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Class	Index Number	Name
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新加坡海星中学

MARIS STELLA HIGH SCHOOL
SEMESTRAL ASSESSMENT ONE
SECONDARY THREE

MATHEMATICS

06 May 2016

2 hours 30 minutes

Additional Materials:

Writing paper (3 sheets)

Graph paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, submit Section A and B **separately**.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use			
	Section A	Section B	
Subtotal			100
Presentation			
Unit			
Rounding off			

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

SECTION A (56 MARKS)

Answer **ALL** the questions in the spaces provided.

1. Given that $-1 \leq a < 3$ and $-5 \leq b \leq -1$, where a and b are integers, find

(a) the greatest value of ab ,

Answer (a) [1]

(b) the smallest possible value of $\frac{a}{b^2}$, given that $\frac{a}{b^2}$ is a perfect cube.

Answer (b) [1]

2. Simplify $\frac{(p^{-1}q^2)^5 \div (2pr^{-4})}{(3qr)^3 (2p^{-2}q^3)^0}$, leaving your answer in positive index notation.

Answer [3]

3. (a) Factorise $169 + 52q + 4qr - r^2$ completely.

Answer (a) [2]

- (b) Make x the subject of the equation $2x^2 = \frac{y-x^2}{2y} + 1$.

Answer (b) $x =$ [3]

4. The curve $y = 2(x+2)(x+k)$ cuts the y -axis at $(0, -8)$.

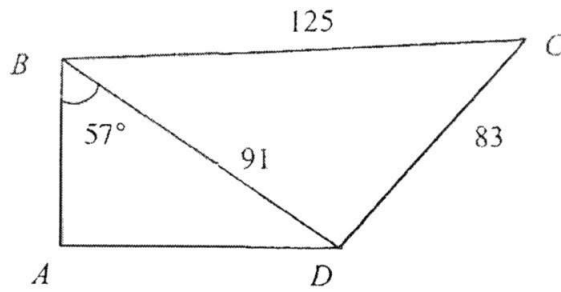
- (a) Find the value of k .

Answer (a) $k =$ [2]

- (b) Using the value of k found in part (a), find the coordinates of the points where the curve cuts the x -axis.

Answer (b) [2]

5. The figure below represents a plot of land $ABCD$. B is due north of A , D is due east of A . $BC = 125$ m, $CD = 83$ m and $BD = 91$ m. Angle $ABD = 57^\circ$.



- (a) Find angle BDC .

Answer (a) $^\circ$ [3]

- (b) Find the bearing of D from C .

Answer (b) $^\circ$ [2]

6. (a) Solve the inequality $\frac{x-35}{3} < 5-3x \leq 2(4-x)$. Represent your answer on the number line below.



Answer (a) [4]

- (b) Write down the largest prime number which satisfies

$$\frac{x-35}{3} < 5-3x \leq 2(4-x).$$

Answer (b) $x =$ [1]

7. A hard disk has a memory capacity of 2 terabytes.

- (a) If a low-resolution photograph takes up about 250 kilobytes, how many million photographs can be stored in the hard disk?

Answer (a) million [2]

- (b) If the hard disk is to store 25 video clips with capacity of 273 megabytes each, how much capacity is left in the disk? Give your answer in standard form, correct to 3 significant figures.

Answer (b) bytes [3]

8. (a) Express $\frac{5}{12x+9} - \frac{2x+1}{9-16x^2}$ as a single fraction.

Answer (a) [3]

- (b) Hence, solve $\frac{5}{12x+9} - \frac{2x+1}{9-16x^2} = 1$.

Answer (b) $x =$ or [3]

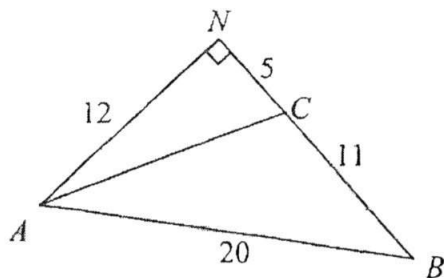
9. (a) Solve $\frac{25^x}{5} - \sqrt{\frac{1}{625}} = 0$.

Answer (a) $x = \dots\dots\dots$ [3]

(b) Find the value of m for which $\sqrt{a^2} \sqrt{a^{\frac{1}{3}}} = a^m$.

Answer (b) $m = \dots\dots\dots$ [3]

10. In the diagram, ABN is a right-angled triangle, and BCN is a straight line.
 $AN = 12$ cm, $CN = 5$ cm, $BC = 11$ cm and $AB = 20$ cm.



Calculate

- (a) the length of AC ,

Answer (a) cm [1]

- (b) $\cos \angle ACB$,

Answer (b) [2]

- (c) angle BAC .

Answer (c)° [3]

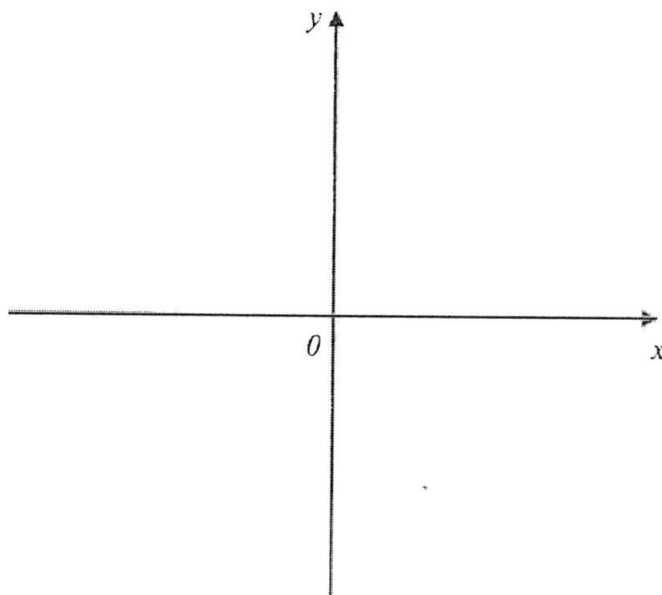
11. (a) Express $-x^2 + 6x - 3$ in the form $-(x+b)^2 + c$.

Answer (a) [3]

(b) Hence, solve the equation $x^2 + 3 = 6x$.

Answer (b) $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

(c) Sketch the graph $y = -x^2 + 6x - 3$ on the axes provided, labeling your turning point, x-intercepts and y-intercept clearly. [3]

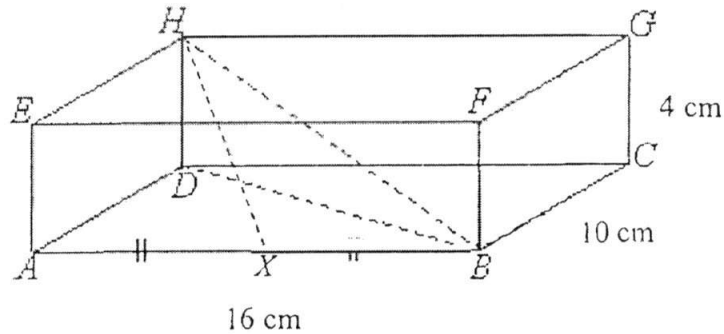


END OF SECTION A

SECTION B (44 MARKS)

Answer **ALL** the questions on the writing papers provided.

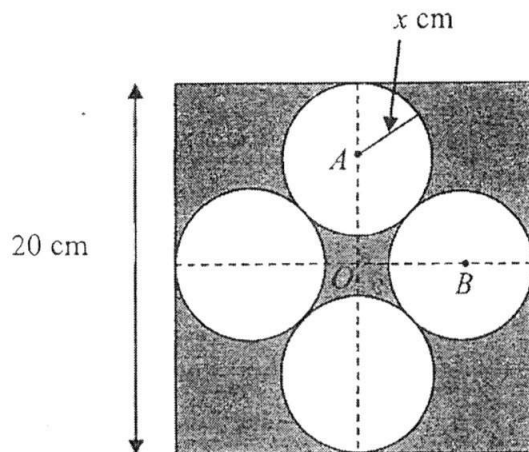
12. The diagram below shows a cuboid of dimensions 16 cm by 10 cm by 4 cm. Point X lies on AB such that $AX = XB$ and angle $HBX = 33.9^\circ$.



Calculate

- (a) the length of BH , [3]
 (b) angle HBD , [2]
 (c) HX , [2]
 (d) angle DHX . [2]

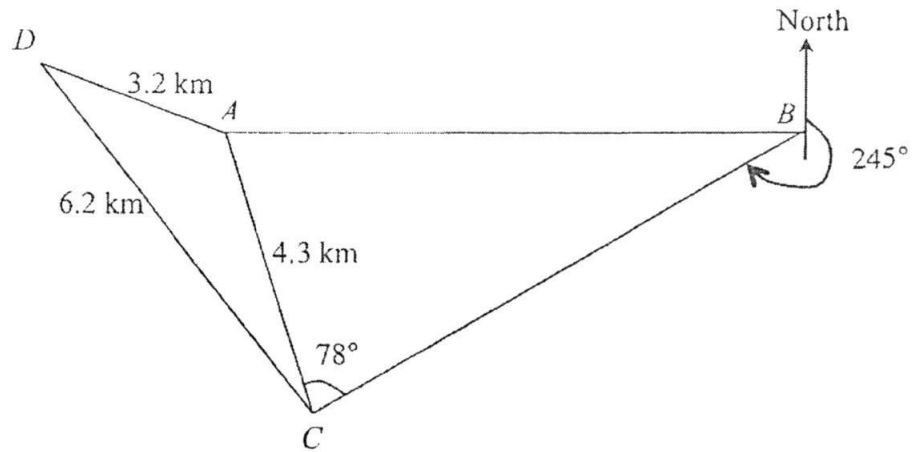
- 13.



The diagram shows a square of length 20 cm. O is the intersection of the diagonals of the square. Four smaller identical circles, touching each other, with radius x cm are drawn as shown. A and B are the centres of two of the smaller circles.

- (a) Express the length of OA and of AB in terms of x . [2]
 (b) Use Pythagoras' Theorem to form an equation in x and show that it can be simplified to $x^2 + 20x - 100 = 0$. [3]
 (c) Solve the equation $x^2 + 20x - 100 = 0$, giving your answers correct to 2 decimal places. [3]
 (d) Calculate the area of the shaded region. [2]

14.



In the diagram, A , B , C and D are four points on level ground. It is given that $AC = 4.3$ km, $AD = 3.2$ km, $CD = 6.2$ km, angle $ACB = 78^\circ$, the bearing of C from B is 245° and B is due east of A . Calculate

- (a) angle ACD , [3]
- (b) the bearing of B from C , [2]
- (c) the length of AB , [3]
- (d) the area of triangle ADC . [2]

Alex walked from D to C along the path DC .

- (e) He stopped at point X when he is at the shortest distance to point A . Calculate AX . [2]
- (f) At point X , Alex saw a building standing vertically at point A . Given that the height of the building is 980 m tall, calculate the angle of elevation of the top of building when viewed by Alex. [2]

15. **Answer the whole of this question on a piece of graph paper.**

The variables x and y are connected by the equation $y = 2x^2 - 5x - 3$. Some corresponding values are given in the following table.

x	-2	-1	0	0.5	1	2	3	4
y	a	4	-3	-5	-6	-5	0	9

- (a) Calculate the value of a . [1]
- (b) Taking 2 cm to represent 1 unit on the x -axis and 2 cm to represent 5 units on the y -axis, draw the graph of $y = 2x^2 - 5x - 3$ for the range $-2 \leq x \leq 4$. [3]
- (c) From your graph, find
- (i) the value(s) of x when $y = 5$, [2]
- (ii) the minimum value of y . [1]
- (d) By adding suitable line(s) on the same graph paper, find
- (i) the values of x for which $2x^2 - 5x - 3 = 0$, [1]
- (ii) the solutions for the equation $2x^2 - 7x = 0$. [3]

END OF PAPER

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	<p>新加坡海星中学</p> <p>MARIS STELLA HIGH SCHOOL</p> <p>SEMESTRAL ASSESSMENT ONE</p> <p>SECONDARY THREE</p>
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<p>MATHEMATICS [Solution]</p> <p><i>Additional Materials:</i> Writing paper (3 sheets) Graph paper (1 sheet)</p>	<p>06 May 2016</p> <p>2 hours 30 minutes</p>
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SECTION A (56 MARKS)

Answer **ALL** the questions in the spaces provided.

1. Given that $-1 \leq a < 3$ and $-5 \leq b \leq -1$, where a and b are integers, find

- (a) the greatest value of ab ,

[Solution]

$$\begin{aligned} \text{the greatest possible value of } ab &= (-1)(-5) \\ &= 5 \end{aligned}$$

Answer (a)5..... [1]

- (b) the smallest possible value of $\frac{a}{b^2}$, given that $\frac{a}{b^2}$ is a perfect cube.

[Solution]

$$\begin{aligned} \text{the smallest possible value of } \frac{a}{b^2} &= \frac{-1}{(-1)^2} \\ &= -1 \end{aligned}$$

Answer (b)-1..... [1]

-
2. Simplify, leaving your answer in positive index notation.

[Solution]

$$\begin{aligned} &\frac{(p^{-1}q^2)^5 \div (2pr^{-4})}{(3qr)^3 (2p^{-2}q^3)^0} \\ &= \frac{p^{-5}q^{10}}{(27q^3r^3)(2pr^{-4})} \\ &= \frac{q^7r}{54p^6} \end{aligned}$$

Answer $\frac{q^7r}{54p^6}$ [3]

3. (a) Factorise $169 + 52q + 4qr - r^2$ completely.

[Solution]

$$\begin{aligned} &169 + 52q + 4qr - r^2 \\ &= 169 - r^2 + 52q + 4qr \\ &= (13 - r)(13 + r) + 4q(13 + r) \\ &= (13 + r)(13 - r + 4q) \end{aligned}$$

Answer $(13 + r)(13 - r + 4q)$ [2]

- (b) Make x the subject of the equation $2x^2 = \frac{y - x^2}{2y} + 1$.

[Solution]

$$\begin{aligned} 2x^2 &= \frac{y - x^2}{2y} + 1 \\ 4x^2y &= y - x^2 + 2y \\ 4x^2y + x^2 &= y + 2y \\ x^2(4y + 1) &= 3y \\ x^2 &= \frac{3y}{4y + 1} \\ x &= \pm \sqrt{\frac{3y}{4y + 1}} \end{aligned}$$

Answer $x = \pm \sqrt{\frac{3y}{4y + 1}}$ [3]

4. The curve $y = 2(x + 2)(x + k)$ cuts the y -axis at $(0, -8)$.

- (a) Find the value of k .

[Solution]

$$\begin{aligned} &\text{At } (0, -8), \\ -8 &= 2(0 + 2)(0 + k) \\ 4k &= -8 \\ k &= -2 \end{aligned}$$

Answer (a) $k = \dots -2 \dots$ [2]

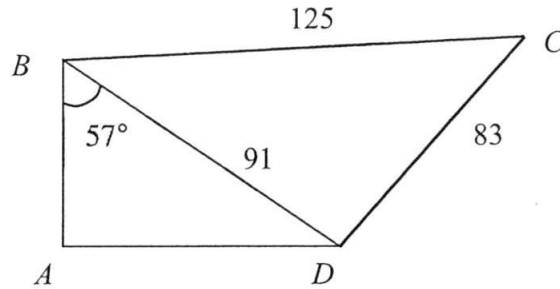
- (b) Using the value of k found in part (a), find the coordinates of the points where the curve cuts the x -axis.

[Solution]

$$\begin{aligned} &\text{When } y = 0, \\ 0 &= 2(x + 2)(x - 2) \\ x &= -2 \text{ or } 2 \\ &(-2, 0) \text{ and } (2, 0) \end{aligned}$$

Answer (b) $(-2, 0)$ and $(2, 0)$ [2]

5. The figure below represents a plot of land $ABCD$. B is due north of A , D is due east of A . $BC = 125$ m, $CD = 83$ m and $BD = 91$ m. Angle $ABD = 57^\circ$.



- (a) Find angle BDC .

[Solution]

By cosine rule,

$$125^2 = 91^2 + 83^2 - 2(91)(83)\cos \angle BDC$$

$$\cos \angle BDC = \frac{125^2 - 91^2 - 83^2}{-2(91)(83)}$$

$$\angle BDC = \cos^{-1}\left(-\frac{5}{166}\right)$$

$$\approx 91.726^\circ$$

$$= 91.7^\circ \text{ (to 1 d.p.)}$$

Answer (a) 91.7 ° [3]

- (b) Find the bearing of D from C .

[Solution]

Bearing from D from C

$$= (91.726^\circ - 57^\circ) + 180^\circ$$

$$\approx 214.726^\circ$$

$$= 214.7^\circ \text{ (to 1 d.p.)}$$

Answer (b) 214.7 ° [2]

6. (a) Solve the inequality $\frac{x-35}{3} < 5-3x \leq 2(4-x)$. Represent your answer on the number line below.

[Solution]

$$\frac{x-35}{3} < 5-3x \leq 2(4-x)$$

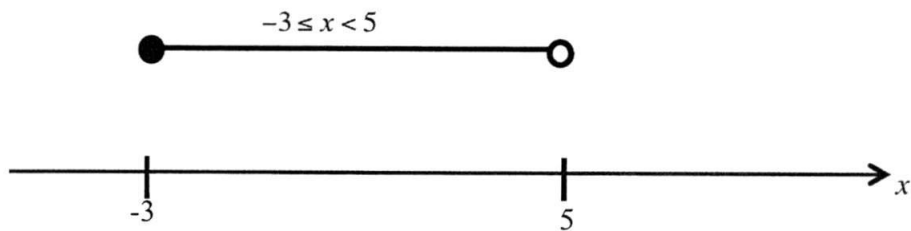
$$\frac{x-35}{3} < 5-3x \quad \text{and} \quad 5-3x \leq 2(4-x)$$

$$x-35 < 15-9x \quad 5-3x \leq 8-2x$$

$$10x < 50 \quad -x \leq 3$$

$$x < 5 \quad x \geq -3$$

$$\therefore -3 \leq x < 5$$



$-3 \leq x < 5$
Answer (a) [4]

- (b) Write down the largest prime number which satisfies

$$\frac{x-35}{3} < 5-3x \leq 2(4-x).$$

Answer (b) $x = \dots^3 \dots$ [1]

7. A hard disk has a memory capacity of 2 terabytes.

- (a) If a low-resolution photograph takes up about 250 kilobytes, how many million photographs can be stored in the hard disk?

[Solution]

no. of photographs

$$\begin{aligned} &= \frac{2 \times 10^{12}}{250 \times 10^3} \\ &= \frac{2 \times 10^{12}}{2.5 \times 10^5} \\ &= 0.8 \times 10^7 \\ &= 8 \times 10^6 \\ &= 8 \text{ million} \end{aligned}$$

Answer (a)⁸ million [2]

- (b) If the hard disk is to store 25 video clips with capacity of 273 megabytes each, how much capacity is left in the disk? Give your answer in standard form, correct to 3 significant figures.

[Solution]

capacity left

$$\begin{aligned} &= 2 \times 10^{12} - (25 \times 273 \times 10^6) \\ &= 2 \times 10^{12} - 6825 \times 10^6 \\ &= 2 \times 10^{12} - 6.825 \times 10^9 \\ &= 2000 \times 10^9 - 6.825 \times 10^9 \\ &= 1993.175 \times 10^9 \text{ bytes} \\ &= 1.993175 \times 10^{12} \text{ bytes} \\ &= 1.99 \times 10^{12} \text{ bytes (to 3 s.f.)} \end{aligned}$$

Answer (b)^{1.99 × 10¹²} bytes [3]

8. (a) Express $\frac{5}{12x+9} - \frac{2x+1}{9-16x^2}$ as a single fraction.

[Solution]

$$\begin{aligned} & \frac{5}{12x+9} - \frac{2x+1}{9-16x^2} \\ &= \frac{5}{3(4x+3)} - \frac{2x+1}{(3+4x)(3-4x)} \\ &= \frac{5(3-4x) - (2x+1)(3)}{3(3+4x)(3-4x)} \\ &= \frac{15 - 20x - 6x - 3}{3(3+4x)(3-4x)} \\ &= \frac{-26x+12}{3(3+4x)(3-4x)} \end{aligned}$$

$$\text{Answer (a) } \dots\dots\dots \frac{-26x+12}{3(3+4x)(3-4x)} \dots\dots\dots [3]$$

- (b) Hence, solve $\frac{5}{12x+9} - \frac{2x+1}{9-16x^2} = 1$.

[Solution]

$$\begin{aligned} & \frac{5}{12x+9} - \frac{2x+1}{9-16x^2} = 1 \\ & \frac{-26x+12}{3(3+4x)(3-4x)} = 1 \\ & -26x+12 = 3(9-16x^2) \\ & 48x^2 - 26x - 15 = 0 \\ & x = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(48)(-15)}}{2(48)} \\ & = \frac{26 \pm \sqrt{3556}}{96} \\ & x = 0.892 \text{ or } x = -0.350 \text{ (to 3 s.f.)} \end{aligned}$$

$$\text{Answer (b) } x = \dots\dots\dots 0.892 \dots\dots\dots \text{ or } \dots\dots\dots -0.350 \dots\dots\dots [3]$$

9. (a) Solve $\frac{25^x}{5} - \sqrt{\frac{1}{625}} = 0$.

[Solution]

$$\frac{25^x}{5} - \sqrt{\frac{1}{625}} = 0$$

$$\frac{5^{2x}}{5} - \frac{1}{25} = 0$$

$$5^{2x-1} = 5^{-2}$$

By comparing indices,

$$2x - 1 = -2$$

$$x = -\frac{1}{2}$$

$$-\frac{1}{2}$$

Answer (a) $x = \dots\dots\dots$ [3]

(b) Find the value of m for which $\sqrt{a^2} \sqrt{a^{\frac{1}{3}}} = a^m$.

[Solution]

$$\sqrt{a^2} \sqrt{a^{\frac{1}{3}}} = a^m$$

$$\sqrt{a^2 a^{\frac{1}{3}}} = a^m$$

$$\sqrt{a^{\frac{13}{6}}} = a^m$$

$$a^{\frac{13}{12}} = a^m$$

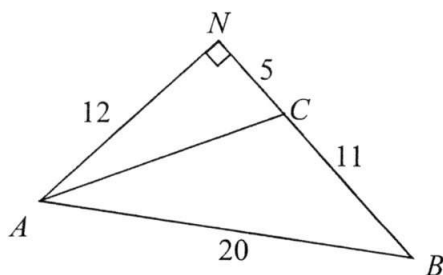
By comparing indices,

$$m = 1\frac{1}{12}$$

$$1\frac{1}{12}$$

Answer (b) $m = \dots\dots\dots$ [3]

10. In the diagram, ABN is a right-angled triangle, and BCN is a straight line.
 $AN = 12$ cm, $CN = 5$ cm, $BC = 11$ cm and $AB = 20$ cm.



Calculate

- (a) the length of AC ,

[Solution]

By Pythagoras' Theorem,

$$\begin{aligned} AC &= \sqrt{12^2 + 5^2} \\ &= 13 \text{ cm} \end{aligned}$$

Answer (a)13..... cm [1]

- (b) $\cos \angle ACB$,

[Solution]

$$\cos \angle ACN = \frac{5}{13}$$

$$\begin{aligned} \cos \angle ACB &= -\cos \angle ACN \\ &= -\frac{5}{13} \end{aligned}$$

Answer (b) $-\frac{5}{13}$ [2]

- (c) angle BAC .

[Solution]

$$\begin{aligned} \frac{\sin \angle BAC}{11} &= \frac{\sin \angle ACB}{20} \\ \sin \angle BAC &= \frac{11 \times \frac{12}{13}}{20} \\ &= \frac{33}{65} \end{aligned}$$

$$\begin{aligned} \angle BAC &= \sin^{-1} \frac{33}{65} \\ &\approx 30.51024^\circ \\ &= 30.5^\circ \text{ (to 1 d.p.)} \end{aligned}$$

Answer (c)30.5..... $^\circ$ [3]

11. (a) Express $-x^2 + 6x - 3$ in the form $a(x+b)^2 + c$.

[Solution]

$$\begin{aligned} & -x^2 + 6x - 3 \\ & = -(x^2 - 6x) - 3 \\ & = -\left(x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right) - 3 \\ & = -((x-3)^2 - 9) - 3 \\ & = -(x-3)^2 + 6 \end{aligned}$$

Answer (a) $\dots\dots\dots -(x-3)^2 + 6$ [3]

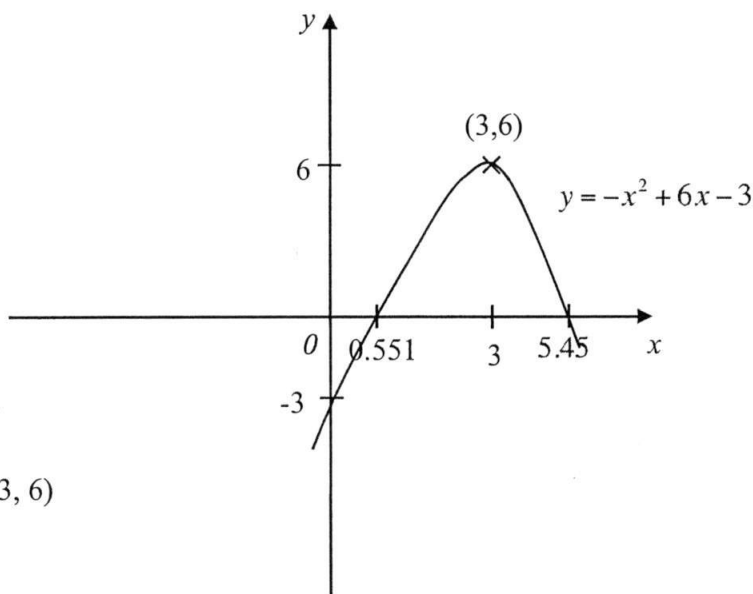
- (b) Hence, solve the equation $x^2 + 3 = 6x$.

[Solution]

$$\begin{aligned} x^2 + 3 &= 6x \\ -x^2 + 6x - 3 &= 0 \\ -(x-3)^2 + 6 &= 0 \\ (x-3)^2 &= 6 \\ x-3 &= \pm\sqrt{6} \\ x &= 3 \pm \sqrt{6} \\ &\approx 0.55051 \text{ or } 5.4495 \\ &= 0.551 \text{ or } 5.45 \end{aligned}$$

Answer (b) $x = \dots\dots\dots 0.551$ or $\dots\dots\dots 5.45$ [3]

- (c) Sketch the graph $y = -x^2 + 6x - 3$ on the axis provided, labeling your turning point and x -intercepts and y -intercept clearly. [3]



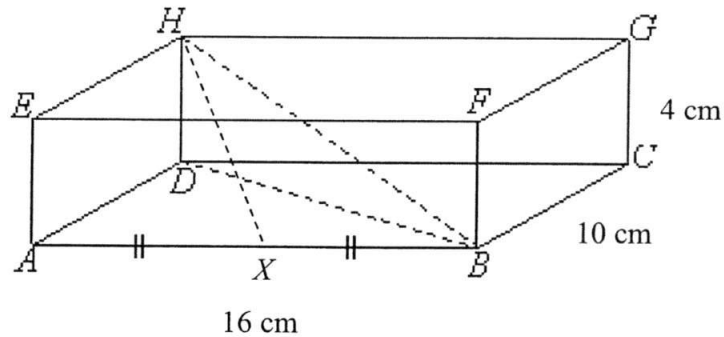
When $x = 0$,
 $y = -3$
 max. point $(3, 6)$

END OF SECTION A

SECTION B (44 MARKS)

Answer ALL the questions on the writing papers provided.

12. The diagram below shows a cuboid of dimensions 16 cm by 10 cm by 4 cm. Point X lies on AB such that $AX = XB$ and angle $HBX = 33.9^\circ$.



Calculate

- (a) the length of BH , [3]
(b) angle HBD , [2]
(c) HX , [2]
(d) angle DHX . [2]

[Solution]

$$\begin{aligned} \text{(a) Length of } BD &= \sqrt{16^2 + 10^2} \\ &= \sqrt{356} \\ &\approx 18.868 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Length of } BH &= \sqrt{4^2 + 18.868^2} \\ &= \sqrt{16 + 356} \\ &= \sqrt{372} \\ &\approx 19.287 \\ &= 19.3 \text{ cm (to 3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{(b) } \tan \angle HBD &= \frac{4}{\sqrt{356}} \\ \angle HBD &= \tan^{-1} \frac{4}{\sqrt{356}} \\ &\approx 11.969^\circ \\ &= 12.0^\circ \text{ (to 1 d.p.)} \end{aligned}$$

(c) By cosine rule,

$$HX^2 = BH^2 + XB^2 - 2(BH)(XB)\cos \angle HBX$$

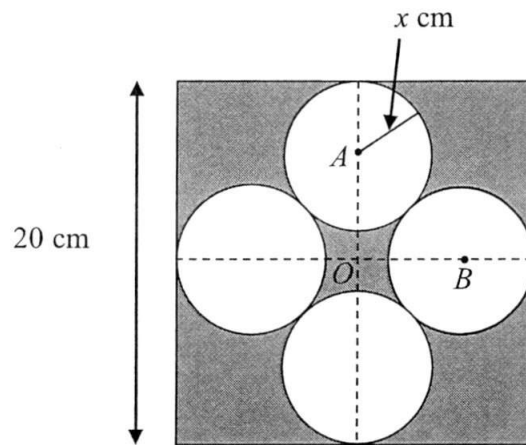
$$HX^2 = (\sqrt{372})^2 + 8^2 - (\sqrt{372})(8)\cos 33.9^\circ$$

$$HX \approx 13.411$$

$$= 13.4 \text{ cm (to 3 s.f.)}$$

$$\begin{aligned} \text{(d) } \cos \angle DHX &= \frac{4}{13.411} \\ \angle DHX &= \cos^{-1} \frac{4}{13.411} \\ &\approx 72.647^\circ \\ &= 72.6^\circ \text{ (to 1 d.p.)} \end{aligned}$$

13.



The diagram shows a square of length 20 cm. O is the intersection of the diagonals of the square. Four smaller identical circles, touching each other, with radius x cm are drawn as shown. A and B are the centres of two of the smaller circles.

- (a) Express the length of OA and of AB in terms of x . [2]
- (b) Use Pythagoras' Theorem to form an equation in x and show that it can be simplified to $x^2 + 20x - 100 = 0$. [3]
- (c) Solve the equation $x^2 + 20x - 100 = 0$, giving your answers correct to 2 decimal places. [3]
- (d) Calculate the area of the shaded region. [2]

[Solution]

$$\begin{aligned} \text{(a) } OA &= \frac{20}{2} - x \\ &= (10 - x) \text{ cm} \end{aligned}$$

$$AB = 2x \text{ cm}$$

$$\begin{aligned} \text{(b) By Pythagoras' Theorem,} \\ (10 - x)^2 + (10 - x)^2 &= (2x)^2 \\ 2(100 - 20x + x^2) &= 4x^2 \\ 2x^2 + 40x - 200 &= 0 \\ x^2 + 20x - 100 &= 0 \text{ (shown)} \end{aligned}$$

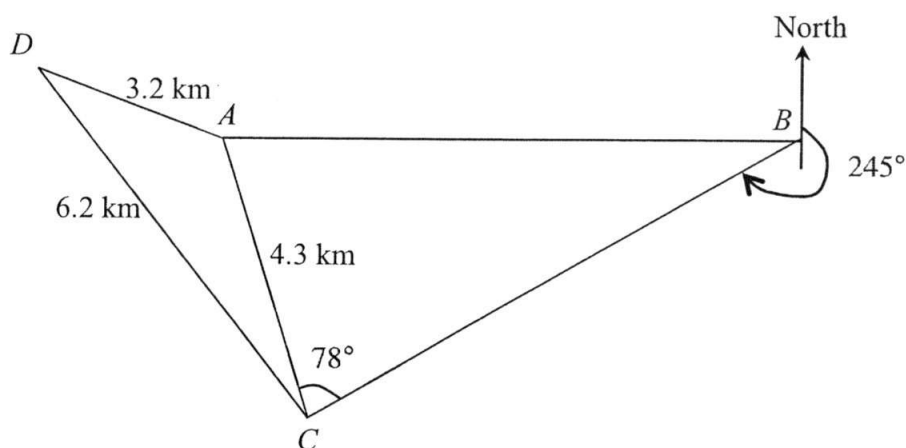
$$\begin{aligned} \text{(c) } x &= \frac{-20 \pm \sqrt{(20)^2 - 4(1)(-100)}}{2} \\ &= \frac{-20 \pm \sqrt{800}}{2} \\ &\approx -24.142 \text{ or } 4.1421 \\ &= -24.14 \text{ or } 4.14 \text{ (to 2 d.p.)} \end{aligned}$$

(d) $x = -24.1$ rejected as $x > 0$

$$\text{Area of shaded region} = 20^2 - 4\pi(4.1421)^2$$

$$= 184 \text{ cm}^2 \text{ (to 3 s.f.)}$$

14.



In the diagram, A , B , C and D are four points on level ground. It is given that $AC = 4.3$ km, $AD = 3.2$ km, $CD = 6.2$ km, angle $ACB = 78^\circ$, the bearing of C from B is 245° and B is due east of A . Calculate

- (a) angle ACD , [3]
- (b) the bearing of B from C , [2]
- (c) the length of AB , [3]
- (d) the area of triangle ADC . [2]

Alex walked from D to C along the path DC .

- (e) He stopped at point X when he is at the shortest distance to point A . Calculate AX . [2]
- (f) At point X , Alex saw a building standing vertically at point A . Given that the height of the building is 980 m tall, calculate the angle of elevation of the top of building when viewed by Alex. [2]

[Solution]

(a) By cosine rule,

$$3.2^2 = 6.2^2 + 4.3^2 - 2(6.2)(4.3)\cos\angle ACD$$

$$\cos\angle ACD = \frac{3.2^2 - 6.2^2 - 4.3^2}{-2(6.2)(4.3)}$$

$$\angle ACD = \cos^{-1}\left(\frac{4669}{5332}\right)$$

$$\approx 28.877^\circ$$

$$= 28.9^\circ \text{ (to 1 d.p.)}$$

$$\begin{aligned}
 \text{(b) Bearing from } B \text{ to } C \\
 &= 245^\circ - 180^\circ \\
 &= 065^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } \angle ABC &= 270^\circ - 245^\circ \\
 &= 25^\circ
 \end{aligned}$$

By sine rule,

$$\begin{aligned}
 \frac{\sin 25^\circ}{4.3} &= \frac{\sin 78^\circ}{AB} \\
 AB &= \frac{4.3 \sin 78^\circ}{\sin 25^\circ} \\
 &\approx 9.9523 \\
 &= 9.95 \text{ km (to 3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) Area of } \triangle ADC &= \frac{1}{2}(4.3)(6.2)\sin \angle ACD \\
 &= \frac{1}{2}(4.3)(6.2)\sin 28.877^\circ \\
 &\approx 6.4375 \\
 &= 6.44 \text{ km}^2 \text{ (to 3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(e) } \sin \angle ACD &= \frac{AX}{4.3} \\
 AX &= (\sin 28.877^\circ)(4.3) \\
 &\approx 2.0766 \\
 &= 2.08 \text{ km (to 3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f) } \tan \angle TXA &= \frac{0.98}{2.0766} \\
 \angle TXA &= \tan^{-1}\left(\frac{0.98}{2.0766}\right) \\
 &\approx 25.264^\circ \\
 &= 25.3^\circ \text{ (to 1 d.p.)}
 \end{aligned}$$

15. Answer the whole of this question on a piece of graph paper.

The variables x and y are connected by the equation $y = 2x^2 - 5x - 3$. Some corresponding values are given in the following table.

x	-2	-1	0	0.5	1	2	3	4
y	a	4	-3	-5	-6	-5	0	9

- (a) Calculate the value of a . [1]
- (b) Taking 2 cm to represent 1 unit on the x -axis and 2 cm to represent 5 units on the y -axis, draw the graph of $y = 2x^2 - 5x - 3$ for the range $-2 \leq x \leq 4$. [3]
- (c) From your graph, find
- (i) the value(s) of x when $y = 5$, [2]
- (ii) the minimum value of y . [1]
- (d) By adding suitable line(s) on the same graph paper, find
- (i) the value(s) of x for which $2x^2 - 5x - 3 = 0$, [1]
- (ii) the solutions for the equation $2x^2 - 7x = 0$. [3]

END OF PAPER

B1: smoothness & scale

B1: All pts. plot correctly

B1: labelling

姓名 _____ ()

科目 / Subject: _____

Name: _____

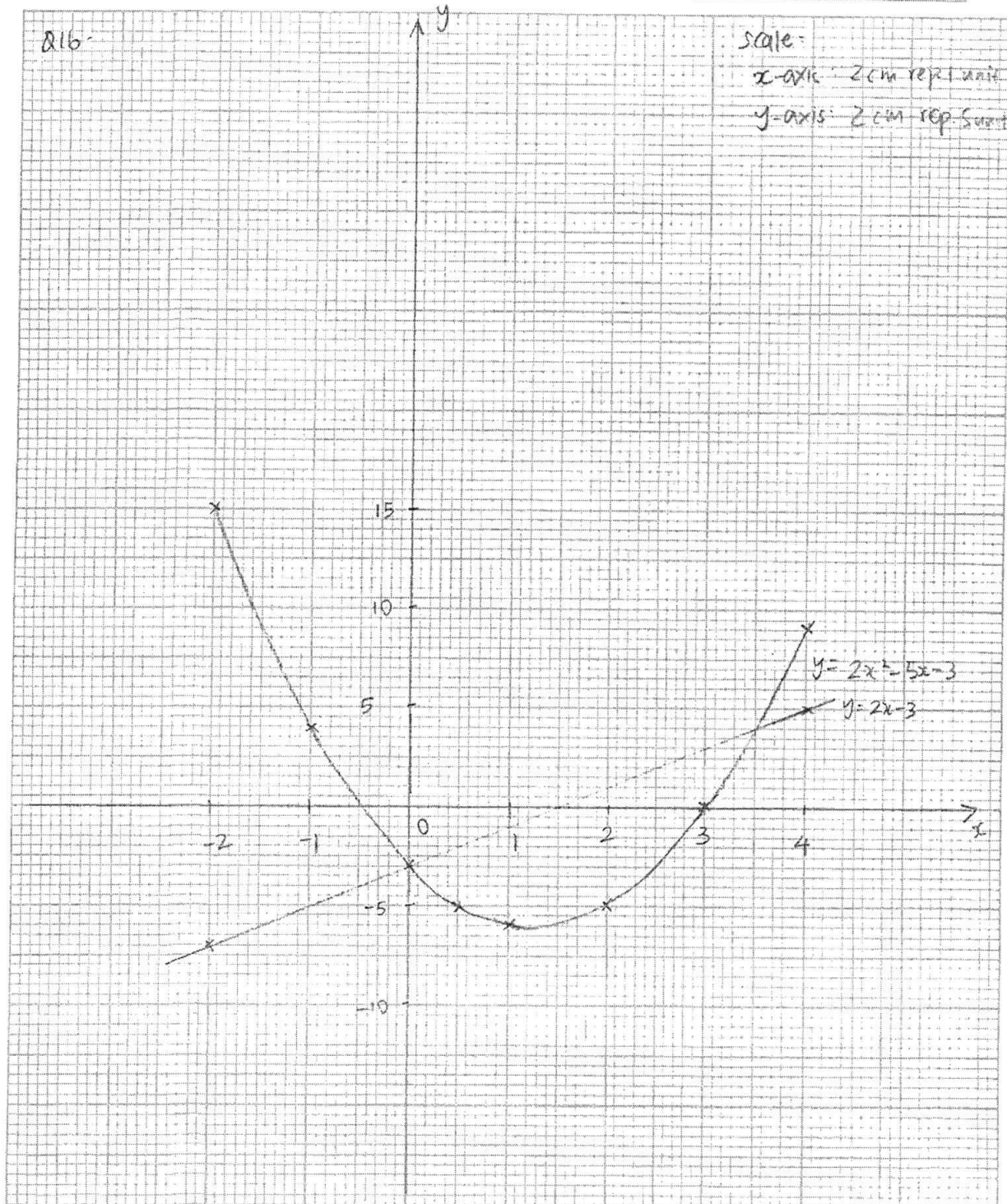
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Q16-

Scale:

x-axis: 2cm rep. 1 unit

y-axis: 2cm rep. 5 units



$$(a) a = 15 \quad -B1$$

$$(c)(i) \text{ when } y=5, x = -1.1 \text{ or } x = 3.6 \quad (\pm 0.05) \quad -B1, B1$$

$$(ii) \text{ min. value of } y = -6 \quad (\pm 0.25) \quad -B1$$

$$(d)(i) \quad 2x^2 - 5x - 3 = 0$$

$$x = -0.3 \text{ or } x = 3 \quad -B1$$

$$(ii) \quad 2x^2 - 7x = 0$$

$$2x^2 - 5x - 2x - 3 = -3$$

$$2x^2 - 7x - 3 = -3 \quad -M1$$

$$\text{Plot } y = 2x - 3 \quad -B1$$

$$\text{From the graph, } x = 0 \text{ or } x = 3.5 \quad -A1$$

For
Examiner's
UseFor
Examiner's
Use

1 (a) Calculate $\frac{22.7 \times 2016}{1956 + \sqrt{60}}$.

Write down the first five digits on your calculator.

Answer..... [1]

(b) Write your answer to part (a) correct to 2 decimal places.

Answer..... [1]

2 The Land Transport Authority announced in a recent news that there were 14 major breakdowns on the MRT network in 2015 which was an increase of 40 percent from the previous year.

How many major breakdowns were there in 2014?

Answer..... [2]

3 These are the first four terms in a sequence.

60 57 54 51

(a) Write down the next two terms.

Answer..... [1]

(b) Write down an expression, in terms of n , for the n^{th} term in the sequence.

Answer..... [1]

[Turn over]

For
Examiner's
Use

- 4 John thought of five positive numbers.
The median is 11 and the mode is 15.
The biggest number is five times of the smallest number.
The mean of the five numbers is 10.4.
Find the five numbers.

For
Examiner's
Use

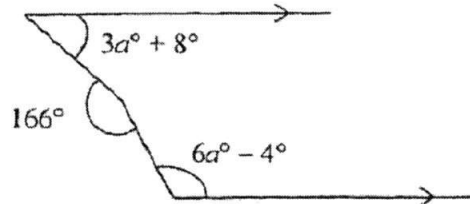
Answer [2]

- 5 If y is inversely proportional to \sqrt{x} and $y = 0.8$ when $x = 16$,
find the value of y when $x = 100$.

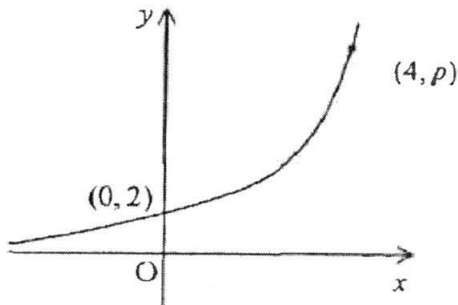
Answer $y = \dots\dots\dots$ [2]

For
Examiner's
Use

- 6 Calculate the value of
- a
- .

For
Examiner's
UseAnswer $a = \dots\dots\dots$ [2]

- 7 The sketch shows the graph of
- $y = k(3^x)$
- . The points
- $(0, 2)$
- and
- $(4, p)$
- lie on the graph. Find the values of
- k
- and of
- p
- .

Answer $k = \dots\dots\dots$ [1] $p = \dots\dots\dots$ [1]

{Turn over}

For
Examiner's
Use

For
Examiner's
Use

8 The numbers 60 and 2016, written as the product of their prime factors, are
 $60 = 2^2 \times 3 \times 5$ and $2016 = 2^5 \times 3^2 \times 7$.

(a) Find the highest common factor and lowest common multiple of
60 and 2016.

Answer HCF = LCM = [2]

(b) Given that $60k$ is a perfect cube, write down the smallest possible integer
value of k .

Answer $k =$ [1]

(c) Find the smallest positive integer value of n for which $60n$ is a multiple of
2016.

Answer $n =$ [1]

9 $\xi = \{\text{first 17 natural numbers}\}$
 $A = \{1, 4, 9, 16\}$
 $B = \{5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17\}$

(a) Describe the set A in words.

Answer [1]

(b) List the elements in B' .

Answer..... [1]

(c) Express $\{9, 16\}$ in terms of A and B in set notation.

Answer..... [1]

For
Examiner's
UseFor
Examiner's
Use

- 10 There are 22 boys and x girls in a group. The probability of selecting a girl from the group is $\frac{3}{14}$.

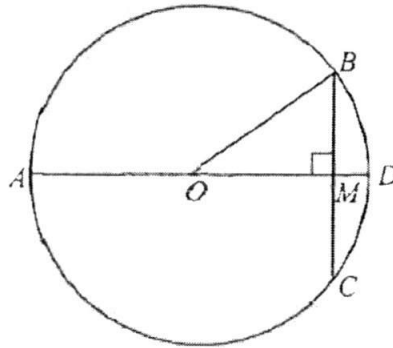
(i) Find the value of x .

Answer $x = \dots\dots\dots$ [1]

(ii) Using your value of x found in part (i), find the extra number of boys that have to join the group so that the probability of selecting a boy from the group will be $\frac{5}{6}$.

Answer $\dots\dots\dots$ [2]

- 11 The diagram shows a circle with centre O and radius 41 cm. $BC = 18$ cm and $\angle BMO = 90^\circ$. $AOMD$ is a straight line.



(a) Find AM .

Answer $\dots\dots\dots$ cm [2]

(b) Write down the exact value of $\cos \hat{A}OB$.

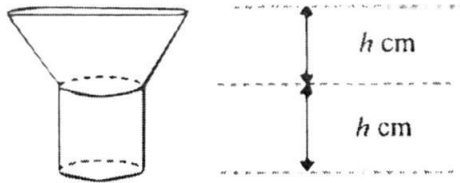
Answer $\dots\dots\dots$ [1]

[Turn over]

For
Examiner's
Use

For
Examiner's
Use

- 12 The diagram shows a container which is made up of a frustum and a cylinder of the same height. It is initially full of water. The volumes of the frustum and the cylinder are in the ratio of 3 : 1. Water is leaking through a hole at the bottom of the container at a constant rate. The container is completely empty in 16 minutes.

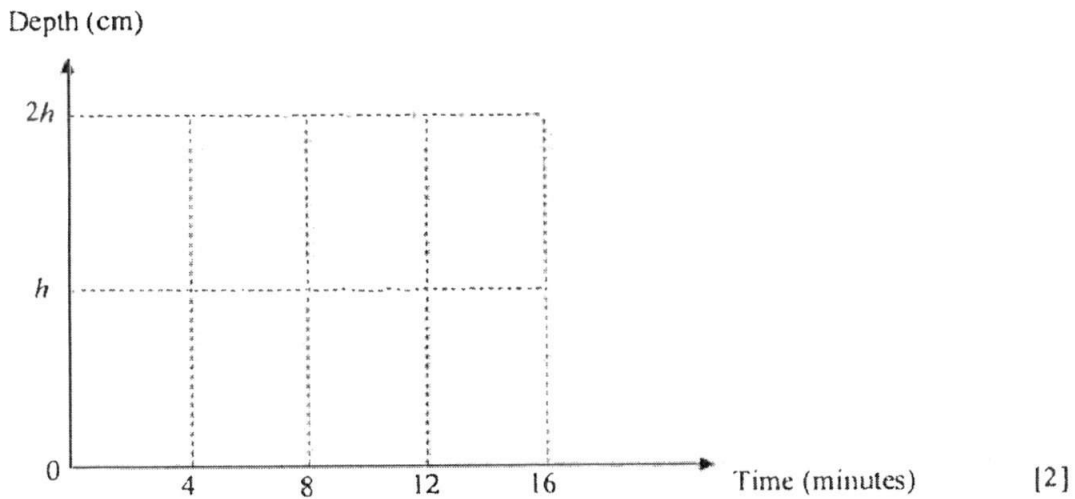


- (a) Find the time taken for the depth of the water to be h cm.

Answer.....minutes [1]

- (b) On the axes in the answer space, sketch the graph showing how the depth of the water, h cm, in the container varies over the 16 minutes.

Answer



For
Examiner's
Use

- 13 (a) Express $x^2 - 10x - 13$ in the form $(x - a)^2 + b$.

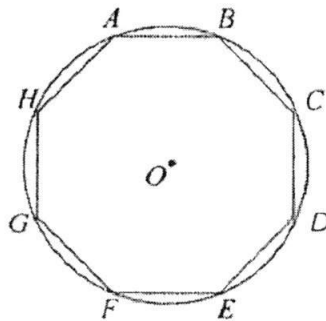
For
Examiner's
Use

Answer [1]

- (b) Hence solve the equation $x^2 - 10x - 13 = 0$.

Answer $x = \dots$ or \dots [2]

- 14 A regular octagon $ABCDEFGH$ fits exactly inside a circle of centre O and radius 6 cm.



Find the area of the circle not covered by the octagon.

Answer cm^2 [4]

[Turn over]

For
Examiner's
Use15 (a) Factorise $a^2 - 2ab + b^2 - 4b^2c^2$ completely.For
Examiner's
Use

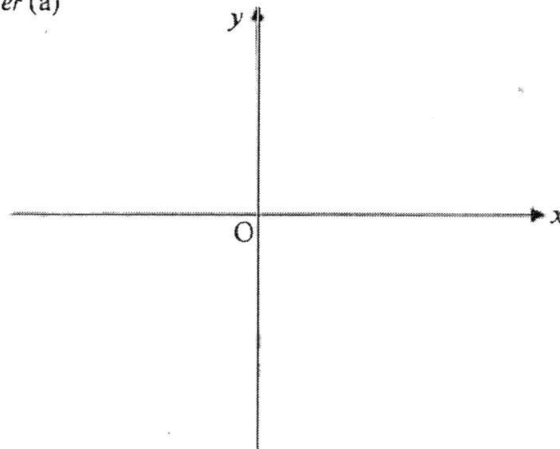
Answer [2]

(b) Given that $T = 2\pi\sqrt{\frac{L}{g}}$, make L the subject of the formula.

Answer [2]

16 (a) Sketch the graph of $y = 2(x - 3)(x + 1)$.

Answer (a)



[2]

(b) Write down the equation of the line of symmetry of $y = 2(x - 3)(x + 1)$.

Answer [1]

(c) Write down the coordinates of the turning point.

Answer(.....) [1]

(d) Write down another quadratic equation, other than $y = 2(x - 3)(x + 1)$ that has the same roots.

Answer [1]

For
Examiner's
Use

17 Solve the following equations.

(a) $16^x \div 4^{3x} = 0.25$

For
Examiner's
Use

Answer $x = \dots\dots\dots$ [2]

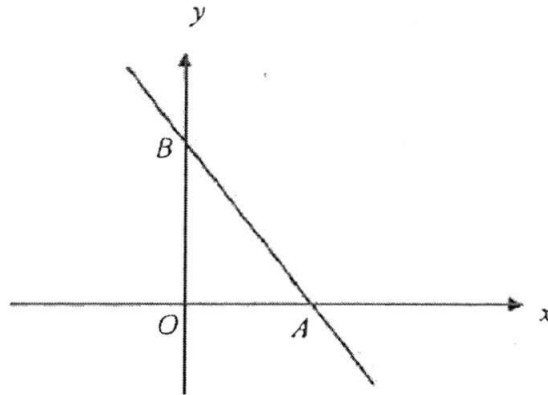
(b) $\sqrt[3]{25} = 125$

Answer $p = \dots\dots\dots$ [2]

[Turn over]

For
Examiner's
Use

- 18 The diagram shows a straight line $4y + 5x = 20$ passing through the points A and B .



- (a) Find the coordinates of A and of B .

Answer A (.....) [1]

B (.....) [1]

- (b) Find the area of $\triangle OAB$ and hence find the shortest distance from O to the line AB .

Answer $Area = \dots\dots\dots$ units² [1]

Shortest distance = $\dots\dots\dots$ units [2]

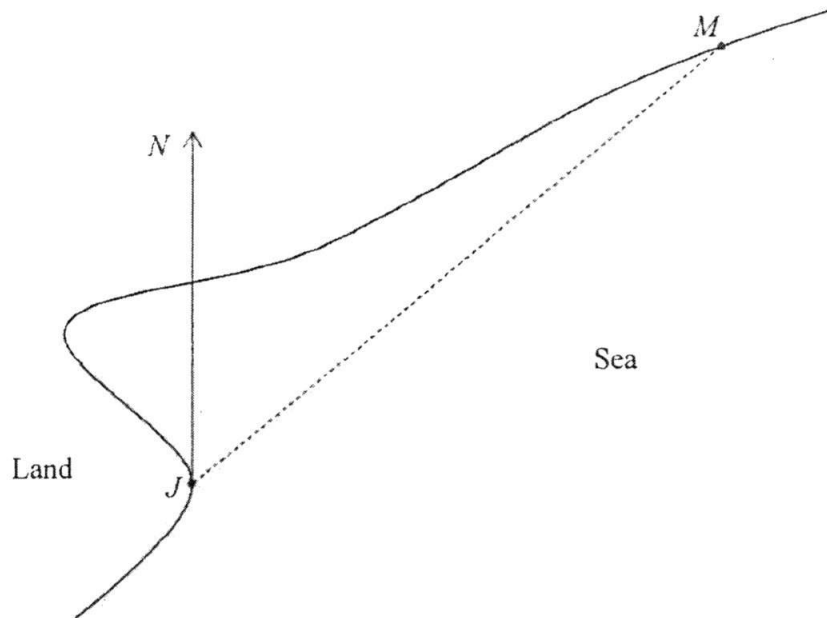
[Turn over]

For
Examiner's
Use

For
Examiner's
Use

For
Examinee
Use

- 19 The scale drawing shows a jetty, J , and a man, M .
The scale is 1 cm to 10 km.



- (a) Measure the bearing of M from J .

Answer.....° [1]

- (b) A boat is 85 km from J on a bearing of 110° .
Mark and label on the diagram the position, B , of the boat. [1]

State the bearing of J from B .

Answer.....° [1]

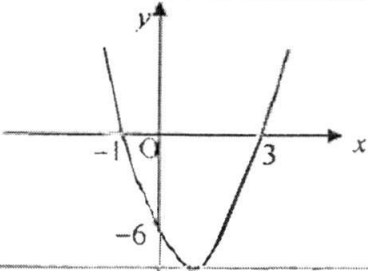
- (c) The boat travels in a straight line towards J at an average speed of 35 km/h.
Calculate the travelling time of the boat.
Give your answer in hours and minutes, to the nearest minute.

Answer.....hmins [2]

End of paper

Answers

1	(a)	23.304
	(b)	23.30
2		10
3	(a)	48, 45
	(b)	$63 - 3n$ or $60 - 3(n - 1)$
4		5, 8, 11, 15, 15
5		0.32
6		18
7		$k = 2; p = 162$
8	(a)	HCF = 12 L.C.M = 10080
	(b)	$k = 450$
	(c)	$n = 168$
9	(a)	Squares numbers / Perfect squares
	(b)	1, 2, 3, 4, 6, 12
	(c)	$A \cap B$
10	(i)	6
	(ii)	8
11	(a)	81 cm
	(b)	$\frac{40}{41}$
12	(a)	12
	(b)	
13	(a)	$(x - 5)^2 - 38$
	(b)	$x = 11.2$ or 1.16
14		11.3 cm^2
15	(a)	$(a - b + 2bc)(a - b - 2bc)$
	(b)	$L = \frac{gT^2}{4\pi^2}$

16	(a)	
16	(b)	$x = 1$
	(c)	$(1, -8)$
	(d)	$y = (x+1)(x-3) / y = -(x+1)(x-3) / y = 5(x+1)(x-3)$
17	(a)	1
	(b)	$\frac{2}{3}$
18	(a)	$A(4, 0) \quad B(0, 5)$
	(b)	10 units ² Shortest distance = 3.12 units
19	(a)	$050^\circ (\pm 1^\circ)$
	(b)	290°
	(c)	2 h 26 mins