NAME:		INDEX NO:		CLASS:	

NORTH VIEW SECONDARY SCHOOL
End-of-Year Examination 2016
Sec 3 Express

ADDITIONAL MATHEMATICS **4047**

11 Oct 2016

Additional Materials: Answer Paper

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Set by: Mdm Lee YP
Vetted by: Mr Chia PC

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\dfrac{7x+2}{x(x-2)^2}$ in partial fractions. [4]

2 The length and width of a rectangle are $\dfrac{2\sqrt{3}}{2-\sqrt{3}}$ cm and $\dfrac{6}{\sqrt{12}}$ cm respectively.
Find the perimeter of the rectangle, expressing your answer in the form $p+q\sqrt{3}$, where p and q are integers. [4]

3 Solve the simultaneous equations

$$3x-y=3,$$
$$2y^2=3xy+10.$$ [5]

4 By using the substitution $y=4^x$ or otherwise, solve the equation

$$4^{2x+1}=33(4^x)-8.$$ [5]

5 Given that $\dfrac{2}{\alpha}+\dfrac{2}{\beta}=-1$ and $\dfrac{4}{\alpha\beta}=\dfrac{2}{3}$, find the quadratic equation whose roots are

(i) $\dfrac{2}{\alpha}$ and $\dfrac{2}{\beta}$, [2]

(ii) α and β. [3]

6 Given that $f(x)=|x-2|+5x$,

(i) find $f(-1)$, [1]

(ii) find the value of x for which $f(x)=4$. [4]

7 Given that $P(x) = 4x^3 - 13x - 6$,

(i) find the remainder when $P(x)$ is divided by $(x+1)$, [1]

(ii) show that $P(x)$ is divisible by $(x-2)$, [1]

(iii) factorize $P(x)$ completely, [3]

(iv) hence, or otherwise, solve the equation $4x^3 = 13x + 6$. [2]

8 **(a)** Solve the quadratic inequality $(2x+1)(2x-1) > 8$. [3]

(b) **(i)** Find the range of values of m for which the equation

$$2x^2 + x + m = mx + 1$$

has no real roots. [4]

(ii) Hence state, giving a reason, what can be deduced about the curve $y = 2x^2 + x + 7$ and the line $y = 7x + 1$. [1]

9 **(a)** Find the term independent of x in the expansion of $\left(\dfrac{1}{2x^3} - x\right)^{12}$. [3]

(b) Find, in ascending powers of x, the first three terms in the expansion of

(i) $(2-x)^5$,

(ii) $\left(1 + \dfrac{1}{2}x\right)^6$.

Hence, find the coefficient of x in the expansion of $(2-x)^5\left(1 + \dfrac{1}{2}x\right)^6$. [5]

10 **(a)** The equation of a circle, C_1, is $x^2+y^2-6x+2ky+17=0$.
Find the values of k if the radius of C_1 is $\sqrt{41}$. [4]

(b) **(i)** Another circle, C_2, has centre (2, 5).
Given that the line $x=8$ is a tangent to C_2, find the equation of C_2. [2]

(ii) Find the possible values of c if $y=c$ is a tangent to C_2. [2]

11 **(a)** Find the value of n for which $\sin\frac{5\pi}{3}+\cot\frac{7\pi}{6}=n\sqrt{3}$. [3]

(b) Find the values of x, between $0°$ and $360°$, which satisfy

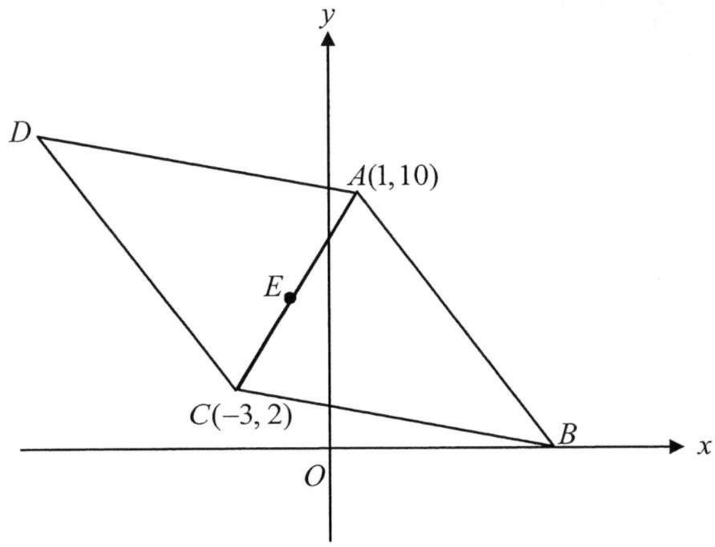
$$\sec x=-\sqrt{2}.$$ [3]

(c) Given that $\cos A=\frac{1}{\sqrt{5}}$ and A is acute, find the exact value of

(i) $\tan(90°-A)$, [2]

(ii) $\operatorname{cosec}(-A)$. [2]

12 **Solutions to this question by accurate drawing will not be accepted.**



The point $A(1,10)$ and $C(-3,2)$ are opposite vertices of a rhombus $ABCD$.
The point B lies on the x-axis and E is the midpoint of AC.

Find

(i) the coordinates of E, [2]

(ii) the equation of BD, [3]

(iii) the coordinates of B, [1]

(iv) the area of the rhombus $ABCD$. [3]

The line $px+qy=0$ is parallel to the diagonal BD.

(v) Express q in terms of p. [2]

2016, S3, NorthView, SA2, AMath / Worked Solutions.

① $\frac{7x+2}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$= \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$

Let $x=0$

$2 = A(-2)^2$

$2 = 4A$

$A = \frac{1}{2}$

Let $x=2$

$7(2)+2 = 2C$

$16 = 2C$

$C = 8$

Let $x=3$

$7(3)+2 = A + 3B + 8(3)$

$23 = \frac{1}{2} + 3B + 24$

$3B = -1.5$

$B = -\frac{1}{2}$

Ans: $\frac{7x+2}{x(x-2)^2} = \frac{1}{2x} - \frac{1}{2(x-2)} + \frac{8}{(x-2)^2}$ ✕

③ $3x - y = 3$ —— ①

$2y^2 = 3xy + 10$ —— ②

From ①, $y = 3x - 3$ —— ③

Sub ③ into ②:

$2(3x-3)^2 = 3x(3x-3) + 10$

$2(9x^2 - 18x + 9) = 9x^2 - 9x + 10$

$18x^2 - 36x + 18 = 9x^2 - 9x + 10$

$9x^2 - 27x + 8 = 0$

By quadratic formula, $x = \frac{1}{3}$; $x = \frac{8}{3}$

$\left(x = \frac{-(-27) \pm \sqrt{(-27)^2 - 4(9)(8)}}{2(9)}\right)$ $y = 3(\frac{1}{3}) - 3 = -2$; $y = 3(\frac{8}{3}) - 3 = 5$

Ans: $x = \frac{1}{3}$, $y = -2$

$x = \frac{8}{3}$, $y = 5$ ✕

(4) $4^{2x+1} = 33(4^x) - 8$

Let $y = 4^x$

$$4^{2x} \cdot 4 - 33(4^x) + 8 = 0$$

$$4(4^x)^2 - 33(4^x) + 8 = 0.$$

$$4y^2 - 33y + 8 = 0$$

By quadratic formula, $y = \frac{1}{4}$; $y = 8$

$4^x = \frac{1}{4} = 4^{-1}$; $4^x = 8$

$x = -1$

$x \lg 4 = \lg 8$

$x = \frac{\lg 8}{\lg 4} = 1.5$

Ans: $x = -1$

$x = 1.5$ ※

⑦ $P(x) = 4x^3 - 13x - 6$

i) When divided by $(x+1)$

$R = P(-1) = 4(-1)^3 - 13(-1) - 6$

$= 3$

ii) When divided by $(x-2)$

$R = f(2) = 4(2)^3 - 13(2) - 6$

$= 0$ (shown)

$$
\begin{array}{r}
4x^2 + 8x + 3 \\
x-2 \overline{\big) 4x^3 + 0x^2 - 13x - 6} \\
-(4x^3 - 8x^2) \\
\hline
8x^2 - 13x \\
-(8x^2 - 16x) \\
\hline
3x - 6 \\
3x - 6 \\
\hline
0
\end{array}
$$

iii) $P(x) = (x-2)(4x^2 + 8x + 3)$

$= (x-2)(2x+1)(2x+3)$

iv) $4x^3 = 13x + 6$

$4x^3 - 13x - 6 = 0$

$(x-2)(2x+1)(2x+3) = 0$

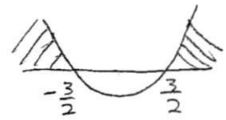
$x = 2, -\frac{1}{2}, -\frac{3}{2}$ ##

⑧ a) $(2x+1)(2x-1) > 8$

$4x^2 - 1 > 8$

$4x^2 - 9 > 0$

$(2x+3)(2x-3) > 0$



$x < -\frac{3}{2}$ or $x > \frac{3}{2}$ ##

8 bi) $2x^2 + x + m = mx + 1$

$2x^2 + x - mx + m - 1 = 0$

$2x^2 + (1-m)x + (m-1) = 0$

$a = 2$
$b = 1-m$
$c = m-1$

For no real roots,

$b^2 - 4ac < 0$

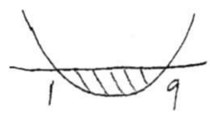
$(1-m)^2 - 4(2)(m-1) < 0$

$1 - 2m + m^2 - 8(m-1) < 0$

$1 - 2m + m^2 - 8m + 8 < 0$

$m^2 - 10m + 9 < 0$

$(m-1)(m-9) < 0$



$1 < m < 9$

8bii) $\left.\begin{array}{l} y = 2x^2 + x + 7 \\ y = 7x + 1 \end{array}\right\}$ $m = 7$
Curve does not intersect the line.

9i) $(2-x)^5 = 2^5 + \binom{5}{1}(2)^4(-x) + \binom{5}{2}(2)^3(-x)^2 + \cdots$

$= 32 - 80x + 80x^2 + \cdots$

ii) $\left(1 + \frac{1}{2}x\right)^6 = 1^6 + \binom{6}{1}(1)^5\left(\frac{1}{2}x\right) + \binom{6}{2}(1)^4\left(\frac{1}{2}x\right)^2 + \cdots$

$= 1 + 3x + \frac{15}{4}x^2 + \cdots$

$(2-x)^5\left(1+\frac{1}{2}x\right)^6 \Rightarrow$ Coeff of $x = (32)(3) + (-80)(1)$

$= 16$

10a). $C_1 : x^2 + y^2 - 6x + 2ky + 17 = 0$

$$x^2 - 6x + \left(\tfrac{6}{2}\right)^2 + y^2 + 2ky + \left(\tfrac{2k}{2}\right)^2 - \left(\tfrac{6}{2}\right)^2 - \left(\tfrac{2k}{2}\right)^2 + 17 = 0$$

$$(x^2-3)^2 + (y+k)^2 - 3^2 - k^2 + 17 = 0$$

$$(x^2-3)^2 + (y+k)^2 = 3^2 + k^2 - 17$$

$$= k^2 - 8$$

$$= r^2$$

$$\therefore k^2 - 8 = r^2 = 41^2$$

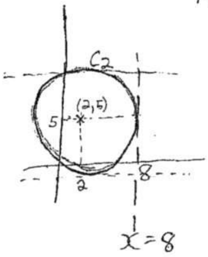
$$k^2 - 8 = 1681$$

$$k^2 = 1689$$

$$k = \pm\sqrt{1689}$$

2.5980762

10bi)



Radius $C_2 = 8 - 2 = 6$

Eq^n $C_2 : (x-2)^2 + (y-5)^2 = 6^2$

ii) $y = C$ is tangent to C_2

$$\left.\begin{aligned} 5+6 &= 11 \\ 5-6 &= -1 \end{aligned}\right\} \quad C = 11 \text{ or } -1$$

11a) $\sin\frac{5\pi}{3} + \cot\frac{7\pi}{6} = n\sqrt{3}$

$\pi\,\text{rad} = 180°$; $\sin 300° = \sin(-60°)$

$= -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

$$\sin 300° + \frac{\cos 210°}{\sin 210°} = n\sqrt{3}$$

$\cos 210° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$; $\sin 210° = -\sin 30°$

$= -\frac{1}{2}$

$$-\frac{\sqrt{3}}{2} + \frac{\frac{-\sqrt{3}}{2}}{-\frac{1}{2}} = n\sqrt{3}$$

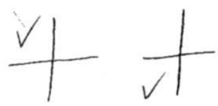
$$-\frac{\sqrt{3}}{2} + \sqrt{3} = n\sqrt{3}$$

$$\frac{1}{2}\sqrt{3} = n\sqrt{3} \quad \Rightarrow n = \frac{1}{2}$$

11b) $\sec x = -2$

$\frac{1}{\cos x} = -2$

$\cos x = -\frac{1}{2}$

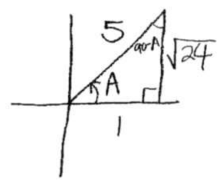


Basic $\measuredangle = \cos^{-1}\frac{1}{2} = 60^\circ$

$x = 180^\circ - 60^\circ, \quad 360^\circ - 60^\circ$

$= 120^\circ, \quad 300^\circ$ ※

c) $\cos A = \frac{1}{5}$, A acute (1st quadrant)



(i) $\tan(90^\circ - A) = \frac{1}{\sqrt{24}}$

(ii) $\text{cosec}(-A) = \frac{1}{\sin(-A)} = \frac{1}{-\sin A}$

$= \frac{1}{-\frac{\sqrt{24}}{5}}$

$= -\frac{5}{\sqrt{24}}$ ※

12i) Coord of E $= \left(\frac{1-3}{2}, \frac{10+2}{2}\right) = (-1, 6)$

ii) Grad AC $= \frac{10-2}{1-(-3)} = 2$

Grad BD $= \frac{-1}{2} = -\frac{1}{2}$

$y = -\frac{1}{2}x + C$

Sub $E(-1,6)$: $6 = -\frac{1}{2}(-1) + C$

$C = 5\frac{1}{2}$

Eqn BD : $y = -\frac{1}{2}x + 5\frac{1}{2}$ ※

iii) Sub $y=0$ into BD : $0 = -\frac{1}{2}x + 5\frac{1}{2}$

$x = 11$

Coord of B $= (11, 0)$ ※

12 iv)

To find coord. of D: Let $D = (x, y)$

$$\left(\frac{x+11}{2}, \frac{y+0}{2} \right) = (-1, 6)$$

$x + 11 = -2$; $y = 12$

$x = -13$

$D = (-13, 12)$

Area of ABCD $= \frac{1}{2} \begin{vmatrix} 1 & -13 & -3 & 11 & 1 \\ 10 & 12 & 2 & 0 & 10 \end{vmatrix}$

$= \frac{1}{2} \left[(12 - 26 - 0 + 110) - (-130 - 36 + 22 + 0) \right]$

$= 70 \text{ units}^2$ ※

$px + qy = 0$

$qy = -px$

$y = -\frac{p}{q} x$

v). Grad BD $= -\frac{1}{2}$

$-\frac{p}{q} = -\frac{1}{2}$

$\frac{p}{q} = \frac{1}{2}$

$2p = q$ ※

(2) $L = \frac{2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$; $W = \frac{6}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$

$$= \frac{2\sqrt{3}(2+\sqrt{3})}{4-3} \qquad = \frac{6\sqrt{12}}{12}$$

$$= 4\sqrt{3} + 6. \qquad = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2}.$$

Perimeter $= 2L + 2W = 2(4\sqrt{3}+6) + 2\left(\frac{2\sqrt{3}}{2}\right)$

$$= 8\sqrt{3} + 12 + 2\sqrt{3}$$

$$= 10\sqrt{3} + 12 \text{ cm}$$ #

(5) $\frac{2}{\alpha} + \frac{2}{\beta} = -1$; $\frac{4}{\alpha\beta} = \frac{2}{3} \Rightarrow 2\alpha\beta = 12$

$\alpha\beta = 6$

$$\frac{2\beta + 2\alpha}{\alpha\beta} = -1$$

$$2(\alpha+\beta) = -\alpha\beta$$

$$\alpha+\beta = \frac{-\alpha\beta}{2} = \frac{-6}{2} = -3$$

(i) Sum roots $= \frac{2}{\alpha} + \frac{2}{\beta} = -1$ (given)

product roots $= \left(\frac{2}{\alpha}\right)\left(\frac{2}{\beta}\right) = \frac{4}{\alpha\beta} = \frac{2}{3}$ (given)

Eq$^{\underline{n}}$: $x^2 - (-1)x + \frac{2}{3} = 0$

$x^2 + x + \frac{2}{3} = 0$ #

(ii) Sum roots $= \alpha+\beta = -3$

product roots $= \alpha\beta = 6$

Eq$^{\underline{n}}$: $x^2 - (-3)x + 6 = 0$

$x^2 + 3x + 6 = 0$ #

⑥ $f(x) = |x-2| + 5x$

(i) $f(-1) = |-1-2| + 5(-1)$

$= |-3| - 5$

$= 3 - 5$

$= -2$ ※

(ii) $f(x) = 4$

$|x-2| + 5x = 4$

$|x-2| = 4 - 5x$

$x - 2 = 4 - 5x$; $x - 2 = -(4-5x)$

$6x = 6$ $\quad$ $x - 2 = 5x - 4$

$x = 1$ (reject) $\quad$ $4x = 2$

$x = \frac{1}{2}$ ※