NORTH VISTA SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2021

90

NAME: ______________________( ) **CLASS:** __________

SUBJECT: ADDITIONAL MATHEMATICS **DATE: 8 OCTOBER**

LEVEL/ STREAM: SECONDARY 3 EXPRESS **TIME : 2 HOURS 15 MINUTES**

CODE : 4049

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use	
Category	**Question No.**
Accuracy	
Brackets	
Fractions	
Units	
Others	
Marks Deducted	

This question paper consists of **21** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\dfrac{53-5x-7x^2}{(3x-1)(x^2+10)}$ in partial fractions. [6]

[Turn over

2 **(a)** Variables x and y are connected by the equation $y = ax^2 + by^3$, where a and b are constants. Explain clearly how a and b can be calculated when a graph of $\frac{x^2}{y}$ against y^2 is drawn. [3]

(b) The mass, m grams, of a radioactive substance decreases with time, t years.
The observation started on 1 January 2021.
Measured values of m and t are shown in the table below.

***m* (grams)**	53.6	35.9	24.1	16.2
***t* (years)**	10	20	30	40

It is known that m and t are related by the equation $m = m_0 e^{-kt}$, where m_0 and k are constants.

(i) Draw the graph of $\ln m$ plotted against t, using a scale of 4 cm for 10 units on the t-axis and a scale of 2 cm for 1 unit on the $\ln m$-axis. [3]

[Turn over

Use your graph to estimate

2 **(b)** **(ii)** the value of k, [2]

(iii) the mass of the substance when the observations began, [2]

(iv) the year for the substance to reach half its original mass. [3]

3 **(a)** In the expansion of $\left(2x^2 - \frac{5}{x^3}\right)^9$, determine whether the term independent of of x exists. [4]

[Turn over

3 **(b)** **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $\left(3-\frac{x}{2}\right)^8$. Give the terms in their simplest form. [3]

(ii) Hence, find the coefficient of x^3 in the expansion of $\left(3-\frac{x}{2}\right)^8\left(2x^2-7x+9\right)$. [2]

4 The points $A(5,17)$ and $B(12,10)$ lie on a circle. The centre of the circle lies on the y-axis.

(i) Find the equation of the perpendicular bisector of AB. [5]

(ii) Find the equation of the circle. [3]

[Turn over

4 **(iii)** Does the point $C(-7,4)$ lie on the circle, inside the circle or outside the circle? Explain clearly by showing all your working. [2]

The circle gets reflected about a vertical line that passes through the point $(-9,6)$.

(iv) Find the equation of the reflected circle. [2]

5 **(i)** Show that $x+1$ is a factor of $2x^3+11x^2+22x+13$. [1]

(ii) Jacob said that $y=2x^3+11x^2+22x+13$ is a cubic graph, therefore the graph will cut the x-axis at 3 points.
Do you agree? Explain clearly by showing all your working. [4]

[Turn over

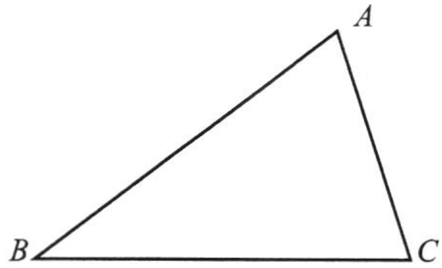
6 **(a)** Find the values of x and y which satisfy the equations

$$5^x = \frac{1}{5(25)^y}$$

$$2^{0.5x} = 4\sqrt{2^{3y}} .$$

[4]

6 **(b)** The diagram shows a triangular field ABC. The area of the field is $\frac{380}{25-7\sqrt{5}}$ m^2 and the shortest distance from A to BC is $(4\sqrt{5}-2)$ m.



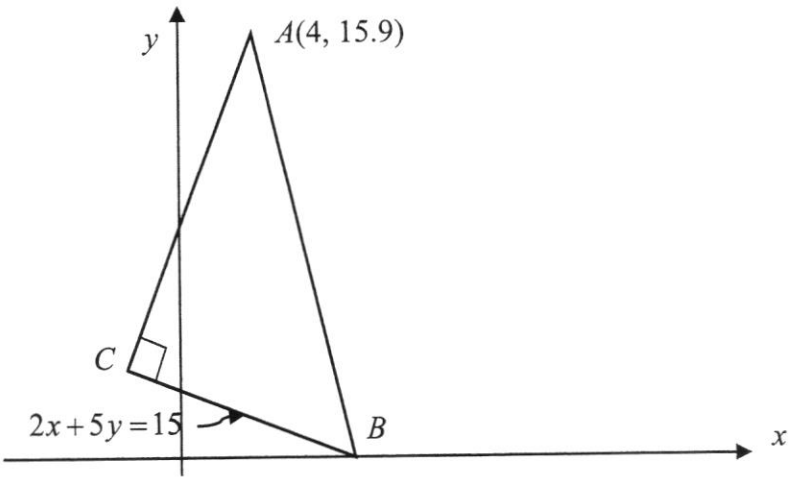
Without using a calculator, obtain an expression for BC in the form $a+b\sqrt{5}$, where a and b are integers. [4]

7 (a) Use the substitution of $u = 3^x$ to solve the equation $3^{2x+1} - 8(3^x) = 3$. [4]

7 **(b)** Solve the equation $\log_4(x+4)-\frac{1}{\log_{2x-3}4}=\log_4\frac{2}{3}+1$. [5]

[Turn over

8



The diagram shows a triangle ABC in which the point A is $(4,15.9)$, the point B lies on the x-axis and the angle ACB is $90°$. The equation of BC is $2x+5y=15$.

(i) Find the coordinates of C. [5]

(ii) Given that *ABCD* is a parallelogram, find the coordinates of *D*. [3]

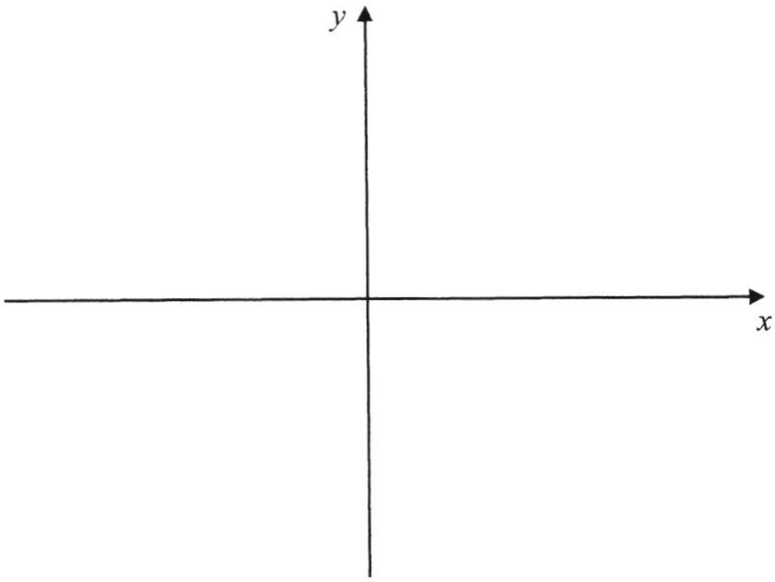
(iii) The point *E* is $(-5,-9)$. Find the area of triangle *BCE*. [2]

[Turn over

9 **(i)** Given that $ax^2 - 3x + c$ is always positive, what conditions must apply to the constants a and c? [3]

(ii) Give an example of values of a and c which satisfy the conditions found in part **(i)**. [2]

10 **(i)** Sketch the graph of $y = \frac{1}{5^x}$. [2]



(ii) In order to solve the equation $x = \log_5\left(\frac{1}{2-3x}\right)$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = \frac{1}{5^x}$.

Find the equation of the straight line and the number of solution(s). [4]

[Turn over

11 The equation of a curve is $y = 4x^2 + (3p-1)x + 1 + q$, where p and q are constants.
The line $y - px = q$ is a tangent to the curve at the point A.

(i) Find the negative value of p. [4]

(ii) Using this value of p, and given that the curve passes through $(-3, 56.5)$, find the coordinates of A. [3]

End of Paper

3E AM EOY 2021 Marking Scheme

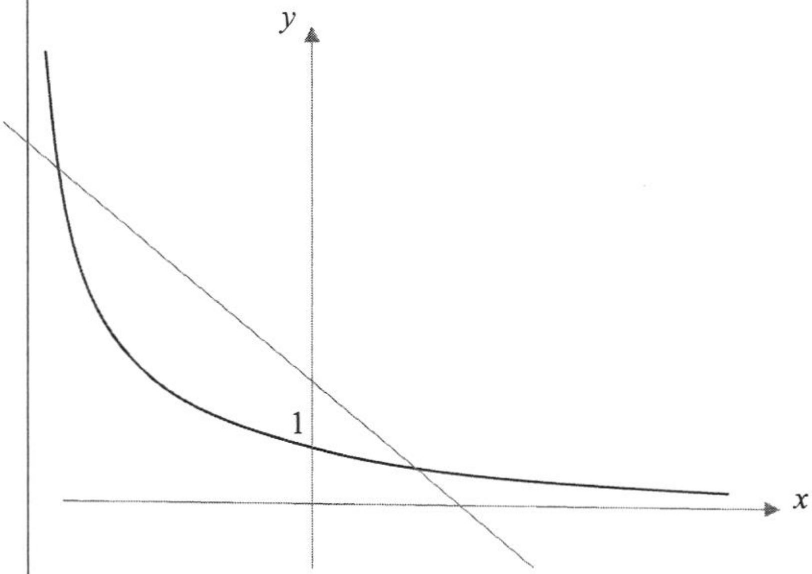
Qn	
1	$\frac{53-5x-7x^2}{(3x-1)(x^2+10)}=\frac{A}{3x-1}+\frac{Bx+C}{x^2+10}$
	$53-5x-7x^2=A(x^2+10)+(Bx+C)(3x-1)$
	$A=5, B=-4, C=-3$
	$\frac{53-5x-7x^2}{(3x-1)(x^2+10)}=\frac{5}{3x-1}-\frac{4x+3}{x^2+10}$
2(a)	$\frac{x^2}{y}=-\frac{by^2}{a}+\frac{1}{a}$
	Vertical-intercept= $\frac{1}{a}$, so a can be found
	Gradient = $-\frac{b}{a}$ so b can now be found
(b)(i)	Scale; Points; Line
(ii)	$-k$ = gradient using two points
	$k=0.0414$
(iii)	$\ln m$ = 4.3 to 4.5
	$m=73.7$ to 90
(iv)	$\ln\frac{e^{4.4}}{2}=4.4-0.041379t$
	$t=16.751\approx 17$ years
	Year 2021 + 17 = 2038

3(a)	$T_{r+1}=\binom{9}{r}\left(2x^2\right)^{9-r}\left(-\frac{5}{x^3}\right)^r$
	$=\binom{9}{r}(2)^{9-r}(-5)^r\left(x^{18-5r}\right)$
	For independent term,
	$18-5r=0$
	$r=3.6$
	Since r is not a whole number, it is not possible to get a term independent of x.
(b)(i)	$6561-8748x+5103x^2$
(ii)	$T_4=\binom{8}{3}(3)^5\left(\frac{-x}{2}\right)^3=-1701x^3$ $-8748(2)+5103(-7)=9(-1701)=-68526$
4(i)	$m_{AB}=-1$
	$m_{\perp}=1$
	$M_{AB}=(8.5,13.5)$
	$y-13.5=1(x-8.5)$
	$y=x+5$
(ii)	$Centre=(0,5)$
	Radius = 13 units
	$x^2+(y-5)^2=169$ or 13^2 or $x^2+y^2-10y-144=0$
(iii)	Distance of C from centre = 7.07 or $5\sqrt{2}$ or $\sqrt{50}$ units
	Since distance of C from centre is less than the radius, the point C lies inside the circle.

(iv)	Centre(-18,5)
	$(x+18)^2+(y-5)^2=169$ or 13^2 $x^2+y^2+36x-10y+11=0$
5(i)	Let $f(x)=2x^3+11x^2+22x+13$
	$f(-1)=2(-1)^3+11(-1)^2+22(-1)+13=0$ Therefore, $x+1$ is a factor.
(ii)	By long division, comparing coefficient or synthetic division,
	$2x^3+11x^2+22x+13=(x+1)(2x^2+9x+13)$
	To cut the x-axis,
	$(x+1)(2x^2+9x+13)=0$
	$x=-1$ or $2x^2+9x+13=0$
	I do not agree, since $b^2-4ac=-23<0$, therefore there is only 1 real root so the graph only cuts th
6(a)	$x=-2y-1$ or $x+2y=-1$ o.e
	$0.5x=2+1.5y$ or $x=4+3y$ o.e
	$x=1, y=-1$
(b)	$BC=\dfrac{760}{(25-7\sqrt{5})(4\sqrt{5}-2)}$ Or $BC=\dfrac{380}{(25-7\sqrt{5})(2\sqrt{5}-1)}$
	$=\dfrac{760}{114\sqrt{5}-190}$ Or $=\dfrac{380}{57\sqrt{5}-95}$
	$=\dfrac{760}{114\sqrt{5}-190}\times\dfrac{114\sqrt{5}+190}{114\sqrt{5}+190}$ Or $=\dfrac{380}{57\sqrt{5}-95}\times\dfrac{57\sqrt{5}+95}{57\sqrt{5}+95}$
	$=3\sqrt{5}+5$
	Alternatively:

	$Area = \frac{380(25+7\sqrt{5})}{(25-7\sqrt{5})(25+7\sqrt{5})} = 25+7\sqrt{5}$
	$BC = \frac{(25+7\sqrt{5})(2\sqrt{5}+1)}{(2\sqrt{5}-1)(2\sqrt{5}+1)}$ o.e
	$BC = \frac{50\sqrt{5}+25+70+7\sqrt{5}}{19}$ o.e
	$BC = \frac{57\sqrt{5}+95}{19} = 3\sqrt{5}+5$
7(a)	$3u^2 - 8u - 3 = 0$
	$(3u+1)(u-3) = 0$
	$3^x = -\frac{1}{3}(rejected)$ or $3^x = 3$, $x = 1$
(b)	$\log_4(x+4) - \frac{1}{\log_{2x-3} 4} = \log_4 \frac{2}{3} + 1$
	$\log_4(x+4) - \frac{1}{\frac{\log_4 4}{\log_4(2x-3)}} = \log_4 \frac{2}{3} + 1$ Or $\log_4(x+4) - \frac{\log_4(2x-3)}{l\log_4 4} = \log_4 \frac{2}{3} + 1$
	$\log_4(x+4) - \log_4(2x-3) = \log_4 \frac{2}{3} + 1$
	$\log_4 \frac{x+4}{2x-3} = \log_4 \frac{8}{3}$ OR $\log_4 \frac{3(x+4)}{2(2x-3)} = 1$

	$\frac{x+4}{2x-3}=\frac{8}{3}$ OR $\frac{x+4}{2x-3}=4$
	$3(x+4)=8(2x-3)$ OR $x+4=4(2x-3)$
	$x=\frac{36}{13}$
8(i)	$m_{AC}=2.5$
	Equation of AC: $y=2.5x+5.9$
	$2x+5(2.5x+5.9)=15$
	$x=-1, y=3.4$
	C(-1,3.4)
(ii)	$M_{AC}=(1.5,9.65)$
	$B(7.5,0)$
	$M_{BD}=\left(\frac{x+7.5}{2},\frac{0+y}{2}\right)=(1.5,9.65)$
	$D(-4.5,19.3)$
	Alternatively :
	Equation of AD : $y=-\frac{2}{5}x+17.5$
	Equation of CD : $y=-4\frac{19}{35}x-1\frac{1}{7}$
	$D(-4.5,19.3)$
(iii)	Area $=\frac{1}{2}\begin{vmatrix} 7.5 & -1 & -5 & 7.5 \\ 0 & 3.4 & -9 & 0 \end{vmatrix}$
	$=0.5((7.5)(3.4)+(-1)(-9))-(7.5(-9)+-5(3.4))=59.5$ units^2
9(i)	For minimum graph, $a>0$

	For graph to lie above the x axis,
	$(-3)^2 - 4ac < 0$
	$ac > \frac{9}{4}$; $4ac > 9$; $a > \frac{9}{4c}$; $c > \frac{9}{4a}$
(ii)	Any suitable pairs
10(i)	
(ii)	$5^x = \frac{1}{2-3x}$
	$2-3x = \frac{1}{(5^x)}$
	Equation of line: $y = 2-3x$
	No of solution = 2

11(i)	$q+px=4x^2+(3p-1)x+1+q$
	$4x^2+(2p-1)x+1=0$
	$(2p-1)^2-4(4)(1)=0$
	$(2p-1)^2=16$
	$p=2.5(rej)$ or $p=-1.5$
(ii)	$q+px=4x^2+(3p-1)x+1+q$
	$y=4x^2-5.5x+1+q$
	$56.5=4(-3)^2-5.5(-3)+1+q$
	$q=3$
	$4x^2-5.5x+1+3=-1.5x+3$
	$(2x-1)^2=0$ $x=\frac{1}{2}$
	$A(\frac{1}{2},2\frac{1}{4})$