Register No.	Class

Name : ____________________

BENDEMEER SECONDARY SCHOOL
2021 END OF YEAR EXAMINATION
SECONDARY 3 EXPRESS
Additional Mathematics
4049

DATE : **13 October 2021**
DURATION : **2 hours 15 minutes**
TOTAL : **90 Marks**

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers in the spaces provided on the question paper.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
90

This document consists of **18** printed pages including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$

$$\sec^2 A=1+\tan^2 A.$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

$$a^2=b^2+c^2-2bc\cos A.$$

$$\Delta=\frac{1}{2}bc\sin A.$$

1 Given $\frac{\sqrt{6}}{6^{x+2}} = 3^{x-1}$, find the exact value of 18^x. [4]

2 Express $\dfrac{x^3+x+3}{x^2(x+1)}$ in partial fractions. [6]

3 James deposited \$540 000 in a bank at the beginning of 1980 and the bank gave a compound interest of 1.6% per annum. After a period of t years, the amount of money that Peter has in the bank was given by 540 000 $(1.016)^t$.

(i) Calculate the amount of money James has at the beginning of 1992. [1]

(ii) Calculate the year that James would have a million dollars in his bank account. [4]

4 **(i)** Prove the identity $\sin 2x - \tan x \cos 2x = \tan x$. [4]

(ii) Hence, without using a calculator, find the value of tan (22.5°). [3]

5 The expression $ax^3 + bx^2 - 11x + 3$ is exactly divisible by $x^2 + 2x - 3$.

(i) Show that $a = 3$ and $b = 5$. [4]

(ii) Factorise the expression $ax^3 + bx^2 - 11x + 3$ completely. [2]

5 **(iii)** Hence, or otherwise, solve the equation [3]
$a(x-1)^3 + b(x-1)^2 - 11x + 14 = 0$

6 The equation of a curve is given by $y = 2x^2 - kx + k$, where k is a constant.

(i) Find the smallest integer value of k for which the curve lies entirely above the x-axis. [3]

(ii) When $k = 3$, find the values of m, for which the line $y - mx = 0$ is tangent to the curve. Give your answers in the form $a + b\sqrt{6}$, where a and b are integers. [4]

7 **(a)** Given that $\cos A = -\frac{1}{2}$ and $\tan B = -\frac{5}{12}$, where A and B are in the same quadrant.

Find, without using a calculator, the exact value of $\sin (A - B)$. [3]

(b) Solve the equation $2sin\ x = \cos(x - \frac{\pi}{4})$ for $0 \leq x \leq 2\pi$. [4]

8 **(a)** Find the term independent of x in the expansion of $\left(x^3 - \frac{1}{2x}\right)^{12}$. [3]

(b) **(i)** The first four terms in the expansion of $(1-3x)^n$ in ascending powers of x are $1 + px + 252x^2 - 1512x^3$.
Find the value of each of the constant n and p. [4]

8 **(b)** **(ii)** Hence find the coefficient of x^3 in the expansion $(2-x)(1-3x)^n$. [2]

9 **(a)** Solve the equation $log_5(3x^2 + 7) = 1 + log_{\sqrt{5}}(x + 1)$. [5]

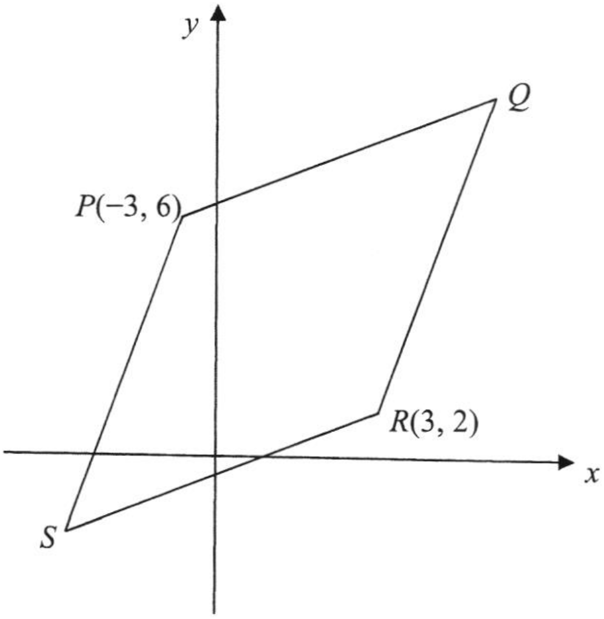
(b) Solve $2e^x - \frac{12}{e^x} + 5 = 0$. [4]

10 **(a)** Write down the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ in radians as a multiple of π. [1]

(b) Solve the equation $sin(2x - 50^o) = -\cos 60^o$ for $0^o \leq x \leq 360^o$. [3]

(c) Prove the identity $\frac{cosx}{cosecx + 1} + \frac{cosx}{cosecx - 1} = 2tanx.$ [3]

11



In the rhombus $PQRS$, the points P and R are $(-3,\ 6)$ and $(3,\ 2)$ respectively. The vertex Q of the rhombus lies on the line $3y - 7x - 2 = 0$.

(i) Show that the x-coordinate of Q is 4. [4]

(ii) Find the equation of the perpendicular bisector of PQ. [3]

(iii) Find the coordinates of S. [2]

(iv) Find the area of the rhombus $PQRS$. [2]

12 It is given that $y = 2cos^2x - sin^2x$.

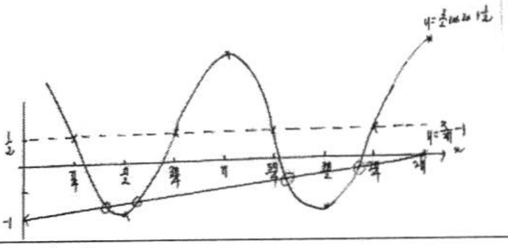
(i) By expressing y in the form of $acos2x + b$, show that $a = \frac{3}{2}$ and $b = \frac{1}{2}$. [3]

(ii) Sketch the graph of $y = 2cos^2x - sin^2x$ for $0 \leq x \leq 2\pi$ radian, showing clearly the turning points and the intercepts with $y = \frac{1}{2}$. [3]

12 **(iii)** By drawing a suitable line on the same axes, state the number of solutions to the equation $4\pi cos^2x - 2\pi sin^2x = x - 2\pi$. [3]

End of paper

BENDEMEER SECONDARY SCHOOL
2021 END OF YEAR EXAMINATION
Secondary Three Express
Additional Mathematics Answer Key 4049

1	$\frac{\sqrt{6}}{12}$	9(a)	x = 0.193, -5.18 (rejected)
2	$\frac{x^2+x+3}{x^2(x+1)} = 1 - \frac{2}{x} + \frac{3}{x^3} + \frac{1}{x+1}$	9(b)	0.405
3(i)	\$653 308.42	10(a)	$-\frac{\pi}{6}$
3(ii)	2019	10(b)	10°, 130°, 190°, 310°
4(i)	Proof	10(c)	proof
4(ii)	$\sqrt{2}-1$	11(i)	proof
5(i)	Proof	11(ii)	$y = -\frac{7}{4}x + \frac{71}{8}$
5(ii)	$(x-1)(x+3)(3x-1)$	11(iii)	S (- 4, -2)
5(iii)	$x = 2, -2, \frac{4}{3}$	11(iv)	52 units^2
6(i)	Smallest integer k is 1	12(i)	Proof
6(ii)	$m = -3 + 2\sqrt{6}$ or $-3 - 2\sqrt{6}$	12(ii)	
7(a)	$\frac{1}{26}(-12\sqrt{3}+5)$	12(iii)	4 solutions
7(b)	x = 0.500 or 3.64		
8(a)	$-\frac{55}{128}$		
8(b)(i)	n = 8, p = -24		
8(b)(ii)	-3276		

Final.

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1 Given $\frac{\sqrt{6}}{6^{x+2}} = 3^{x-1}$, find the exact value of 18^x. [4]

$$\frac{6^{\frac{1}{2}}}{6^x \cdot 36} = \frac{3^x}{3} \quad [M_1, M_1]$$

$$18^x = \frac{3 \times 6^{\frac{1}{2}}}{36} \quad [M1]$$

$$= \underline{\underline{\frac{\sqrt{6}}{12}}} \quad [A1]$$

2 Express $\frac{x^3+x+3}{x^2(x+1)}$ in partial fractions. [6]

$$\begin{array}{r}
1 \\
x^3+x^2 \,\big)\, \overline{x^3+0x^2+x+3} \\
-\ \underline{x^3+x^2} \\
\underline{-x^2+x+3}
\end{array}$$

[M1]

$$\frac{x^3+x+3}{x^2(x+1)} = 1 + \frac{-x^2+x+3}{x^2(x+1)}$$ [M1]

$$\frac{-x^3+x+3}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$ [M1]

$$-x^2+x+3 = Ax(x+1) + B(x+1) + Cx^2$$

let $x=-1$,

$1 = C(-1)^2 \therefore \underline{\underline{C=1}}$

comparing coefficient of x^2:

$-1 = A+C$

$\underline{\underline{A=-2}}$

comparing coefficient of x:

$1 = A+B$

$\underline{\underline{B=3}}$

[M2] (M1 - at least 2 correct)

$$\therefore \frac{x^3+x+3}{x^2(x+1)} = \underline{\underline{1 - \frac{2}{x} + \frac{3}{x^2} + \frac{1}{x+1}}}$$ [A1]

3 James deposited \$540 000 in a bank at the beginning of 1980 and the bank gave a compound interest of 1.6% per annum. After a period of t years, the amount of money that Peter has in the bank was given by

$540\,000\,(1.016)^t$.

(i) Calculate the amount of money James has at the beginning of 1992. [1]

$$\text{Amount of money} = 540\,000\,(1{\cdot}016)^{12}$$

$$= \underline{\underline{\$653308{\cdot}42}}\ (2dp) \quad [B1]$$

(ii) Calculate the year that James would have a million dollar in his bank account. [4]

$$1000\,000 = 540\,000\,(1{\cdot}016)^t \quad [M1]$$

$$\frac{1000\,000}{540\,000} = (1{\cdot}016)^t$$

$$t = \frac{lg\left(\frac{1000\,000}{540\,000}\right)}{lg\,(1{\cdot}016)} \quad [M1]$$

$$= \underline{38{\cdot}819\text{ years}} \quad [M1]$$

$\therefore$ He will get his million in the year $\underline{\underline{2019}}$ [A1]

[4]

4 (i) Prove the identity $\sin 2x - \tan x \cos 2x = \tan x$.

$$LHS = \sin 2x - \frac{\sin x \cos 2x}{\cos x}$$ [M1] sin2x

$$= \frac{2\sin x \cos^2 x - (2\cos^2 x - 1)\sin x}{\cos x}$$ [M1] cos2x

[M1] - combine into single fraction correctly

$$= \frac{\sin x}{\cos x}$$ (A1)

$$= \underline{\tan x = RHS}$$

(ii) Hence, without using a calculator, find the value of tan (22.5°). [3]

$$\sin(45°) - \tan(22.5°)\cos 45° = \tan(22.5°)$$

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}(\tan 22.5°) = \tan 22.5°$$ [M1]

$$\tan 22.5° = \frac{\sqrt{2}}{\sqrt{2}+2} \times \frac{\sqrt{2}-2}{\sqrt{2}-2}$$ [M1]

$$= \frac{2 - 2\sqrt{2}}{-2}$$

$$= \underline{\underline{\sqrt{2} - 1}}$$ [A1]

5 The expression $ax^3 + bx^2 - 11x + 3$ is exactly divisible by $x^2 + 2x - 3$.

(i) Show that $a = 3$ and $b = 5$. [4]

$x^2+2x-3 = (x-1)(x+3)$

sub $x=1$, $a+b-11+3=0$

$a+b=8$ —— ① [M1]

sub $x=-3$, $-27a+9b+33+3=0$

$3a-b=4$ —— ② [M1]

①+② $4a=12$ [M1]

$\underline{\underline{a=3}}$

$\therefore \underline{\underline{b=5}}$ } [AG1]

(ii) Factorise the expression $ax^3 + bx^2 - 11x + 3$ completely. [2]

No mark if no working shown.

$3x^3+5x^2-11x+3$

$$\begin{array}{r} 3x-1 \\ x^2+2x-3\ \overline{)\,3x^3+5x^2-11x+3} \\ -\ \ 3x^3+6x^2-9x \\ \hline -x^2-2x+3 \\ -\ \ -x^2-2x+3 \\ \hline \underline{0} \end{array}$$

[M1]

$\therefore 3x^3+5x^2-11x+3$

$= (x^2+2x-3)(3x-1)$ [A1]

$= \underline{\underline{(x-1)(x+3)(3x-1)}}$

Solve [1 mark]

[3]

5 (iii) Hence, or otherwise, solve the equation
$a(x-1)^3 + b(x-1)^2 - 11x + 14 = 0$

$x - 1 = 1, -3, \frac{1}{3}$

$\therefore x = 2, -2, \frac{4}{3}$ [A1]

[M1] - values of $x-1$

[M1] - making use of x by $x-1$.

6 The equation of a curve is given by $y = 2x^2 - kx + k$, where k is a constant.

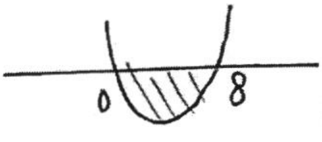
(i) Find the smallest integer value of k for which the curve lies entirely above the x-axis. [3]

For $b^2 - 4ac < 0$

$(-k)^2 - (4)(2)(k) < 0$ [M1]

$k^2 - 8k < 0$

$k(k-8) < 0$



[M1]

$0 < k < 8$

$\therefore$ The smallest integer value of k is $\underline{\underline{1}}$ [A1]

(ii) When $k = 3$, find the values of m, for which the line $y - mx = 0$ is tangent to the curve. Give your answers in the form $a + b\sqrt{6}$, where a and b are integers. [4]

$y = mx$ — ①

$y = 2x^2 - 3x + 3$ — ②

① = ②

$2x^2 - 3x + 3 = mx$

$2x^2 + (-3-m)x + 3 = 0$ [M1]

For $b^2 - 4ac = 0$,

$(-3-m)^2 - (4)(2)(3) = 0$ [M1] values must be substituted correctly.

$(3+m)^2 = 24$

$3 + m = \pm 2\sqrt{6}$

$\therefore m = \underline{\underline{-3+2\sqrt{6}}}$ or $\underline{\underline{-3-2\sqrt{6}}}$ [A1, A1]

If answers are correct, but not in surd form. A1 only.

7 (a) Given that $\cos A = -\frac{1}{2}$ and $\tan B = -\frac{5}{12}$, where A and B are in the same quadrant. Find, without using a calculator, the exact value of $\sin(A - B)$. [3]

$$\sin(A-B)$$
$$= \sin A \cos B - \cos A \sin B$$
$$= \underbrace{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)}_{(M1)} - \underbrace{\left(-\frac{1}{2}\right)\left(\frac{5}{13}\right)}_{(M1)}$$
$$= \frac{-12\sqrt{3}}{26} + \frac{5}{26}$$
$$= \frac{1}{26}\left(-12\sqrt{3}+5\right) \quad (A1)$$

(b) Solve the equation $2\sin x = \cos(x - \frac{\pi}{4})$ for $0 \le x \le 2\pi$. [4]

$$2\sin x = \cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}$$
$$2\sin x = \cos x\left(\frac{1}{\sqrt{2}}\right) + \sin x\left(\frac{1}{\sqrt{2}}\right) \quad [M1]$$
$$\left(2 - \frac{1}{\sqrt{2}}\right)\sin x = \frac{1}{\sqrt{2}}\cos x$$
$$\left(\frac{2\sqrt{2}-1}{\sqrt{2}}\right)\sin x = \frac{1}{\sqrt{2}}\cos x$$
$$\tan x = \frac{\sqrt{2}}{4-\sqrt{2}} \quad [M1]$$

base angle, $\alpha = 0.500474$ (6 sf)

$x = 0.500$ rad or $x = \pi + \alpha$
(3sf) $= 3.64$ rad (3sf)

$\therefore x = 0.500$ rad or 3.64 [A1]
(A1)

8 (a) Find the term independent of x in the expansion of $\left(x^3 - \frac{1}{2x}\right)^{12}$. [3]

$T_{r+1} = {}^{12}C_r \, (x^3)^{12-r} \left(-\frac{1}{2x}\right)^r$

$= \binom{12}{r}\left(-\frac{1}{2}\right)^r x^{36-4r}$

$\therefore\ 36-4r=0$ [M1]

$r=9$

$T_{10} = \binom{12}{9}(x^3)^3\left(-\frac{1}{2x}\right)^9$ [M1]

$\underline{T_{10} = (220)\left(-\frac{1}{512}\right)}$

$\therefore$ term independent of x = $\underline{\underline{-\frac{55}{128}}}$ [A1]

(b) (i) The first four terms in the expansion of $(1-3x)^n$ in ascending powers of x are $1+px+252x^2-1512x^3$.

Find the value of each of the constant n and p. [4]

$(1-3x)^n = 1+px+252x^2-1512x^3+\cdots$

$1-3nx+\frac{n(n-1)}{2}(-3x)^2+\cdots = 1+px+252x^2+\cdots$

comparing coefficient of x^2

$\frac{9n(n-1)}{2} = 252$ [M1]

$9n^2-9n=504$

$n^2-n-25=0$

$(n-8)(n+7)=0$ [M1]

$\underline{\underline{n=8}}$ or $\underline{n=-7}$ (rejected) [A1]

$p=-3(8)$

$\underline{\underline{p=-24}}$ [A1]

8 **(b)** **(ii)** Hence find the coefficient of x^3 in the expansion $(2-x)(1-3x)^n$. [2]

$(2-x)(1-3x)^n = (2-x)(1-24x+252x^2-1512x^3+\dots)$ [M1]

coefficient of $x^3 = (2)(-1512)+(-1)(252)$

$= \underline{\underline{-3276}}$ [A1]

9 **(a)** Solve the equation $log_5(3x^2+7) = 1 + log_{\sqrt{5}}(x+1)$. [5]

$$\log_5(3x^2+7) = 1 + \frac{\log_5(x+1)}{\log_5 5^{\frac{1}{2}}} \quad [M1]$$

$$\log_5(3x^2+7) - 2\log_5(x+1) = 1 \quad [M1]$$

$$\log_5 \frac{3x^2+7}{(x+1)^2} = 1 \quad [M1]$$

$$3x^2+7 = 5(x+1)^2$$

$$3x^2+7 = 5x^2+10x+5$$

$$2x^2+10x-2=0$$

$$x^2+5x-1=0 \quad [M1]$$

$$x = \frac{-5 \pm \sqrt{5^2-4(1)(-1)}}{2(1)} = \underline{\underline{0.193}} \text{ or } \underline{-5.19} \quad [A1]$$

(3sf) (rejected)

(b) Solve $2e^{x} - \frac{12}{e^{x}} + 5 = 0$. [4]

Let $u = e^x$

$$2u - \frac{12}{u} + 5 = 0$$

$$2u^2+5u-12=0 \quad [M1]$$

$$(2u-3)(u+4)=0$$

[M1]

$$u = \frac{3}{2} \text{ or } u = -4$$

(NA)

$$\therefore e^x = \frac{3}{2} \quad [M1]$$

$$x = \ln\left(\frac{3}{2}\right) \quad [A1]$$

$$= \underline{\underline{0.405 \ (3sf)}}$$

10 (a) Write down the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ in radians as a multiple of π. [1]

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

∴ The principal value = $\underline{\underline{-\frac{\pi}{6}}}$ [B1]

(b) Solve the equation $sin(2x - 50^o) = -\cos 60^o$ for $0^o \le x \le 360^o$. [3]

$\sin(2x-50^\circ) = -\cos 60^\circ$

$= -\frac{1}{2}$

base angle, $\alpha = \underline{30^\circ}$ [M1]

$0^\circ \le x \le 360^\circ$

$\underline{50^\circ \le 2x-5 \le 670^\circ}$

3rd quad

$2x-50^\circ = 180^\circ+\alpha$

$\underline{\underline{x = 130^\circ}}$

4th quad

$2x-50^\circ = 360^\circ-\alpha$

$\underline{\underline{x = 190^\circ}}$

[A2, 1, 0]

$2x-5 = 360^\circ + (180^\circ+\alpha)$

$\underline{\underline{x = 310^\circ}}$

$2x-50^\circ = -30^\circ$

$\underline{\underline{x = 10^\circ}}$

At least 2 correct to get A1

∴ $x = \underline{10^\circ, 130^\circ, 190^\circ, 310^\circ}$

(c) Prove the identity $\frac{cosx}{cosecx+1} + \frac{cosx}{cosecx-1} = 2tanx.$ [3]

$$LHS = \frac{\cos x(\operatorname{cosec}x-1)}{(\operatorname{cosec}x+1)(\operatorname{cosec}x-1)} + \frac{\cos x(\operatorname{cosec}x+1)}{(\operatorname{cosec}x+1)(\operatorname{cosec}x-1)} \quad [M1]$$

$$= \frac{\cot x - \cos x + \cot x + \cos x}{\operatorname{cosec}^2 x - 1}$$

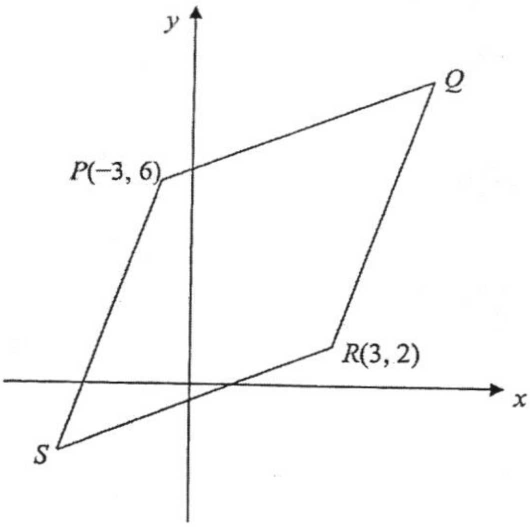
$$= \frac{2\cot x}{\cot^2 x} \quad [M1]$$

$$= \frac{2}{\cot x}$$

$$= 2\tan x \quad (AG1)$$

$= \underline{RHS}$ (shown)

11



In the rhombus $PQRS$, the points P and R are $(-3,\ 6)$ and $(3,\ 2)$ respectively. The vertex Q of the rhombus lies on the line $3y - 7x - 2 = 0$.

(i) Show that the x-coordinate of Q is 4. [4]

gradient of PR $= \frac{6-2}{-3-3}$

$= \frac{4}{-6}$

$= -\frac{2}{3}$ [M1]

midpoint of PR $= \left(\frac{-3+3}{2}, \frac{6+2}{2}\right)$

$= (0, 4)$ [M1]

Equation of QS: $y - 4 = \frac{3}{2}x$

$\underline{y = \frac{3}{2}x + 4}$ [M1]

$y = \frac{3}{2}x + 4$ — ①

$3y - 7x - 2 = 0$ — ②

[M1]

$3\left(\frac{3}{2}x + 4\right) - 7x - 2 = 0$

$\frac{9}{2}x + 12 - 7x - 2 = 0$

$\frac{5}{2}x = 10$

$\underline{\underline{x = 4}}$ (shown)

(ii) Find the equation of the perpendicular bisector of PQ. [3]

sub $x=4$, $y=\frac{3}{2}(4)+4$

$=10$

$\therefore$ Q is $(4,10)$

gradient of PQ $=\frac{10-6}{4-(-3)}$

$=\frac{4}{7}$

midpoint of PQ $=\left(\frac{-3+4}{2}, \frac{6+10}{2}\right)$ [M1]

$=\left(\frac{1}{2}, 8\right)$

$\therefore$ equation of bisector:

$y-8=-\frac{7}{4}\left(x-\frac{1}{2}\right)$ [M1]

$y=-\frac{7}{4}x+\frac{71}{8}$ [A1]

(iii) Find the coordinates of S. [2]

$(0,4)=\left(\frac{x+4}{2}, \frac{y+10}{2}\right)$ [M1]

$x=-4$, $y=-2$

$\therefore S(-4,-2)$ [A1]

(iv) Find the area of the rhombus $PQRS$. [2]

area of PQRS

$=\frac{1}{2}\begin{vmatrix} -3 & -4 & 3 & 4 & -3 \\ 6 & -2 & 2 & 10 & 6 \end{vmatrix}$ [M1]

$=\frac{1}{2}[52-(-52)]$

$=52$ units^2 [A1]

12 It is given that $y = 2cos^2x - sin^2x$.

(i) By expressing y in the form of $acos2x + b$, show that $a = \frac{3}{2}$ and $b = \frac{1}{2}$. [3]

$y = 2\cos^2 x - \sin^2 x$

$= 2(1-\sin^2 x) - \sin^2 x$ [M1]

$= 2 - 2\sin^2 x - \sin^2 x$

$= 2 - 3\sin^2 x$ [M1]

$= \frac{3}{2}(1-2\sin^2 x) + \frac{1}{2}$

$= \frac{3}{2}\cos 2x + \frac{1}{2}$ [A1]

(ii) Sketch the graph of $y = 2cos^2x - sin^2x$ for $0 \leq x \leq 2\pi$ radian, showing clearly the turning points and the intercepts with $y = \frac{1}{2}$. [3]

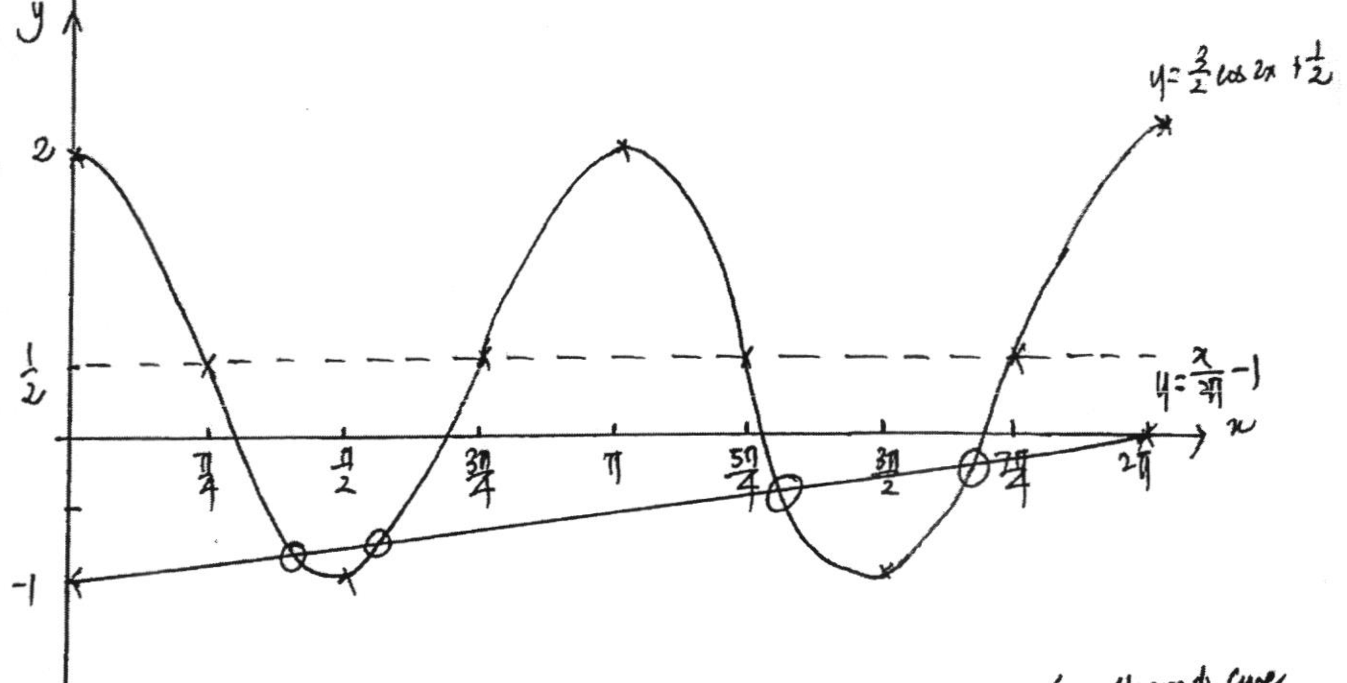
$y = \frac{3}{2}\cos 2x + \frac{1}{2}$

period $= \frac{2\pi}{2}$

$= \pi$

Max value $= 2$

min value $= -1$



G1 - shape of curve

G1 - turning point

G1 - intercept with $y = \frac{1}{2}$

12 (ii) By drawing a suitable line on the same axes, state the number of solutions to the equation $4\pi cos^2x - 2\pi sin^2x = x - 2\pi$. [3]

$$2\pi\,(2\cos^2x - \sin^2x) = x - 2\pi$$

$$2\cos^2x - \sin^2x = \frac{1}{2\pi}(x - 2\pi) \quad (M1)$$

$$\frac{3}{2}\sin 2x + \frac{1}{2} = \frac{x}{2\pi} - 1$$

new $y = \frac{x}{2\pi} - 1$ [B1]

$\therefore$ There are <u>4 solutions</u>. [A1]

End of paper