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BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2015

SUBJECT : Mathematics

LEVEL : Secondary 3 Express

PAPER : 4048 / 01

DURATION : 1 hour 15 minutes

SETTER : Mrs Rose Ang

DATE : 6 October 2015

CLASS :	NAME :	REG NO :
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READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is **50**.

For Examiner's Use
50

This paper consists of 12 printed pages (including this cover page)

81 - 82 [Turn over

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 The population in Singapore in June 2010 was 5 077 000.

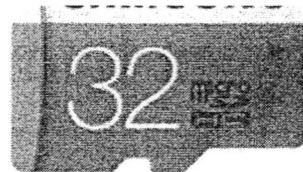
- (a) Express 5 077 000 in standard form. Correct your answer to 3 significant figures.

Answer (a) [1]

- (b) The population in Singapore in June 2014 was 5.47 million. Find the percentage increase in population over this period of time. Correct your answer to 1 decimal place.

Answer (b) [1]

- (c) Joel bought a memory card that has a storage space of 32 gigabytes. If each uncompressed digital photo takes up 48 megabytes of space, how many digital photos can be stored in the memory card?



Answer (c) [2]

2 Simplify $\frac{2}{\sqrt[3]{8p^6}} \times \frac{2^{-2}}{p^4}$, giving your answer in positive index form.

83

Answer [2]

- 3 (a) Given that $2^y = 5$ and $3^y = 13$, find the exact value of 12^y

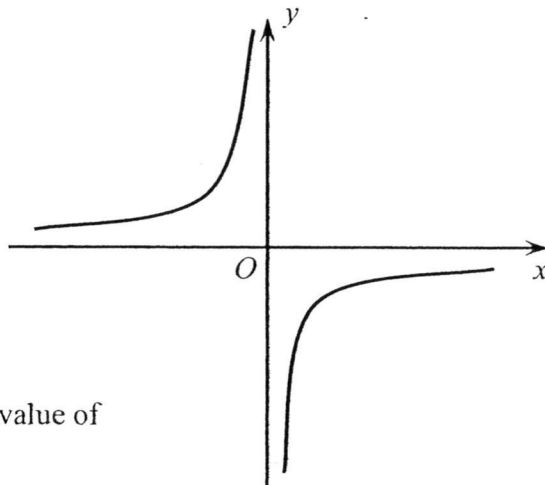
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Answer (a) [2]

- (b) Solve the equation $3^0(2^{15} - 2^{14}) = 4(2^x)$.

Answer (b) $x =$ [2]

- 4 The diagram shows the sketch for the graph of $y = \frac{k}{x^n}$.



Write down a possible value of

- (a) k ,
(b) n .

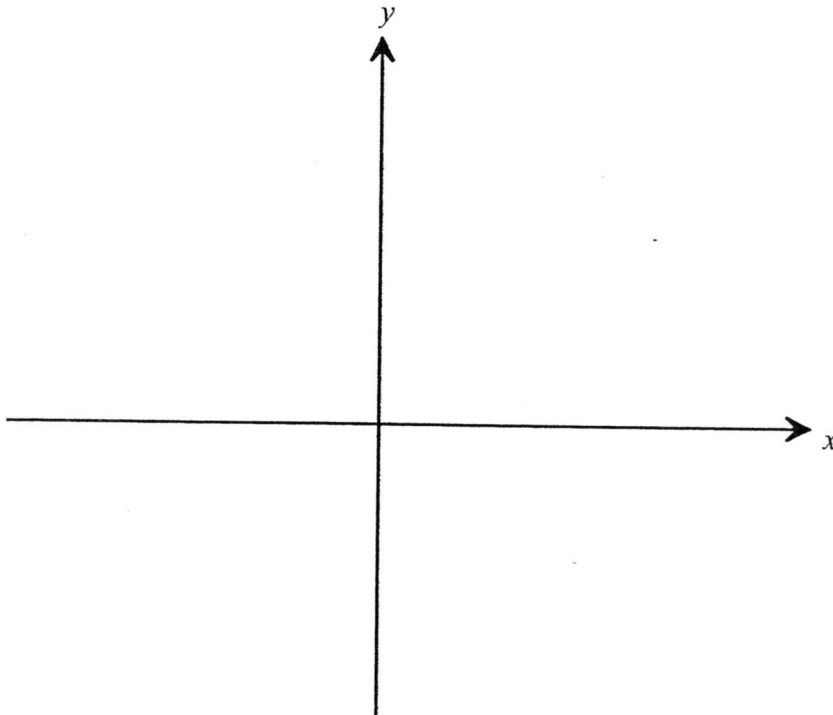
Answer (a) $k =$ [1]
(b) $n =$ [1]

- 6 (a) Express $x^2 - 6x + 2$ in the form $(x + a)^2 + b$.

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Examiners'
use only

Answer : (a) [1]

- (b) Sketch the graph of $y = x^2 - 6x + 2$, indicating clearly the y -intercept and the coordinates of the turning point.



[2]

5 Solve the following equations.

(a) $2x^2 - x = 3$

(b) $3 - 2x - 4x^2 = 0$, correct your answer to 2 decimal places.

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Answer (a) $x =$ [2]

(b) $x =$ [2]

[Turn over

7 Find the range of values of x for which

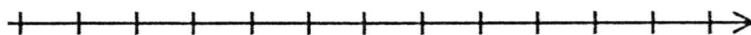
- (a) (i) $x - 1 > 1 - x$
 (ii) $6 \leq 10 - x \leq 10$

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Answer : (a)(i)..... [1]

(ii) [1]

- (b) On the number line below, illustrate the solution to both inequalities in (a)(i) and (a)(ii).



[1]

- (c) Hence, state the least integer that satisfies both inequalities in (a)(i) and (a)(ii).

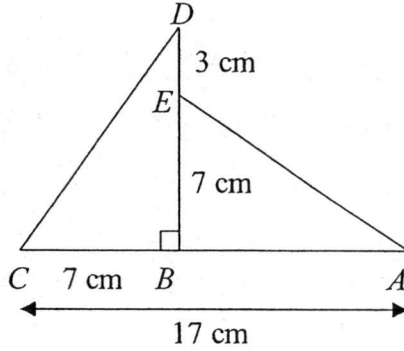
Answer : (c) [1]

85

[Turn over

- 8 (a) In the figure, $DE = 3$ cm, $EB = 7$ cm, $BC = 7$ cm and $AC = 17$ cm.

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use only



Show that $\triangle ABE$ is congruent to $\triangle DBC$.

Answer :

[2]

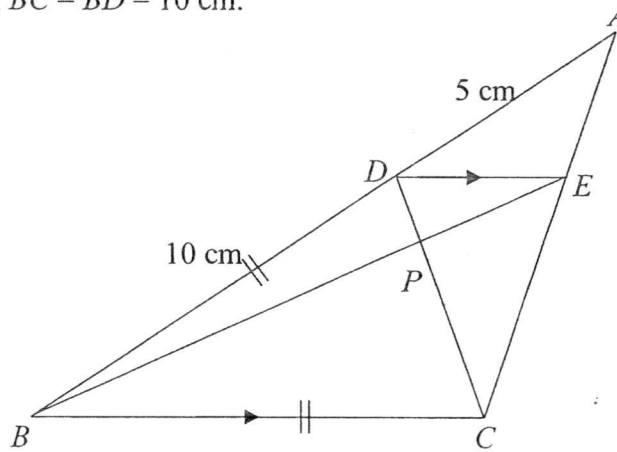
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- (b) In the diagram, DE is parallel to BC . DC and BE meet at the point P . $AD = 5$ cm, $BC = BD = 10$ cm.



- (i) Find the length of DE .

Answer : (b)(i)cm [1]

- (ii) Show that $\triangle DPE$ is similar to $\triangle CPB$.

Answer :

[2]

.....

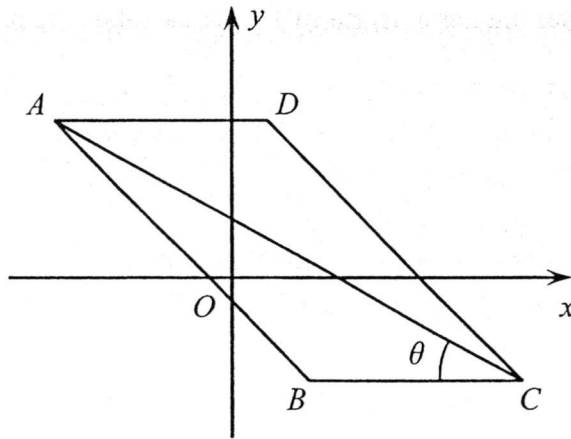
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- 9 The diagram shows a parallelogram $ABCD$ with vertices $A(-4,5)$ and $B(2,-3)$. AC is parallel to the line $5y = -4x$ and BC is parallel to the x -axis.

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use only



- (a) Find the equation of AC .

Answer : (a) [2]

- (b) Find the length of BC .

Answer : (b) [2]

- (c) Find the angle θ , in degrees.

Answer : (c) [2]

- (d) Find the area of triangle ABC .

86

Answer : (d) [1]

10 The masses of two similar jugs are in the ratio of 27 : 64.

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Examiners'
use only*

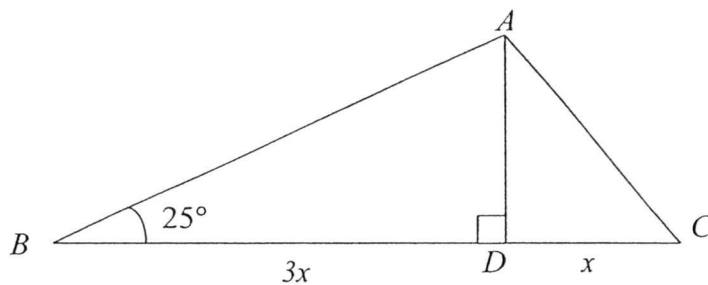
- (a) Given that the smaller jug has a height of 15 cm, calculate the height of the bigger jug.

Answer : (a)cm [2]

- (b) Given that the base area of the bigger jug is 54 cm^2 , find the base area of the smaller jug.

Answer : (b) cm^2 [2]

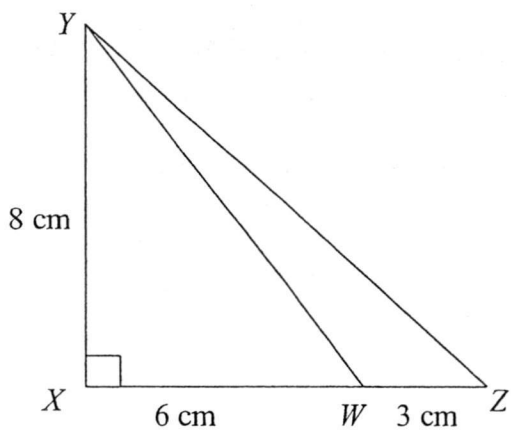
- 11 (a) In triangle ABC , $\angle ABC = 25^\circ$, $BD = 3x \text{ cm}$ and $DC = x \text{ cm}$. Given that B, D and C lie on a straight line and $\angle ADB = 90^\circ$, calculate $\angle CAD$.



Answer : (a) [3]

- 11 (b) In the right-angled triangle XYZ , $XY = 8$ cm and W is a point on XZ such that $WZ = 3$ cm and $XW = 6$ cm.

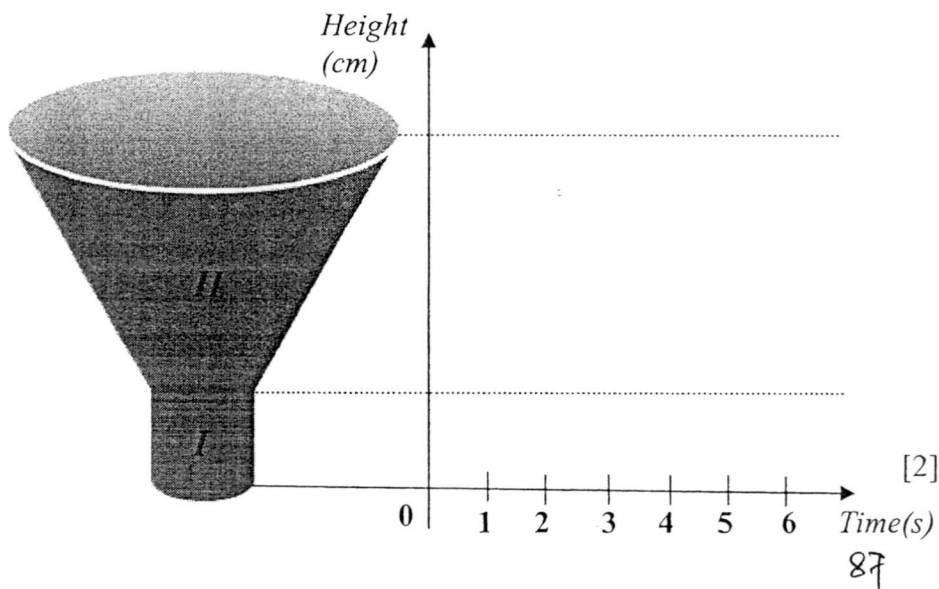
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Find the **exact** value of $\cos \angle YWZ$.

Answer : (b) [2]

- 12 (a) The container below consists of two sections, *I* and *II*, which have a volume ratio of 1 : 5. Water is poured into the container at a constant rate. It takes a total of 6 seconds to completely fill the container. On the axes below, sketch the graph of the height of the water level against time.

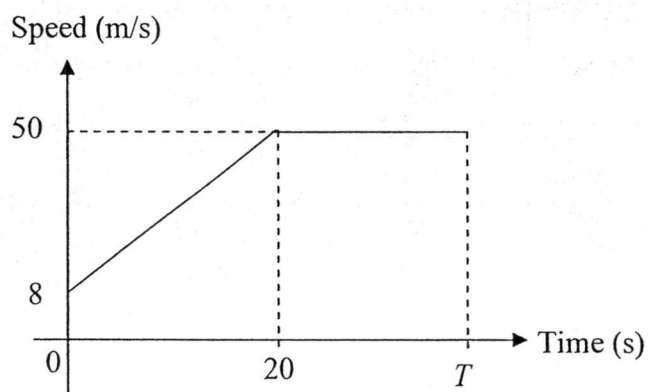


[2]

[Turn over

- 12 (b) The diagram below shows the speed-time graph for the first T seconds of a particle.

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- (i) Calculate the speed when the time is 18 seconds.

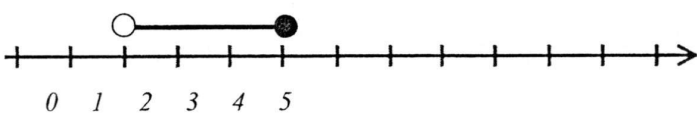
Answer : (b)(i)m/s [2]

- (ii) Calculate the value of T , if the distance travelled in the first T seconds is 1.6 km.

$T =$
Answer : (b)(ii)s [2]

~ End of Paper ~

Answer Key

1	a	5.08×10^6	b. 7.7%	c. 666
2		$\frac{1}{4p^6}$		
3	a	325	b. 12	
4	a	Any negative value	b. 1 or 3 or 5	
5	a	$x = \frac{3}{2}$ or $x = -1$	b. $x = 0.65$ or $x = -1.15$	
6	a	$(x-3)^2 - 7$		
7	ai	$x > 1$		
	aii	$0 \leq x \leq 4$		
	b			
	c	2		
8	a	$BE = BC = 7 \text{ cm (S)}$ $\angle ABE = \angle DBC = 90^\circ (A)$ $AB = DB = 10 \text{ cm (S)}$ $\triangle ABE \cong \triangle DBC \text{ (SAS Property)}$		
	b	$3\frac{1}{3}$		
	c	$\angle DPE = \angle CPB \text{ (vert. opp. } \angle\text{s) [B1]}$ $\angle DEP = \angle CBP \text{ (alt. } \angle\text{s, } DE \parallel BC) \text{ [B1]}$ $\angle EDP = \angle BCP \text{ (alt. } \angle\text{s, } DE \parallel BC) \text{ [any 2 statements]}$		
9	a	$y = -\frac{4}{5}x + \frac{9}{5}$		
	b	4 units		
	c	$\theta = 38.7^\circ$		
	d	16		
10	a	20		
	b	$30\frac{3}{8}$		
11	a	$\angle CAD = 35.6^\circ$		
	b	$\cos \angle YWZ = -\frac{3}{5}$		
12	bi	$v = 45.8$		
	bii	40.4 s		



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1 The population in Singapore in June 2010 was 5 077 000.

- (a) Express 5 077 000 in standard form. Correct your answer to 3 significant figures.

$$5.08 \times 10^6 \text{ [B1]}$$

Answer (a) [1]

- (b) The population in Singapore in June 2014 was 5.47 million. Find the percentage increase in population over this period of time. Correct your answer to 1 decimal place.

$$\frac{5.47 \times 10^6 - 5.08 \times 10^6}{5.08 \times 10^6} \times 100\% \text{ [B1]}$$

$$= 7.7\%$$

Answer (b) [1]

- (c) Joel bought a memory card that has a storage space of 32 gigabytes. If each uncompressed digital photo takes up 48 megabytes of space, how many digital photos can be stored in the memory card ?

$$\frac{32 \times 10^9}{48 \times 10^6} \text{ [M1]}$$

Ans : 666 photos [A1]

Answer (c) [2]

2 Simplify $\frac{2}{\sqrt[3]{8p^6}} \times \frac{2^{-2}}{p^4}$, giving your answer in positive index form.

$$\frac{2}{2p^2} \times \frac{2^{-2}}{p^4} \text{ [M1]}$$

$$= \frac{1}{4p^6} \text{ [A1]}$$

Answer [2]

]

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- 3 (a) Given that $2^x = 5$ and $3^y = 13$, find the exact value of 12^y

$$12^y = (2^2 \times 3)^y \quad [\text{M1}]$$

$$12^y = (2^x)^2 \times 3^y = 25 \times 13 = 325 \quad [\text{A1}]$$

Answer (a) [2]

- (b) Solve the equation $3^0(2^{15} - 2^{14}) = 4(2^x)$.

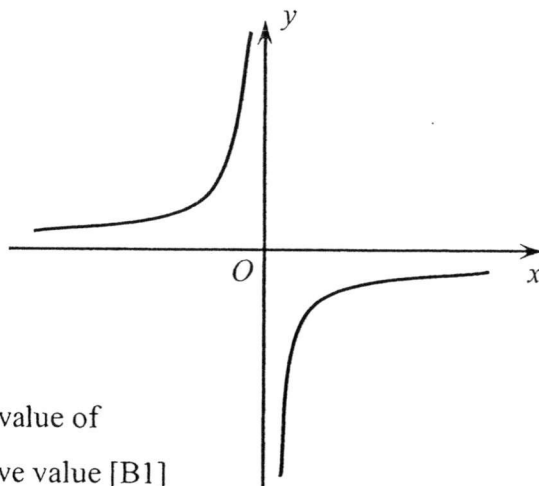
$$2^{14}(2 - 1) = 4(2^x) \quad [\text{M1}]$$

$$2^{14} = 2^{x+2}$$

$$x = 12 \quad [\text{A1}]$$

Answer (b) [2]

- 4 The diagram shows the sketch for the graph of $y = \frac{k}{x^n}$.



Write down a possible value of

- (a) k , any negative value [B1]
 (b) n , 1, 3 or 5 [B1]

Answer (a) [1]

(b) [1]

5 Solve the following equations.

(a) $2x^2 - x = 3$

(b) $3 - 2x - 4x^2 = 0$, correct your answer to 2 decimal places.

(a) $2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$ [M1]

$x = \frac{3}{2}$ or $x = -1$ [A1]

(b) $3 - 2x - 4x^2 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(-4)(3)}}{-8}$ [M1]

$x = 0.65$ or $x = -1.15$ (to 2 d.p.) [A1]

Answer (a) [2]

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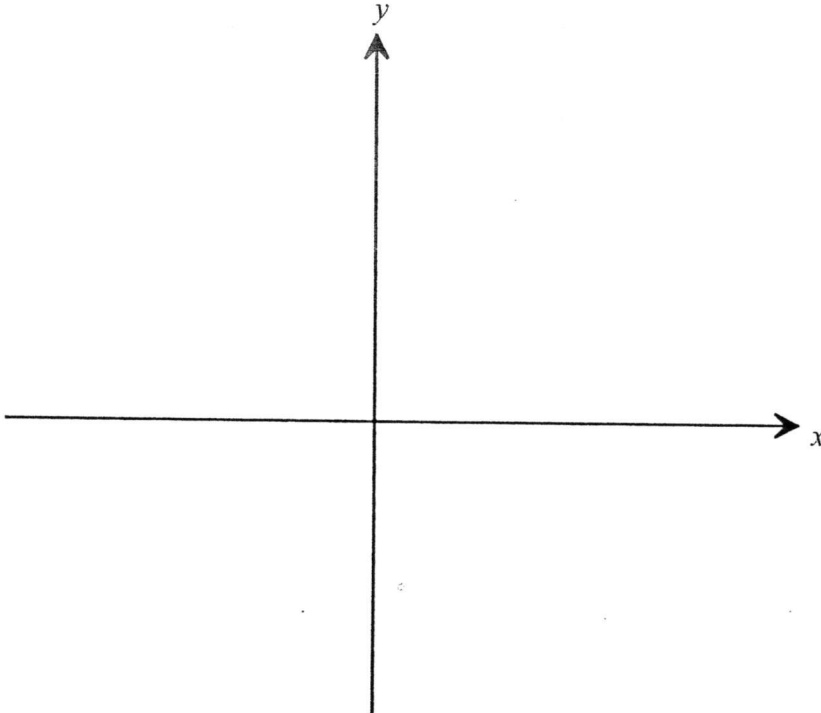
(b) [2]

- 6 (a) Express $x^2 - 6x + 2$ in the form $(x + a)^2 + b$.

$$\begin{aligned} & x^2 - 6x + 2 \\ &= x^2 - 6x + 9 - 9 + 2 \\ &= (x - 3)^2 - 7 \quad \text{[B1]} \end{aligned}$$

Answer : (a) [1]

- (b) Sketch the graph of $y = x^2 - 6x + 2$, indicating clearly the y -intercept and the coordinates of the turning point.



[G1 – y-intercept, G1 – turning point]

[2]

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use only

7 Find the range of values of x for which

(a) (i) $x - 1 > 1 - x$

(ii) $6 \leq 10 - x \leq 10$

(i) $x > 1$ [B1]

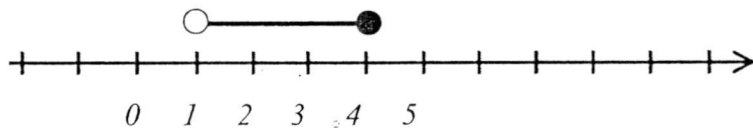
(ii) $-4 \leq -x \leq 0$

$0 \leq x \leq 4$ [B1]

Answer : (a)(i)..... [1]

(ii) [1]

(b) On the number line below, illustrate the solution to both inequalities in (a)(i) and (a)(ii).



[1]

(c) Hence, state the least integer that satisfies both inequalities in (a)(i) and (a)(ii).

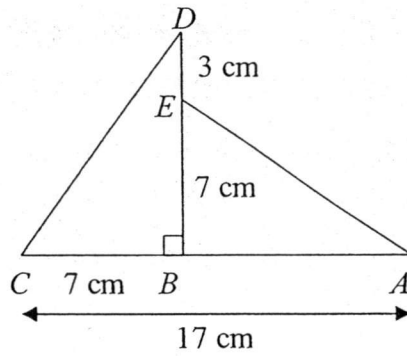
2 [B1]

Answer : (c) [1]

92

[Turn over

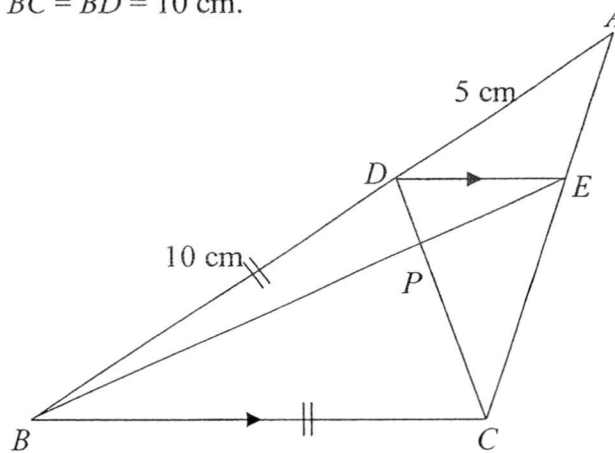
- 8 (a) In the figure, $DE = 3$ cm, $EB = 7$ cm, $BC = 7$ cm and $AC = 17$ cm.



Show that $\triangle ABE$ is congruent to $\triangle DBC$.

Answer : $BE = BC = 7$ cm (S) } [2]
 $\angle ABE = \angle DBC = 90^\circ$ (A) } [M1]
 $AB = DB = 10$ cm (S) }
 $\triangle ABE \cong \triangle DBC$ (SAS Property) [A1]

- (b) In the diagram, DE is parallel to BC . DC and BE meet at the point P . $AD = 5$ cm, $BC = BD = 10$ cm.



- (i) Find the length of DE .

$$\frac{DE}{10} = \frac{5}{15} \Rightarrow DE = 3\frac{1}{3} \text{ [B1]}$$

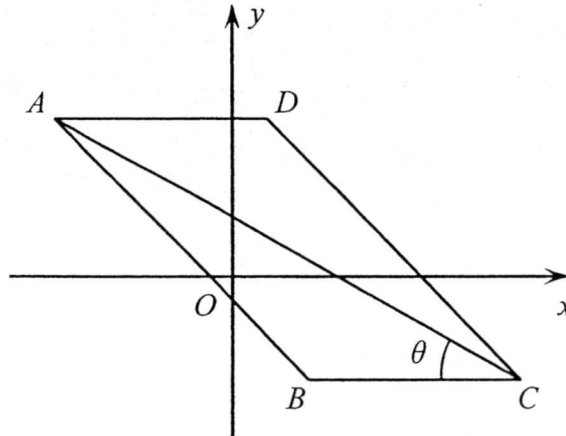
Answer : (b)(i)cm [1]

- (ii) Show that $\triangle DPE$ is similar to $\triangle CPB$.

Answer : $\angle DPE = \angle CPB$ (vert. opp. \angle s) [B1]
 $\angle DEP = \angle CBP$ (alt. \angle s, $DE \parallel BC$) [B1]
 $\angle EDP = \angle BCP$ (alt. \angle s, $DE \parallel BC$) [any 2 statements] [2]

- 9 The diagram shows a parallelogram $ABCD$ with vertices $A(-4, 5)$ and $B(2, -3)$. AC is parallel to the line $5y = -4x$ and BC is parallel to the x -axis.

For
Examiners'
use only



- (a) Find the equation of AC .

$$\text{Gradient of } AC = -\frac{4}{5} \quad [\text{M1}]$$

$$y = -\frac{4}{5}x + c$$

$$5 = -\frac{4}{5}(4) + c$$

$$c = \frac{9}{5}$$

$$y = -\frac{4}{5}x + \frac{9}{5} \quad [\text{A1}]$$

Answer : (a) [2]

- (b) Find the length of BC .

$$-3 = -\frac{4}{5}x + \frac{9}{5} \quad [\text{M1}]$$

$$x = 6 \Rightarrow B(2, -3) \text{ and } C(6, -3)$$

$$\text{Length of } BC = 4 \text{ units } [\text{A1}]$$

Answer : (b) [2]

- (c) Find the angle θ , in degrees.

$$\tan \theta = \frac{8}{10} \quad [\text{M1}] \Rightarrow \theta = 38.7^\circ \quad [\text{A1}]$$

Answer : (c) [2]

- (d) Find the area of triangle ABC .

$$\frac{1}{2}(4)(8) = 16 \text{ units}^2 \quad [\text{B1}]$$

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Answer : (d) [1]

11 The masses of two similar jugs are in the ratio of 27 : 64.

- (a) Given that the smaller jug has a height of 15 cm, calculate the height of the bigger jug.

$$\frac{l_{big}}{15} = \frac{4}{3} \text{ [M1]}$$

$$l_{big} = 20 \text{ [A1]}$$

Answer : (a)cm [2]

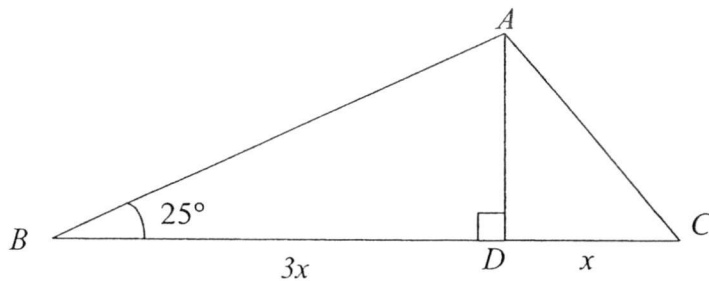
- (b) Given that the base area of the bigger jug is 54 cm^2 , find the base area of the smaller jug.

$$\frac{Area_{small}}{54} = \left(\frac{3}{4}\right)^2 \text{ [M1]}$$

$$Area_{small} = 30\frac{3}{8} \text{ cm}^2$$

Answer : (b)cm² [2]

- 12 (a) In triangle ABC , $\angle ABC = 25^\circ$, $BD = 3x \text{ cm}$ and $DC = x \text{ cm}$. Given that B , D and C lie on a straight line and $\angle ADB = 90^\circ$, calculate $\angle CAD$.



$$\tan 25^\circ = \frac{AD}{3x} \Rightarrow AD = 3x \tan 25^\circ \text{ [M1]}$$

$$\tan \angle CAD = \frac{AD}{x}$$

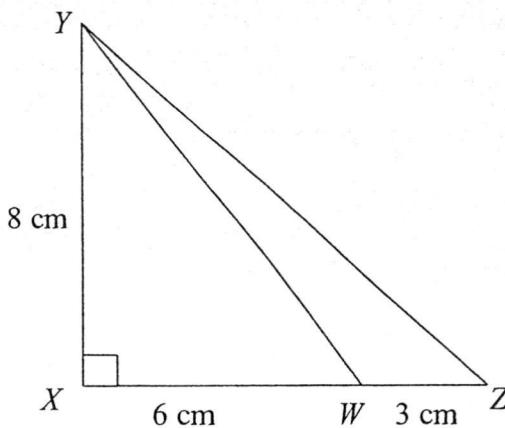
$$\tan \angle CAD = \frac{x}{3x \tan 25^\circ} \text{ [M1]}$$

$$\angle CAD = 35.6^\circ \text{ [A1]}$$

Answer : (a) [3]

- 12 (b) In the right-angled triangle XYZ , $XY = 8$ cm and W is a point on XZ such that $WZ = 3$ cm and $XW = 6$ cm.

For
Examiners
use only



Find the **exact** value of $\cos \angle YWZ$.

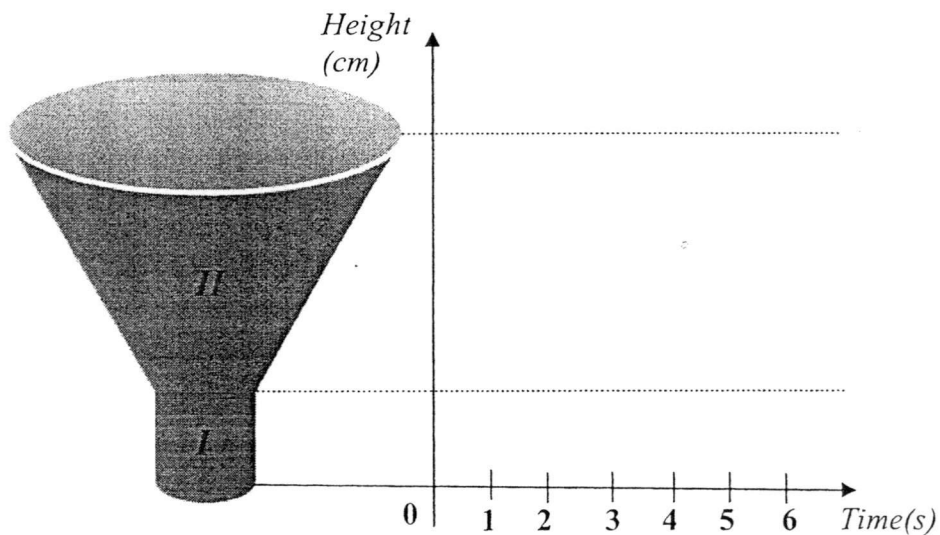
$$YW = \sqrt{8^2 + 6^2}$$

$$YW = 10 \quad [\text{M1}]$$

$$\cos \angle YWZ = -\frac{3}{5} \quad [\text{A1}]$$

Answer : (b) [2]

- 13 (a) The diagram below shows the cross-section of an empty container. Water is poured into the container at a constant rate. Sketch the graph of the height of the water level against time on the axes in the answer space below.



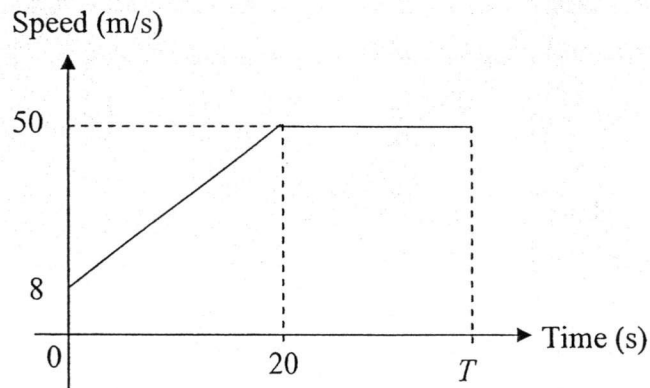
[G1 for each part]

9/11 [2]

[Turn over

- 13 (b) The diagram below shows the speed-time graph for the first T seconds of a particle.

*For
Examiners
use only*



- (i) Calculate the speed when the time is 18 seconds. [2]

Let the speed at 18 s be x m/s

$$\frac{50 - v}{20 - 18} = \frac{42}{20} \quad [\text{M1}]$$

$$v = 45.8 \text{ m/s} \quad [\text{A1}]$$

- (ii) Calculate the value of T , if the distance travelled in the first T seconds is 1.6 km. [2]

Distance = 1 600 m

$$\frac{1}{2}(8 + 50)20 + (T - 20)50 = 1600 \quad [\text{M1}]$$

$$40.4 \text{ s} \quad [\text{A1}]$$

~ End of Paper ~

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Curved surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} bc \sin A$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

Standard Deviation =

$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 On 1st January 2010, Carolyn invested \$50 000 in Bank A which paid simple interest of 1.5% per year. On 1st January 2013, Carolyn withdrew all her investment from Bank A and invested everything in Bank B which paid an interest of 1.2% per year compounded half-yearly. On 1st January 2015, Carolyn withdrew all her investment from Bank B. Find the total interest Carolyn earned in her 5 years investment.

[4]

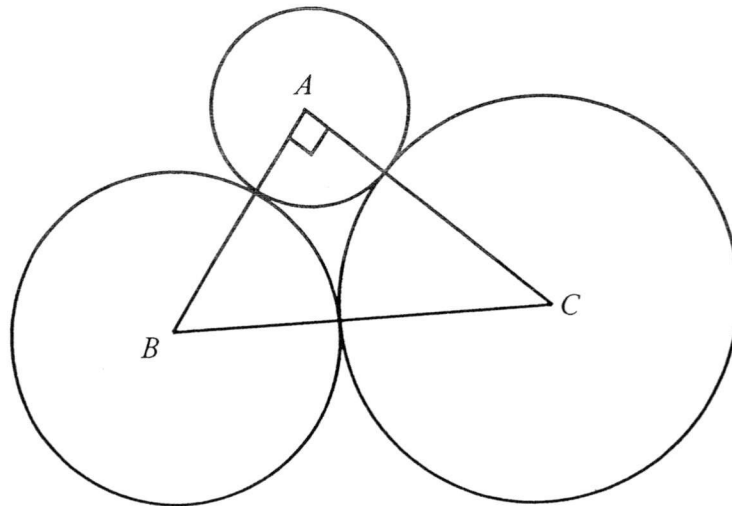
2 (a) Simplify $\frac{(2x^2y)^3 \div (xy^4)^{-2}}{(4x^4y^2)^{\frac{1}{2}}}$.

[2]

(b) Solve the equation $\frac{3}{9x^2 - 6x + 1} - \frac{1}{1 - 3x} = 2$.

[4]

- 3 The diagram shows three circles touching each other. The circles of centres A , B and C have radius $(x - 1)$ cm, 3 cm and 5 cm respectively. The three lines joining the centres of the circles form a right-angled triangle.

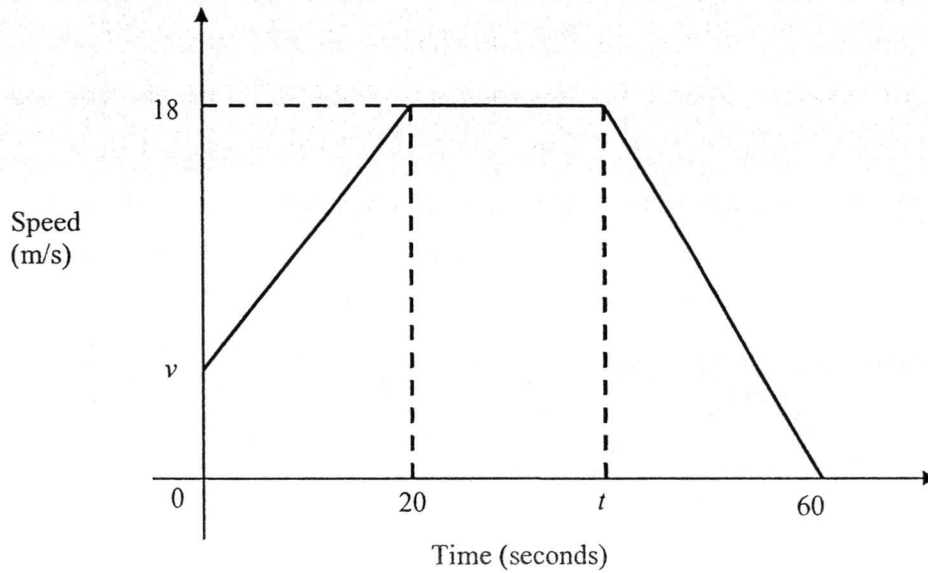


- (a) Form an equation in x and show that it reduces to $x^2 + 6x - 22 = 0$. [2]
- (b) Solve the equation $x^2 + 6x - 22 = 0$, giving both answers correct to two decimal places. [3]
- (c) Find the perimeter of the right-angled triangle. [2]

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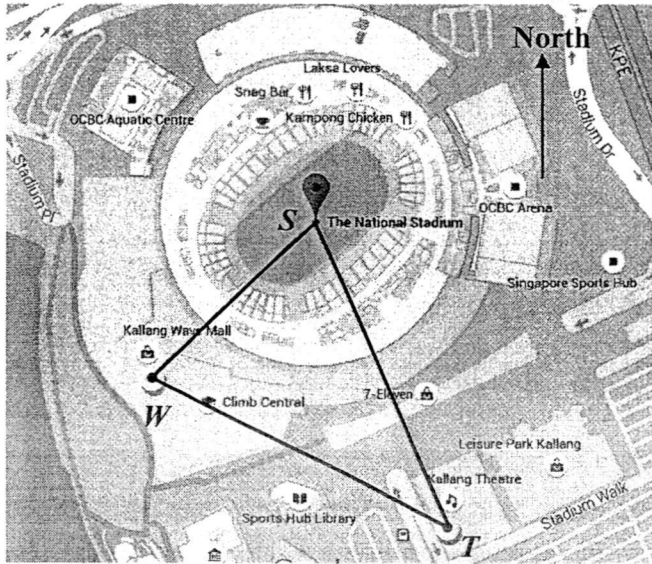
[Turn over

- 4 The diagram is the speed-time graph of a truck's journey for a period of 60 seconds.



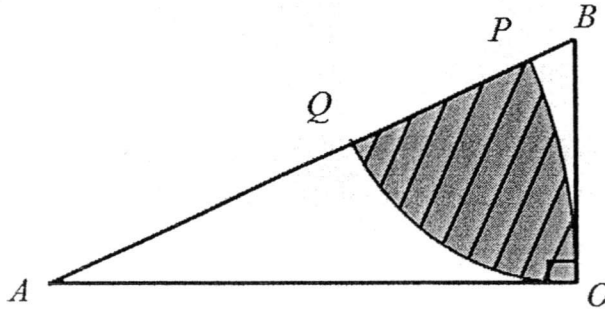
- (a) If the truck accelerates at 0.7 m/s^2 in the first 20 seconds, find the value of v . [2]
- (b) If the truck travels at constant speed of 18 m/s for a distance of 252 m , find the value of t . [2]
- (c) Find the speed of the truck after 50 seconds. [2]
- (d) Calculate the average speed of the truck for the whole journey. [2]

- 5 The diagram below is the map of the Singapore Sports Hub. S , T and W are positions of the National Stadium, Kallang Theatre and Kallang Wave Mall respectively.
 $SW = 200$ m, $ST = 275$ m, angle $WST = 80^\circ$ and the bearing of T from S is 158° .



- (a) Calculate the
- area of triangle WST , [2]
 - length of WT , [2]
 - bearing of W from T , [3]
 - shortest distance from S to WT . [2]
- (b) A Singapore flag is hung 250 m vertically above S . Calculate the greatest possible angle of elevation from a point along WT . [2]

- 6 The diagram shows a right-angled triangle AOB in which OA is 15 cm and OB is 8 cm. Point P lies on AB and OP is an arc of a circle with centre A . Point Q lies on AB and OQ is an arc of a circle with centre B .



- (a) Show that angle ABO is 1.0808 radians, correct to 4 decimal places. [1]
- (b) Calculate the area of the shaded region. [4]

7 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{x^2}{4} + \frac{5}{x} - 3$.

The table below shows some values of x and the corresponding values of y correct to 1 decimal place.

x	0.5	1	1.5	2	2.5	3	4	5	6
y	7.1	2.3	0.9	0.5	0.6	0.9	2.3	p	6.8

(a) Calculate the value of p . [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x -axis for $0 \leq x \leq 6$.

Using a scale of 2 cm to 1 unit, draw a vertical y -axis for $0 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [2]

(c) Use your graph to find the solutions to the equation $\frac{x^2}{4} + \frac{5}{x} = 5$. [2]

(d) By drawing a tangent, find the gradient of the curve at (1, 2.3). [2]

(e) (i) On the same axes, draw the line $y = 5 - x$. [1]

(ii) Hence state the range of values of x for $\frac{x^2}{4} + \frac{5}{x} - 3 \leq 5 - x$. [1]

~~~ End of Paper ~~~

**Answer Keys:**

1. \$3 515.33

2(a)  $4x^6y^{10}$  (b)  $x = 0$  or  $x = \frac{5}{6}$

3(b)  $x = -8.57$  or  $2.57$  (c) 19.1 cm

4(a)  $v = 4$  (b)  $t = 34$  (c) 11.25 m/s (d) 11.8 m/s

5(a) (i) 27 080 m<sup>2</sup> (ii) 311 m (iii) 298.7° (iv) 174 m

5(b) 55.1°

6(b) 29.7 cm<sup>2</sup>

7(a)  $p = 4.3$  (c)  $x = 1.05$  or  $3.85$  (d) gradient = -4.5

7(e) (ii)  $0.7 \leq x \leq 3.5$

2015 Beatty Sec Sch EOY Exam Sec 3E & 3N'O' Mathematics Paper 2

1 Simple interested earned in first 3 years =  $\frac{50000 \times 1.5 \times 3}{100} = \$2250$  ... [M1]

Total Amount after first 3 years =  $50\,000 + 2250 = \$52\,250$

Total Amount at end of 5 years =  $52250 \left(1 + \frac{0.6}{100}\right)^4$  ... [M1]

=  $\$53\,515.33$  ... [A1]

Total interested earned in 5 years =  $\$(53\,515.33 - 50\,000)$

=  $\$3\,515.33$  ... [A1]

2 (a)  $\frac{(2x^2y)^3 \div (xy^4)^{-2}}{(4x^4y^2)^{\frac{1}{2}}}$  =  $\frac{8x^6y^3 \div x^{-2}y^{-8}}{2x^2y}$  ... [M1]

=  $\frac{8x^8y^{11}}{2x^2y}$

=  $4x^6y^{10}$  ... [A1]

(b)  $\frac{3}{9x^2 - 6x + 1} - \frac{1}{1 - 3x} = 2$

$\frac{3}{(3x-1)(3x-1)} - \frac{1}{1-3x} = 2$

$\frac{3}{(3x-1)(3x-1)} + \frac{1}{3x-1} = 2$  ... [M1, factorise both denominators]

$3 + (3x-1) = 2(3x-1)(3x-1)$  ... [M1, multiply throughout by LCM]

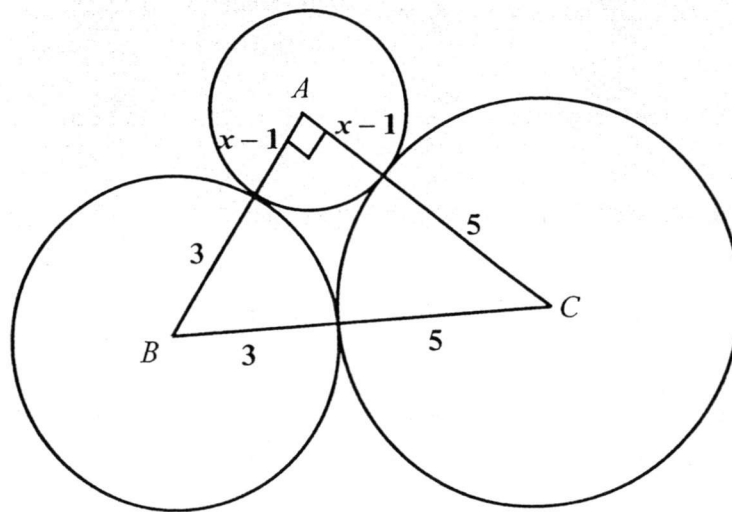
$3x + 2 = 18x^2 - 12x + 2$

$18x^2 - 15x = 0$  ... [M1, expand and simplify]

$3x(6x - 5) = 0$

$x = 0$  or  $x = \frac{5}{6}$  ... [A1]

3



(a) Using Pythagoras' Theorem,

$$((x-1) + 3)^2 + ((x-1) + 5)^2 = (3 + 5)^2 \quad \dots \text{ [M1, apply PT]}$$

$$(x+2)^2 + (x+4)^2 = 8^2$$

$$x^2 + 4x + 4 + x^2 + 8x + 16 = 64$$

$$2x^2 + 12x - 44 = 0$$

$$x^2 + 6x - 22 = 0 \text{ (shown)}$$

... [A1, expand and simplify]

(b)  $x^2 + 6x - 22 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-22)}}{2(1)}$$

... [M1]

$$x = \frac{-6 \pm \sqrt{124}}{2}$$

$$x = -8.5677 \text{ or } 2.5677 \text{ (2 d p)}$$

$$x = -8.57 \text{ or } 2.57 \text{ (2 d p)}$$

... [A1+A1]

(c) Perimeter of right-angled triangle

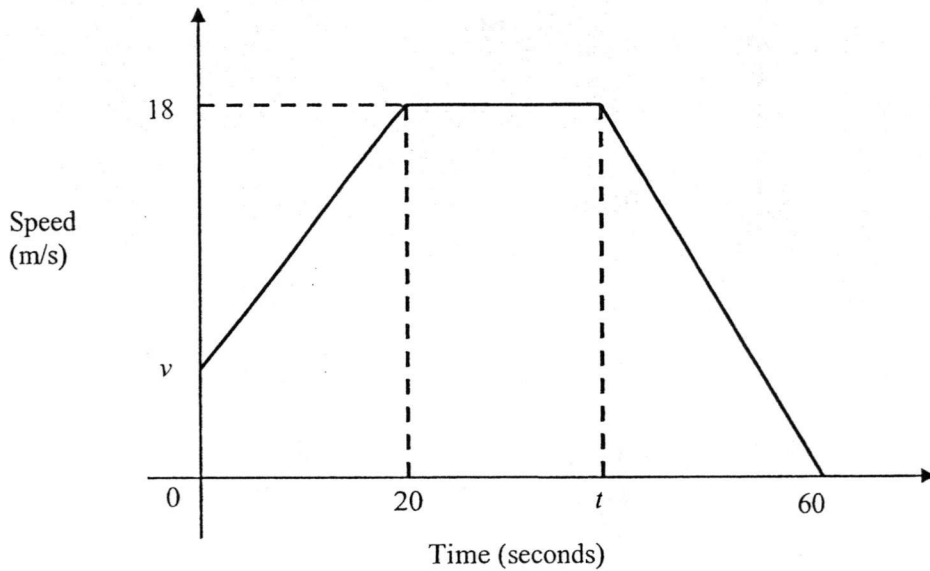
$$= 2(2.5677 - 1) + 6 + 10$$

... [M1]

$$= 19.1354$$

$$= 19.1 \text{ cm (3 s f)}$$

... [A1]



$$(a) \quad \frac{18-v}{20} = 0.7 \quad \dots \text{ [M1]}$$

$$18 - v = 14$$

$$v = 4 \quad \dots \text{ [A1]}$$

$$(b) \quad (t-20) \times 18 = 252 \quad \dots \text{ [M1]}$$

$$t = 34 \quad \dots \text{ [A1]}$$

$$(c) \quad \text{deceleration} = \frac{18}{60-34} = \frac{9}{13} \text{ m/s}^2 \quad \dots \text{ [M1]}$$

$$\text{In 16 seconds, speed decrease by} = \left(\frac{9}{13} \times 16\right) \text{ m/s}$$

$$= 11\frac{1}{13} \text{ m/s}$$

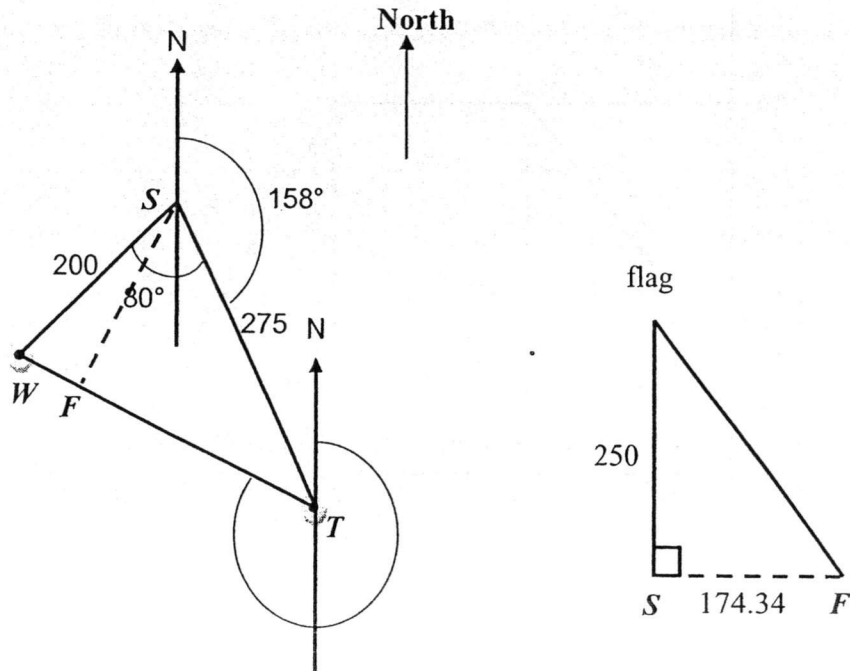
$$\text{Speed of truck after 50 seconds} = \left(18 - 11\frac{1}{13}\right) \text{ m/s}$$

$$= 6\frac{12}{13} \text{ m/s or } 6.92 \text{ m/s (3 s f)} \quad \dots \text{ [A1]}$$

$$(d) \quad \text{Total distance} = \left(\frac{1}{2} \times (4+18) \times 20\right) + 252 + \left(\frac{1}{2} \times 18 \times 26\right) \quad \dots \text{ [M1]}$$

$$= 706 \text{ m}$$

$$\text{Average speed} = \frac{706}{60} = 11.8 \text{ m/s (3 s f) or } 11\frac{23}{30} \text{ m/s} \quad \dots \text{ [A1]}$$



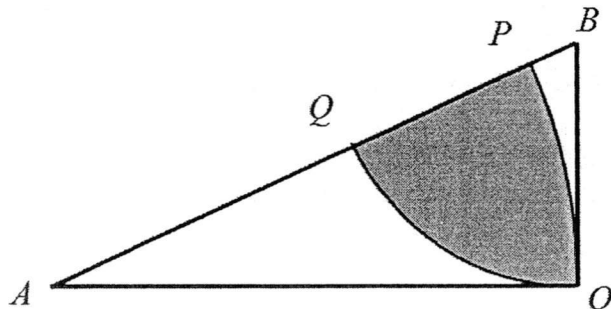
- (a) (i) area of triangle  $WST = \frac{1}{2} \times 200 \times 275 \times \sin 80^\circ$  ... [M1]  
 $= 27\,082.21$   
 $= 27\,100 \text{ m}^2$  (3 s f) ... [A1]
- (ii)  $WT = 200^2 + 275^2 - 2(200)(275)\cos 80^\circ$  ... [M1]  
 $= 310.68$   
 $= 311 \text{ m}$  (3 s f) ... [A1]
- (iii)  $180^\circ - 158^\circ = 22^\circ$  (int angles, // lines)  
 $\frac{\sin \angle WTS}{200} = \frac{\sin 80^\circ}{310.68}$  ... [M1, or use cosine rule]  
 $\sin \angle WTS = \frac{\sin 80^\circ}{310.68} \times 200$   
 $\angle WTS = 39.343^\circ$  ... [A1]  
 Bearing of  $W$  from  $T = 360^\circ - 22^\circ - 39.343^\circ = 298.7^\circ$  (1 d p) ... [A1]
- (iv)  $\frac{1}{2} \times 310.68 \times \text{shortest distance} = 27082.21$  ... [M1, or use CAH]  
 Shortest distance  $= 174.34 = 174 \text{ m}$  (3 s f) ... [A1]

(b)  $\tan \theta = \frac{250}{174.34} \quad \dots \text{ [M1]}$

$\theta = 55.109^\circ$

Hence the greatest possible angle of elevation =  $55.1^\circ$  (1 d p)  $\dots \text{ [A1]}$

6



(a)  $\tan \angle ABO = \frac{15}{8}$   
 $\angle ABO = 1.08083$   
 $\angle ABO = 1.0808$  (4 d p) (shown)  $\dots \text{ [B1]}$

(b) Area of triangle  $AOB = \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2 \quad \dots \text{ [M1]}$

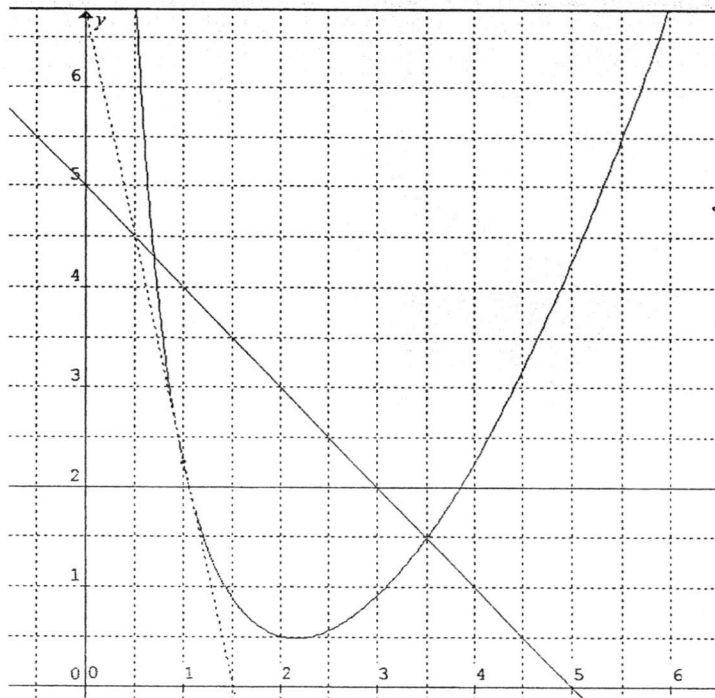
Area of sector  $BOQ = \frac{1}{2} \times 8^2 \times 1.0808 = 34.5856 \quad \dots \text{ [M1]}$

Area of sector  $AOP = \frac{1}{2} \times 15^2 \times \left(\frac{\pi}{2} - 1.0808\right) = 55.1245 \quad \dots \text{ [M1]}$

Area of shaded region =  $(34.5856 + 55.1245) - 60$   
 $= 29.7101$   
 $= 29.7 \text{ cm}^2 \quad \dots \text{ [A1]}$

- 7 (a) when  $x = 5$ ,  $p = \frac{5^2}{4} + \frac{5}{5} - 3 = 4.3$  (1 d p) or 4.25 ... [B1]

(b)



[G1] – all points plotted corrected

[G1] – all points are joined with a smooth curve

- (c) Use the line  $y = 2$  ... [M1]  
 $x = 1.05$  or  $3.85$  (accept  $\pm 0.1$ ) ... [A1]
- (d) Draw tangent at  $(1, 2.3)$  ... [M1]  
 gradient =  $-4.5$  (accept  $\pm 0.3$ ) ... [A1]
- (e) (i) Draw the line  $y = 5 - x$  ... [G1]  
 (ii)  $0.7 \leq x \leq 3.5$  (accept  $\pm 0.1$  for each  $x$  value) ... [B1]