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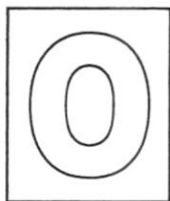
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**FAJAR SECONDARY SCHOOL**  
**2021 END-OF-YEAR EXAMINATIONS**  
**SECONDARY 3 EXPRESS**

8

CANDIDATE  
NAME

CLASS




INDEX  
NUMBER



**ADDITIONAL MATHEMATICS**

**4049**

**30 September 2021**

**2 hours 15 minutes**

Setter: Miss Lily

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your class, index number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

<b>For Examiner's Use</b>	
<b>Total</b>	<b>90</b>

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This document consists of **18** printed pages and **0** blank pages.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Answer **all** the questions.

- 1 (a) Express each of the following expressions  $2x^2 - 4x + 7$  and  $-x^2 - 4x - 1$  in the form  $a(x+b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. [4]

- (b) Use your answer from part (a) to explain why the curves with equations  $y = 2x^2 - 4x + 7$  and  $y = -x^2 - 4x - 1$  will not intersect. [3]

2 Do not use a calculator in this question.

- (a) It is given that  $x - \sqrt{\quad} = x\sqrt{3} + \quad$ . Find  $x$  in the form  $a + b\sqrt{\quad}$  where  $a$  and  $b$  are constants. [3]

- (b) Solve the equation  $\sqrt{x-3} + x = \quad$ . [3]

- 3 (a) Find the set of values of  $b$  for which the curve  $y = 2x^2 + bx + 17$  intersects the line  $y - 7x - 9 = 0$  at two distinct points.

[4]

- (b) Find the range of values of  $k$  for which  $x^2 + k(2x + 1) + 12$  is always positive.

[3]

- 4 (a) The expression  $x^2 + 7x + 3$  has the same remainder when divided by  $(x - p)$  and  $(x + q)$ , where  $p$  and  $q$  are constants and  $p \neq q$ . Find the value of  $p - q$ . [4]

- (b) Given that  $f(x) = 2x^3 + x^2 - 10x - 8$ , show that  $x + 2$  is a factor and hence solve the equation  $f(x) = 0$ .

[5]

5 Express  $\frac{x^3 + 2x^2 - 6}{x^2 - 9}$  in partial fractions.

[5]

6 The coefficient of  $x^2$  in the expansion of  $(a+x)^4 + \left(2 - \frac{x}{4}\right)^5$  is 29.

(a) Find the value of the positive constant  $a$ .

[4]

(b) Hence calculate the coefficient of  $x$  in the expansion of  $(a+x)^4 + \left(2 - \frac{x}{4}\right)^5$ . [2]

7 (a) Given that  $\log_2 x = p$  and  $\log_2 y = q$ , express  $\log_x 4y$  in terms of  $p$  and  $q$ . [3]

(b) Express  $2\log_2 x - \log_2(x-5) = 2$  as a quadratic equation in  $x$  and explain why there are no real solutions. [5]

- 8 A returning resident is suspected to have contracted influenza from his recent travel. Suppose that all the residents who come into contact with him are susceptible to influenza, an epidemic is expected to break out in Hugetown following the expression,

$$N = \frac{15000}{1 + 9999e^{-0.2t}},$$

where  $N$  is the number of residents who contracted influenza,  $t$  days after first contact.

- (a) Find the number of residents who are likely to have contracted influenza after 7 days. [1]

- (b) Explain why the number of infected residents can never exceed 15 000. [1]

The town would be labelled as an influenza red-zone if at least 3000 residents were infected.

- (c) Suppose the spread of influenza continues following the given expression, after how many days would the town be labelled as a red zone? [3]

9 Solve the simultaneous equations

$$\begin{aligned}8^y &= \frac{1}{4^x} \\ y + (\sqrt{3})^x &= \sqrt{\quad}\end{aligned}\quad [5]$$

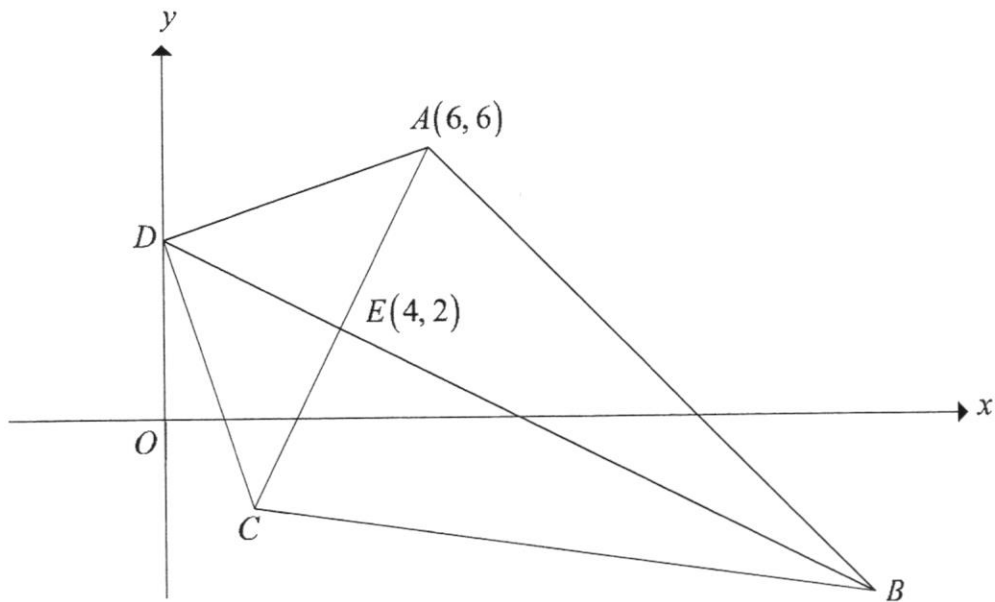
10 A circle,  $C$ , has equation  $x^2 + y^2 - 27x = -41$ .

- (a) Find the radius and the coordinates of the centre of  $C$ . [4]

A straight line that cuts the  $y$ -axis at  $-3$  intersects the circle  $x^2 + y^2 - 27x = -41$  at  $P(2, 3)$ .

- (b) Find the coordinates of the point  $Q$  at which the line meets the circle again. [4]

11



The diagram shows a quadrilateral  $ABCD$  in which  $BD$  is the perpendicular bisector of  $AC$  and cuts  $AC$  at the point  $E$ . The coordinates of  $A$  and  $E$  are  $(6, 6)$  and  $(4, 2)$  respectively.

(a) Find the coordinates of  $C$ .

[2]

$AB$  is parallel to the line passing through the point  $C$  and the origin. The point  $D$  lies on the  $y$ -axis.

Find

(b) the coordinates of  $B$  and of  $D$ ,

[7]

(c) the area of  $ABCD$ .

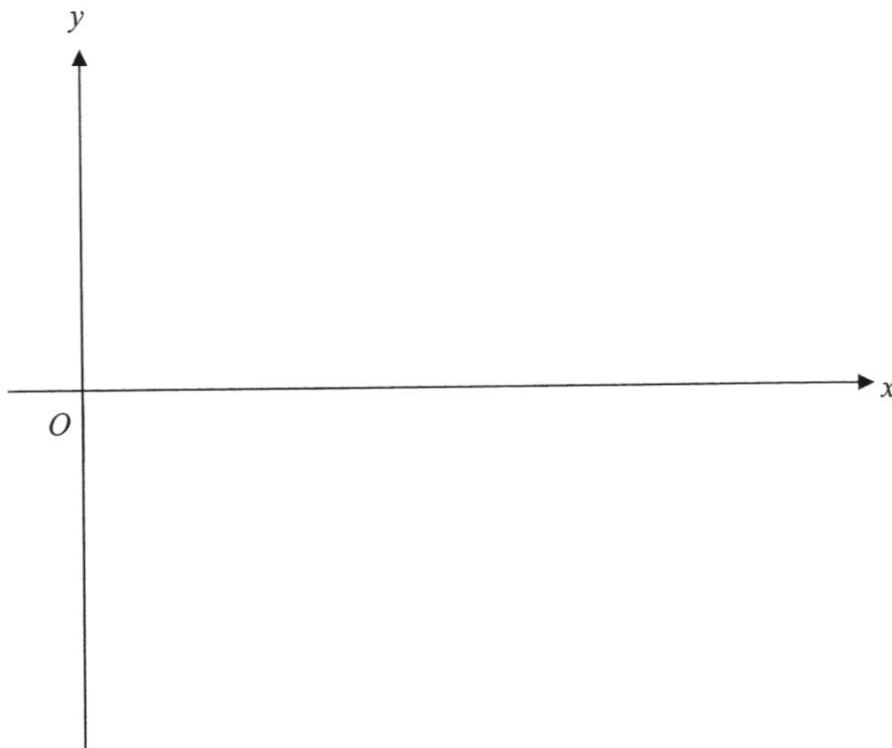
[2]

12 (a) State the period and amplitude of  $y = 2 + 3 \cos 2x$ .

[2]

(b) Sketch the graph of  $y = 2 + 3 \cos 2x$  for  $0 \leq x \leq 2\pi$ .

[2]



- 13 The acute angles  $A$  and  $B$  are such that  $\sin A = \frac{12}{13}$  and  $\tan B = \frac{3}{4}$ .

**Without using a calculator**, find the exact value of

(a)  $\cos A$ ,

[1]

(b)  $\sin(A+B)$ .

[2]

14 (a) Show that  $\frac{\sec x - \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} = \frac{\tan x - 1}{\tan x + 1}$ . [3]

(b) Hence find, for  $0 \leq x \leq 2\pi$ , the values of  $x$  in radians for which

$$\frac{\sec x - \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} = \frac{4}{9}. \quad [3]$$

**END OF PAPER**



Qn. #	Solution	Mark Allocation
2a	$x - \sqrt{\quad} = x\sqrt{3} +$ $x - x\sqrt{3} = + \sqrt{\quad}$ $x(-\sqrt{3}) = + \sqrt{\quad}$ $x = \frac{5+7\sqrt{3}}{4-\sqrt{3}} \left( \times \frac{4+\sqrt{3}}{4+\sqrt{3}} \right)$ $= \frac{20+5\sqrt{3}+28\sqrt{3}+21}{16-3}$ $= \frac{41+33\sqrt{3}}{13}$ $= \frac{41}{13} + \frac{33}{13}\sqrt{3}$	<p>M1 (factorise; (<math>4-\sqrt{3}</math>) seen)</p> <p>M1 (rationalise surds)</p> <p>A1 (in the form <math>a+b\sqrt{3}</math>)</p>
2b	$2\sqrt{x-3} + x = 11$ $\sqrt{x-3} = -x$ $4(x-3) = (11-x)^2$ $4x-12 = 121-22x+x^2$ $x^2 - 26x + 133 = 0$ $(x-7)(x-19) = 0$ $x = 7 \text{ or } x = 19 \text{ (rejected)}$	<p>M1 (square both sides)</p> <p>M1 (factorise)</p> <p>A1 (reject <math>x = 19</math>)</p>
3a	$2x^2 + bx + 17 - 7x - 9 = 0$ $2x^2 + (b-7)x + 8 = 0$ <p>Since curve and line intersect at two distinct points,</p> $b^2 - 4ac > 0$ $(b-7)^2 - 4(2)(8) > 0$ $b^2 - 14b + 49 - 64 > 0$ $b^2 - 14b - 15 > 0$ $(b-15)(b+1) > 0$ $\therefore b < -1 \text{ or } b > 15$	<p>M1 (substitution method)</p> <p>M1 (correct discriminant)</p> <p>M1 (factorise)</p> <p>A1</p>

Qn. #	Solution	Mark Allocation
3b	$x^2 + k(2x+1) + 12 = x^2 + 2kx + k + 12$ <p>Since curve is always positive,</p> $b^2 - 4ac < 0$ $(2k)^2 - 4(1)(k+12) < 0$ $4k^2 - 4k - 48 < 0$ $k^2 - k - 12 < 0$ $(k-4)(k+3) < 0$ $\therefore -3 < k < 4$	<p>M1 (correct discriminant)</p> <p>M1 (factorise)</p> <p>A1</p>
4a	<p>Let <math>f(x) = x^2 + 7x + 3</math></p> $f(p) = (p)^2 + 7(p) + 3$ $f(p) = p^2 + 7p + 3 \quad - (1)$ $f(-q) = (-q)^2 + 7(-q) + 3$ $f(-q) = q^2 - 7q + 3 \quad - (2)$ $(1) = (2)$ $p^2 + 7p + 3 = q^2 - 7q + 3$ $p^2 - q^2 + 7p + 7q = 0$ $(p+q)(p-q) + 7(p+q) = 0$ $(p+q)(p-q+7) = 0$ $p+q = 0 \quad \text{or} \quad p-q+7 = 0$ $\text{(reject)} \quad p-q = -7$	<p>M1 (either equation seen)</p> <p>M1 (substitution method)</p> <p>M1 (factorise)</p> <p>A1 (no mark award if student did not reject <math>p+q=0</math>)</p>



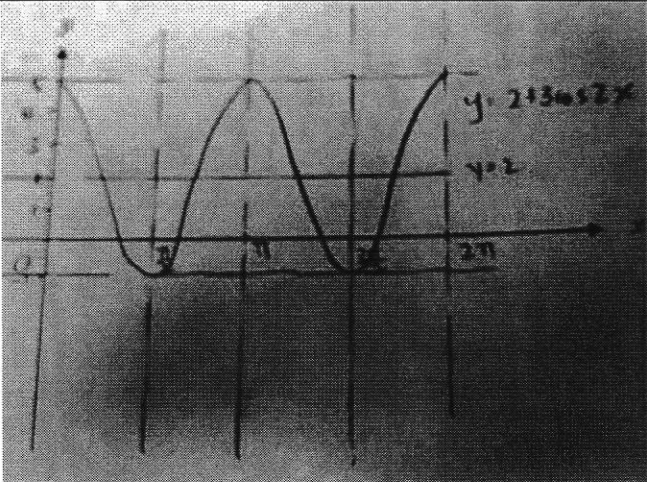
Qn. #	Solution	Mark Allocation
5	<p>By long division, <math>\frac{x^3 + 2x^2 - 6}{x^2 - 9} = x + 2 + \frac{9x + 12}{x^2 - 9}</math></p> $\frac{9x + 12}{x^2 - 9} = \frac{9x + 12}{(x - 3)(x + 3)} = \frac{A}{x - 3} + \frac{B}{x + 3}$ $9x + 12 = A(x + 3) + B(x - 3)$ <p>When <math>x = 3</math>,</p> $9(3) + 12 = A(3 + 3)$ $A = \frac{13}{2}$ <p>When <math>x = -3</math>,</p> $9(-3) + 12 = B(-3 - 3)$ $B = \frac{5}{2}$ $\frac{9x + 24}{x^2 - 9} = \frac{13}{2(x - 3)} + \frac{5}{2(x + 3)}$ $\frac{x^3 + 2x^2 - 6}{x^2 - 9} = x + 2 + \frac{13}{2(x - 3)} + \frac{5}{2(x + 3)}$	<p>M1 (long division)</p> <p>M1 (factorise <math>x^2 - 9 = (x - 3)(x + 3)</math>)</p> <p>M1 (<math>A</math> found, ecf)</p> <p>M1 (<math>B</math> found, ecf)</p> <p>A1</p>
6a	$(a + x)^4 + \left(2 - \frac{x}{4}\right)^5$ $= a^4 + \binom{4}{1}a^3x + \binom{4}{2}a^2x^2 + \dots + 2^5 + \binom{5}{1}2^4\left(-\frac{x}{4}\right) + \binom{5}{2}2^3\left(-\frac{x}{4}\right)^2 + \dots$ $= a^4 + 4a^3x + 6a^2x^2 + 80 - 20x + 5x^2 + \dots$ <p>Comparing coefficient of <math>x^2</math>,</p> $6a^2 + 5 = 29$ $6a^2 = 24$ $a^2 = 4$ $a = 2 \text{ (reject } a = -2)$	<p>M1</p> $a^4 + \binom{4}{1}a^3x + \binom{4}{2}a^2x^2 + \dots$ <p>seen</p> <p>M1</p> $2^5 + \binom{5}{1}2^4\left(-\frac{x}{4}\right) + \binom{5}{2}2^3\left(-\frac{x}{4}\right)^2 + \dots$ <p>seen</p> <p>M1</p> <p>A1 (rejected <math>a = -2</math>)</p>
6b	<p>Coefficient of <math>x</math></p> $= 4a^3 - 20$ $= 4(2)^3 - 20$ $= 12$	<p>M1</p> <p>A1</p>

Qn. #	Solution	Mark Allocation
7a	$\log_x 4y = \frac{\log_2 4y}{\log_2 x}$ $= \frac{\log_2 4 + \log_2 y}{\log_2 x}$ $= \frac{2+q}{p}$	M1 (change base to 2) M1 (product law) A1
7b	$2\log_2 x - \log_2 (x-5) = 2$ $\log_2 \frac{x^2}{(x-5)} = 2$ $\frac{x^2}{(x-5)} = 2^2$ $x^2 = 4(x-5)$ $x^2 - 4x + 20 = 0$ <p>Discriminant  <math>= b^2 - 4ac</math>  <math>= (-4)^2 - 4(1)(20)</math>  <math>= -64 &lt; 0</math></p> <p>Since discriminant <math>&lt; 0</math>, the equation has no real solutions.</p>	M1 (quotient law)  M1 (change log to exponential) M1 (form quadratic equation)  M1 (find value of discriminant / used quadratic formula)  A1 (show that when discriminant $< 0$ , there are no real solutions / concluded that no answer from quadratic formula)
8a	$N = \frac{15000}{1 + 9999e^{-0.2(7)}} = 6.080942101 = 6$ <p>6 residents are likely to have contracted influenza by Day 7.</p>	B1
8b	<p>As <math>t \rightarrow \infty</math>,</p> $9999e^{-0.2t} \rightarrow 0$ $\frac{15000}{1 + 9999e^{-0.2t}} \rightarrow 15000$ <p>The population of residents in Hugetown is only 15 000.</p>	B1

Qn. #	Solution	Mark Allocation
8c	$\frac{15000}{1+9999e^{-0.2t}} = 3000$ $1+9999e^{-0.2t} = 5$ $e^{-0.2t} = \frac{4}{9999}$ $-0.2t = \ln \frac{4}{9999}$ $t = 39.1197300293$ $\therefore \text{Day 40}$	<p>M1 (3000 seen)</p> <p>M1 (take ln)</p> <p>A1</p>
9	$2^{3y} = 2^{-2x} \qquad 3^{3y} \div 3^{\frac{1}{2}x} = 3^4 \left( 3^{\frac{1}{2}} \right)$ $3y = -2x \qquad 3^{3y - \frac{1}{2}x} = 3^{4 + \frac{1}{2}}$ $y = -\frac{2}{3}x \quad - (1) \qquad 3y - \frac{1}{2}x = \frac{9}{2} \quad - (2)$ <p>Sub (1) into (2),</p> $3 \left( -\frac{2}{3}x \right) - \frac{1}{2}x = \frac{9}{2}$ $-\frac{5}{2}x = \frac{9}{2}$ $x = -\frac{9}{5}$ <p>Sub <math>x = -\frac{9}{5}</math> into (1),</p> $y = -\frac{2}{3} \left( -\frac{9}{5} \right)$ $y = \frac{6}{5}$ <p>Therefore, <math>x = -\frac{9}{5}</math> and <math>y = \frac{6}{5}</math>.</p>	<p>M1 (law of indices)</p> <p>M1 (both equations)</p> <p>M1 (substitution method)</p> <p>A1</p> <p>A1</p>

Qn. #	Solution	Mark Allocation
10a	$x^2 + y^2 + 2gx + 2fy + c = 0, r = \sqrt{g^2 + f^2 - c}$ <p>Given <math>x^2 + y^2 - 27x + 41 = 0</math>,  Comparing coefficient of <math>x</math>,  <math>2g = -27</math>  <math>g = -\frac{27}{2}</math></p> <p>Comparing coefficient of <math>y</math>,  <math>-2f = 0</math>  <math>f = 0</math></p> <p>Comparing constant, <math>c = 41</math></p> <p>Centre <math>\left(\frac{27}{2}, 0\right)</math> and  <math>r = \sqrt{\left(-\frac{27}{2}\right)^2 - 41}</math>  <math>= 11.9 \text{ units}^2 \text{ (3 s.f.)}</math></p>	<p>M1 (either <math>g, f</math> or <math>c</math> found)</p> <p>A1 (centre)</p> <p>M1 (radius)</p> <p>A1</p>
10b	<p>Let the equation of the straight line be <math>y = mx + c</math>.  Line passes through <math>(0, -3)</math> and <math>P(2, 3)</math>.</p> $m = \frac{3+3}{2-0}$ $= 3$ $y = 3x - 3$ <p><math>x^2 + y^2 - 27x + 41 = 0</math> - (1)  <math>y = 3x - 3</math> - (2)  Sub (2) into (1).  <math>x^2 + (3x - 3)^2 - 27x + 41 = 0</math>  <math>x^2 + 9x^2 - 18x + 9 - 27x + 41 = 0</math>  <math>10x^2 - 45x + 50 = 0</math>  <math>2x^2 - 9x + 10 = 0</math>  <math>(2x - 5)(x - 2) = 0</math>  <math>x = \frac{5}{2}</math> or <math>x = 2</math> (reject; <math>x</math>-coordinate of <math>P</math>)  <math>y = \frac{9}{2}</math> Therefore, <math>Q\left(\frac{5}{2}, \frac{9}{2}\right)</math></p>	<p>M1 (equation of line)</p> <p>M1 (substitution method)</p> <p>M1 (quadratic equation form)</p> <p>A1 (coordinates of <math>Q</math> found)</p>

Qn. #	Solution	Mark Allocation
11a	<p>Let the coordinates of <math>C(x, y)</math>.</p> <p><math>E</math> is the midpoint of <math>AC</math>.</p> $\left(\frac{x+6}{2}, \frac{y+6}{2}\right) = (4, 2)$ $\frac{x+6}{2} = 4 \quad \text{or} \quad \frac{y+6}{2} = 2$ $x = 2 \quad \quad \quad y = -2$ <p>Therefore, <math>C(2, -2)</math>.</p>	<p>M1 (Midpoint)</p> <p>A1</p>
11b	<p>Let the equation of <math>AB</math> be <math>y = m_{AB} + c</math>.</p> $m_{OC} = \frac{-2-0}{2-0}$ $= -1$ $m_{AB} = m_{OC} = -1$ <p>Sub <math>A(6, 6)</math> and <math>m_{AB} = -1</math>,</p> $y - 6 = -(x - 6)$ $y = -x + 12.$ <p>Let the equation of <math>BD</math> be <math>y = m_{BD} + c</math>.</p> $m_{AC} = \frac{6+2}{6-2}$ $= 2$ $m_{BD} = -\frac{1}{m_{AC}} = -\frac{1}{2}$ <p><math>E</math> lies on line <math>BD</math></p> $y - 2 = -\frac{1}{2}(x - 4)$ $y = -\frac{1}{2}x + 4.$ <p><math>B</math> lies on <math>AB</math> and <math>BD</math>,</p> $y = -x + 12. \quad - (1)$ $y = -\frac{1}{2}x + 4. \quad - (2)$ $(1) = (2),$	<p>M1 (parallel gradient)</p> <p>M1 (equation of <math>AB</math>)</p> <p>M1 (perpendicular gradient)</p> <p>M1 (equation of <math>BD</math>)</p>

Qn. #	Solution	Mark Allocation
	$-x + 12 = -\frac{1}{2}x + 4$ $-\frac{1}{2}x = -8$ $x = 16$ $y = -16 + 12 = -4$ $B(16, -4).$ <p><math>D</math> lies on <math>y</math> axis and <math>BD</math>,</p> <p>When <math>x = 0</math>, <math>y = -\frac{1}{2}(0) + 4 = 4</math></p> $D(0, 4).$	<p>M1 (substitution)</p> <p>A1 (B)</p> <p>A1 (D)</p>
11c	$\text{Area of } ABCD = \frac{1}{2} \begin{vmatrix} 0 & 2 & 16 & 6 & 0 \\ 4 & -2 & -4 & 6 & 4 \end{vmatrix}$ $= \frac{1}{2}  112 - (-48) $ $= 80 \text{ units}^2.$	<p>M1 (Simplification)</p> <p>A1</p>
12a	$\text{Period} = \frac{2\pi}{2}$ $= \pi$ <p>amplitude = 3</p>	<p>B1</p> <p>B1</p>
12b		<p>B1 correct shape and correct max/min value</p> <p>B1 correct number of cycles</p>
13a	$\cos A = \frac{5}{13}$	B1
13b	$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $= \frac{12}{13} \left(\frac{4}{5}\right) + \frac{5}{13} \left(\frac{3}{5}\right)$ $= \frac{63}{65}$	<p>M1</p> <p>A1</p>

Qn. #	Solution	Mark Allocation
14a	$\begin{aligned} \text{LHS} &= \frac{\sec x - \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} \\ &= \frac{\frac{1}{\cos x} - \frac{1}{\sin x}}{\frac{1}{\cos x} + \frac{1}{\sin x}} \\ &= \frac{\frac{\sin x - \cos x}{\sin x \cos x}}{\frac{\sin x + \cos x}{\sin x \cos x}} \\ &= \frac{\sin x - \cos x}{\sin x + \cos x} \\ &= \frac{\cos x}{\sin x + \cos x} \\ &= \frac{\tan x - 1}{\tan x + 1} \end{aligned}$	<p>M1 (reciprocal for either <math>\sec x</math> or <math>\operatorname{cosec} x</math>)</p> <p>M1 (common denominator)</p> <p>A1 (shown)</p>
14b	$\begin{aligned} \frac{\tan x - 1}{\tan x + 1} &= \frac{4}{9} \\ 9 \tan x - 9 &= 4 \tan x + 4 \\ 5 \tan x &= 13 \\ \tan x &= \frac{13}{5} \\ \alpha &= 1.2036\dots \\ x &= 1.2036\dots, \pi + 1.2036\dots \\ x &\approx 1.20, 4.35 \end{aligned}$	<p>M1</p> <p>M1 (basic angle)</p> <p>A1 (both values)</p>