1 The straight line $x + y = 12$ meets the curve $y = \frac{10}{x} + x$ at two points A and B.

(i) Calculate the coordinates of A and of B. [4]

(ii) Hence, find the equation of the perpendicular bisector of AB. [4]

2 Solve the following simultaneous equations: [4]

$$3^{x-1} = 9\left(3^{y}\right)$$
$$2^{x} + 2^{y} = 9$$

3 (a) Find the value of c for which the line $y = 2x + c$ is a tangent to the curve

$y = 2x^2 - 6x + 5$. [3]

(b) Show that the equation $(p+2)x^2 - 2\left(\sqrt{2p+1}\right)x + 4 = 0$ cannot have real roots for all values of x as long as $p > 0$. [3]

4 (a) Solve the equation $9^{x+1} = 10\left(3^{x}\right) - 1$. [4]

(b) Given that $k = \frac{1}{\sqrt{2}}$ and that $p = \frac{1+k}{1-k}$, prove that $p = 3 + 2\sqrt{2}$. [4]

5 (a) Express $\left(\frac{4\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}\right)$ in the form a + b$\sqrt{c}$ where a, b and c are constants.

[3]

(b) Given that $\left(\frac{\sqrt{40}}{3} - \frac{1}{\sqrt{10}}\right)\left(\frac{30}{\sqrt{5}}\right) = k\sqrt{2}$, find the integer value of k. [4]

6 It is given that $x^4 - x^3 - 7x^2 + x + 6 = \left(x^2 - 2\right)Q(x) + ax - 4$ for all values of x where $Q(x)$ is a polynomial in x and a is a constant. Find

(i) $Q(x)$, [4]

(ii) the value of constant a , [1]

(iii) the reminder when $x^4 - x^3 - 7x^2 + x + 6$ is divided by $x + 3$. [2]

Start Question 7 on a fresh sheet of Answer Paper.

Hand in Questions 7 to Question 11 separately from Question 1 to Question 6.

7 **(i)** Find the range of values of m for which the line $y = mx - 4$ meets the curve

$y = 2x^2 + 2x + m$. [5]

(ii) Using the smallest positive integer value of m found in **(i)**, find the coordinates of the point of intersection between the line and the curve in **(i).** [3]

8 **(a)** Solve the quadratic inequality $2x^2 - 8x + 14 \geq 3x^2 - 8x + 10$. [3]

(b) Find the range of values of x for which $\frac{3(x+1)^2}{4} < x + 2 \leq 2x(x-1)$. [5]

9 The roots of the equation $2x^2 - x + 6 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [4]

(ii) Factorise $\alpha^3 + \beta^3$ completely and show that $\alpha^3 + \beta^3 = -\frac{35}{8}$. [4]

(iii) Hence, form a quadratic equation whose roots are α^3 and β^3. [2]

10 If $5x^3 + Ax^2 + x + 3 = (x+1)(Bx^2 + Cx) + D$, find the values of A, B, C and D. [5]

11 **(i)** Given that $f(x) = x^3 + 2x^2 - 17x + 6$,

(a) show that $x - 3$ is a factor, [1]

(b) solve the equation $f(x) = 0$, giving your answers to 2 decimal places where appropriate. [3]

(ii) Hence, or otherwise, express $\frac{11}{x^3 + 2x^2 - 17x + 6}$ in partial fractions. [5]

~End of Paper~

Have you checked your work?

Answer Key

1 (i) A = (1, 11) B= (5, 7) (ii) y = x+6

2 x = 3 , y = 0

3 (a) c = - 3 (b) D = -4 (p+9) < 0

4 (a) x = -2 or x = 0 (b) $3+2\sqrt{2}$

5 (a) $4\sqrt{10}+12$ (b) k = 17

6 (i) $Q(x)=x^2-x-5$ (ii) $a=-1$ (iii) $f(-3)=48$

7 (i) $m\le -2$ *or* $m\ge 14$ (ii) $(3,\ 38)$

8 (a) $-2 < x \le 2$ (b) $-1\frac{2}{3} < x \le -\frac{1}{2}$

9 (i) $-5\frac{3}{4}$ (ii) $-\frac{35}{8}$ (proved)

(iii) $8x^2+35x+216=0$

10 (a) A = 6, B = 5 , C = 1, D = 3

11 (i) (a) f(3) = 0 (b) x = 3 or x = -5.37 or x = 0.37

(ii) $\frac{1}{2(x-3)}-\frac{x+8}{2(x^2+5x-2)}$

Sec 3 Express A. Maths MY 2016

1(i)
$$y = 12 - x \quad — (1)$$
$$y = \frac{10}{x} + x \quad — (2)$$

$$\therefore \frac{10}{x} + x = 12 - x \qquad M1$$
$$10 + x^2 = 12x - x^2$$
$$2x^2 - 12x + 10 = 0$$
$$x^2 - 6x + 5 = 0 \qquad M1$$
$$(x-5)(x-1) = 0$$
$$x = 1 \text{ or } x = 5 \qquad A1$$
$$\therefore y = 11 \text{ or } y = 7 \qquad A1$$

(4)

$$\therefore A \equiv (1, 11) \quad B = (5, 7)$$

(ii) Grad of $AB = \frac{11-7}{1-5}$
$$= \frac{4}{-4}$$
$$= -1 \qquad 1$$

Mid pt $\equiv \left(\frac{1+5}{2}, \frac{11+7}{2}\right)$
$$= (3, 9) \qquad 1$$

$\therefore$ Eqn of $\perp$ bisector is $\frac{y-9}{x-3} = 1$ 1
$$y - 9 = x - 3$$
$$y = x + 6 \qquad 1$$

(4)

(8)

2)

$$3^{x-1} = 9(3^{y})$$

$$3^{x-1} = 3^{2}(3^{y})$$

$$= 3^{2+y}$$

$$\therefore x-1 = 2+y$$

$$x = 3+y \quad \text{— ①} \qquad M1$$

$$2^{x} + 2^{y} = 9$$

$$\therefore 2^{3+y} + 2^{y} = 9 \qquad M1$$

$$8(2^{y}) + 2^{y} = 9$$

$$9(2^{y}) = 9$$

$$2^{y} = 1$$

$$y = 0 \qquad 1$$

$$\therefore x = 3$$

Ans: $x = 3, \; y = 0$ 1

4

3a)

$$y = 2x + c \quad — ①$$
$$y = 2x^2 - 6x + 5 \quad — ②$$

sub ① → ② $\quad 2x^2 - 6x + 5 = 2x + c$

$$2x^2 - 8x + (5 - c) = 0 \qquad 1$$

since only 1 root, $D = 0$

$$b^2 - 4ac = 0$$
$$64 - 4(2)(5-c) = 0 \qquad 1$$
$$8 - (5 - c) = 0$$
$$3 + c = 0$$
$$\underline{\underline{c = -3}} \qquad 1$$

3

b) $(p+2)x^2 - 2(\sqrt{2p+1})x + 4 = 0$

$$D = b^2 - 4ac$$
$$= 4(2p+1) - 4(p+2)(4) \qquad 1$$
$$= 4[2p + 1 - 4p - 8]$$
$$= 4[-2p - 7]$$
$$= -4[p + 7] \qquad 1$$
$$\underline{\underline{< 0}} \text{ if } p > 0 \qquad 1$$

∴ It cannot have real roots for all values of x as long as $p > 0$

proved.

3

6

4 a) $9^{x+1} = 10(3^x) - 1$

$9(3^x)^2 = 10(3^x) - 1$ 1

Let $y = 3^x$

$9y^2 - 10y + 1 = 0$ 1

$(9y-1)(y-1) = 0$

$y = \frac{1}{9}$ or $y = 1$

$\therefore 3^x = 3^{-2}$ or $3^x = 3^0$ 1

$\therefore x = -2$ or $x = 0$ 1 (4)

b) $p = \frac{1+k}{1-k}$

$= \frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}$ 1

$= \frac{\sqrt{2}+1}{\sqrt{2}} \div \frac{\sqrt{2}-1}{\sqrt{2}}$

$= \frac{\sqrt{2}+1}{\sqrt{2}-1}$ 1

$= \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}$ 1

$= \frac{2+2\sqrt{2}+1}{1}$ 1

$= 3+2\sqrt{2}$ proved. (4)

(8)

5 a) $$\frac{4\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{4\sqrt{2}(2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})}$$ 1

$$= \frac{8\sqrt{10}+24}{20-18}$$

$$= \frac{8\sqrt{10}+24}{2}$$ 1

$$= \underline{\underline{4\sqrt{10}+12}}$$ 1

3

(b) $$\left(\frac{\sqrt{40}}{3} - \frac{1}{\sqrt{10}}\right)\frac{30}{\sqrt{5}} = k\sqrt{2}$$

$$\left(\frac{20-3}{3\sqrt{10}}\right)\left(\frac{30}{\sqrt{5}}\right) = k\sqrt{2}$$ 1

$$\left(\frac{17}{3\sqrt{10}}\right)\left(\frac{30}{\sqrt{5}}\right) = k\sqrt{2}$$ 1

$$\frac{170}{5\sqrt{2}} = k\sqrt{2}$$

$$\frac{170}{10} = k$$ 1

$$\therefore \underline{\underline{k = 17}}$$ 1

4

7

6) $x^4 - x^3 - 7x^2 + x + 6 = (x^2-2)\,Q(x) + ax - 4$

$$
\begin{array}{r l}
 & x^2 - x - 5 \\
\cline{2-2}
x^2-2 \,\big) & x^4 - x^3 - 7x^2 + x + 6 \\
-) & x^4 \qquad - 2x^2 \\
\cline{1-2}
 & -x^3 - 5x^2 + x \\
-) & -x^3 \qquad + 2x \\
\cline{1-2}
 & -5x^2 - x + 6 \\
-) & -5x^2 \qquad + 10 \\
\cline{1-2}
 & -x - 4 \\
\cline{2-2}
\end{array}
$$

M1

M1

M1

$\therefore x^4 - x^3 - 7x^2 + x + 6 = (x^2-2)(x^2-x-5) - x - 4$

(i) $Q(x) = \underline{\underline{x^2 - x - 5}}$ A1

(ii) $a = \underline{\underline{-1}}$ A1 (5)

(iii) let $f(x) = x^4 - x^3 - 7x^2 + x + 6$

$f(-3) = 81 + 27 - 63 - 3 + 6$ M1

$= \underline{\underline{48}}$ A1 (2)

(7)

7 (i)

$$y = mx - 4 \quad —①$$

$$y = 2x^2 + 2x + m \quad —②$$

sub ① → ② $\quad 2x^2 + 2x + m = mx - 4$

$$2x^2 + (2-m)x + m + 4 = 0 \qquad M1$$

Meeting the curve $\Rightarrow \quad D \geq 0 \qquad 1$

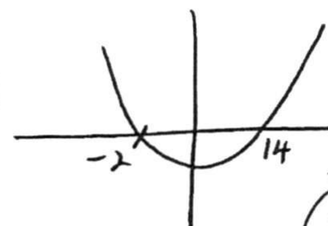
$$\therefore (2-m)^2 - 4(2)(m+4) \geq 0$$

$$4 - 4m + m^2 - 8m - 32 \geq 0$$

$$m^2 - 12m - 28 \geq 0 \qquad 1$$

$$(m - 14)(m+2) \geq 0 \qquad 1$$

$$\therefore \underline{\underline{m \leq -2 \quad \text{or} \quad m \geq 14}} \qquad 1$$



5

(ii) Let $m = 14 \qquad 1$

$$y = 14x - 4 \quad —①$$

$$y = 2x^2 + 2x + 14 \quad —②$$

$$2x^2 + 2x + 14 = 14x - 4$$

$$2x^2 - 12x + 18 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0 \qquad 1$$

$$x = 3$$

$$\therefore y = 14(3) - 4$$

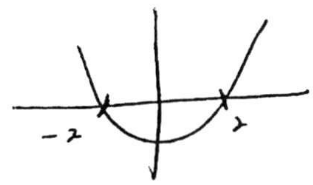
$$= 38$$

8 $\quad \therefore$ The pt is $\underline{\underline{(3, 38)}}$. 1 $\qquad$ 3

8 a) $2x^2 - 8x + 14 \geq 3x^2 - 8x + 10$

$x^2 - 4 \leq 0$ 1

$(x-2)(x+2) \leq 0$



1

$\therefore -2 \leq x \leq 2$ 1

(3)

b) $\dfrac{3(x+1)^2}{4} < x + 2 \leq 2x(x-1)$

$\frac{3}{4}(x+1)^2 < x+2$ and $x+2 \leq 2x(x-1)$

$3(x^2+2x+1) < 4x+8$ and $x+2 \leq 2x^2 - 2x$

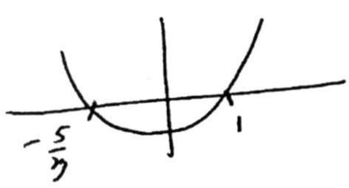
$3x^2 + 6x + 3 < 4x + 8$ 1

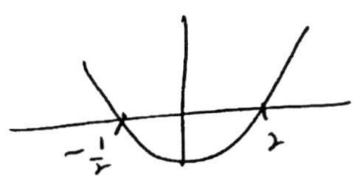
$2x^2 - 3x - 2 \geq 0$ 1

$3x^2 + 2x - 5 < 0$

$(2x+1)(x-2) \geq 0$

$(3x+5)(x-1) < 0$

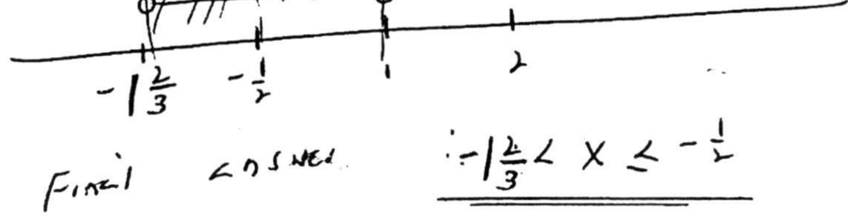




$x \leq -\frac{1}{2}$ or $x \geq 2$ 1

1

$\therefore -\frac{5}{3} < x < 1$



(8)

Final answer $\therefore -1\frac{2}{3} < x \leq -\frac{1}{2}$ 1

(5)

9)

$$2x^2 - x + 6 = 0$$

$\alpha + \beta = \frac{1}{2}$ B1

$\alpha\beta = 3$ B1

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ M1

$= \frac{1}{4} - 6$

$= -5\frac{3}{4}$ A1 (4)

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ B1

$= \left(\frac{1}{2}\right)\left(-5\frac{3}{4} - 3\right)$ 1

$= \left(\frac{1}{2}\right)\left(-8\frac{3}{4}\right)$ 1

$= -4\frac{3}{8}$

$= -\frac{35}{8}$ proved. 1 (4)

(ii) $\alpha^3\beta^3 = (\alpha\beta)^3$

$= 27.$ 1

$\therefore$ The eqn is

$$x^2 - \left(-\frac{35}{8}\right)x + 27 = 0$$

$\therefore \underline{x^2 + \frac{35}{8}x + 27 = 0}$ 1 (2)

(10)

$\underline{8x^2 + 35x + 216 = 0}$

10) $5x^3 + Ax^2 + x + 3 = (x+1)(Bx^2 + Cx) + D$

When $x = 0$ $\quad \underline{3 = D}$ B1

Equating coeff of x^3 : $\underline{5 = B}$ B1

When $x = -1$: $-5 + A - 1 + 3 = D$

$A - 3 = 3$

$\underline{A = 6}$ B1

When $x = 1$: $5 + A + 1 + 3 = 2(B + C) + D$

$5 + 6 + 1 + 3 = 2(5 + C) + 3$ M1

$15 = 10 + 2C + 3$

$= 13 + 2C$

$2 = 2C$

$\underline{C = 1}$ A1

5

Ans : $\underline{A = 6, \ B = 5, \ C = 1, \ D = 3}$

11 i) $f(x) = x^3 + 2x^2 - 17x + 6$

a) $f(3) = 27 + 18 - 51 + 6$

$= 0$

$\therefore$ $x-3$ is a factor.

b)

	1	2	-17	6	
+	0	3	15	-6	3
	1	5	-2	$\boxed{0}$	

$\therefore f(x) = (x-3)(x^2+5x-2)$

For $(x-3)(x^2+5x-2) = 0$

$x = 3$ or $x = \dfrac{-5 \pm \sqrt{25-4(-2)}}{2}$

$x = 3$ or $x = -5.37$ or 0.37 (2 d p)

(ii) $\dfrac{11}{x^3+2x^2-17x+6} = \dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+5x-2}$

$\therefore 11 = A(x^2+5x-2) + (Bx+C)(x-3)$

when $x = 3$, $11 = 22A \Rightarrow A = \frac{1}{2}$

Equating coeff of x^2 : $0 = A + B \Rightarrow B = -\frac{1}{2}$

when $x = 0$ $11 = -2A - 3C$

$\Rightarrow C = -4$

$\therefore \dfrac{11}{x^3+2x^2-17x+6} = \dfrac{1}{2(x-3)} + \dfrac{-\frac{1}{2}x - 4}{x^2+5x-2}$

$= \dfrac{1}{2(x-3)} - \dfrac{\frac{1}{2}x+4}{x^2+5x-2}$

$= \dfrac{1}{2(x-3)} - \dfrac{x+8}{2(x^2+5x-2)}$