EXP

PUNGGOL SECONDARY SCHOOL
SECONDARY 3
EXPRESS
END-OF-YEAR EXAMINATION

QUESTION & ANSWER BOOKLET

NAME

CLASS

INDEX NUMBER

Additional Mathematics

4049

-

4 October 2021

2 hours 15 minutes

READ THESE INSTRUCTIONS CAREFULLY

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces and graph paper provided.
Staple your answer on graph paper to the last page of this question and answer booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Total

/ 90

Parent's Signature

This paper consists of **15** printed pages and **1** blank page.

Setter(s) :	Vetter :
Mdm Ho Wei Ling	Ms Jillian Khong

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots\dots\dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots\dots\dots\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the values of x and y which satisfy the equations.

$$4^{x+y} = \sqrt{64}$$

$$\frac{9^y}{3^x} = \left(\frac{1}{3}\right)^{-3}$$ [4]

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2 Given that $\cos A = -\frac{1}{a}$ for $\pi \leq A \leq 2\pi$, express in terms of a,

(i) cosec A, [3]

(ii) tan $(-A)$ [1]

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3 Find the set of values of the constant a for which the curve $y = x^2 + (2a + 1)x + 4$ lies entirely above the line $y = x$. [4]

4 The amount of petrol, P litres, consumed by a car, which is travelling at a speed of x km/h can be modelled by the equation

$$P = \frac{1}{1000}x^2 - \frac{7}{50}x + 10\frac{9}{10}.$$

Mr Tan claims that the car consumes the least amount of petrol when it travels at 73 km/h. Explain whether Mr Tan's claim is true. [4]

5 Express $\frac{8x^2 - 7x + 10}{2x^2 - x - 3}$ in partial fractions. [5]

6 Given that $y = a \sin 2x + b$, where a and b are positive integers,

(i) state the period of y, [1]

Given that the maximum and minimum values of y are 3 and -1 respectively, find

(ii) the value of a and of b. [2]

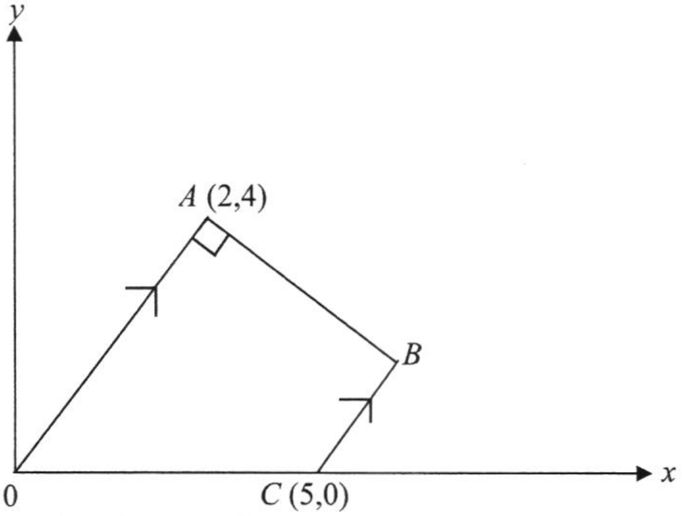
(iii) Using the values of a and b found in **part (ii)**, sketch the graph of y for $0 \leq x \leq 360°$. [2]

7 The area of a quadrilateral is $(12\sqrt{2}+22)$ cm^2.

(i) In the case where the quadrilateral is a rectangle with width $(4-\sqrt{2})$ cm, Find, **without using a calculator**, the length of the rectangle in the form $(a+b\sqrt{2})$ cm. [4]

(ii) In the case where the quadrilateral is a square with length $(3\sqrt{2}+c)$ cm, find, **without using a calculator**, the value of the constant c. [3]

8 **Answers to this question by accurate drawing will not be accepted.**



In the trapezium *OABC*, the point *A* has coordinates (2,4) and the point *C* has coordinates (5,0). The sides *OA* and *CB* are parallel, and *AB* is perpendicular to *OA*.

(i) Find the coordinates of *B*. [6]

(ii) Given that *CB* is produced to point *D* such that *OADC* is a parallelogram, find the coordinates of *D*. [1]

9 In 2010, a certain type of bacteria was found at the bottom of a seabed. It was known to grow with time, such that its population P, after t years is given by $P = 50\ 000\ e^{kt}$, where k is a constant.

(i) Given that the population doubled in two years, show that $k = \frac{1}{2}\ln 2$. [1]

Hence, find

(ii) the year in which the population first exceeds 450 000, [3]

(iii) the size of the population of bacteria in 2015, giving your answer correct to the nearest 10 000. [2]

(iv) Sketch the graph of P against t. [2]

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10 The expression $6x^3 + ax^2 + bx + 2$ has a factor of $x + 2$ and leaves a remainder of 36 when divided by $x - 1$.

(a) Show that $a = 17$ and $b = 11$. [4]

(b) Hence, solve the equation $6x^3 + 17x^2 + 11x + 2 = 0$. [3]

(c) Hence, explain why there are no real roots to the equation

$$6x^4 + 17x^2 + 13 = 2\left(1 - \frac{1}{x^2}\right).$$ [3]

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11 **(a)** **(i)** Write down the first three terms in the expansion of $\left(1 + \frac{2}{3}x\right)^{10}$, in ascending powers of x. [3]

(ii) Hence find the coefficient of x^2 in the expansion of

$$(1 - 2x + 3x^2)\left(1 + \frac{2}{3}x\right)^{10}.$$ [3]

(b) Find the term independent of x in the expansion $\left(\frac{x}{2}-\frac{4}{3x^2}\right)^9$. [4]

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12 A rectangle of area q m^2 has sides of p m and $(Ap + B)$ m, where A and B are constants and p and q are variables. Corresponding values of p and q are shown in the table below.

p	50	100	150	200	250
q	3700	11 000	21 600	36 400	54 500

(i) On the grid found on the next page, plot $\frac{q}{p}$ against p and draw a straight line graph. The vertical scale should start at 0 and have a scale of 2 cm to 20 units. [3]

(ii) Use your graph to estimate the value of A and of B. [4]

(iii) On the same grid, draw the straight line representing the equation $q = p^2$ and explain the significance of the value of p given by the point of intersection of the two lines. [3]

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13 **(a)** Solve the following equations.

(i) $2^x\left(5^{2x}\right) = 7\left(3^x\right)$ [3]

(ii) $\log_4 (6-5x) - 4\log_{16} x = 1$ [4]

(b) It is given that $\lg 2z - \lg (z+y) = \lg y$

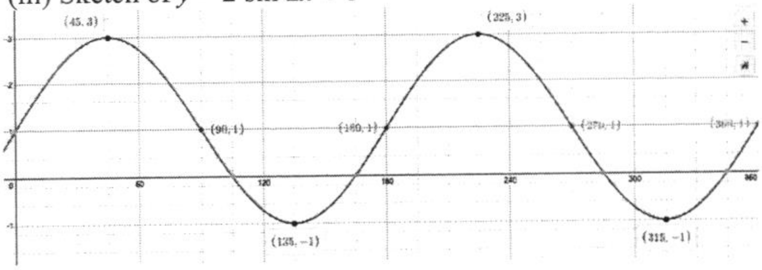
(i) Express z in terms of y. [3]

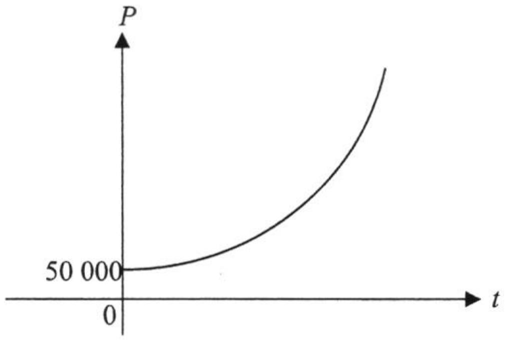
(ii) State the range of values of z and explain clearly why $0 < y < 2$. [2]

---------- **End of Paper** ---------

Marking Scheme

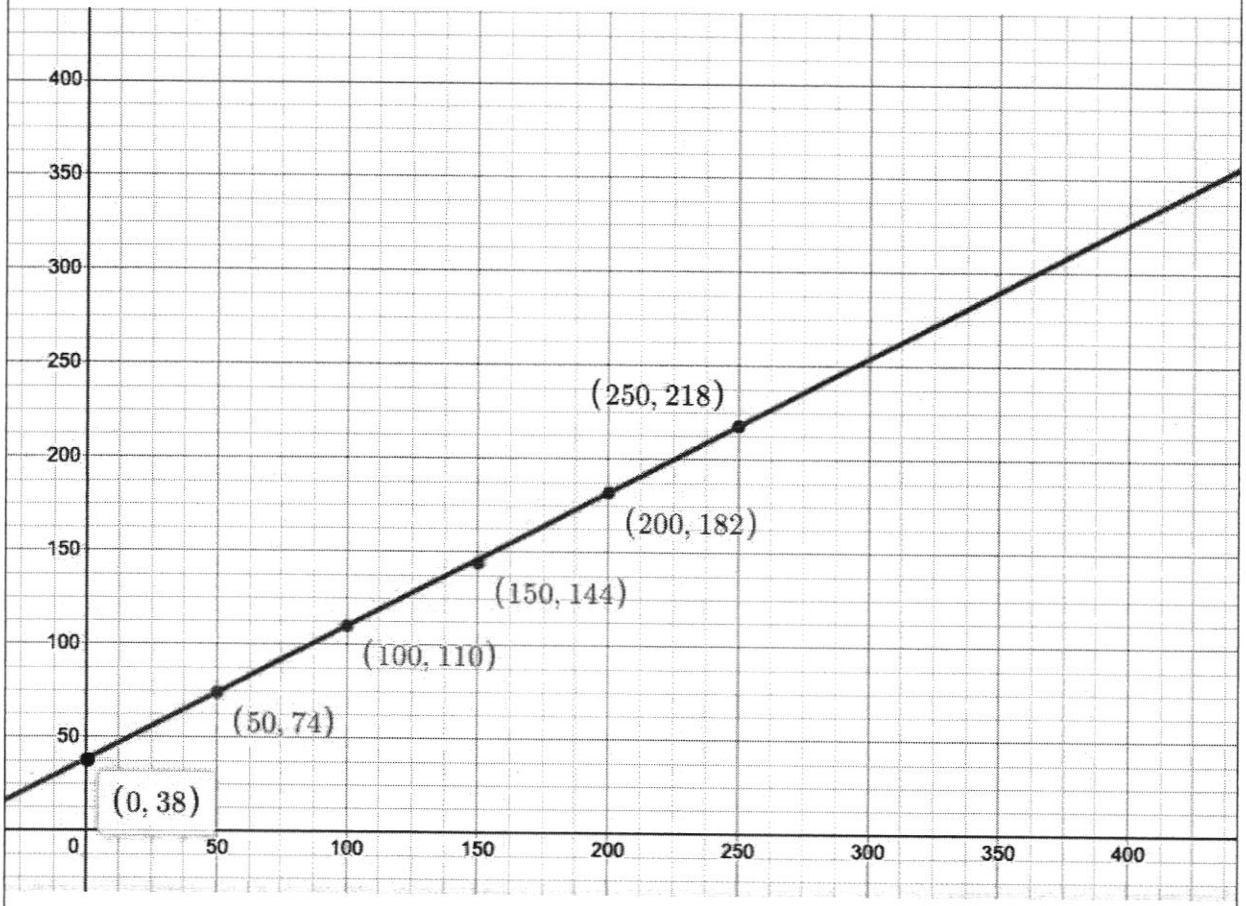
Qn.	Working Steps	Marks	Remarks
1	$4^{x+y} = \sqrt[3]{64}$ → $x + y = 1$ --- (1) $\frac{9^y}{3^x} = \left(\frac{1}{3}\right)^{-3}$ → $2y - x = 3$ --- (2) Solve the simultaneous equations; $x = -\frac{1}{3}; y = \frac{4}{3}$	M1 M1 A1A1	Obtain equation using common base Obtain equation using common base
2	$\cos A = -\frac{1}{a}$ for $\pi \leq A \leq 2\pi$ A exists in 3rd quadrant. Obtain $\sqrt{a^2 - 1}$ for hypotenuse. (i) $\operatorname{cosec} A = \frac{1}{\sin A}$ $= -\frac{a}{\sqrt{a^2-1}}$ (ii) $\tan(-A) = -\sqrt{a^2 - 1}$	B1 B1 B1 B1	
3	$x^2 + (2a + 1)x + 4 > x$ $x^2 + 2ax + 4 > 0$ $(2a)^2 - 4(1)(4) < 0$ $4a^2 - 16 < 0$ $a^2 - 4 < 0$ $(a - 2)(a + 2) < 0$ $-2 < a < 2$	M1 M1M1 A1	Show substitution of equation M1 – Discriminant; M1 - Inequality sign
4	$P = \frac{1}{1000}x^2 - \frac{7}{50}x + 10\frac{9}{10}$ $P = \frac{1}{1000}(x^2 - 140x) + 10\frac{9}{10}$ $P = \frac{1}{1000}[(x - 70)^2 - 70^2] + 10\frac{9}{10}$ $P = \frac{1}{1000}(x - 70)^2 + 6$ Since coefficient of x^2 is positive, minimum point of P is (70, 6). Therefore Mr Tan's claim is not correct, the minimum fuel usage is when he drives at 70 km/h.	M1 A1A1 B1	Show completing of square. A1- Explain why the point is minimum. A1 - Obtaining minimum point Correct Justification
5	$\frac{8x^2 - 7x + 10}{2x^2 - x - 3} = 4 + \frac{-3x + 22}{(2x - 3)(x + 1)}$ Let $\frac{-3x+22}{(2x-3)(x+1)} = \frac{A}{2x-3} + \frac{B}{x+1}$ $-3x + 22 = A(x + 1) + B(2x - 3)$ Sub $x = -1, B = -5$ Sub $x = \frac{3}{2}, A = 7$ $\frac{8x^2 - 7x + 10}{2x^2 - x - 3} = 4 + \frac{7}{(2x - 3)} - \frac{5}{x + 1}$	M1 M1 M1A1 A1	Use of Long Division or Inspection Correct expression in partial fractions Use of Substitution/Comparing Method

6	(i) Period = 180° (ii) $a = 2$ and $b = 1$ (iii) Sketch of $y = 2 \sin 2x + 1$ 	B1 B1B1 B2	B1 – Correct shape and 2 complete cycles B1 – Fully correct curve that starts and ends at y = 1.
7	(i) $\frac{12\sqrt{2}+22}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}}$ $= \frac{48\sqrt{2}+24+88+22\sqrt{2}}{4^2-2}$ $= \frac{70\sqrt{2}+112}{14}$ $= 5\sqrt{2}+8$ (ii) $(3\sqrt{2}+c)^2 = 18 + 6\sqrt{2}c + c^2$ Comparing $12\sqrt{2}+22 = 18 + 6\sqrt{2}c + c^2$ $6c = 12$ $c = 2$	M1 M1 A1A1 M1 M1 A1	Show rationalizing by multiplying by conjugate.
8	(i) Gradient of BC = Gradient of OA $= \frac{4-0}{2-0}$ $= 2$ Equation of BC: $y = 2x - 10$ Gradient of AB = $-\frac{1}{2}$ Equation of AB: $y = -\frac{1}{2}x + 5$ $2x - 10 = -\frac{1}{2}x + 5$ $x = 6, y = 2$ B(6, 2) (ii) Let D be (x, y). Midpoint of AC = Midpoint of OD $\left(\frac{x}{2}, \frac{y}{2}\right) = \left(\frac{2+5}{2}, \frac{4+0}{2}\right)$ $x = 7, y = 4$ D(7, 4)	M1 A1 M1 A1 M1 A1 B1	Solving simultaneous equations
9	(i) $2 = e^{2k}$ $\ln 2 = 2k$ $k = \frac{1}{2}\ln 2$	B1	

	(ii) $450000=50000e^{\frac{1}{2}ln\,2t}$ $\ln 9 = (\frac{1}{2}\ln 2)t$ $t = 6.339$ ∴ Year 2017 (iii) $P=50000e^{\frac{5}{2}ln2}$ $= 280\ 000$ (iv) 	M1 A1 B1 M1 A1 B2	B1 – Increasing Function Shape B1 – Curve starts at (0, 50 000).
10	(a) Let $f(x) = 6x^3 + ax^2 + bx + 2$ Since $(x + 2)$ is a factor of $f(x)$, $6(-2)^3 + a(-2)^2 + b\,(-2) + 2 = 0$ $2a - b = 23$ ---- (1) Since $f(x)$ leaves a remainder of 36 when divided by $(x - 1)$; $6(1)^3 + a(1)^2 + b\,(1) + 2 = 36$ $a + b = 28$ ---- (2) Solve simultaneous equations; $a = 17, b = 11$	M1 M1 A2	
10	(b) Show method of long division or by inspection $6x^3 + 17x^2 + 11x + 2 = 0$ $(x + 2)(6x^2 + 5x + 1) = 0$ $(x + 2)(3x + 1)(2x + 1) = 0$ $x = -2, -\frac{1}{3}, -\frac{1}{2}$	M1 B2	
10	(c) $6x^4 + 17x^2 + 13 = 2(1 - \frac{1}{x^2})$ $6x^6 + 17x^4 + 11x^2 + 2 = 0$ $x^2 = -2, -\frac{1}{3}, -\frac{1}{2}$ ➔ No solution for x as there are no real values of square root of negative values.	M1 A1, B1	

<table>
<tr><td>11</td><td>(a)(i) $\left(1+\frac{2}{3}x\right)^{10}$</td><td></td><td></td></tr>
<tr><td></td><td>$= 1 + {}^{10}C_1\left(\frac{2}{3}x\right)^1 + {}^{10}C_2\left(\frac{2}{3}x\right)^2 + \cdots$</td><td>M1</td><td></td></tr>
<tr><td></td><td>$= 1 + \frac{20}{3}x + 20x^2 + \cdots$</td><td>A2</td><td>A1 – For any two correct terms</td></tr>
<tr><td></td><td>(a)(ii) $(1 - 2x + 3x^2)\left(1+\frac{2}{3}x\right)^{10}$</td><td></td><td></td></tr>
<tr><td></td><td>$= (1 - 2x + 3x^2)(1 + \frac{20}{3}x + 20x^2 + \cdots)$</td><td></td><td></td></tr>
<tr><td></td><td>$=\ldots + 1(20x^2) + (-2x)\left(\frac{20}{3}x\right) + 3x^2 + \cdots$</td><td>M1</td><td></td></tr>
<tr><td></td><td>$=\ldots +20x^2 - \frac{40}{3}x + 3x^2 + \cdots$</td><td>M1</td><td></td></tr>
<tr><td></td><td>Coefficient of $x^2 = \frac{29}{3} = 9\frac{2}{3}$</td><td>A1</td><td></td></tr>
<tr><td></td><td>(b) General term of $\left(\frac{x}{2}-\frac{4}{3x^2}\right)^9 = {}^9C_r\left(\frac{x}{2}\right)^{9-r}\left(-\frac{4}{3x^2}\right)^r$</td><td>M1</td><td></td></tr>
<tr><td></td><td>$\frac{x^{9-r}}{x^{2r}} = x^0$</td><td></td><td></td></tr>
<tr><td></td><td>$9 - 3r = 0$</td><td></td><td></td></tr>
<tr><td></td><td>$r = 3$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Term independent of $x = {}^9C_3\left(\frac{x}{2}\right)^6\left(-\frac{4}{3x^2}\right)^3$</td><td>M1</td><td></td></tr>
<tr><td></td><td>$= {}^9C_3\left(\frac{x}{2}\right)^6\left(-\frac{4}{3x^2}\right)^3$</td><td></td><td></td></tr>
<tr><td></td><td>$= 84\left(\frac{1}{64}\right)\left(-\frac{64}{27}\right)$</td><td></td><td></td></tr>
<tr><td></td><td>$= -3\frac{1}{9}$</td><td>A1</td><td></td></tr>
<tr><td>12</td><td>(i)
<table>
<tr><td>p</td><td>50</td><td>100</td><td>150</td><td>200</td><td>250</td></tr>
<tr><td>q</td><td>3700</td><td>11 000</td><td>21 600</td><td>36 400</td><td>54 500</td></tr>
<tr><td>$\frac{q}{p}$</td><td>74</td><td>110</td><td>144</td><td>182</td><td>218</td></tr>
</table></td><td>B1</td><td></td></tr>
<tr><td></td><td>Plotting of points – B1
Straight line drawn – B1</td><td>B2</td><td></td></tr>
<tr><td></td><td>(ii)
$q = p(Ap + B)$</td><td></td><td></td></tr>
<tr><td></td><td>$\frac{q}{p} = Ap + B$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Gradient = A = 0.72</td><td>M1A1</td><td></td></tr>
</table>

	(Accept 0.65 to 0.77) Intercept with vertical axis = $B = 38$ (accept 36 to 40)	B1	



12	(iii) $\frac{q}{p} = p$ Draw the line of $\frac{q}{p} = p$, passes through (0, 0) and (50, 50) Intersect at $p = 136$, where the length of both sides are equal ie p and $Ap + B$ are equal. Thus, it is a square of side p m.	B1 B1 B1	Derive the equation. Draw the graph on the grid.
13	(a)(i) $2^x(5^{2x}) = 7(3^x)$ $50^x = 7(3^x)$ $\left(\frac{50}{3}\right)^x = 7$ $x = \frac{\ln 7}{ln\left(\frac{50}{3}\right)}$ $x = 0.692$ (3 *sf*)	M1 M1 A1	

	(a)(ii) $\log_4(6-5x)-4\log_{16}x=1$		
	$\log_4(6-5x)-\frac{4\log_4 x}{\log_4 16}=\log_4 4$	M1	Use of change of base law.
	$\log_4(6-5x)-2\log_4 x=\log_4 4$		
	$\log_4\frac{(6-5x)}{x^2}=\log_4 4$	M1	
	$\frac{(6-5x)}{x^2}=4^1$	M1	
	$4x^2+5x-6=0$ $(4x-3)(x+2)=0$ $x=0.75$ or $x=-2$ (NA)	A1	
	(b)(i) $\lg 2z-\lg(z+y)=\lg y$		
	$\lg\left(\frac{2z}{z+y}\right)=lgy$	M1	
	$\frac{2z}{z+y}=y$	M1	
	$2z=zy+y^2$ $2z-zy=y^2$ $z(2-y)=y^2$ $z=\frac{y^2}{2-y}$	A1	
	(ii) For $\lg 2z$ to be defined, $z>0$.	B1	
	Since $z>0$ and $y^2\geq 0$, therefore $2-y>0$ $y<2$ For $\lg y$ to be defined, $y>0$, thus $0<y<2$.	B1	