Calculator Model:		
Class	Full Name	Index Number

bowen
SECONDARY

I believe, therefore I am

MID YEAR EXAMINATION 2016

O

4048/ 01

MATHEMATICS
Paper 1

1 hour 30 minutes

Secondary 3 Express
06 May 2016

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate,
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answers to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

At the end of the test, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

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For Examiner's Use
60

Setter: Mrs Jane Cheng

This document consists of **13** printed pages, including this cover page.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = \text{P}\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of a triangle ABC $= \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

For Examiner's Use

1 Simplify

(a) $5a^3 \div 7a^2b$,

Answer (a).....[1]

(b) $(2x^4)^3 \times \frac{1}{64x^4}$.

Answer (b).....[1]

2 Simplify the following, leaving your answers in positive index.

(a) $\left(\frac{3x^0y^3}{4}\right)^3 \div \left(\frac{3x^{-2}y^5}{2}\right)^2$,

Answer (a).....[2]

(b) $\frac{12c^{\frac{2}{3}}}{5a^{\frac{1}{2}}b^{-\frac{1}{3}}} \times \frac{3a^{\frac{3}{2}}b}{8c^{\frac{1}{3}}}$.

Answer (b).....[2]

For Examiner's Use

3 **(a)** Express 859 nanograms in grams, giving your answer in standard form.

Answer (a)..... g [1]

(b) The diameter of a circular organism is 5 micrometres.

(i) Express 5 micrometres in metres.

(ii) Find the area, in square metres, of the circular organism, giving your answer in standard form correct to 3 significant figures.

Answer (b) (i) ..... m [1]

(ii) ..... m^2 [2]

4 Solve

(a) $3^{5(x-1)} = 27$,

Answer (a) x =.....[2]

(b) $216^{x-1} = \dfrac{1}{36^{x+4}}$.

Answer (b) x = ...[2]

For Examiner's Use

For Examiner's Use

5 Given that $p = 4.52 \times 10^8$ and $q = 6.12 \times 10^7$, evaluate the following, giving your answers in standard form correct to 3 significant figures.

(a) $2p - q$,

Answer (a).....[2]

(b) $\frac{5p}{2q}$.

Answer (b).....[2]

6 Ms Chong deposits \$42, 000 in ACBC Bank which pays an interest rate of 3% p.a compounded monthly. Calculate how much interest she can get at the end of one year.

Answer \$.....[2]

For Examiner's Use

For Examiner's Use

7 Solve the following equations, giving your answers correct to 2 decimal places.

(a) $\frac{2}{x+5} - \frac{3}{x-5} = 4$,

Answer (a) x =.....or.....[4]

(b) Solve $(4x-3)(3x+2) = 5x+1$.

Answer (b) x =.....or.....[4]

For Examiner's Use

For Examiner's Use

8 **(a)** Express $y = x^2 - 6x + 2$ in the form of $y = (x-h)^2 + k$.

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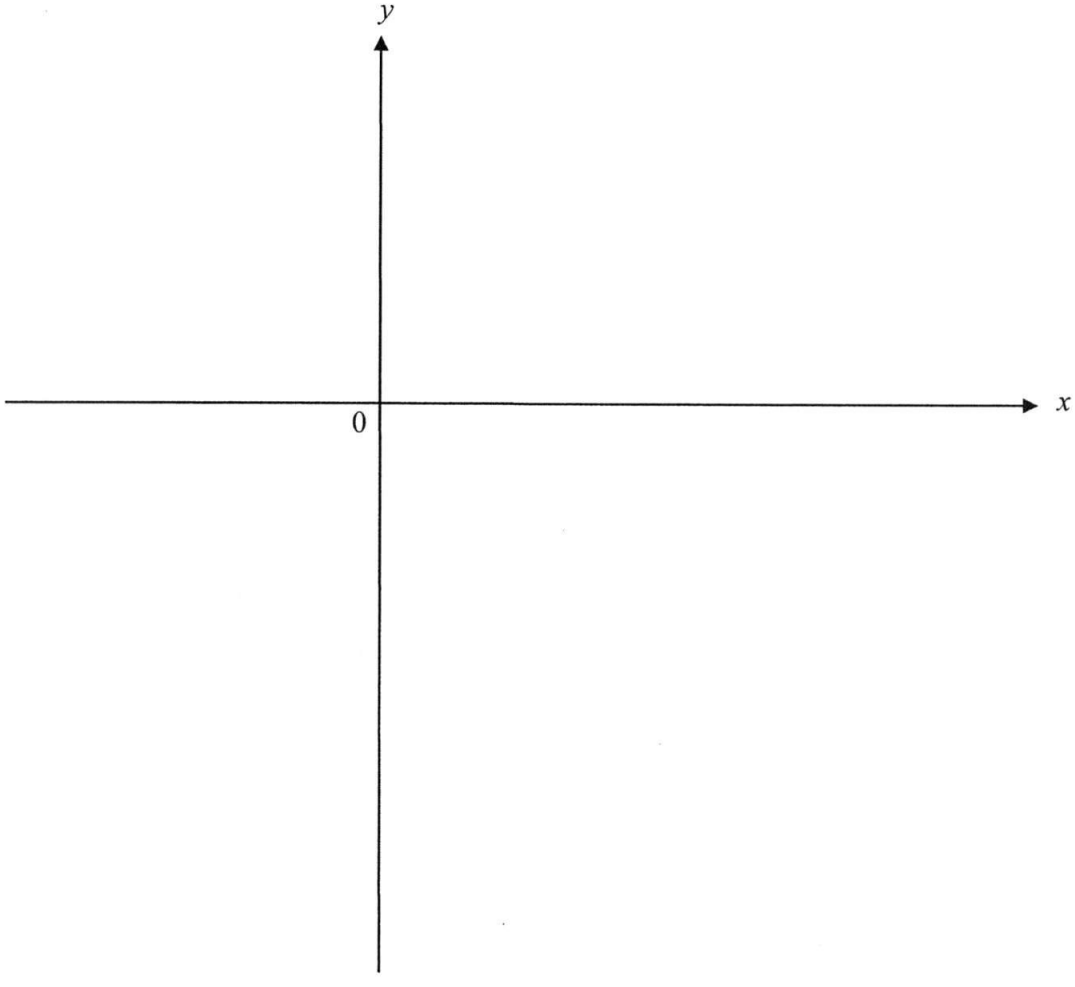
Answer (a).....[2]

(b) Sketch the graph of $y = x^2 - 6x + 2$ indicating clearly its intercepts with the axes and its turning point. [2]

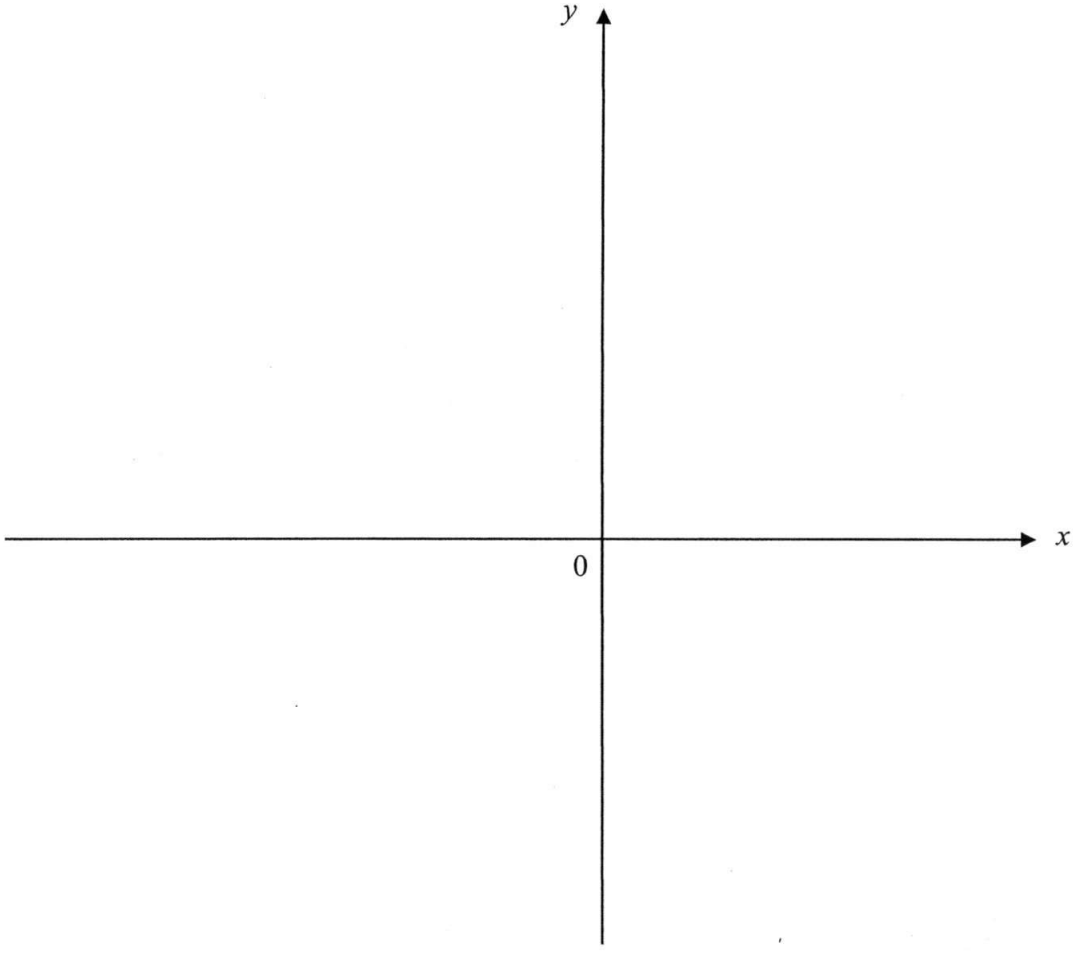


For Examiner's Use

9 **(a)** Sketch the graph of $y = -x^2 - 3x + 4$ indicating clearly its intercepts with the axes and its turning point. [2]

For Examiner's Use

(b) Write down the equation of the line of symmetry of the graph.



Answer (b).....[1]

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10 It is given that x and y are integers such that $1 \le x \le 9$ and $-3 \le y < 0$.
Find

(a) the largest possible value of $x - y$,

Answer (a) .....[1]

(b) the least possible value of $\frac{y}{x}$,

Answer (b) .....[1]

(c) the largest possible value of $\frac{1}{x^2 + y^2}$.

Answer (c) .....[1]

11 Given that $\frac{x-11}{2} < \frac{3-2x}{5} \le \frac{x+7}{3}$, find

(a) the range of values of x that satisfy the inequality and represent your solutions on a number line.

(b) the least integer value of x.

(c) the greatest prime value of x.

Answer (a) .....[4]

Number Line

Answer (b) .....[1]

Answer (c) .....[1]

For Examiner's Use

For Examiner's Use

12 A sequence of 5 diagrams is shown below.

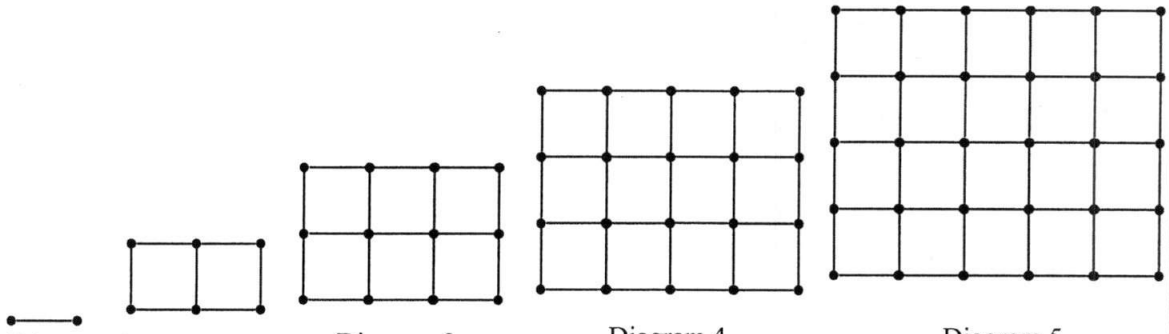


Diagram 1 Diagram 2 Diagram 3 Diagram 4 Diagram 5

The number of dots and lines in each of the diagrams are shown in the table below.

Diagram number	1	2	3	4	5	6
Number of dots	2	6	12	20	30	p
Number of lines	1	7	17	31	49	q

(a) Find the value of p and of q.

(b) Write down an expression for the number of dots in diagram n.

(c) The number of lines in diagram n is $2n^2 - 1$.
Find the diagram number which has 287 lines.

Answer (a) p = ...[1]

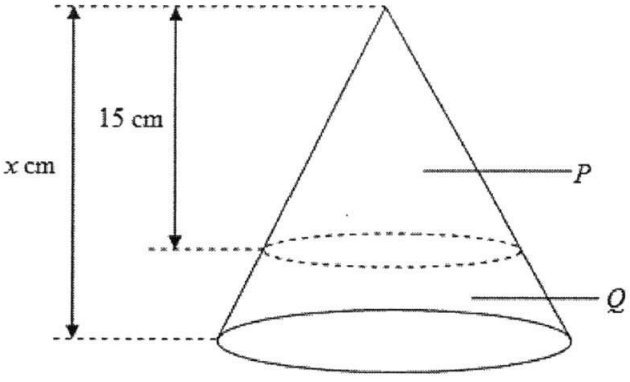
q = ...[1]

Answer (b).....[1]

Answer (c).....[2]

For Examiner's Use

13 The following diagram shows a solid cone that is cut up into 2 sections, P and Q, such that section P is a cone similar to the original cone. The curved surface area of cone P and the original cone is 160 cm^2 and 250 cm^2 respectively.



(a) If the height of cone P is 15 cm, calculate the height, x cm, of the original cone.

(b) Given that the mass of cone P is 12.8 kg, find the mass of frustum Q.

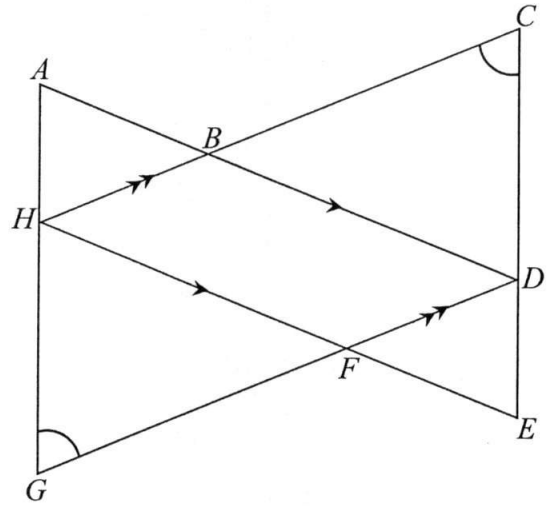
Answer (a) ..... cm [2]

Answer (b) ..... kg [2]

For Examiner's Use

For Examiner's Use

14 In the diagram below, ΔADG and ΔEHC overlap to form a parallelogram $BDFH$. Given further that $AG = EC$ and $\angle HCE = \angle DGA$, prove that ΔADG is congruent to ΔEHC. State your reasons clearly.



Answer .....

.....

.....

.....

.....[3]

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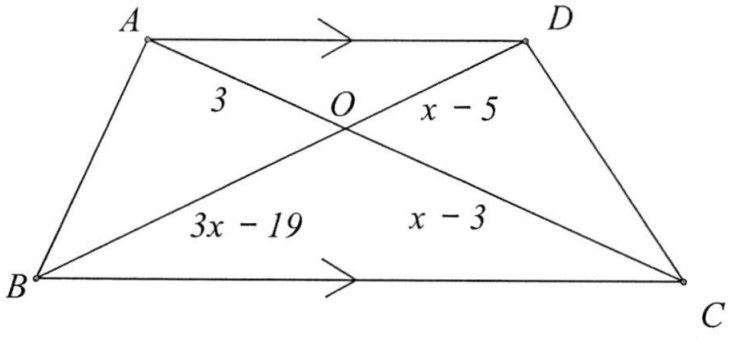
15 $ABCD$ is a trapezium in which AD is parallel to BC.
The diagonals AC and BD intersect at O.



(a) Name the triangle which is similar to ΔAOD.

Answer (a) .....[1]

(b) If $AO = 3\ cm$, $BO = (3x - 19)\ cm$, $CO = (x - 3)\ cm$ and $DO = (x - 5)\ cm$, find the values of x.

Answer (b) x = ... or .....[3]

END OF PAPER

Answer Key

1(a)	$\frac{5a}{7b}$	9(a)	$y=(-x+1))(x+4)$ x-intercepts at $x=-4$ and $x=1$ y-intercept = 4 Coordinates of maximum point = (-1.5, 6.25)
(b)	$\frac{x^8}{8}$	(b)	Equation of line of symmetry $x=-1.5$
2(a)	$\frac{3x^4}{16y}$	10(a)	12
(b)	$\frac{9ab^{\frac{2}{3}}c^{\frac{1}{3}}}{10}$	(b)	-3
3(a)	8.59×10^{-7} g	(c)	1
(b)(i)	$5\times10^{-6}=0.000005$ m	11(a)	$-2\frac{4}{11}\le x<6\frac{7}{9}$
(ii)	7.86×10^{-11} m^2	(b)	2
4(a)	$x=\frac{8}{5}$	(c)	5
(b)	$x=-1$	12(a)	p = 30 + 12 = 42 q = 49 + 22 = 71
5(a)	8.43×10^8	12(b)	Diagram 1 = 1 x 2 = 2 Diagram 2 = 2 x 3 = 6 Diagram 3 = 3 x 4 =12 Diagram n = n(n + 1) = n^2 + n
(b)	1.85×10	12(c)	n = 12
6	\$1277.47	13(a)	Ratio of the length = 5 : 4 height original cone x = $18\frac{3}{4}cm$
		(b)	Ratio of the volume of original cone to the volume of cone P = 125 : 64 Mass of original cone = 25 *kg* Mass of the frustum = 12.2 *kg*
7(a)	4.21 or -4.46	14	$\angle HCE=\angle DGA$ (given) $AG=CE$ (given) $\angle CHE=\angle DGA$ (opp. $\angle$ of a parallelogram) $\Delta ADG\equiv EHC$ (AAS or SAA or ASA)
7(b)	1.05 or -0.554	15(a)	ΔCOB

8(a)	$y = (x-3)^2 - 7$	(b)	$\frac{3}{x-3} = \frac{x-5}{3x-19}$ $x^2 - 17x + 72 = 0$ $(x-8)(x-9) = 0$ $x = 8$ *or* $x = 9$
(b)	y-intercept = 2 x-intercepts at x=0.35, x=5.65 Coordinates of Minimum point = (3, -7)	16(a)	$x = 10$
		(b)	$2x^2 - 7x + 3 = 0$

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Marking Scheme

MID YEAR EXAMINATION

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For Examiner's Use

1 Simplify

(a) $5a^3 \div 7a^2b$,

$$= \frac{5a^3}{7a^2b} = \frac{5a}{7b} \quad \text{---------- [A1]}$$

Answer (a).....[1]

(b) $(2x^4)^3 \times \frac{1}{64x^4}$.

$$= \frac{8x^{12}}{64x^4} = \frac{1}{8}x^8 \quad \text{---------- [A1]}$$

Answer (b).....[1]

2 Simplify the following, leaving your answers in positive index.

(a) $\left(\frac{3x^0y^3}{4}\right)^3 \div \left(\frac{3x^{-2}y^5}{2}\right)^2$,

$$= \frac{3^3x^0y^9}{4^3} \div \frac{3^2x^{-4}y^{10}}{2^2}$$

$$= \frac{3^3x^0y^9}{4^3} \times \frac{2^2}{3^2x^{-4}y^{10}} \quad \text{---------- [M1]}$$

$$= \frac{3x^4y^{-1}}{4^2} = \frac{3x^4}{16y} \quad \text{---------- [A1]}$$

Answer (a).....[2]

(b) $\frac{12c^{\frac{2}{3}}}{5a^{\frac{1}{2}}b^{-\frac{1}{3}}} \times \frac{3a^{\frac{3}{2}}b}{8c^{\frac{1}{3}}}$.

$$= \frac{9}{10}a^{\frac{3}{2}-\frac{1}{2}}b^{1+\frac{1}{3}}c^{\frac{2}{3}-\frac{1}{3}} \quad \text{---------- [M1]}$$

$$= \frac{9}{10}ab^{\frac{4}{3}}c^{\frac{1}{3}} \quad \text{---------- [A1]}$$

Answer (b).....[2]

For Examiner's Use

3 **(a)** Express 859 nanograms in grams, giving your answer in standard form.

$859\times10^{-9}=8.59\times10^{-7}$ ---------- [A]

Answer (a)..... g [1]

(b) The diameter of a circular organism is 5 micrometres.

(i) Express 5 micrometres in metres.

(ii) Find the area, in square metres, of the circular organism, giving your answer in standard form correct to 3 significant figures.

(i) $5\times10^{-6}=0.000005m$ ---------- [A1]

(ii) Ares = $\pi r^2=3.142\times(5\times10^{-6})^2$ ---------- [M1]

$=78.55\times10^{-12}$

$=7.86\times10^{-11}$ ---------- [A1]

Answer (b) (i) ..... m [1]

(ii) ..... m^2 [2]

4 Solve

(a) $3^{5(x-1)}=27$,

$3^{5x-5}=3^3$ ---------- [M1]

$5x-5=3$

$x=\frac{8}{5}$ ---------- [A1]

Answer (a) x =.....[2]

(b) $216^{x-1}=\frac{1}{36^{x+4}}$.

$6^{3(x-1)}=6^{-2(x+4)}$ ---------- [M1]

$3x-3=-2x-8$

$5x=-5$

$x=-1$ ---------- [A1]

Answer (b) x = ...[2]

For Examiner's Use

5 Given that $p = 4.52 \times 10^8$ and $q = 6.12 \times 10^7$, evaluate the following. Give your answers in standard form correct to 3 significant figures.

(a) $2p - q$,

$= 2 \times 4.52 \times 10^8 - 6.12 \times 10^7$

$= 9.04 \times 10^8 - 0.612 \times 10^8$ ---------- [M1]

$= 8.428 \times 10^8$

$= 8.43 \times 10^8$ ---------- [A1]

Answer (a).....[2]

(b) $\frac{5p}{2q}$.

$= \frac{5 \times 4.52 \times 10^8}{2 \times 6.12 \times 10^7}$ ---------- [M1]

$= 1.8464 \times 10$

$= 1.85 \times 10$ ---------- [A1]

Answer (b).....[2]

6 Ms Chong deposits \$42, 000 in ACBC Bank which pays an interest rate of 3% p.a compounded monthly. Calculate how much interest she can get at the end of one year.

$$\text{Total amount} = \text{P}\left(1 + \frac{r}{100}\right)^n$$

$$= 42000 \times (1 + \frac{\frac{3}{12}}{100})^{12} \text{ ---------- [M1]}$$

$$= 42000 \times (1 + 0.0025)^{12}$$

$$= \$\ 43277.47019$$

$\text{Interest} = \$\ 43277.47019 - 42000$

$= \$\ 1277.47$ ---------- [A1]

Answer \$.....[2]

For Examiner's Use

7 Solve the following equations, giving your answers correct to 2 decimal places.

(a) $\frac{2}{x+5}-\frac{3}{x-5}=4$,

$$\frac{2(x-5)-3(x+5)}{(x+5)(x-5)}=4$$

$$2x-10-3x-15=4(x^2-25)$$
$$4x^2+x-75=0 \quad \text{---------- [M1]}$$

Students are expected to use the quadratic formula:

$a=4 \qquad b=1 \qquad c=-75$

$$x=\frac{-1\pm\sqrt{1^2-4(4)(-75)}}{2(4)} \quad \text{---------- [M1]}$$

$x=4.2069$
$=4.21(2d.p)$ [A1]

$x=-4.4569$
$=-4.46(2d.p)$ [A1]

Answer (a) $x=$.....or.....[4]

(b) Solve $(4x-3)(3x+2)=5x+1$.

$$12x^2+8x-9x-6=5x+1$$
$$12x^2-6x-7=0 \quad \text{---------- [M1]}$$

$a=12 \qquad b=-6 \qquad c=-7$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(12)(-7)}}{2(12)} \quad \text{---------- [M1]}$$

$x=1.05363$
$=1.05(2d.p)$ [A1]

$x=-0.5536$
$=-0.55(2d.p)$ [A1]

Answer (b) $x=$.....or.....[4]

For Examiner's Use

8 **(a)** Express $y = x^2 - 6x + 2$ in the form of $y = (x-h)^2 + k$.

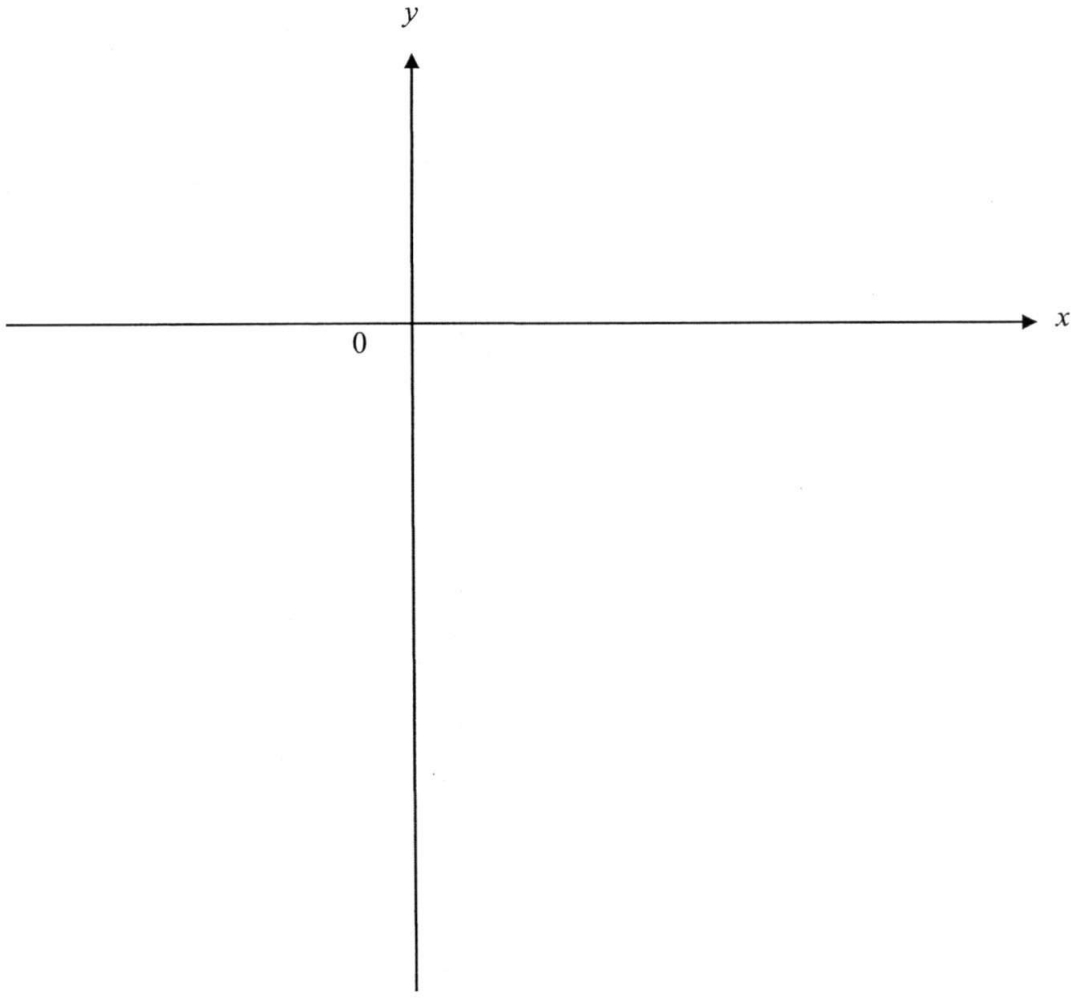
$$y = x^2 - 6x + (\frac{-6}{2})^2 + 2 - (\frac{-6}{2})^2 \quad \text{---------- [M1]}$$

$$y = (x-3)^2 - 7 \quad \text{---------- [A1]}$$

Answer (a).....[2]

(b) Sketch the graph of $y = x^2 - 6x + 2$ indicating clearly its intercepts with the axes and its turning point. [2]

Shape of the graph [A1]
y-intercept = 2
x-intercepts at x=0.35, x=5.65
Coordinates of Minimum point = (3, -7) ---------- [A1]



For Examiner's Use

For Examiner's Use

9 **(a)** Sketch the graph of $y = -x^2 - 3x + 4$ indicating clearly its intercepts with the axes and its turning point. [2]

(b) Write down the equation of the line of symmetry of the graph.

For Examiner's Use

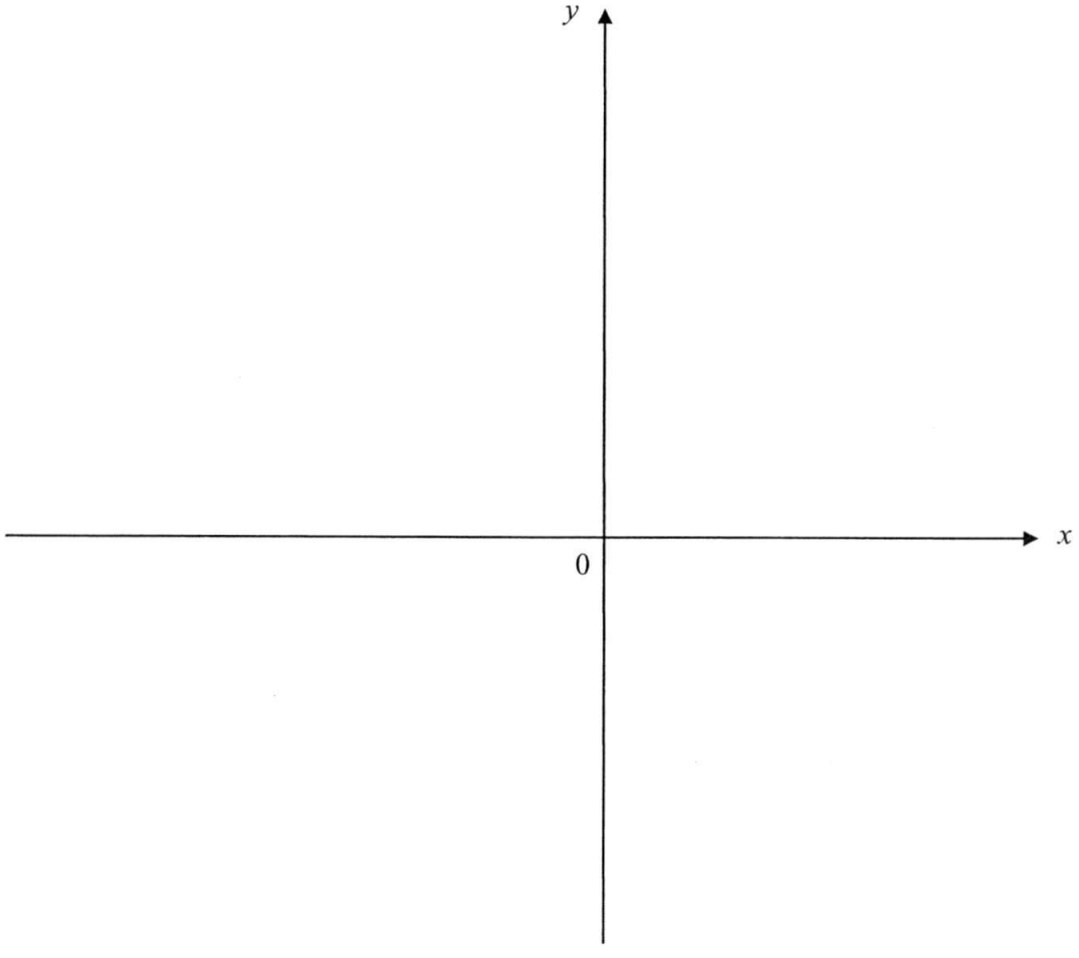
(a) $y = (-x + 1)(x + 4)$

Shape of graph [A1]

x-intercepts at $x = -4$ and $x = 1$

y-intercept = 4

Coordinates of maximum point = (-1.5, 6.25) ---------- [A1]



Answer (b) Equation of line of symmetry $x = -1.5$ [A1]

For Examiner's Use

10 It is given that x and y are integers such that $1 \le x \le 9$ and $-3 \le y < 0$.
Find

(a) the largest possible value of $x-y$,

$9-(-3)=12$ ---------- [A1]

Answer (a) .....[1]

(b) the least possible value of $\frac{y}{x}$,

$\frac{-3}{1}=-3$ ---------- [A1]

Answer (b) .....[1]

(c) the largest possible value of $\frac{1}{x^2+y^2}$.

$\frac{1}{1^2+0^2}=1$ ---------- [A1]

Answer (c) .....[1]

11 Given that $\frac{x-11}{2} < \frac{3-2x}{5} \le \frac{x+7}{3}$, find

(a) the range of values of x that satisfy the inequality and represent your solutions on a number line.

(b) the least integer value of x.

(c) the greatest prime value of x.

(a) $\frac{x-11}{2} < \frac{3-2x}{5}$

$5(x-11) < 2(3-2x)$
$5x-55 < 6-4x$
$9x < 61$ [M1]
$x < 6\frac{7}{9}$

$\frac{3-2x}{5} \le \frac{x+7}{3}$

$3(3-2x) \le 5(x+7)$
$9-6x \le 5x+35$
$-26 \le 11x$ [M1]
$-2\frac{4}{11} \le x$

Answer (a) $-2\frac{4}{11} \le x < 6\frac{7}{9}$ .....[A1]

Number Line _____

[A1]

Answer (b) .....-2.....[1]

Answer (c) .....5.....[1]

For Examiner's Use

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12 A sequence of 5 diagrams is shown below.

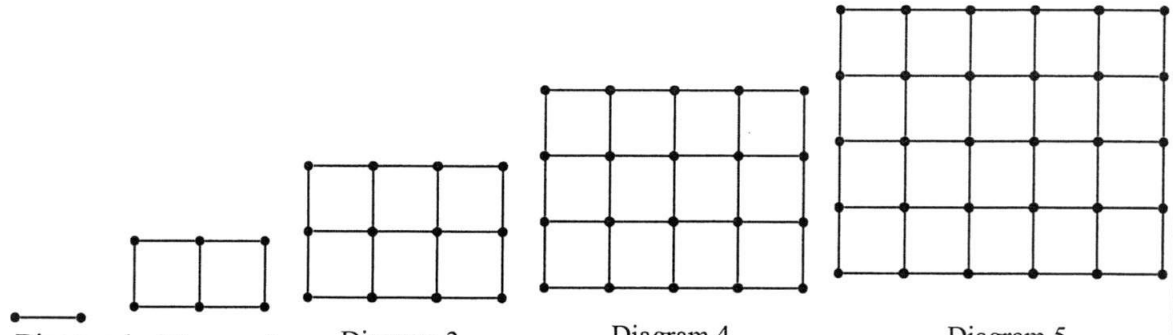

Diagram 1 Diagram 2 Diagram 3 Diagram 4 Diagram 5

The number of dots and lines in each of the diagrams are shown in the table below.

Diagram number	1	2	3	4	5	6
Number of dots	2	6	12	20	30	p
Number of lines	1	7	17	31	49	q

(a) Find the value of p and of q.

(b) Write down an expression for the number of dots in diagram n.

(c) The number of lines in diagram n is $2n^2 - 1$.
Find the diagram number which has 287 lines.

(a) $p = 30 + 12 = 42$ ----------[A1]
$q = 49 + 22 = 71$ ----------[A1]

(b) Diagram 1 = 1 x 2 = 2
Diagram 2 = 2 x 3 = 6
Diagram 3 = 3 x 4 =12
Diagram n = n(n + 1) = $n^2 + n$ ---------- [A1]

(c) $2n^2 - 1 = 287$
$2n^2 - 288 = 0$ ---------- [M1]
$n^2 = 144$
$n = 12$
[A1]

Answer (a) p = ...[1]

q = ...[1]

Answer (b).....[1]

Answer (c).....[2]

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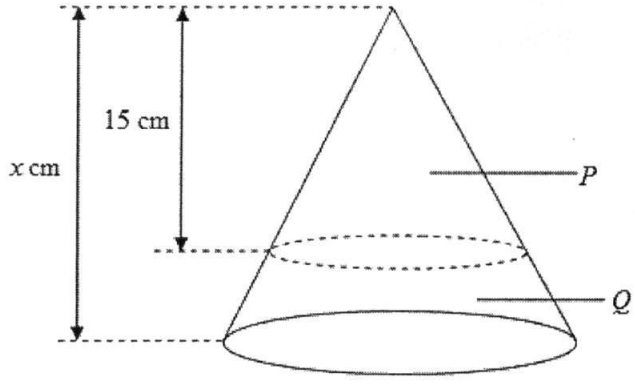
For Examiner's Use

13 The following diagram shows a solid cone that is cut up into 2 sections, P and Q, such that section P is a cone similar to the original cone. The curved surface area of cone P and the original cone is 160 cm^2 and 250 cm^2 respectively.



(a) If the height of cone P is 15 cm, calculate the height, x cm, of the original cone.

(b) Given that the mass of cone P is 12.8 g, find the mass of section Q.

(a) Ratio of the length $= \sqrt{\frac{250}{160}}$ ---------- [M1]

$= 5 : 4$

$\frac{x}{15} = \sqrt{\frac{250}{160}}$

$x = \frac{5}{4} \times 15$ ---------- [A1]

$x = \frac{75}{4} = 18\frac{3}{4} cm$

(b) Ratio of the mass of the original cone to the mass of cone P

$\frac{M_1}{M_2} = \left(\frac{5}{4}\right)^3$ ---------- [M1]

$= 125 : 64$

Mass of original cone

$= \frac{125 \times 12.8}{63} = 25kg$

Mass of Section Q = $25 - 12.8 = 12.2kg$ ---------- [A1]

Answer (a) ..... cm [2]

Answer (c) ..... kg [2]

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14 In the diagram below, ΔADG and ΔEHC overlap to form a parallelogram $BDFH$. Given further that $AG = EC$ and $\angle HCE = \angle DGA$, prove that ΔADG is congruent to ΔEHC. State your reasons clearly.

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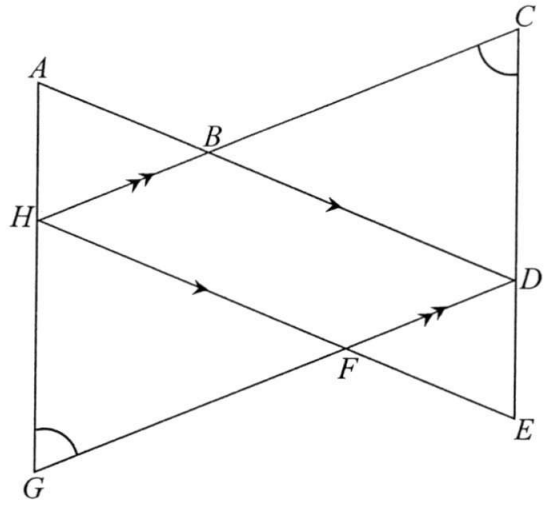


$\angle HCE = \angle DGA$ (given))

$AG=EC$ (given)) ---------- [M1]

$\angle CHE = \angle DGA$ (opp. $\angle$ s of a parallelogram) ---------- [M1]

Hence $\Delta ADG \equiv \Delta EHC$ (AAS or SAA or ASA) ---------- [A1]

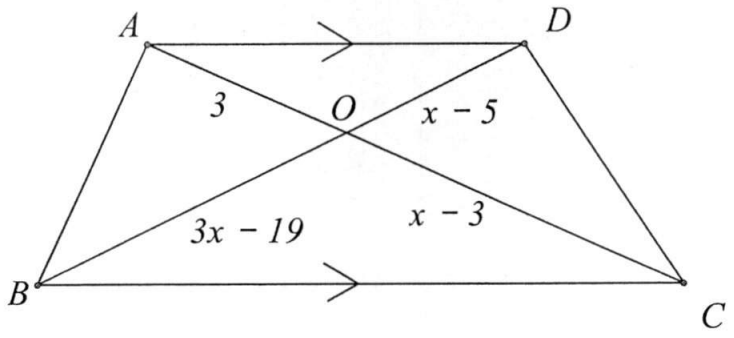
Answer .....

.....

.....

.....[3]

15 $ABCD$ is a trapezium in which AD is parallel to BC.
The diagonals AC and BD intersect at O.



(a) Name the triangle which is similar to ΔAOD.

Answer (a) ..... ΔCOB .....[1]

(b) If $AO = 3\ cm$, $BO = (3x - 19)\ cm$, $CO = (x - 3)\ cm$ and $DO = (x - 5)\ cm$, find the values of x.

$$\frac{3}{x-3} = \frac{x-5}{3x-19}$$

$$x^2 - 17x + 72 = 0$$

$$(x - 8)(x - 9) = 0$$

$$x = 8 \text{ or } x = 9$$

Answer (b) x = ... or .....[3]

END OF PAPER

Answer Key

1(a)	$\frac{5a}{7b}$	9(a)	$y=(-x+1))(x+4)$ x-intercepts at $x=-4$ and $x=1$ y-intercept = 4 Coordinates of maximum point = (-1.5, 6.25)
(b)	$\frac{x^8}{8}$	(b)	Equation of line of symmetry $x=-1.5$
2(a)	$\frac{3x^4}{16y}$	10(a)	12
(b)	$\frac{9}{5}ab^{\frac{4}{3}}c^{\frac{1}{3}}$	(b)	-3
3(a)	8.59×10^{-7} g	(c)	1
(b)(i)	$5\times10^{-6}=0.000005$ m	11(a)	$-2\frac{4}{11}\le x<6\frac{7}{9}$
(ii)	7.96×10^{-11}m^2	(b)	2
4(a)	$x=\frac{8}{5}$	(c)	5
(b)	$x=-1$	12(a)	$p=30+12=42$ $q=49+22=71$
5(a)	8.43×10^8	12(b)	Diagram 1 = 1 x 2 = 2 Diagram 2 = 2 x 3 = 6 Diagram 3 = 3 x 4 =12 Diagram n = n(n + 1) = n^2 + n
(b)	1.85×10	12(c)	n = 12
6	\$1277.47	13(a)	Ratio of the length = 5 : 4 height original cone x = $18\frac{3}{4}cm$
		(b)	Ratio of the volume of original cone to the volume of cone P = 125 : 64 Mass of original cone = 25 *kg* Mass of the frustum = 12.2 *kg*
7(a)	4.21 or -4.46	14	$\angle HCE=\angle DGA$ (given) $AG=CE$ (given) $\angle CHE=\angle DGA$ (opp. $\angle$ of a parallelogram) $\Delta ADG\equiv EHC$ (AAS or SAA or ASA)
(b)	1.05 or -0.55		

8(a)	$y = (x-3)^2 - 7$	15(a)	ΔCOB
(b)	y-intercept = 2 x-intercepts at x=0.35, x=5.65 Coordinates of Minimum point = (3, -7)	(b)	$\frac{3}{x-3} = \frac{x-5}{3x-19}$ $x^2 - 17x + 72 = 0$ $(x-8)(x-9) = 0$ $x = 8$ *or* $x = 9$

Calculator Model :

Class	Full Name	Index Number

bowen
SECONDARY
I believe, therefore I am

MID YEAR EXAMINATION 2016

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4048/02

MATHEMATICS
Paper 2

Secondary 3 Express
10th May 2016 **2 hours**

Additional Materials: Writing Papers
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use
80

This document consists of 7 printed pages, including this cover page.

Setter: Ms Melissa Chong

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Find the value of $\dfrac{\sqrt{6.43\times10^{4}-2.78\times10^{-2}}}{4.2^{\frac{2}{7}}}$, correct to 2 decimal places. [1]

(b) In Singapore, the number of HDB households in 2013 was 1.175×10^{6}. The average number of people living in each household was 3.47.

(i) Estimate the total number of people living in HDB households in Singapore in 2013. [1]

Leave your answer in standard form, correct to 3 significant figures.

(ii) If the number of households increases by 1.8% per year, find the total number of HDB households in Singapore in 2015. [2]

Leave your answer in standard form, correct to 3 significant figures.

(c) Simplify the following and express your answers in positive index notation.

(i) $\left(\dfrac{3p^{2}}{2pq^{0}}\right)^{2}$ [2]

(ii) $\dfrac{x^{2}}{4y}\div\dfrac{6x^{-3}}{y^{3}}$ [2]

(iii) $(2a^{2}b^{-\frac{2}{3}})^{3}\times\sqrt[3]{8ab}$ [2]

2 **(a)** **(i)** Solve $5^{2x-3}=1$. [1]

(ii) Given that $3^{x+3}\times\left(\dfrac{1}{9}\right)^{x}=27^{4}$, find the value of x. [2]

(b) It is given that $W=\dfrac{1}{2}m(v^{2}-u^{2})$.

(i) Find W when $m=3$, $u=4$ and $v=10$. [1]

(ii) Express u in terms of W, m and v. [2]

(c) Factorise completely $18p^{2}-8$. [2]

3 **(a)** Solve the equation $\frac{3x-7}{4}+\frac{1}{x}=\frac{x}{4}$, giving your answers to 3 decimal places. [3]

(b) Given that $\frac{x+2y}{x+5y}=\frac{3}{7}$, find the value of $\frac{3y}{2x}$ [3]

(c) **(i)** Express x^2-7x+3 in the form $(x-a)^2+b$ by completing the square. [2]

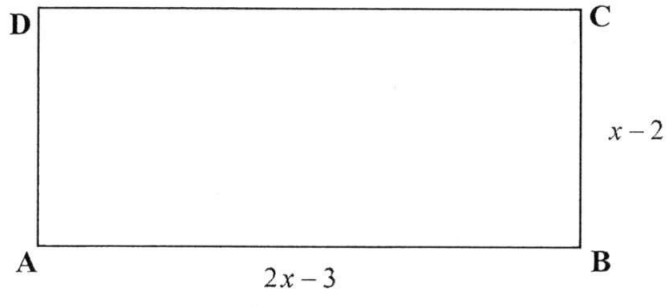
(ii) **Hence,** solve the equation $x^2-7x+3=0$, giving your answer correct to 2 decimal places. [3]

4 **(a)** Jonathan invested some money in a bank which pays simple interest at a rate of 4.5% per annum. He would be able to receive \$15 735.75 in total (including interest) 5 years later. How much money did Jonathan invest in the bank initially? [2]

(b) At the same time, Jonathan also invested \$8 000 in another bank that pays compound interest at a rate of 2.5% per annum compounded half-yearly. How much money will Jonathan get back at the end of 3 years? [2]

5 **(a)** Solve the simultaneous inequalities $3(2x-1)<2(7+5x)$ and $\frac{x-1}{3}\leq\frac{x-4}{7}$. [3]

(b) The diagram shows a rectangle **ABCD** whereby its perimeter is at most 40 cm.

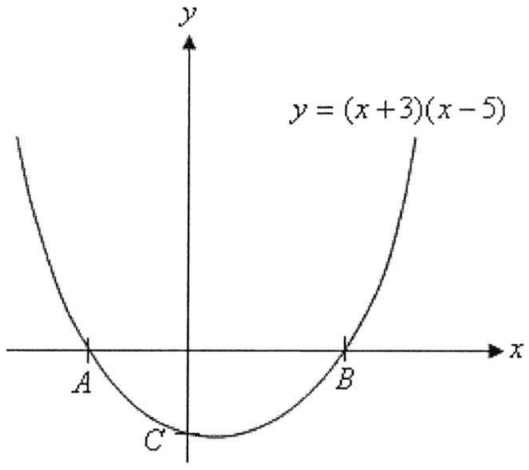


(i) Form an inequality in terms of x. [1]

(ii) Solve the inequality. [1]

(iii) Determine the greatest possible length of **AB** if x is an integer. [1]

6 The curve $y=(x+3)(x-5)$ cuts the x-axis at A and B, and the y-axis at C.



Find

(a) the coordinates of A and B, [2]

(b) the coordinates of C, [1]

(c) the equation of the line of symmetry, [1]

(d) the coordinates of the minimum point of the curve. [1]

7 Alice and Betty started cycling together for a 10 km journey at their respective constant speed.

Alice rode at a speed of x km/h, while Betty's speed was 1 km more than Alice's.

(a) Write down an expression in terms of x for the time, in hours, Alice took to complete the entire journey. [1]

(b) Write down an expression in terms of x for the time, in hours, Betty took to complete the entire journey. [1]

(c) Given that Betty finished the journey 15 minutes earlier than Alice, form an equation in x and show that it reduces to $x^2 + x - 40 = 0$. [3]

(d) Solve the equation $x^2 + x - 40 = 0$, giving both answers correct to 2 decimal places. [4]

(e) Find the time in hours and minutes, which Alice took to complete the 10 km journey. [3]

Answer the whole of this question on a sheet of graph paper.

8 The following table gives the corresponding values of x and y which are connected by the equation $y=8+2x-x^2$

x	−3	−2	−1	0	1	2	3	4	5
y	−7	0	p	8	9	8	5	0	−7

(a) Find the value of p. [1]

(b) Draw the graph of $y=8+2x-x^2$. [3]

Using a scale of 2 cm to represent 1 unit on the x-axis, draw a horizontal x-axis for $-3 \leq x \leq 5$ and a scale of 2 cm to represent 2 units on the y-axis, draw a vertical y-axis for $0 \leq y \leq 10$.

On your axes, plot the points given in the table and join them with a smooth curve.

(c) State the equation of the line of symmetry. [1]

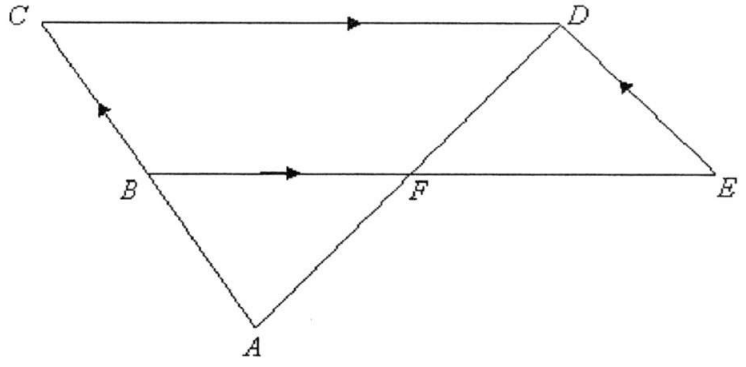
(d) For the range of $-3 \leq x \leq 5$, use your graph to

(i) solve the equation $8+2x-x^2=0$ [2]

(ii) find the values of x when $y=2$, [2]

(iii) find solutions of the equation $4+2x-x^2=0$ by drawing a suitable straight line on the same axes as your graph. [3]

9 **(a)** In the figure below, *BCDE* is a parallelogram. *CB* is extended to meet *DF* extended at *A*. *B* is the mid-point of *AC*.



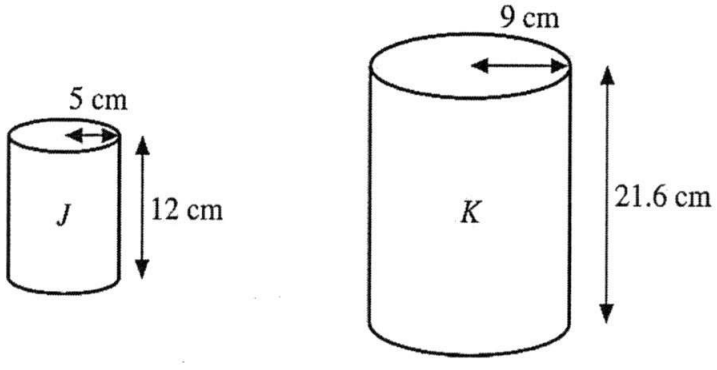
(i) Prove, stating the reasons clearly, that ΔDEF and ΔABF are congruent. [2]

(ii) Prove, stating the reasons clearly, that ΔDEF and ΔACD are similar. [2]

(iii) If $CD = 18$ cm, find FE. [2]

(iv) Given that the area of ΔDEF is 21 cm^2, find the area of the quadrilateral *BCDF*. [2]

(b) The diagram shows two geometrically similar cylinders with their dimensions. These cylinders are made with the same material.



(i) Find the ratio of the total surface area of cylinder **J** that of cylinder **K**. [1]

(ii) The cost of painting cylinder **J** is $10.45, find the cost of painting cylinder **K**. [1]

(iii) The containers are completely filled with water. Given that cylinder **K** holds 2.5 litres of water, calculate the capacity of cylinder **J**, correcting your answer to 2 decimal places. [2]

END OF PAPER 2

Answers to Sec 3 MYE Paper 2 2016				
1	(a)		168.28 [A1]	[1]
	(b)	(i)	$1.175\times10^6\times3.47$ $=4077250$ $=4.08\times10^6$ ----[A1]	[1]
		(ii)	$1.175\times10^6\times\frac{101.8}{100}$ $=1196150(2014)$ ----[M1] $1196150\times\frac{101.8}{100}$ $=1.22\times10^6(3sf)$ ----[A1]	[2]
	(c)	(i)	$(\frac{3p^2}{2pq^0})^{-2}$ $=(\frac{2pq^0}{3p^2})^2$ ----[M1] $=\frac{4}{9p^2}$ ----[A1]	[2]
		(ii)	$\frac{3x^2}{4y}\div\frac{6x^{-3}}{y^3}$ $=\frac{3x^2}{4y}\times\frac{y^3}{6x^{-3}}$ $=\frac{3x^2x^3y^3}{24y}$ ----[M1] $=\frac{x^5y^2}{8}$ ----[A1]	[2]
		(iii)	$(2a^2b^{-\frac{2}{3}})^3\leftarrow\sqrt[3]{8ab}$ $=(8a^6b^{-2})\leftarrow2a^{\frac{1}{3}}b^{\frac{1}{3}}$ ----[M1] $=16a^{\frac{19}{3}}b^{-\frac{5}{3}}$ $=\frac{16a^{\frac{19}{3}}}{b^{\frac{5}{3}}}$ ----[A1]	[2]
2	(a)	(i)	$5^{2x-3}=1$ $5^{2x-3}=5^0$ $2x-3=0$ $2x=3$ $x=\frac{3}{2}$ ----[A1]	[1]

		(ii)	$3^{x+3} \leftrightarrow (\frac{1}{9})^x = 27^4$ $3^{x+3} \times 3^{-2x} = 3^{12}$ ----[M1] $x+3-2x=12$ $-x=9$ $x=-9$ ----[A1]	[2]
	(b)	(i)	$W=126$	[1]
		(ii)	$W=\frac{1}{2}m(v^2-u^2)$. $2W=mv^2-mu^2$ $mu^2=mv^2-2W$ ----[M1] $u=\pm\sqrt{\frac{mv^2-2W}{m}}$ ----[A1]	[2]
	(c)		$18p^2-8$ $=2(9p^2-4)$----[M1] $=2(3p-2)(3p+2)$----[A1]	[2]
3	(a)		$\frac{3x-7}{4}+\frac{1}{x}=\frac{x}{4}$ $\frac{x(3x-7)+4}{4x}=\frac{x}{4}$ $\frac{3x^2-7x+4}{4x}=\frac{x}{4}$ $4(3x^2-7x+4)=4x^2$ $12x^2-28x+16=4x^2$ $8x^2-28x+16=0$ ----[M1] $x=\frac{28\pm\sqrt{(-28)^2-4(8)(16)}}{2(8)}$ $=\frac{28\pm\sqrt{272}}{16}$ $\therefore x=\frac{28+\sqrt{272}}{16}$ *or* $\frac{28-\sqrt{272}}{16}$ $=2.781$ *or* $0.719(3dp)$ ----[A2]	[3]
	(b)		$\frac{x+2y}{x+5y}=\frac{3}{7}$	[3]

			$7(x+2y)=3(x+5y)$ $7x+14y=3x+15y$ $4x=y$----[M1] $\frac{x}{y}=\frac{1}{4}$ $\frac{y}{x}=4$----[M1] $\frac{3y}{2x}$ $=(\frac{3}{2})4$ $=6$----[A1]	
	(c)	(i)	x^2-7x+3 $=x^2-7x+(\frac{-7}{2})^2+3-(\frac{-7}{2})^2$----[M1] $=(x-\frac{7}{2})^2-\frac{37}{4}$----[A1]	[2]
		(ii)	$(x-\frac{7}{2})^2-\frac{37}{4}=0$ $(x-\frac{7}{2})^2=\frac{37}{4}$----[M1] $x-\frac{7}{2}=\sqrt{\frac{37}{4}} or -\sqrt{\frac{37}{4}}$ $x=\sqrt{\frac{37}{4}}+\frac{7}{2}$ *or* $-\sqrt{\frac{37}{4}}+\frac{7}{2}$ $=6.54$ *or* 0.46----[A2]	[3]
4	(a)		$\frac{P(4.5)(5)}{100}=15735.75-P$----[M1] $P(22.5)=100(15735.75-P)$ $22.5P=1573575-100P$ $122.5P=1573575$ $P=12845.51(2dp)$----[A1]	[2]
	(b)		Amount $=8000(1+\frac{2.5/2}{100})^6$----[M1] $=8619.07(2dp)$----[A1]	[2]
5	(a)		$3(2x-1)<2(7+5x)$	[3]

			$6x-3<14+10x$ $-4x<17$ $x>-\frac{17}{4}$ $x>-4\frac{1}{4}$----[M1] $\frac{x-1}{3}\leq\frac{x-4}{7}$ $7x-7\leq 3x-12$ $7x-3x\leq -12+7$ $4x\leq -5$ $x\leq -\frac{5}{4}$----[M1] $\therefore -4\frac{1}{4}<x\leq -\frac{5}{4}$----[A1]	
	(b)	(i)	$2x-3+2x-3+x-2+x-2\leq 40$----[A1]	[1]
		(ii)	$6x-10\leq 40$ $6x\leq 50$ $x\leq\frac{50}{6}$ $x\leq 8\frac{1}{3}$----[A1]	[1]
		(iii)	Greatest possible x = 8 Greatest possible AB = 2(8)-3 = 13cm ----[A1]	[1]
6	(a)	(i)	When y = 0, $(x+3)(x-5)=0$ $x+3=0$ *or* $x-5=0$ $x=-3$ *or* 5 $\therefore A(-3,0), B(5,0)$----[A2]	[2]
		(ii)	When x = 0, $(0+3)(0-5)=y$ $y=(3)(-5)$ $=-15$ $\therefore C(0,-15)$----[A1]	[1]
		(iii)	x = 1	[1]
		(iv)	x-coordinate of minimum point = 1 When x = 1,	[1]

			$(1+3)(1-5)=y$ $y=(4)(-4)$ $=-16$ $\therefore (1,-16)----[A1]$	
7		(i)	$\frac{10}{x}h$	[1]
		(ii)	$(\frac{10}{x+1})h$	[1]
		(iii)	$\frac{10}{x+1}+\frac{1}{4}=\frac{10}{x}----[M1]$ $\frac{40+x+1}{4(x+1)}=\frac{10}{x}$ $\frac{41+x}{4x+4}=\frac{10}{x}$ $x(41+x)=10(4x+4)$ $x^2+41x=40x+40----[M1]$ $x^2+x-40=0(shown)----[A1]$	[3]
		(iv)	$x^2+x-40=0$ $x=\frac{-1\pm\sqrt{1^2-4(1)(-40)}}{2}----[M1]$ $=\frac{-1\pm\sqrt{161}}{2}----[M1]$ $=5.8442 or -6.8442$ $=5.84 or -6.84(2dp)----[A2]$	[4]
		(v)	Reject x = -6.84 Time taken by Alice $=\frac{10}{5.8442}----[M1]$ $=1.71109h----[M1]$ $=1h\ 43\min----[A1]$	[3]
8			refer to graph paper	

9	(a)	(i)	DE = CB (opp. sides of //gram)
			Since CB = AB (B is midpoint of AC), DE = AB.
			Angle DFE = Angle AFB (vert. opp. angles)
			Angle DEF = Angle ABF (alt. angles)
			ΔDEF and ΔABF are congruent (ASA – 1m)
				[2]

1m

		(ii)	Angle DEF = Angle ACD (opp. angles of //gram) Angle EDF = Angle CAD (alt. angles) } 1m ΔDEF and ΔACD are similar (AAA – 1m).	[2]
		(iii)	Since ΔDEF and ΔACD are similar, $\frac{CD}{EF} = \frac{AC}{DE} = \frac{2}{1}$ ----[M1] $\frac{18}{EF} = \frac{2}{1}$ $EF = 9cm$ ----[A1]	[2]
		(iv)	$\frac{Area\ of\ DEF}{Area\ of\ ACD} = (\frac{DE}{AC})^2 = (\frac{1}{2})^2$ $\frac{21}{Area\ of\ ACD} = \frac{1}{4}$ $Area\ of\ ACD = 84cm^2$ ----[M1] Since area of ABF = area of DEF = 21cm^2, area of BCDF = (84 – 21)cm^2- = 63 cm^2---[A1]	[2]
	(b)	(i)	25:81 ----[A1]	[1]
		(ii)	($10.45÷25)x 81 =$33.85----[A1]	[1]
		(iii)	Ratio of volume = 125:729----[M1] Capacity of cylinder J = (2.5÷729)x125 =0.43 litres of water ----[A1]	[2]