Class	Register No	Name

Bukit Merah Secondary School
Mid-Year Examination 2015
Secondary 3 Express

E

MATHEMATICS

4048 / 01

Paper 1

13 May 2015

Candidates answer on the Question Paper.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used when appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Calculator Model:

For Examiner's Use

This document consists of **16** printed pages.

27

[TURN OVER

Mathematical formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin c$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

For Examiner's use

1 **(a)** Calculate $\dfrac{58.4^2 \times 3\pi}{\sqrt{23.9 \times 0.123}}$, giving your answer in 4 significant figures.

(b) Arrange the following numbers in ascending order,

$\sqrt{0.96}$, -0.923, $0.\dot{9}\dot{7}$, $-\dfrac{12}{13}$, $-\dfrac{1}{0.96}$.

Answer *(a)* .... **[1]**

(b) .... **[2]**

2 **(a)** The atomic radius of hydrogen is about 52.5 picometres.
Express 52.5 picometres in metres, giving your answer in standard form.

(b) The length of a micro-organism X is 4.2×10^{-5} m and the length of a micro-organism Y is 4.4×10^{-6} m. Giving your answer in standard form, find the **mean** length of the two micro-organisms X and Y.

Answer *(a)* .... m **[1]**

(b) .... m **[2]**

>8

For Examiner's use

For Examiner use

3 A tank is $\frac{1}{5}$ full. After 55 litres of petrol is pumped into it, it is $\frac{3}{4}$ full. Find the total capacity of the tank.

Answer .... [2]

4 Factorise the following expressions completely.

(a) $x^2 - 16a^2$.

(b) $3u^2 - vd + 3ud - uv$.

Answer *(a)* .... [1]

(b) .... [2]

5 Given that x and y are integers, where $-7 \le x < -1$ and $-4 \le y < 7$, find

(a) the greatest possible value of $-2y - x$,

(b) the least possible value of $-\frac{x^2}{y}$.

Answer *(a)* .... [1]

(b) .... [1]

For Examiner's use

6 **(a)** Simplify the expressions and leave your answers in positive index notation.

(i) $\dfrac{\sqrt[3]{p} \times p^2}{p^{-4}}$,

(ii) $(4a^3b^2)^3 \div (4a^3c^{-5})^{-1}$.

(b) Given that $3^{2-m} \times 81^m = 1$, find the value of m.

Answer *(a)* *(i)* .... **[1]**

(ii) .... **[2]**

(b) .... **[2]**

7 The price of a Certificate of Entitlement (COE) in Jan 2013 was 75% more than in Jan 2012. In 2013, the price of the COE was $92 000.
Find the price of the COE in Jan 2012, giving your answer to the nearest hundred dollars.

Answer $.......................... **[2]**

29

For Examiner's use

For Examiner use

8 **(a)** Study the number sequence below and find the values of p and q.

$$\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, p, q.$$

(b) Write down an expression, in terms of n, for the nth term of the sequence

12, 23, 34, 45, 56, …

Answer *(a)* $p =$ .... **[1]**

$q =$ .... **[1]**

(b) .... **[1]**

9 Solve the following equations.

(a) $(2x+5)^2 = 64$,

(b) $(x-3)(x-2) = 30$.

Answer *(a)* $x =$ .... or............ **[2]**

(b) $x =$ .... or............ **[2]**

[TURN OVER

For Examiner's use

10 **(a)** Express $x^2 - 3x - 5$ in the form $(x-a)^2 + b$.

(b) **Hence** solve the equation $x^2 - 3x - 5 = 0$, giving your answers correct to 2 decimal places.

Answer *(a)* .... **[2]**

(b) $x =$ .... or............. **[2]**

11 A concert ticket sale consists of 80 VIP tickets and 4520 general tickets.
A VIP ticket costs $100 more than a general ticket.
Find the **minimum** price of a VIP ticket, correct to 2 decimal places, such that the total selling price would be at least $1 million.

Answer $.................................. **[3]**

For Examiner's use

12 Solve the simultaneous equations

$$5x + 4y + 10 = 0,$$
$$y + 3x = 1.$$

Answer $x =$ ....

$y =$ .... **[3]**

13 A map is drawn to a scale of 1: 200 000.

(a) Find the actual distance, in kilometres, represented by 8 cm on the map.

(b) The actual area of a park is 2.5 km^2. Find, in square centimetres, the area on the map representing the park.

Answer *(a)* .... km **[1]**

(b) .... cm^2 **[2]**

For Examiner's use

14 **(a)** Solve the equation $\dfrac{2}{x-1} = \dfrac{2(x+1)}{3x+3}$.

(b) Express $\dfrac{3}{(y-1)^2} - \dfrac{2}{y-1}$ as a single fraction in its simplest form.

Answer *(a)* $x =$ .... [3]

(b) .... [2]

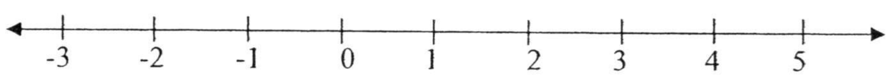
For Examiner's use

15 **(i)** Solve the inequality $-5 \leq 4 - 3x < 7$.

(ii) Show your solution on the number line in the answer space below.

(iii) List all the prime numbers which satisfy the inequality.

Answer (ii):



[1]

Answer *(i)* .... **[2]**

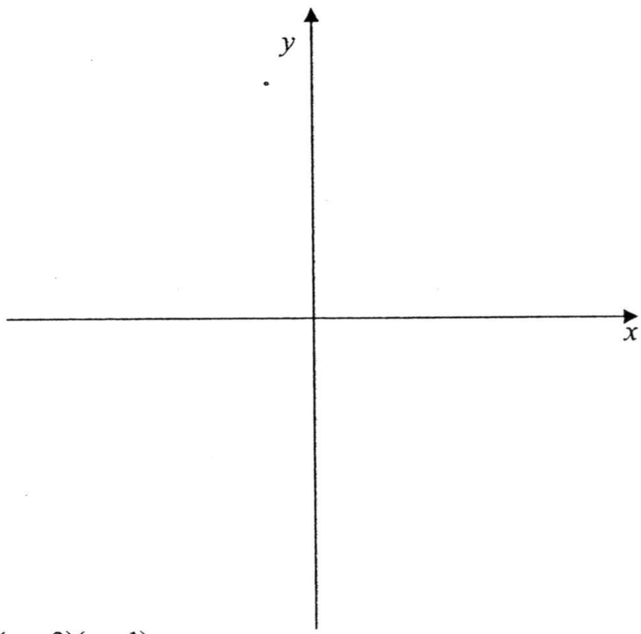
(iii) .... **[1]**

For Examiner use

For Examiner's use

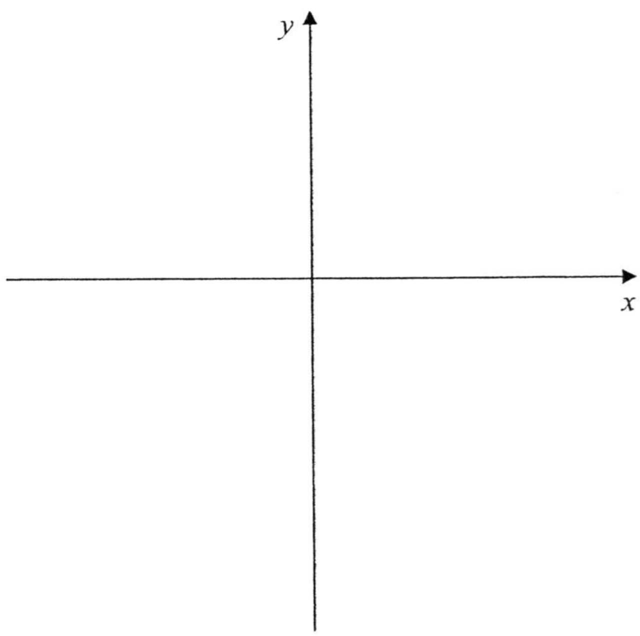
16 **(a)** **(i)** Sketch the graph of $y = 2 - (x+3)^2$.

(ii) Write down the equation of the line of symmetry of $y = 2 - (x+3)^2$.



[2]

(b) Sketch the graph of $y = (x+3)(x-1)$.



[2]

Answer *(a)* *(ii)* .... **[1]**

For Examiner's use

17 (a) Given that y is directly proportional to $(x + 3)^2$ and $y = 15$ when $x = 2$. Express y in terms of x.

(b) A shipyard can repair a ship with 55 workers working 8 hours a day for 30 days. A special request was put in so that the same repair can be completed in just 22 days. If the workers work for 10 hours each day, find the number of workers it takes to fulfil this special request?

Answer *(a)* .... [2]

(b) .... [3]

For Examiner's use

For Examiner use

18 **(a)** Solve the equation $\frac{3x}{4} - \frac{3(2x-1)}{5} = 6$.

(b) One solution of the equation $3x^2 + kx - 8 = 0$, where k is a constant, is $x = \frac{2}{3}$.

Find

(i) the value of k,

(ii) the second solution of the equation.

Answer *(a)* $x =$ .... **[3]**

(b) *(i)* $k =$ .... **[1]**

(ii) $x =$.................................... **[1]**

For Examiner's use

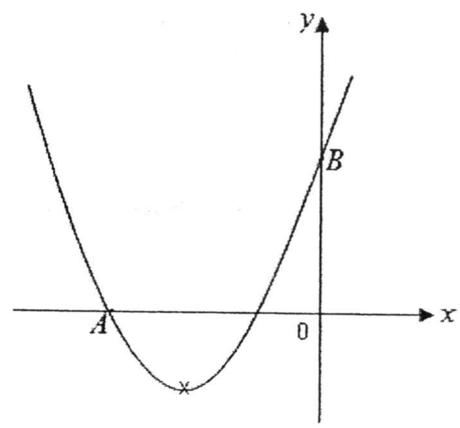
19 The figure below shows a quadratic graph $y = x^2 + 7x + 10$. The graph cuts the x-axis at point A and the y-axis at point B.

(a) Find the coordinates of A and B.

(b) State the minimum point of the graph.

(c) Find the length of AB.

(d) Calculate the perpendicular distance from the origin to the line AB.



Answer (a) A (...... , ...)

B (...... , ...) **[2]**

(b) (...... , ...) **[1]**

(c) .... units **[2]**

(d) .... units **[2]**

For Examiner's use

20 The points A, B and C are shown in the diagram.

A is on the y–axis, B is the point $(n, 2n)$ and C is vertically below B.

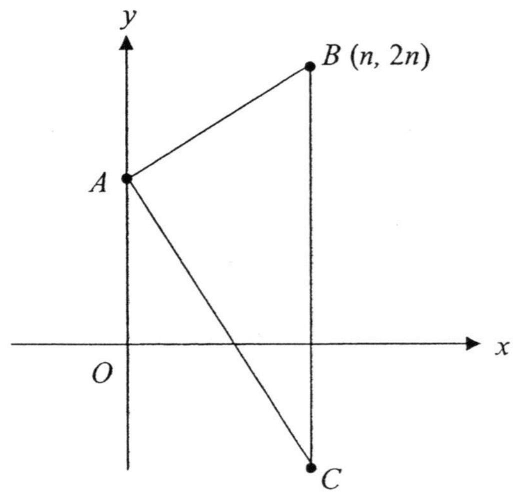
(a) The equation of line AB is $y = x + 4$.

(i) the coordinates of A,

(ii) the coordinates of B.

(b) The gradient of AC is -2. Find the coordinates of C.

(c) Calculate the area of the triangle ABC.



Answer *(a)* *(i)* A (........ , ...)

(ii) B (........ , ...) **[2]**

(b) C (........ , ...) **[1]**

(c) .... units^2 **[1]**

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For Examiner's use

21 Mary wants to invest \$8 000 in a savings account for 2 years.

(i) In one account, the rate of compound interest is fixed at r % per annum.
At the end of the 2 years, there will be \$8 560 in this account.
Calculate the value of r to 1 decimal place.

(ii) In another savings account, the interest of 3% per annum is compounded every half-yearly. Calculate the amount Mary will have in this account at the year of 2 years, giving your answer to 2 decimal places.

Answer *(i)* r =............................. [3]

(ii) \$............................... [2]

End-of-Paper 1

Answer **all** the questions

1 **(a)** Calculate $\frac{58.4^2 \times 3\pi}{\sqrt{23.9 \times 0.123}}$, giving your answer in 4 significant figures.

(b) Arrange the following numbers in ascending order,

$\sqrt{0.96}$, -0.923, $0.\dot{9}\dot{7}$, $-\frac{12}{13}$, $-\frac{1}{0.96}$.

Solution:

(a) ≈ 18747.58
$= 18750 \quad [B1]$

(b) -1.041, -0.9231, -0.923, 0.979795, 0.979797

$-\frac{1}{0.96}$, $-\frac{12}{13}$, -0.923, $\sqrt{0.96}$, $0.\dot{9}\dot{7}$ [B2: deduct 1 mark until 0 marks]

2 (a) The atomic radius of hydrogen is about 52.5 picometres.
Express 52.5 picometres in metres, giving your answer in standard form.

(b) The length of a micro-organism *X* is 4.2×10^{-5} m and the length of a micro-organism *Y* is 4.4×10^{-6} m. Giving your answer in standard form, find the **mean** length of the two micro-organisms *X* and *Y*.

Solution:
a) 52.5×10^{-12} m = 5.25×10^{-11} m [B1]

b) 2.32×10^{-5} m [M1A1 or B2; B1 for 0.0000232]

3 A tank is $\frac{1}{5}$ full. After 55 litres of petrol is pumped into it, it is $\frac{3}{4}$ full. Find the total capacity of the tank.

Solution:

$\frac{3}{4} - \frac{1}{5} = \frac{11}{20}$ of the tank
11 parts = 55 litres [M1]
20 parts = $\frac{20}{11} \times 55 = 100 litres$ [A1]

4 Factorise the following expressions completely.

(a) $x^2 - 16a^2$,

(b) $3u^2 - vd + 3ud - uv$

Solution:

a) $x^2 - 16a^2 = (x - 4a)(x + 4a)$ [B1]

b) $3u^2 - vd + 3ud - uv$

$= 3u^2 + 3ud - vd - uv$

$= 3u(u + d) - v(u + d)$ [M1]

$= (u + d)(3u - v)$ [A1]

5 Given that x and y are integers, where $-7 \le x < -1$ and $-4 \le y < 7$, find

(a) the greatest possible value of $-2y - x$,

(b) the least possible value of $-\frac{x^2}{y}$.

Solution:

a) greatest value of $(-2y - x) = -2 \times (-4) - (-7) = 15$ [B1]

b) least value of $-\frac{x^2}{y} = -\frac{(-7)^2}{1} = -49$ [B1]

6 **(a)** Simplify the expressions and leave your answers in positive index notation.

(i) $\frac{\sqrt[3]{p} \times p^2}{p^{-4}}$,

(ii) $(4a^3b^2)^3 \div (4a^3c^{-5})^{-1}$.

(b) Given that $3^{2-m} \times 81^m = 1$, find the value of m.

Solution:

ai)	ii) $(4a^3b^2)^3 \div (4a^3c^{-5})^{-1}$
$\frac{\sqrt[3]{p} \times p^2}{p^{-4}}$	$= 4^3a^9b^6 \div 4^{-1}a^{-3}c^5$ [M1]
$= p^{\frac{1}{3}+2-(-4)}$	$= \frac{256a^{12}b^6}{c^5}$ [A1]
$= p^{\frac{19}{3}}$ [B1]	P/S: Accept 4^4 as final answer.

b) $3^{2-m} \times 81^{m} = 1$

$3^{2-m} \times 3^{4m} = 3^0$ [M1] P/S: Accept RHS = 1.

$3^{2+3m} = 3^0$

$2 + 3m = 0$

$m = -\frac{2}{3}$ [A1]

7 The price of a Certificate of Entitlement (COE) in Jan 2013 was 75% more than in Jan 2012. In 2013, the price of the COE was \$92 000.
Find the price of the COE in Jan 2012, giving your answer to the nearest hundred dollars.

Solution:

In 2013, \$92 000 = 175% of the price in 2012.

In 2012, the price = $\frac{100}{175} \times \$92000 = \$52571 \approx \$52\ 600$. [M1A1]

8 **(a)** Study the number sequence below and find the values of p and q.

$$\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, p, q.$$

(b) Write down an expression, in terms of n, for the nth term of the sequence

12, 23, 34, 45, 56, ...

Solution:

a) $p = \frac{7}{9}$, $q = \frac{4}{5}$ [B2]

b) nth term $= 11n + 1$ [B1]

9 Solve the following equations.

(a) $(2x+5)^2 = 64$

(b) $(x-3)(x-2) = 30$

Solution:

a) $(2x+5)^2 = 64$

$2x + 5 = \pm 8$ [M1]

$x = \frac{8-5}{2}$ or $x = \frac{-8-5}{2}$

$x = \frac{3}{2}$ or $x = -\frac{13}{2}$ [A1]

b) $(x-3)(x-2) = 30$

$x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0$ [M1]

$x = 8$ *or* $x = -3$ [A1]

10 **(a)** Express $x^2 - 3x - 5$ in the form $(x-a)^2 + b$

(b) **Hence** solve the equation $x^2 - 3x - 5 = 0$, giving your answers correct to 2 decimal places.

Solution:

a) $x^2 - 3x - 5$

$= x^2 - 3x + (-\frac{3}{2})^2 - (-\frac{3}{2})^2 - 5$ [B2]

$= (x - \frac{3}{2})^2 - \frac{29}{4}$ P/S: 1 mark for **a** and 1 mark for **b**

b) $(x - \frac{3}{2})^2 - \frac{29}{4} = 0$

$(x - \frac{3}{2})^2 = \frac{29}{4}$

$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}}$ [M1]

$x = \sqrt{\frac{29}{4}} + \frac{3}{2}$, $-\sqrt{\frac{29}{4}} + \frac{3}{2}$

$= 4.19, \ -1.19$ [A1]

11 A concert ticket sale consists of 80 VIP tickets and 4520 general tickets.
A VIP ticket costs \$100 more than a general ticket.
Find the **minimum** price of a VIP ticket, correct to 2 decimal places, such that the total selling price would be at least \$1 million.

Solution:

Let x be the price of a VIP ticket.

$80x + 4520(x - 100) \geq 1\ 000\ 000$ [M1, to include the inclusive sign or any other method]

$4600x - 452000 \geq 1\ 000\ 000$

$4600x \geq 1\ 452\ 000$

$x \geq 315.65$ [A1 ft]

Min price is \$315.66 [B1]

12	Solve the simultaneous equations $$5x + 4y + 10 = 0,$$ $$y + 3x = 1.$$ Solution: (Any acceptable method) [M1] $x = 2,\ y = -5.$ [A1], [A1]
13	A map is drawn to a scale of 1: 200 000. **(a)** Find the actual distance, in kilometres, represented by 8 cm on the map. **(b)** The actual area of a park is 2.5 km^2. Find, in square centimetres, the area on the map representing the park. Solution: (a) 1: 200 000 1 cm : 200 000 cm 1 cm : 2 km Actual distance = 8×2 $= 16$ km [B1] (b) 1 cm^2 : 4 km^2. [M1] Area on the map $= \frac{2.5}{4}$ $= 0.625$ cm^2 [A1]
14	**(a)** Solve the equation $\frac{2}{x-1} = \frac{2(x+1)}{3x+3}$. **(b)** Express $\frac{3}{(y-1)^2} - \frac{2}{y-1}$ as a single fraction in its simplest form. a) $\frac{2}{x-1} = \frac{2(x+1)}{3x+3}$ $6x + 6 = 2(x^2 - 1)$ [M1] $2x^2 - 6x - 8 = 0$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ [M1] $x = 4$ or -1(NA) [A1] This method is acceptable: a) $\frac{2}{x-1} = \frac{2(x+1)}{3(x+1)}$ [M1] $6 = 2(x-1)$ [M1] $2x = 6 + 2$ $x = 4$ [A1]

b) $\frac{3}{(y-1)^2}-\frac{2}{y-1}$

$=\frac{3-2(y-1)}{(y-1)^2}$ [M1]

$=\frac{5-2y}{(y-1)^2}$ [A1]

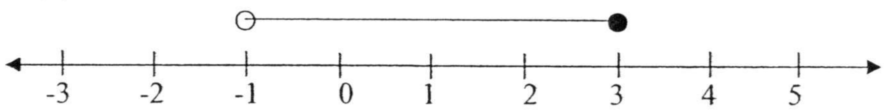
15 **(i)** Solve the inequality $-5 \le 4-3x < 7$.

(ii) Show your solution on the number line in the answer space below.

(iii) List all the prime numbers which satisfy the inequality.

a) $-5 \le 4-3x < 7$		OR	a) $-5 \le 4-3x < 7$		
$-9 \le -3x < 3$	[M1]		$-5 \le 4-3x$ and $4-3x < 7$		
$3 \ge x > -1$	[A1]		$3x \le 9$ and $3x > -3$		
			$x \le 3$ and $x > -1$		[M1]
				$3 \ge x > -1$	[A1]

Answer (ii):



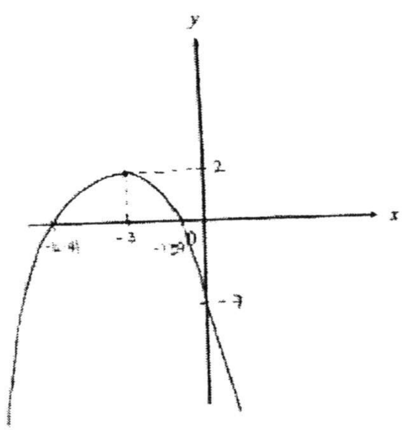
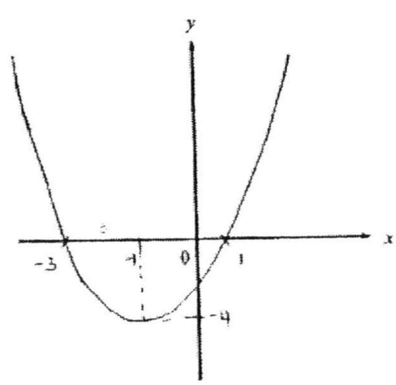
[1]

i) The prime numbers are 2, 3 [B1 ft]

16 **(a)** (i) Sketch the graph of $y=2-(x+3)^2$.

(ii) Write down the equation of the line of symmetry of $y=2-(x+3)^2$.

(b) Sketch the graph of $y=(x+3)(x-1)$.

(a)	(b)
 (-3, 2) [B1] and (0, -7) [B1]	 (-3, 0) & (1, 0) [B1]. (0,-3) [B1]
(a)(ii) x = -3 [B1]	

17 (a) Given that y is directly proportional to $(x + 3)^2$ and $y = 15$ when $x = 2$. Express y in terms of x.

(b) A shipyard can repair a ship with 55 workers working 8 hours a day for 30 days. A special request was put in so that the same repair can be completed in just 22 days. If the workers work for 10 hours each day, find the number of workers it takes to fulfil this special request?

Solution:

(a) $y = k(x + 3)^2$, k is a constant

when $x = 2$, $y = 15$

$15 = k(25)$

$k = \frac{3}{5}$ [M1]

$\therefore y = \frac{3}{5}(x + 3)^2$ [A1]

(b)

Effort taken to overhaul the ship $= 55 \times 8 \times 30 = 13200$ man-hours [M1]

Number of workers required for special request

$= 13200 \div (22 \times 10) = 60$ [M1, A1]

18 **(a)** Solve the equation $\frac{3x}{4} - \frac{3(2x-1)}{5} = 6$.

(b) One solution of the equation $3x^2 + kx - 8 = 0$, where k is a constant, is $x = \frac{2}{3}$.

Find

(i) the value of k,

(ii) the second solution of the equation.

Solution:

(a)

$3x(5) - 12(2x-1) = 6(20)$ [M1]

$15x - 24x = 120 - 12$ [M1]

$-9x = 108$

$x = -12$ [A1]

OR

$\frac{5(3x) - 12(2x-1)}{20} = 6$ [M1]

$15x - 24x = 120 - 12$ [M1]

$-9x = 108$

$x = -12$ [A1]

(b)(i) $3\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) - 8 = 0$ $\frac{2}{3}k = \frac{20}{3}$ $k = 10$ [B1]	(ii) $3x^2 + 10x - 8 = 0$ $(3x-2)(x+4) = 0$ $x = -4$ or $x = \frac{2}{3}$ The second solution is – 4. [B1]

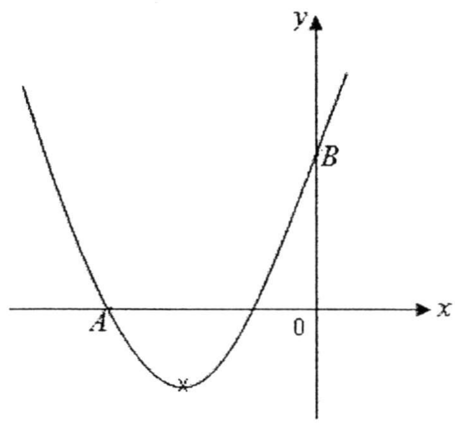
19 The figure below shows a quadratic graph $y = x^2 + 7x + 10$. A line passes through the x-axis at point A and the y-axis at point B.

(a) Find the coordinates of A and B.

(b) State the minimum point of the graph.

(c) Find the length of AB.

(d) Calculate the perpendicular distance from the origin (0, 0) to the line AB.



a) When $y = 0$, $x = -2$ or $x = -5$ $A(-5,0)$ [B1] When $x = 0$, $B(0, 10)$ [B1]	c) Length of AB $= \sqrt{(0-(-5))^2 + (10-0)^2}$ [M1] $= 11.180$ $= 11.2$ units (correct to 3 sig. fig.) [A1]
b) Minimum point $= \left(-\frac{7}{2}, -\frac{9}{4}\right)$ [B1]	d) $= \dfrac{2 \times \frac{1}{2} \times 10 \times 5}{\sqrt{125}}$ [M1] $= 4.47$ units [A1]

20 The points A, B and C are shown in the diagram.

A is on the y–axis, B is the point $(n, 2n)$ and C is vertically below B.

(a) The equation of line AB is $y = x + 4$.

(i) the coordinates of A,

(ii) the coordinates of B.

(b) The gradient of AC is -2.

Find the coordinates of C.

(c) Calculate the area of the triangle ABC.

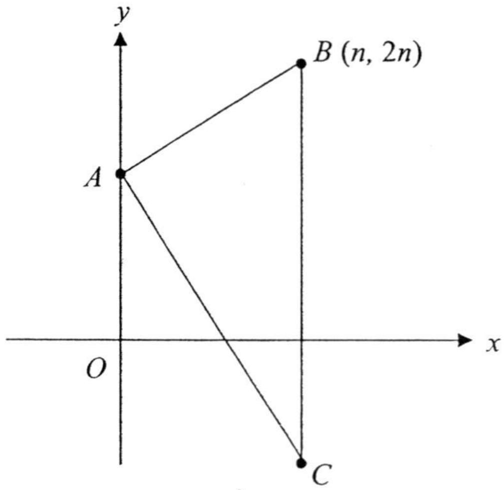
Solution:

(a)(i) (0, 4) [B1]

(a)(ii) (4, 8) [B1]

(b) (4, −4) [B1]

(c) 24 units^2[B1]



21 Mary wants to invest \$8 000 in a savings account for 2 years.

(i) In one account, the rate of compound interest is fixed at r% per annum.

At the end of the 2 years, there will be \$8 560 in this account.

Calculate the value of r to 1 decimal place.

(ii) In another savings account, the interest of 3% per annum is compounded every half-yearly. Calculate the amount Mary will have in this account at the year of 2 years, giving your answer to 2 decimal places.

Solution:

a) $A = P(1+\frac{r}{100})^2$

$8\,560 = 8\,000(1 + \frac{r}{100})^2$ [M1]

$(1+\frac{r}{100})^2 = \frac{8560}{8000}$

$(1+\frac{r}{100}) = \sqrt{\frac{8560}{8000}}$ [M1]

$\frac{r}{100} = \sqrt{\frac{8560}{8000}} - 1 = 0.034$

$r = 3.4\%$ [A1]

b) $A = P(1+\frac{r}{100})^4$

$A = 8\,000(1 + \frac{1.5}{100})^4$ [M1]

$A = \$8\,490.91$ [A1]

Class	Register No	Name

Bukit Merah Secondary School
Mid-Year Examination 2015
Secondary 3 Express

E

MATHEMATICS

4048 / 02

Paper 2

15 May 2015

Additional Materials: Writing papers (4 sheets)
Graph paper (1 sheet)

1 hour 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used when appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

This document consists of **6** printed pages.

40

[TURN OVER

Mathematical formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin c$$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

[TURN OVER

Answer **all** questions.

1 **(a)** Simplify $\frac{5a}{6b^3} \div \frac{3a^5}{5b^4}$. **[2]**

(b) Simplify $\frac{4x-16}{x^2-5x+4}$. **[2]**

(c) Solve the inequality $\frac{4x-3}{5} - \frac{2x+1}{3} \geq 2$. **[3]**

(d) It is given that $p = \frac{4q^3+r}{7}$.

(i) Make q the subject of the equation. **[2]**

(ii) Find the value of q when $p = -15$ and $r = 3$. **[1]**

---

2 **(a)** Express 10 800 as the product of its prime factors. **[1]**

(b) Given that $\frac{10800}{k} = p^3$, where k and p are integers and p is as large as possible, find the values of k and p. **[2]**

(c) The lowest common multiple of two numbers is 10 800.

The highest common factor of these two numbers is 900.

Both numbers are greater than 900.

Find the two numbers. **[2]**

4.1

3. Alice cycled from her home in Redhill to Clementi, a total distance of 10 km. For the first 6 km, her average speed is y km/h but for the last 4 km, her average speed is 2 km/h less.

(a) Express, in terms of y, the number of hours she took to cycle for

(i) the first 6 km, [1]

(ii) the last 4 km. [1]

(b) If the total journey took 30 minutes, form an equation in y and show that it reduces to $y^2 - 22y + 24 = 0$. [2]

(c) **(i)** Solve the equation $y^2 - 22y + 24 = 0$, giving the answers correct to 2 decimal places. [3]

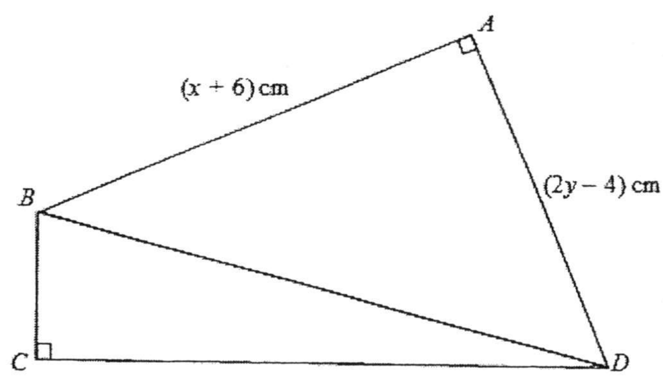
(ii) Hence, find the time taken to cycle the last 4 km, giving your answer in minutes and seconds. [1]

---

4 **(a)** **(i)** Solve the inequality $1 - x < 3x + 5 \leq \frac{4x+15}{2}$. [3]

(ii) Hence, write down the integral values of x that satisfy this inequality. [1]

(b)



$ABCD$ is a quadrilateral with $\angle BAD = \angle BCD = 90^\circ$, $AB = (x+6)$ cm and $AD = (2y-4)$ cm.

(i) Express BD^2 in terms of x and y. [2]

(ii) Given that $BC = (x - 2)$ cm and $CD = (2y)$ cm. Form a second expression for BD^2. Using the results in part (i), show that $y - x = 3$. [3]

(iii) If 3 times of BC is longer than AD by 1 cm, form a second equation in terms of x and y. [2]

(iv) Hence, find the value of x and the value of y. [2]

5. Look at the pattern.

$$T_1 = 2^0 + 1 = 2$$
$$T_2 = 2^1 + 3 = 5$$
$$T_3 = 2^2 + 5 = 9$$
$$T_4 = 2^3 + 7 = 15$$

(a) Write down an expression for T_6. **[1]**

(b) Find an expression, in terms of n, for the term, T_n, of the sequence. **[2]**

(c) Evaluate T_{20}. **[1]**

(d) Explain whether the value of T_n is even or odd for $n > 1$. **[1]**

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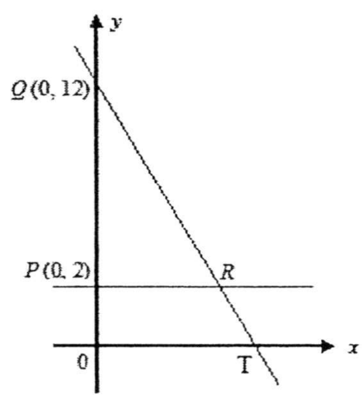
6. In the diagram [*not drawn to scale*], QR and PR are straight lines.



(a) Given that the line QR has the same gradient as the line $y = -2x + 5$,

write down the equation of the line QR. **[2]**

(b) Given that the line PR is parallel to the x- axis and the point $P(0, 2)$.

The line QR crosses the x-axis at the point T. Calculate

(i) the coordinates of the point R, **[2]**

(ii) the coordinates of the point T. **[1]**

(iii) Hence, find the ratio of $QR : RT$. **[2]**

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Answer the whole of this question on a sheet of graph paper.

7. The table below gives some values of x and the corresponding values of y for $y = 5 + 3x - x^2$.

x	-2	-1	0	1	2	3	4	5
y	-5	1	5	7	7	5	p	-5

(a) Calculate the value of p. **[1]**

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 5$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-6 \leq y \leq 8$.
On your axes, plot the points given in the table and join them with a smooth curve. **[3]**

(c) The point $(k, 4)$ lies on the curve. Use your graph to find the value(s) of k. **[2]**

(d) Use your graph to solve the equation $8 + 3x - x^2 = 0$, giving your answers to 1 decimal place. **[2]**

(e) **(i)** On the same axes, draw the line $y = -2x + 6$. **[1]**

(ii) Write down the x – coordinates of the points at which the line intersects the curve. **[1]**

(iii) Find the equation in the form $x^2 + bx + c = 0$ whose solutions are the x – coordinates found in (d)(ii). **[2]**

End-of-Paper 2

Answer **all** questions.

1 **(a)** Simplify $\frac{5a}{6b^3} \div \frac{3a^5}{5b^4}$. **[2]**

(b) Simplify $\frac{4x-16}{x^2-5x+4}$. **[2]**

(c) Solve the inequality $\frac{4x-3}{5} - \frac{2x+1}{3} \geq 2$. **[3]**

(d) It is given that $p = \frac{4q^3 + r}{7}$.

(i) Make q the subject of the equation.

(ii) Find the value of q when $p = -15$ and $r = 3$. **[3]**

Solution:

(a) $\frac{5a}{6b^3} \div \frac{3a^5}{5b^4} = \frac{5a}{6b^3} \times \frac{5b^4}{3a^5}$ [M1]

$= \frac{25b}{18a^4}$ [A1]

(b) $\frac{4x-16}{x^2-5x+4} = \frac{4(x-4)}{(x-4)(x-1)}$ [M1]

$= \frac{4}{(x-1)}$ [A1]

(c) $\frac{4x-3}{5} - \frac{2x+1}{3} \geq 2$

$\frac{3(4x-3)-5(2x+1)}{15} \geq 2$ [M1]

$12x-9-10x-5 \geq 30$ [M1]

$2x \geq 44$

$x \geq 22$ [A1]

OR

$3(4x-3)-5(2x+1) \geq 30$ [M1]

$12x-9-10x-5 \geq 30$ [M1]

$2x \geq 44$

$x \geq 22$ [A1]

(d)(i) $7p = 4q^3 + r$

$7p - r = 4q^3$

$\frac{7p-r}{4} = q^3$ [M1]

$q = \sqrt[3]{\frac{7p-r}{4}}$ [A1]

(ii) When $p = -15$ and $r = 3$, $q = \sqrt[3]{\frac{7(-15)-3}{4}} = -3$ [B1]

43

2 (a) Express 10 800 as the product of its prime factors. [1]

(b) Given that $\frac{10800}{k} = p^3$, where k and p are integers and p is as large as possible, find the values of k and p. [2]

(c) The lowest common multiple of two numbers is 10 800.
The highest common factor of these two numbers is 900.
Both numbers are greater than 900.
Find the two numbers. [2]

Solution:

(a) $10\ 800 = 2^4 \text{ x } 3^3 \text{ x } 5^2$. [B1]

(b) $\frac{10800}{k} = p^3 \Rightarrow \frac{2^4 \times 3^3 \times 5^2}{k} = p^3$

$k = 2 \text{ x } 25 = 50$ [B1]

$p = 6$ [B1]

(c) $\text{LCM} = 10\ 800 = 2^4 \text{ x } 3^3 \text{ x } 5^2$

$\text{HCF} = 2^2 \text{ x } 3^2 \text{ x } 5^2 = 900$

The 2 numbers are $900 \text{ x } 3 = 2\ 700$ and $900 \text{ x } 2^2 = 3\ 600$ [B2]

Check:

$2\ 700 = 2^2 \text{ x } 3^3 \text{ x } 5^2$

$3\ 600 = 2^4 \text{ x } 3^2 \text{ x } 5^2$

$\text{HCF} = 2^2 \text{ x } 3^2 \text{ x } 5^2$

$\text{LCM} = 2^4 \text{ x } 3^3 \text{ x } 5^2$

---

3. Alice cycled from her home in Redhill to Clementi, a total distance of 10 km. For the first 6 km, her average speed is y km/h but for the last 4 km, her average speed is 2 km/h less.

(a) Express, in terms of y, the number of hours she took to cycle for

(i) the first 6 km, [1]

(ii) the last 4 km. [1]

(b) If the total journey took 30 minutes, form an equation in y and show that it reduces to $y^2 - 22y + 24 = 0$. [2]

(c) (i) Solve the equation $y^2 - 22y + 24 = 0$, giving the answers correct to 2 decimal places. [3]

(ii) Hence, find the time taken to cycle the last 4 km, giving your answer in minutes and seconds. [1]

Solutions:

(a)(i) $\frac{6}{y}$ *hour* [B1]

(ii) $\frac{4}{y-2}$ *hour* [B1]

(b) $\frac{4}{y-2}+\frac{6}{y}=\frac{1}{2}$

$\frac{4y+6(y-2)}{y(y-2)}=\frac{1}{2}$ [M1]

$8y+12y-24=y^2-2y$ [M1]

$y^2-22y+24=0$

(c)(i) $y=\frac{-(-22)\pm\sqrt{(-22)^2-4(1)(24)}}{2(1)}$ [M1]

$y=\frac{22\pm\sqrt{388}}{2}$ [M1]

$y=20.849$ or $y=1.15$ (NA) [A1]

$=20.85$ km/h

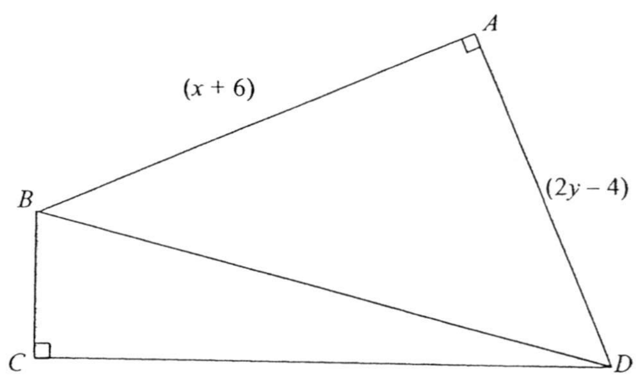
(ii) The time taken to cycle the last 4 km = $\frac{4km}{(20.849-2)km/h}$ = 12.73 min = 12 min and 44 s. [B1]

---

4 **(a)** **(i)** Solve the inequality $1-x<3x+5\leq\frac{4x+15}{2}$. [3]

(ii) Hence, write down the integral values of x that satisfy this inequality. [1]

(b)



$ABCD$ is a quadrilateral with $\angle BAD=\angle BCD=90^o$, $AB=(x+6)$ cm and $AD=(2y-4)$ cm.

(i) Express BD^2 in terms of x and y. [2]

(ii) Given that $BC=(x-2)$ cm and $CD=(2y)$ cm. Find a second expression for BD^2. Using the results in part (i), show that $y-x=3$. [3]

44

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(iii) If 3 times of BC is longer than AD by 1 cm, form a second equation in terms of x and y. **[2]**

(iv) Hence find the value of x and the value of y. **[2]**

Solution:

(a)(i)

$1 - x < 3x + 5$ and $3x + 5 \leq \frac{4x+15}{2}$

$-4x < 4$ $\quad 6x + 10 \leq 4x + 15$

$x > -1$ [M1] $\quad 2x \leq 5$

$x \leq \frac{5}{2}$ [M1]

$\therefore$ The solution is $-1 < x \leq \frac{5}{2}$. [A1]

(ii) The integral values of x are 0, 1 and 2. [B1]

(b)(i) $BD^2 = (x+6)^2 + (2y-4)^2 = x^2 + 4y^2 + 12x - 16y + 36 + 16$ [M1A1]

(ii) $BD^2 = (x-2)^2 + (2y)^2 = x^2 + 4y^2 - 4x + 4$ [M1A1]

$x^2 + 4y^2 + 12x - 16y + 36 + 16 = x^2 + 4y^2 - 4x + 4$

$16x - 16y = -48$

$x - y = -3$

$y - x = 3$ -------- (2) [A1]

(iii) $3BC - AD = 1$

$3(x-2) - (2y-4) = 1$ [M1]

$3x - 6 - 2y + 4 = 1$

$3x - 2y = 3$ [A1] --------- (1)

(iv)

Sub (2) into (1): $3(y-3) - 2y = 3$ [M1]

$y = 12,\ x = 9$ [A1 – for both answers]

---

5. Look at the pattern.

$$T_1 = 2^0 + 1 = 2$$
$$T_2 = 2^1 + 3 = 5$$
$$T_3 = 2^2 + 5 = 9$$
$$T_4 = 2^3 + 7 = 15$$

(a) Write down an expression for T_6. [1]

(b) Find an expression, in terms of n, for the term, T_n, of the sequence. [2]

(c) Evaluate T_{20}. [1]

(d) Explain whether the value of T_n is even or odd for $n > 1$. [1]

Solution:

(a) $T_6 = 2^5 + 11 = 43$ [B1]

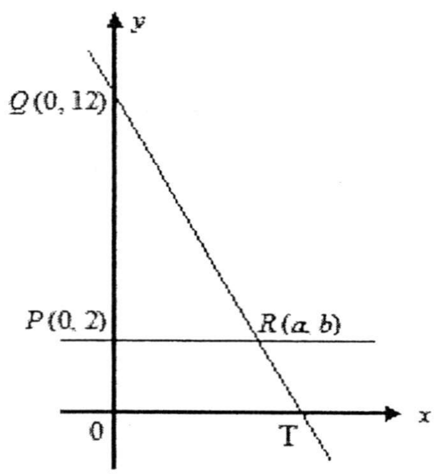
(b) $T_n = 2^{n-1} + [1 + 2(n-1)]$ [B1, B1]
$= 2^{n-1} + [1 + 2n - 2]$
$= 2^{n-1} + 2n - 1$

(c) $T_{20} = 2^{19} + 40 - 1$
$= 524\ 327$ [B1]

(d) 2^{n-1} is even and $2n-1$ is odd and so the sum of an even and odd number odd. [B1]

---

6. In the diagram [*not drawn to scale*], *QR* and *PR* are straight lines.



(a) Given that the line *QR* has the same gradient as the line $y = -2x + 5$, write down the equation of the line *QR*. [2]

(b) Given that the line *PR* is parallel to the *x*- axis and the line QR meets the *x*-axis at the point *T*. Calculate

(i) the coordinates of the point *R*, [2]

(ii) the coordinates of the point *T*. [1]

(iii) Hence, find the ratio of QR: RT. [2]

45

(a) Equation of QR: $y=-2x+12$. [B2: 1 for gradient and 1 for y-intercept]

(b) (i) Sub $(a, 2)$ into equation of QR, [M1]

$2=-2a+12$

$2a=10$

$a=5$

Coordinates of $R=(5, 2)$. [A1]

(ii) Let $y=0$ in $y=-2x+12 \Rightarrow x=6$

Coordinates of $T=(6, 0)$. [B1]

(iii) Length of QR $=\sqrt{(0-5)^2+(12-2)^2}$

$=\sqrt{125}$ units

Length of RT $=\sqrt{(6-5)^2+(0-2)^2}$ [M1 for one or both lengths]

$=\sqrt{5}$ units

QR: RT $=\dfrac{\sqrt{125}}{\sqrt{5}}=\sqrt{\dfrac{125}{5}}=\sqrt{\dfrac{25}{1}}=\dfrac{5}{1}$ [A1: only surds are accepted for this method. The alt method is to use similar triangles]

Answer the whole of this question on a sheet of graph paper.

7. The table below gives some values of x the corresponding values of y for $y=5+3x-x^2$.

x	−2	−1	0	1	2	3	4	5
y	−5	1	5	7	7	5	o	−5

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1unit, draw a horizontal x-axis for $-2 \le x \le 5$.
Using a scale of 1 cm to represent 1unit, draw a vertical y-axis for $-6 \le y \le 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) The point $(k, 4)$ lies on the curve. Use your graph to find the value of k. [1]

(d) Use your graph to solve the equation $8+3x-x^2=0$, giving your answers to 1 decimal place. [2]

(e) **(i)** On the same axes, draw the line $y=-2x+6$. [1]

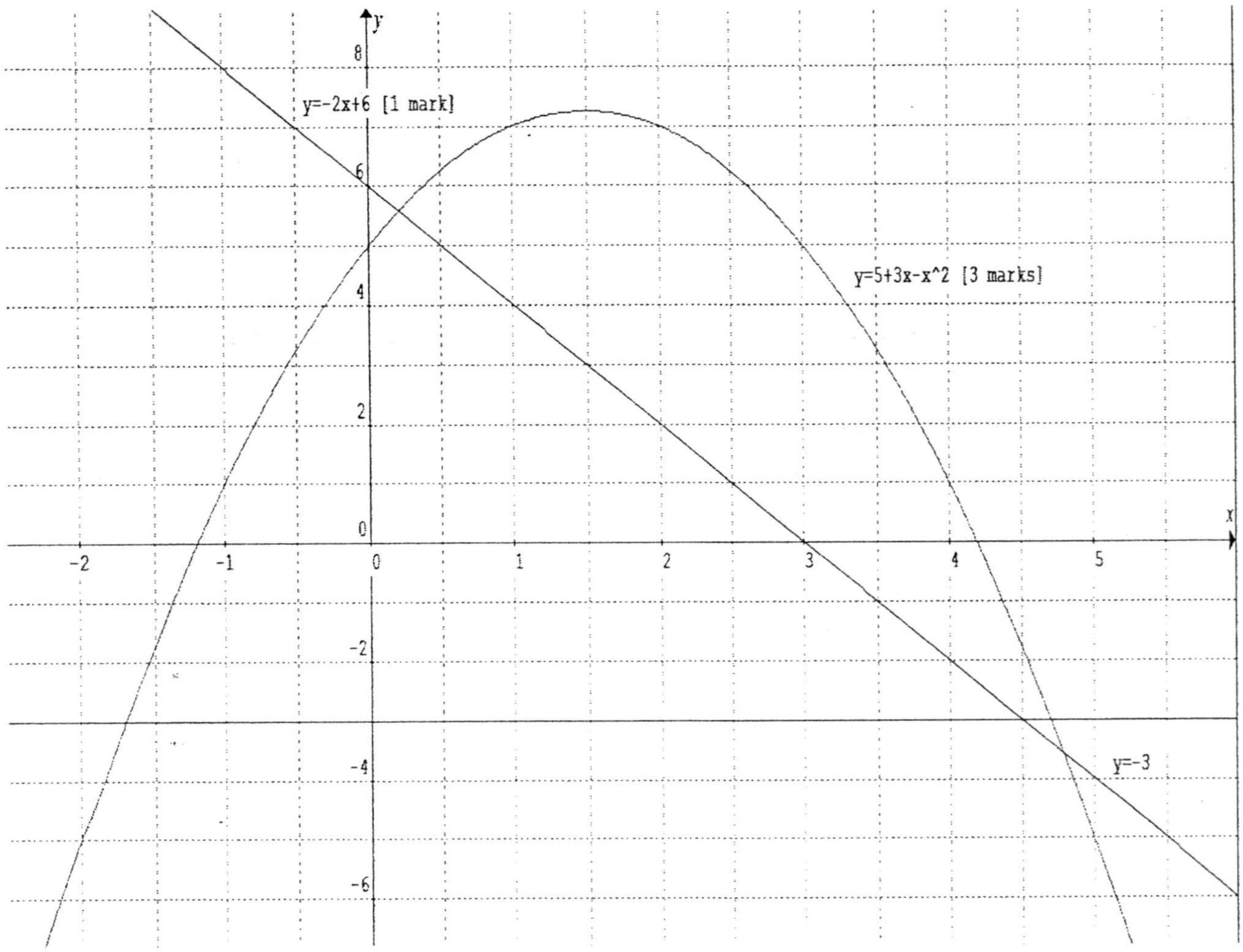
(ii) Write down the x – coordinates of the points at which the line intersects the curve. [1]

(iii) Find the equation in the form $x^2 + bx + c = 0$ whose solutions are the x – coordinates found in (d)(i). [2]

Solution:

a) $p = 1$ [B1]

b) Graph : P2 marks for all the correct points [1 mark for at least 6 correct points]; C1 mark for a smooth curve; deduct 1 mark for wrong scales; deduct 1 mark if axes are not labelled.



c) When $y = 4$, $k = -0.3$ or 3.3 (± 0.1) [B2]

d) Draw $y = -3$, $x = -1.7$ or 4.7 (± 0.1) [B2 for the answers]

e)(i) Line $y = -2x + 6$ [B1]

(ii) $x = 0.2$ or 4.8 (± 0.1) [B1]

(iii) $5 + 3x - x^2 = -2x + 6$ [M1]
$x^2 - 5x + 1 = 0$ [A1]

End-of-Paper 2

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