers' Report
	Midpoint, M of AB $=\left(\frac{4+6}{2},\frac{11+9}{2}\right)=(5,10)$. $M(5,10)$ lies on perpendicular bisector, hence $y-10=x-5$ $y=x+5$	A1	
10ii)	Centre passes through perpendicular bisector as well as line $y=2x$, hence, $y=x+5$ - eq (1) $y=2x$ - eq (2) Sub (2) into (1), $2x=x+5$ $x=5$ $y=10$ Centre $=(5,10)$	M1 – Substitution A1	
10iii)	Let equation of circle be $(x-5)^2+(y-10)^2=r^2$ Using $C(5,10)$ and $A(4,11)$, $r^2=(11-10)^2+(4-5)^2=2$ Therefore, $(x-5)^2+(y-10)^2=2$	M1 – Find r^2 A1	
10iv)	Centre becomes $=(10,5)$ Therefore, new equation is $(x-10)^2+(y-5)^2=2$	M1 – new centre A1	
11a)	Let $f(x)=2x^3-9x^2+3x+4$ $f(1)=0$, therefore $(x-1)$ is a factor of $f(x)$. $2x^3-9x^2+3x+4=(x-1)(ax^2+bx+c)$ Comparing coefficient of x^3, $a=2$. Comparing constant, $c=-4$. Comparing coefficient of x, $3=c-b$ $3=-4-b$	M1 (find factor) M1 M1	115

Qn. #	Solution	Mark Allocation	Markers' Report
	$b=-7$ $2x^3-9x^2+3x+4=(x-1)(2x^2-7x-4)$ $2x^3-9x^2+3x+4=(x-1)(2x+1)(x-4)$ $(x-1)(2x+1)(x-4)=0$ $x=1,\ x=-\frac{1}{2}$ or $x=4$	M1 A1	
11b)	Let $f(x)=6x^3+px^2+qx+10$ Since $f\left(\frac{1}{2}\right)=0$, $6\left(\frac{1}{2}\right)^3+p\left(\frac{1}{2}\right)^2+q\left(\frac{1}{2}\right)+10=0$ $\frac{3}{4}+\frac{p}{4}+\frac{q}{2}+10=0$ $\frac{p}{4}+\frac{q}{2}+10\frac{3}{4}=0$ $p+2q=-43$ - Eq (1) Since $f(-2)=-20$, $6(-2)^3+p(-2)^2+q(-2)+10=-20$ $-48+4p-2q+10=-20$ $4p-2q=18$ $2p-q=9$ - Eq (2) From (1), $p=-43-2q$ - Eq (3) Sub (3) into (2), $2(-43-2q)-q=9$ $-86-4q-q=9$ $-5q=95$ $q=-19$ $p=-5$	M1 – Form Eq (1) M1 – Form Eq (2) M1 – Simplification A1 A1	
12i)	Let the equation of QR be $y=mx+c$.		

Qn. #	Solution	Mark Allocation	Markers' Report
	$m=\frac{6}{2}=3$ Sub $(3, 4)$, $4=3(3)+c$ $c=-5$ $y=3x-5$	M1 – Gradient A1	
12ii)	Let equation of RS be $y=mx+c$. $m=-\frac{1}{3}$ Sub $(10, 5)$, $5=\left(-\frac{1}{3}\right)(10)+c$ $c=8\frac{1}{3}$ $y=-\frac{1}{3}x+8\frac{1}{3}$ To find R, $y=3x-5$ - Eq (1) $y=-\frac{1}{3}x+8\frac{1}{3}$ - Eq (2) Sub (1) into (2), $3x-5=-\frac{1}{3}x+8\frac{1}{3}$ $9x-15=-x+25$ $10x=40$ $x=4$ $y=3(4)-5=7$ $R(4, 7)$	M1 – Gradient M1 – Equation of RS M1 – Simplification A1	
12iii)	$P(10, 4)$ Area of $PQRS$ $=\frac{1}{2}\times\begin{vmatrix}10 & 10 & 4 & 3 & 10\\4 & 5 & 7 & 4 & 4\end{vmatrix}$	M1	116

Qn. #	Solution	Mark Allocation	Markers' Report
	$= 13.5 \text{ units}^2$	A1	
12iv)	$T = \left(4 + \left(\frac{3}{5} \times 6\right),\ 7 - \left(\frac{3}{5} \times 2\right)\right)$ $T = \left(7\frac{3}{5},\ 5\frac{4}{5}\right)$	M1 A1	

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END.