BEATTY SECONDARY SCHOOL
END–OF–YEAR EXAMINATION 2021

SUBJECT : Additional Mathematics

PAPER : 4049

LEVEL : Secondary 3 Express

DURATION : 2 hours 15 minutes

DATE : 11 October 2021

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

This question paper consists of 16 printed pages (including this cover page).

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 Angles A and B are in the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{7}{25}$.

Find the exact value of

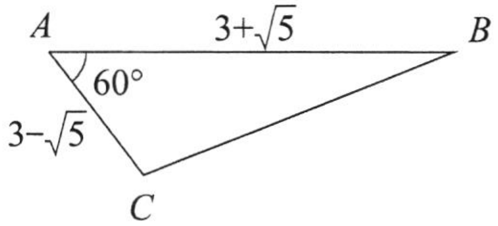
(i) $\tan A$, [2]

(ii) $\operatorname{cosec} B$. [2]

2 Given that $2^{x+1} + 2^{x+2} = 6^x$, find the value of 3^x and hence find the value of x correct to 4 significant figures. [3]

[Turn over

3 In triangle ABC, $AB = \left(3+\sqrt{5}\right)$ m, $AC = \left(3-\sqrt{5}\right)$ m and angle $CAB = 60°$.



(i) Express the length of BC in the form $\left(k\sqrt{6}\right)$m, where k is an integer. [3]

(ii) Express the shortest distance from A to BC produced in the form $\left(p\sqrt{2}\right)$ m, where p is a real number. [3]

4 The line $3y = 6 - 2x$ cuts the curve $x^2 - y^2 = 9$ at points A and B.
Find the coordinates of the midpoint of AB. [5]

[Turn over

5 The equation of a curve is $y = 2x^2 + 12x + 9$.

(i) Find the coordinates of the minimum point of the curve. [3]

(ii) Find the range of values of m such that $y = mx + 1$ cuts the curve at two distinct points. [4]

(iii) Write down the equation of another quadratic curve such that both curves cut each other at only one point. [1]

6 Express $\frac{x^2}{x^2-6x+9}$ in partial fractions. [4]

[Turn over

7 Solve $\log_2 x - 3\log_x 2 = 2$. [4]

8 **(i)** On the same axes, sketch the graphs of

$$y = \lg x \qquad \text{and} \qquad y = -x\,.$$ [2]

(ii) Explain how you would use your graphs to determine the number of solutions to the equation $x10^x = 1$. [3]

9 **(i)** In the binomial expansion of $(2-4x)^7$, explain why the coefficient of every term is divisible by 128. [2]

(ii) Write down, and simplify, the first 3 terms in the expansion of $(2-4x)^7$ in ascending powers of x. [2]

(iii) In the expansion of $(1+kx)(2-4x)^7$, there is no term in x^2.
Find the value of k. [3]

[Turn over

10 The expression $x^3 + ax + b$, where a and b are constants, has a factor of $(x + 4)$ and leaves a remainder of -5 when divided by $(x - 1)$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and of b found in part **(i)**, solve the equation $x^3 + ax + b = 0$, expressing non-integer roots in the form $c \pm \sqrt{d}$, where c and d are integers. [4]

11 The variables x and y are such that when values of xy are plotted against $\frac{1}{x}$, a straight line is obtained. It is given that $y = 1.75$ when $x = 2$ and $y = 1$ when $x = -1$.

Find the value of y when $x = 3$. [5]

[Turn over

12 Some rabbits were introduced into a piece of grassland.
The population, P, of the rabbits is estimated to be $P = 55e^{kt} + 5$, where k is a constant and t is measured in months. After 2 months, the population grew to double of the initial population.

(i) Calculate the value of k. [3]

(ii) Find the greatest integer value of t before the population of rabbits exceeds 1000. [2]

13 $A(-3, 7)$ and $B(-5, 5)$ are two points on a circle with centre C.

The gradient of the tangent to the circle at A is $\frac{3}{4}$.

(i) Find the equation of the normal to the circle at A. [3]

(ii) Show that the coordinates of C are $(3, -1)$. [5]

[Turn over

(iii) Find the equation of the circle. [2]

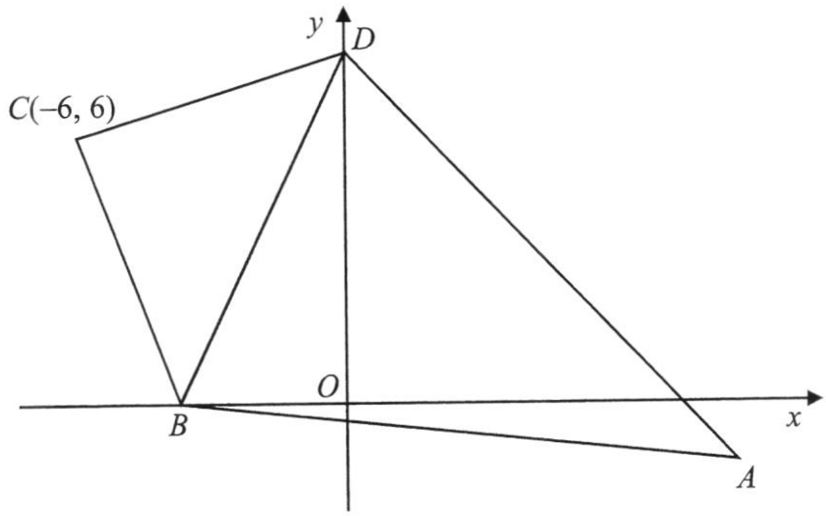
14 **(i)** In the same axes, sketch, for $0° \leq x \leq 360°$, the graphs of

$$y = 2 \sin x \qquad \text{and} \qquad y = \cos\frac{x}{2} - \frac{1}{2} .$$ [4]

(ii) Use your graphs to explain why there is no obtuse angle x such that

$$2\sin x = \cos\frac{x}{2} - \frac{1}{2} .$$ [2]

15 The diagram shows a kite $ABCD$ where $AB = AD$ and $CB = CD$.
The coordinates of point C are $(-6, 6)$.
Point D is on the y-axis and point B is on the x-axis.
$\tan \angle DBO = 2$.



(i) Show that the coordinates of B and of D are $(-4, 0)$ and $(0, 8)$ respectively. [5]

[Turn over

(ii) Given that the area of $ABCD$ is 80 square units, find the coordinates of A. [5]

End of Paper

Answer Key

1(i) $-\frac{3}{4}$ (ii) $\frac{25}{24}$

2. 1.631

3(i) $2\sqrt{6}$ m (ii) $\frac{1}{2}\sqrt{2}$ m

4. $(-2.4, 3.6)$

5(i) $(-3, -9)$ (ii) $m < 4$ or $m > 20$

6. $1+\frac{6}{x-3}+\frac{9}{(x-3)^2}$

7. $x = 8$ or 0.5

8(i)

9(ii) $128-1792x+10752x^2-\ldots$ (iii) $k=6$

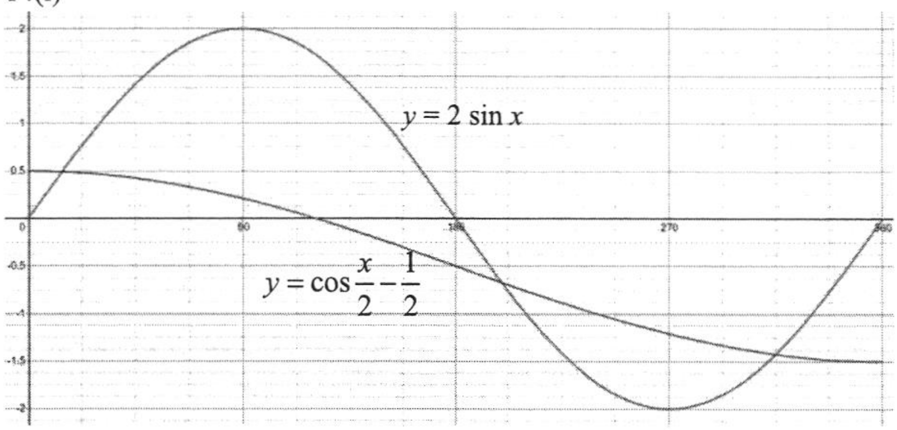
10(i) $a=-14$, $b=8$ (ii) $x=-4$ or $2\pm\sqrt{2}$

11. $y=1$

12(i) $k=0.369$ (ii) 7

13(i) $y=-\frac{4}{3}x+3$ (iii) $(x-3)^2+(y+1)^2=100$

14(i)



15(ii) $A = (10, -2)$

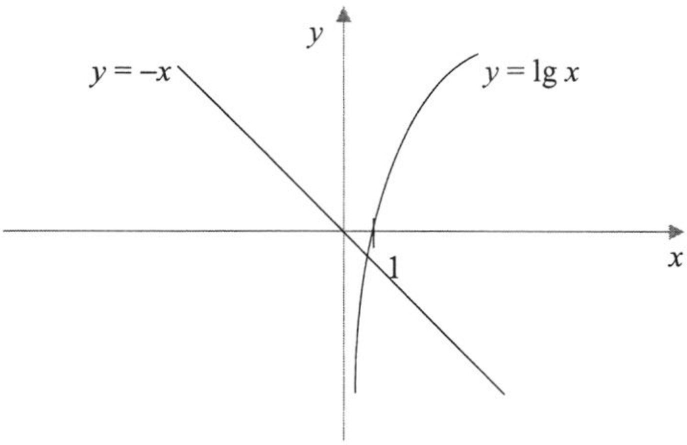
[Turn over

Beatty Secondary School
EOY 2021
3E Additional Mathematics

Qn	Solution
1(i)	A and B are in the 2nd quadrant. For angle A, adj = $-\sqrt{5^2-3^2}=-4$ ... **[M1] Accept "+4"** $\tan A = -\frac{3}{4}$ ... **[A1]**
1(ii)	For angle B, opp = $\sqrt{25^2-7^2}=24$ ... **[M1]** $\operatorname{cosec} B = \frac{1}{\sin B} = \frac{25}{24}$ ... **[A1]**
2.	$2^{x+1}+2^{x+2}=6^x$ $2^x(2^1+2^2)=6^x$ ... **[M1]** $2^x(6)=6^x$ $\frac{6^x}{2^x}=6$ $3^x=6$ ... **[A1]** $x=\frac{\ln 6}{\ln 3}=1.631$ (to 4sf) ... **[A1]**
3(i)	$BC=\sqrt{\left(3-\sqrt{5}\right)^2+\left(3+\sqrt{5}\right)^2-2\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)\cos 60°}$ ... **[M1]** $=\sqrt{9-6\sqrt{5}+5+9+6\sqrt{5}+5-2(9-5)\left(\frac{1}{2}\right)}$ ... **[M1]** $=\sqrt{24}$ $=2\sqrt{6}$ m ... **[A1]**
(ii)	$\frac{1}{2}\left(\ +\sqrt{5}\right)\left(\ -\sqrt{\ }\right) \qquad ° = \frac{1}{2}(h)\left(\ \sqrt{\ }\right)$ ... **[M1]** $(9-5)\left(\frac{\sqrt{3}}{2}\right)=\left(2\sqrt{6}\right)h$ $2\sqrt{3}=\left(2\sqrt{6}\right)h$ ... **[M1]**

	$h=\frac{\sqrt{3}}{\sqrt{6}}$ $=\frac{1}{\sqrt{2}}$ $=\frac{1}{2}\sqrt{2}$ m ... **[A1]**
4.	$3y = 6 - 2x$ $x = 3 - 1.5y$ ... (1) Sub (1) into $x^2 - y^2 = 9$ $(3-1.5y)^2 - y^2 = 9$ ... **[M1]** $9-9y+2.25y^2 - y^2 = 9$ $1.25y^2 - 9y = 0$ $5y^2 - 36y = 0$ $y(5y-36) = 0$ ... **[M1]** $y = 0$ or $y = 7.2$ ... **[A1]** When $y = 0$, $x = 3 - 1.5(0) = 3$ When $y = 7.2$, $x = 3 - 1.5(7.2) = -7.8$ } ...**[A1]** Midpoint of $AB = \left(\frac{3-7.8}{2}, \frac{0+7.2}{2}\right) = (-2.4, 3.6)$ ... **[A1]**
5(i)	$2x^2+12x+9$ $=2(x^2+6x)+9$ $=2[(x+3)^2-9]+9$ ... **[M1]** $=2(x+3)^2-9$ ... **[M1]** Minimum point = (–3, –9) ... **[A1]** -- *Alternatively,* *Let* $2x^2+12x+9=0$. $x=\frac{-12\pm\sqrt{72}}{4}$ ... ***[M1]*** *x-coordinate of min pt* = $\left(\frac{-12+\sqrt{72}}{4}+\frac{-12-\sqrt{72}}{4}\right)\div 2 = -3$ ... ***[M1]*** *y-coordinate of min pt* = $2(-3)^2+12(-3)+9=-9$ *Minimum point = (–3, –9)* ... ***[A1]***

(ii)	$2x^2+12x+9=mx+1$ ... **[M1]** $2x^2+(12-m)x+8=0$ Discriminant >0 $(12-m)^2-4(2)(8)>0$ ... **[M1]** $(12-m)^2>64$ $m^2-24m+80>0$ $(m-4)(m-20)>0$ ... **[M1]** $m<4$ or $m>20$ ... **[A1]**
(iii)	Any quadratic equation in the form $y=a(x+3)^2-9$, where $a\neq 2$ or $y=2x^2+bx+c$, where $b\neq 12$ or $y=ax^2+12x+9$, where $a\neq 2$ or any equation where when it is substituted into original equation, the resulting equation is a perfect square ... **[B1]**
6	By long division, $\frac{x^2}{x^2-6x+9}=1+\frac{6x-9}{x^2-6x+9}$ ... **[M1]** $=1+\frac{6x-9}{(x-3)^2}$ Let $\frac{6x-9}{(x-3)^2}=\frac{A}{x-3}+\frac{B}{(x-3)^2}$ $6x-9=A(x-3)+B$ ... **[M1]** Comparing coefficient of x, $A=6$ ... **[A1]** Let $x=3$ $\quad B=6(3)-9=9$ ... **[A1]** $\frac{x^2}{x^2-6x+9}=1+\frac{6}{x-3}+\frac{9}{(x-3)^2}$
7	$\log_2 x-3\log_x 2=2$ $\log_2 x-\frac{3}{\log_2 x}=2$ ... **[M1]** Let $y=\log_2 x$ $y-\frac{3}{y}=2$

	$y^2-2y-3=0$ $(y-3)(y+1)=0$ **... [M1]** $y=3$ or $y=-1$ **... [A1]** $\log_2 x=3$ or $\log_2 x=-1$ $x=2^3$ or $x=2^{-1}$ $x=8$ or $x=0.5$ **... [A1]**
8(i)	 **... [G1], [G1]**
8(ii)	$\lg x=-x$ **... [M1]** $10^{-x}=x$ $x10^x=1$ **... [A1]** Since the graphs cut at only one point, there is one solution to $x10^x=1$. **[A1]** --- *Alternatively,* $x10^x=1$ $\lg x10^x=\lg 1$ ***... [M1]*** $\lg x+\lg 10^x=0$ $\lg x+x=0$ $\lg x=-x$ ***... [A1]*** *Since the graphs cut at only one point, there is one solution to* $x10^x=1$. ***[A1]***

9(i)	$(2-4x)^7 = 2^7(1-2x)^7$ **... [M1]** $= 128(1-2x)^7$ Since the coefficient of every term in $(1-2x)^7$ is an integer, the coefficient of every term in $128(1-2x)^7$ is a multiple of 128. **... [A1]** ***Accept also if students expand all 8 terms correctly and claim that every coefficient is multiple of 128.*** ***All 8 terms:*** $128-1792x+10752x^2-35840x^3+71680x^4$ $-86016x^5+57344x^6-16384x^7$
9(ii)	$(2-4x)^7$ $= 2^7 - \binom{7}{1}(2^6)(4x) + \binom{7}{2}(2^5)(4x)^2 - \ldots$ **... [M1]** $= 128-1792x+10752x^2-\ldots$ **... [A1]**
9(iii)	$(1+kx)(2-4x)^7$ $= (1+kx)(128-1792x+10752x^2-\ldots)$ Coefficient of $x^2 = 10752 - 1792k$ **... [M1]** $10752 - 1792k = 0$ **... [M1]** $k = 6$ **... [A1]**
10(i)	Let $f(x) = x^3+ax+b$ $f(-4) = 0$ and $f(1) = -5$ $(-4)^3+a(-4)+b=0$ **... [M1]** $-64-4a+b=0$ ... (1) $1^3+a(1)+b=-5$ **... [M1]** $6+a+b=0$ ... (2) (2) – (1) $5a+70=0$ $a=-14$ **... [A1]** When $a=-14$, $6-14+b=0$ $b=8$ **... [A1]**

(ii)	$x^3 - 14x + 8 = (x+4)(x^2 + kx + 2)$ Comparing coefficient of x, $-14 = 2 + 4k$ $k = -4$ ... **[M1] (Accept long division too)** $x^3 - 14x + 8 = (x+4)(x^2 - 4x + 2) = 0$ $x = -4$ ... **[A1]** or $x = \frac{4 \pm \sqrt{4^2 - 4(1)(2)}}{2}$ ... **[M1]** $= \frac{4 \pm \sqrt{8}}{2}$ $= \frac{4 \pm 2\sqrt{2}}{2}$ $= 2 \pm \sqrt{2}$ ... **[A1]**
11	At $(x, y) = (2, 1.75)$ and $(-1, 1)$, $(\frac{1}{x}, xy) = (0.5, 3.5)$ and $(-1, -1)$ ... **[M1]** Gradient $= \frac{3.5+1}{0.5+1}$ ... **[M1]** $= 3$ $xy + 1 = 3\left(\frac{1}{x} + 1\right)$ ... **[M1]** $xy = \frac{3}{x} + 2$ When $x = 3$, $3y = 1 + 2$ ... **[M1]** $y = 1$ ... **[A1]**
12(i)	$P = 55e^{kt} + 5$ To get initial population, let $t = 0$ ⇨ $P = 55e^0 + 5$ ... **[M1]** $= 60$ Let $P = 120$ and $t = 2$ $120 = 55e^{2k} + 5$ $55e^{2k} = 115$ $e^{2k} = \frac{23}{11}$ ... **[M1]**

	$k = \dfrac{\ln \frac{23}{11}}{2}$ ≈ 0.36879 $= 0.369$ (to 3sf) **… [A1]**
12(ii)	$1000 = 55e^{0.36879t} + 5$ $e^{0.36879t} = \dfrac{199}{11}$ $t = \dfrac{\ln \frac{199}{11}}{0.36879}$ **… [M1]** $= 7.85$ (3sf) Greatest integer $t = 7$ **… [A1]**
13(i)	Gradient of normal = $-\dfrac{4}{3}$ **… [M1]** $y - 7 = -\dfrac{4}{3}(x+3)$ **… [M1]** $y = -\dfrac{4}{3}x + 3$ **… [A1]**
13(ii)	Gradient of AB = $\dfrac{5-7}{-5+3} = 1$ Gradient of perpendicular bisector of $AB = -1$ **… [M1]** Midpoint of $AB = \left(\dfrac{-3-5}{2}, \dfrac{7+5}{2}\right) = (-4, 6)$ **… [M1]** $y - 6 = -(x+4)$ $y = -x + 2$ … (1) **… [M1]** --- *Alternatively,* $AC = BC$ $\sqrt{\quad\quad y-7)^2} = \sqrt{\quad\quad y-5)^2}$ ***… [M1]*** $x^2 + 6x + 9 + y^2 - 14y + 49 = x^2 + 10x + 25 + y^2 - 10y + 25$ ***… [M1]*** $4y + 4x = 8$ $y = -x + 2$ … *(1)* ***… [M1]***

Sub (1) into $y=-\frac{4}{3}x+3$

$-x+2=-\frac{4}{3}x+3$ ... **[M1]**

$\frac{1}{3}x=1$

$x=3$

When $x = 3$, $y = -3 + 2 = -1$

Hence, $C = (3, -1)$ (shown) ... **[A1]**

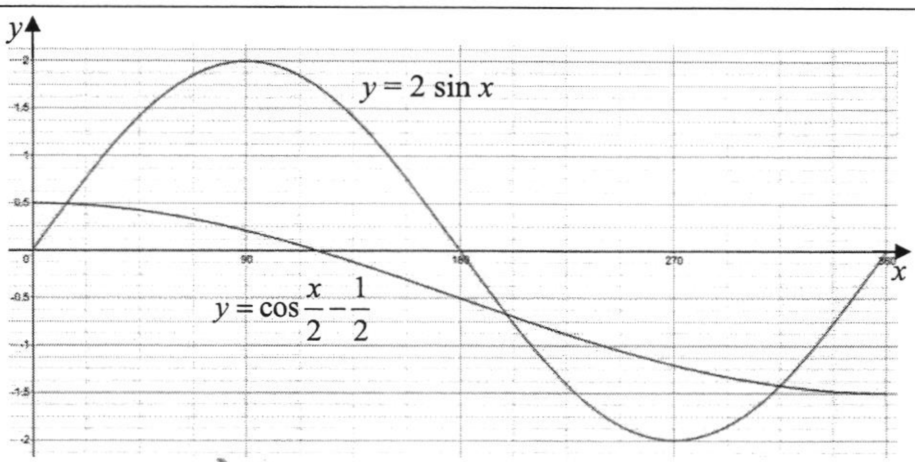
13(iii)

Radius of circle = $\sqrt{(3+3)^2+(-1-7)^2}=10$ ... **[M1]**

Equation of circle is $(x-3)^2+(y+1)^2=100$ ... **[A1]**

Accept also $x^2+y^2-6x+2y-90=0$

14(i)



G1 for correct amplitude for both graphs
G2 for correct number of cycles for both graphs
G1 for correct curves (including labelling of axes)

14(ii)

There is no point of intersection ... **[B1]** between the two graphs where x is between 90° and 180° ...**[B1]**

15(i)

Let $B = (b, 0)$ and $D = (0, d)$

$\tan \angle DBO = 2$

$\dfrac{d-0}{0-b} = 2$ ... **[M1]**

$d = -2b$ ... (1)

$BC = CD$

$\sqrt{b \qquad\qquad -6)^2} = \sqrt{\qquad\qquad\qquad d-6)^2}$ ... **[M1]**

$b^2 + 12b + 36 + 36 = 36 + d^2 - 12d + 36$

$b^2 + 12b = d^2 - 12d$ ... (2)

Sub (1) into (2)

$b^2 + 12b = (-2b)^2 - 12(-2b)$ ... **[M1]**

$b^2 + 12b - 4b^2 - 24b = 0$

$-3b^2 - 12b = 0$

$-3b(b+4) = 0$ ... **[M1]**

$b = 0$ or $b = -4$

(Rej)

When $b = -4$, $d = -2(-4) = 8$

Hence, $B = (-4, 0)$ and $D = (0, 8)$ (shown) ... **[A1]**

15(ii)

Gradient of $BD = \dfrac{8-0}{0+4} = 2$ ... **[M1]**

Gradient of $AC = -\dfrac{1}{2}$

Let $A = (p, q)$

$\dfrac{q-6}{p+6} = -\dfrac{1}{2}$ ... **[M1]**

$2(q-6) = -p-6$

$p = 6 - 2q$ ... (3)

Area = 80

$\dfrac{1}{2}\begin{vmatrix} -6 & -4 & 6-2q & 0 & -6 \\ 6 & 0 & q & 8 & 6 \end{vmatrix} = 80$ ... **[M1]**

$\dfrac{1}{2}\left[-4q + 8(6-2q) + 24 + 48\right] = 80$

	$\frac{1}{2}\left[-20q+120\right]=80$ $q=-2$ **… [A1]** When $q = -2$, $p = 6 - 2(-2) = 10$ Hence, $A = (10, -2)$ **… [A1]**