Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
MID-YEAR EXAMINATION 2016
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS **4047**

Wednesday **04 May 2016** **2 hours**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

This document consists of **5** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Solve the simultaneous equations

$$2^x = 4^{y-1},$$

$$(27^{2y})^{-2} = 81^{\frac{1}{2}x}.$$ [5]

2 The roots of the quadratic equation $2x^2 - 56x + 32 = 0$ are α^2 and β^2 where $\alpha + \beta > 0$ and $\alpha\beta > 0$.

(i) Find the value of $\alpha\beta$. [2]

(ii) Show that $\alpha + \beta = 6$. [2]

(iii) Find the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$. [3]

3 **(a)** Solve the equation $x(5x-1) < 3x$. [2]

(b) Show that the roots of the equation $x^2 - p = 2 - px$ are real and distinct for all values of p. [3]

(c) Find the range of values of k for which the line $y = kx + 3$ meets the curve $2x^2 = xy + 6$. [3]

4 Express $\frac{2x^2 - 3x - 2}{x^2 - 2x - 8}$ in partial fractions. [5]

5 Given that $\sqrt{a + b\sqrt{2}} = \frac{3\sqrt{2}}{2 - \sqrt{2}}$, where a and b are integers, find without using a calculator, the value of a and of b. [4]

6 Given that $f(x) = x^3 + 7x^2 + 8x - 12$.

(i) Show that $(x+3)$ is a factor of $f(x)$. [1]

(ii) Given that $x^3 + 7x^2 + 8x - 12 = (x+3)(Ax^2 + Bx + C)$ where A, B and C are constants, find the values of A, B and C. [3]

(iii) Hence, solve the equation $f(x) = 0$. [2]

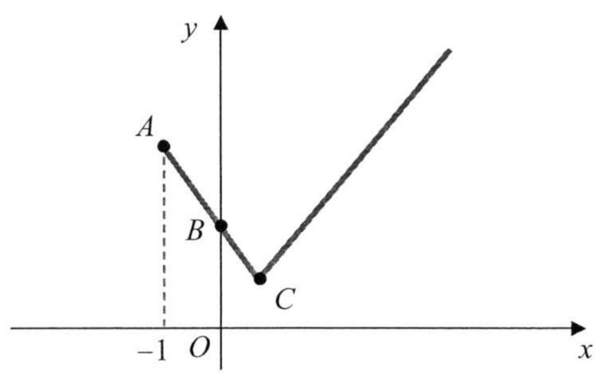
(iv) Use the solutions of $f(x) = 0$, solve the equation $x^6 + 7x^4 + 8x^2 - 12 = 0$. [2]

7 **(a)** Using algebraic identities or otherwise, factorise $f(x) = 64 - (x+1)^3$ completely.

Find the remainder when $f(x)$ is divided by $(1-x)$. [3]

(b) The expression $x^2 - (k+7)x + k^2 - 26$ is exactly divisible by $(x + 2)$ but leaves a remainder when divided by $(x-11)$. Find the value of k. [5]

8 The diagram shows part of the graph of $y = |2x-1| + 3$ for $-1 \le x \le 3$.



(a) State the coordinates of A, B and C. [3]

(b) Solve the equation $|2x-1| + 3 = 8$. [3]

9 **(a)** In the expansion of $(1-2x)^n$ in ascending powers of x, the coefficient of the third term is 24.

(i) Find the value of n, where n is a positive integer. [4]

(ii) Hence find the coefficient of x in the expansion of $\left(3x+\frac{1}{x}\right)(1-2x)^n$. [2]

(b) Find the term independent of x in the expansion of $\left(x^2+\frac{1}{2x}\right)^9$. [4]

10 **(a)** By using an appropriate substitution, solve the equation $2^{x+1}+2^{-x}=3$. [4]

(b) A piece of chicken is removed from the refrigerator and it is then allowed to thaw. Its temperature, T °C after t minutes, is given by the formula $T=28-32e^{-0.2t}$. Find

(i) the initial temperature, [1]

(ii) the value of T when $t = 10$ minutes, [1]

(iii) the value of T as t becomes very large. Explain the significance of this value. [2]

11 The line $2x+y+3=0$ intersects with the curve $y=x^2-6x$ at points R and S.

(i) Find the coordinates of R and S. [4]

(ii) Find the equation of the perpendicular bisector of RS. [4]

(iii) Line l is parallel to RS and passes through $T(5, 3)$. Given that line l cuts the y-axis at U, find the coordinates of the point U. [3]

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Answer Scheme AMKSS 2016 MYE 3E AM

	Answer	Marks
1 **[5]**	$2^x = 2^{2(y-1)} \Rightarrow x = 2y-2$ --------(1) $(3^{6y})^{-2} = (3^4)^{\frac{1}{2}x} \Rightarrow -12y = 2x$ ------(2) Subst (1) into (2) $-12y = 2(2y-2)$ $y = \frac{1}{4}$ $x = -1\frac{1}{2}$	M1 M1 M1 (any mtd) A1 A1
2(i) **[2]**	$\alpha^2\beta^2 = 16$ $\alpha\beta = 4$ or -4 (reject)	M1 A1
2(ii) **[2]**	$\alpha^2+\beta^2 = 28$ $(\alpha+\beta)^2 = \alpha^2+2\alpha\beta+\beta^2$ $= 28+2(4)$ $\alpha+\beta = 6$ or -6 (reject)	M1 A1
2(iii) **[3]**	$\alpha+\frac{1}{\beta}+\beta+\frac{1}{\alpha} = \frac{\beta+\alpha}{\alpha\beta}+(\alpha+\beta)$ $= 6+\frac{6}{4} = \frac{15}{2}$ $\left(\alpha+\frac{1}{\beta}\right)\times\left(\beta+\frac{1}{\alpha}\right) = \alpha\beta+1+1+\frac{1}{\alpha\beta}$ $= 4+2+\frac{1}{4} = \frac{25}{4}$ The equation whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$ is $x^2-\frac{15}{2}x+\frac{25}{4} = 0$ or $4x^2-30x+25=0$.	M1 M1 A1

3a **[2]**	$x(5x-1)<3x$ $5x^2-4x<0$ $x(5x-4)<0$ $0<x<\frac{4}{5}$	 M1 A1
3b **[3]**	$x^2+px-p-2=0$ Discriminant $=p^2-4(1)(-p-2)$ $=p^2+4p+8$ $=(p+2)^2-4+8$ $=(p+2)^2+4>0$ Since discriminant is always positive, hence roots are real for all values of p	M1(correct a, b,c) M1(complete the square) M1 for >0
3c **[3]**	$2x^2=x(kx+3)+6$ $=kx^2+3x+6$ $2x^2-kx^2-3x-6=0$ Since line meets curve, $D\geq 0$, $(-3)^2-4(2-k)(-6)\geq 0$ $9+48-24k\geq 0$ $24k\leq 57$ $k\leq\frac{19}{8}$ $k\leq 2\frac{3}{8}$	M1($D\geq 0$) M1(correct a,b,c) A1
4 **[5]**	$\frac{2x^2-3x-2}{x^2-2x-8}=2+\frac{x+14}{x^2-2x-8}$ by long division $\frac{x+14}{(x-4)(x+2)}=\frac{A}{x-4}+\frac{B}{x+2}$ $x+14=A(x+2)+B(x-4)$ Sub $x=-2$, $-2+14=B(-2-4)\Rightarrow B=-2$ Sub $x=4$, $4+14=A(4+2)\Rightarrow A=3$ $\frac{2x^2-3x-2}{x^2-2x-8}=2+\frac{3}{x-4}-\frac{2}{x+2}$	M1 M1 M1 M1 A1

5 **[4]**	$\sqrt{a+b\sqrt{2}}=\dfrac{3\sqrt{2}}{2-\sqrt{2}}$ $a+b\sqrt{2}=\dfrac{(3\sqrt{2})^2}{(2-\sqrt{2})^2}$ $=\dfrac{9(2)}{4-4\sqrt{2}+2}$ $=\dfrac{18}{6-4\sqrt{2}}$ $=\dfrac{18}{6-4\sqrt{2}}\times\dfrac{6+4\sqrt{2}}{6+4\sqrt{2}}$ $=\dfrac{108+72\sqrt{2}}{36-16(2)}$ $=27+18\sqrt{2}$ Hence $a=27$, $b=18$	M1 M1 M1 A1
6(i) **[1]**	$f(x)=x^3+7x^2+8x-12$ $f(-3)=(-3)^3+7(-3)^2+8(-3)-12=0$ $\therefore (x+3)$ is a factor of $f(x)$	M1(show subst)
6(ii) **[3]**	$x^3+7x^2+8x-12=(x+3)(Ax^2+Bx+C)$ Comparing x^3, $A=1$ Comparing constant, $3C=-12\Rightarrow C=-4$ Comparing x^2, $3A+B=7\Rightarrow B=4$	B1 B1 B1
6(iii) **[2]**	$(x+3)(x^2+4x-4)=0$ $x=-3$ or $x^2+4x-4=0$ $x=\dfrac{-4\pm\sqrt{16+4(4)}}{2}$ $=-2\pm2\sqrt{2}$ = 0.828, -4.83	A1 A1
6(iv) **[2]**	Replace x by x^2, $x^2=-3(NA)$ or $x^2=-2-2\sqrt{2}(NA)$ or $x^2=-2+2\sqrt{2}$ $x=\pm0.910$	M1 A1

7(a) **[3]**	$f(x)=4^3-(x+1)^3$ $=[4-(x+1)][4^2+4(x+1)+(x+1)^2]$ $=(3-x)(x^2+6x+21)$ Remainder $=f(1)=64-2^3=56$	M1 A1 A1
7(b) **[5]**	Let $f(x)=x^2-(k+7)x+k^2-26$. $f(-2)=0$, $4+2k+14+k^2-26=0$ $k^2+2k-8=0$ $(k+4)(k-2)=0$ $k=-4$ or 2 When $k=-4$, $f(x)=x^2-3x-10$ $f(11)=11^2-3(11)-10=78$ When $k=2$, $f(x)=x^2-9x-22$ $f(11)=11^2-9(11)-22=0$ Since f(x) is divisible by $(x+2)$ but not $(x-11)$, therefore $k=-4$	 M1 M1 M1 M1 A1
8a **[3]**	Let $x=-1, y=\|-3\|+3=6 \Rightarrow A(-1,6)$ Let $x=0, y=\|-1\|+3=4 \Rightarrow B(0,4)$ Let $2x-1=0, x=0{,}5, y=\|0\|+3=3 \Rightarrow C(0.5,3)$	B1 B1 B1
8b **[3]**	$\|2x-1\|+3=8$ $\|2x-1\|=5$ $2x-1=5$ or $2x-1=-5$ $x=3$ or $x=-2$	 M1 A1, A1

9ai **[4]**	$(1-2x)^n = 1 + {}^nC_1(-2x) + {}^nC_2(-2x)^2 + \ldots$ $= 1 - 2nx + 2n(n-1)x^2 + \ldots$ $2n^2 - 2n = 24$ $n^2 - n - 12 = 0$ $(n-4)(n+3) = 0$ $n = 4$ or $n = -3 (NA)$	M2 M1 A1
9aii **[2]**	$\left(3x + \frac{1}{x}\right)(1-2x)^n = \left(3x + \frac{1}{x}\right)(1 - 8x + 24x^2 + \ldots)$ $= 3x + 24x + \ldots$ $\therefore$ Coefficient of $x = 27$	M1 A1
9b **[4]**	General term for $\left(x^2 + \frac{1}{2x}\right)^9$ $= {}^9C_r (x^2)^{9-r} \left(\frac{1}{2x}\right)^r$ $== {}^9C_r x^{18-2r} \left(\frac{1}{2}\right)^r x^{-r}$ $= {}^9C_r \left(\frac{1}{2}\right)^r x^{18-3r}$ For term indept of x, let $18 - 3r = 0 \Rightarrow r = 6$ Hence term is ${}^9C_6 \left(\frac{1}{2}\right)^6 = \frac{21}{16} = 1\frac{5}{16}$	 M1 M1 M1 A1
10a **[4]**	$2^{x+1} + 2^{-x} = 3$ $2(2^x) + \frac{1}{2^x} = 3$ Let $u = 2^x$, $2u + \frac{1}{u} = 3$ $2u^2 - 3u + 1 = 0$ $(2u-1)(u-1) = 0$ $u = \frac{1}{2}$ or $u = 1$ $2^x = 2^{-1}$ $2^x = 2^0$ $x = -1$ $x = 0$	 M1 M1 A1,A1
10bi	When $t = 0$, $T = 28 - 32e^0 = -4°C$	B1
10bii	When $t = 10$, $T = 28 - 32e^{-0.2(10)} = 23.7°C$	B1
10 biii **[2]**	$t \to \infty$, $\frac{32}{e^{0.2t}} \to 0$ $\therefore T = 28°C$ $28°C$ is the room temperature.	B1 B1

11i **[4]**	$2x+y+3=0 \Rightarrow y=-2x-3$ ------(1) $y=x^2-6x$ -------(2) (1)=(2) $x^2-6x=-2x-3$ $x^2-4x+3=0$ $(x-1)(x-3)=0$ $x=1$ or $x=3$ $y=-5$ $\quad y=-9$ $R(1,-5)$ $\quad S(3,-9)$	M1 M1 A1,A1
11ii **[4]**	Midpoint of $RS=\left(\frac{1+3}{2},\frac{-5+(-9)}{2}\right)=(2,-7)$ Grad of $RS=\frac{-9-(-5)}{3-1}=-2$ Grad of perpendicular bisector $=\frac{1}{2}$ Eqn of perpendicular bisector $y-(-7)=\frac{1}{2}(x-2)$ $y+7=\frac{1}{2}x-1$ $y=\frac{1}{2}x-8$	M1 M1 M1 (use midpt & correct grad) A1
11iii **[3]**	$m_l=m_{RS}=-2$ Eqn of line *l*: $y-3=-2(x-5)$ $y=-2x+13$ Cuts the *y*-axis, let $x=0$, $y=13$ $\therefore U(0,13)$	M1 M1 A1