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**2022 End of Year Examination**  
**Secondary Three Express**

**MATHEMATICS**  
4052/01

06 Oct 2022  
1 hour 45 minutes  
1130h – 1315h

**Name:** \_\_\_\_\_ (     )     **Class:** \_\_\_\_\_

**READ THESE INSTRUCTIONS FIRST**

Write your full name, class and index number on all work you hand in.  
 Write in dark blue or black pen.  
 You may use a pencil for any diagrams or graphs.  
 Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 70.

| <b>FOR MARKER'S USE</b> |                  |              |
|-------------------------|------------------|--------------|
|                         | Marks<br>Awarded | Max<br>Marks |
| Total                   |                  | 70           |

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**This question paper consists of 14 printed pages including the cover page.**

**Setter: Mr Tan Qi Guang**

- 1 Solve the following simultaneous equations.

$$2x - y = 2$$

$$x + 3y = 15$$

*Answer*  $x = \dots\dots\dots$

$y = \dots\dots\dots$  [3]

---

- 2 Simplify the following, giving your answer in positive index form.

(a)  $(3a^3)^2$

*Answer*  $\dots\dots\dots$  [1]

(b)  $6pq \times 4p^3q$

*Answer*  $\dots\dots\dots$  [1]

(c)  $\sqrt[3]{a^4} \div a^{\frac{1}{3}} \times a^0$

*Answer*  $\dots\dots\dots$  [2]

3

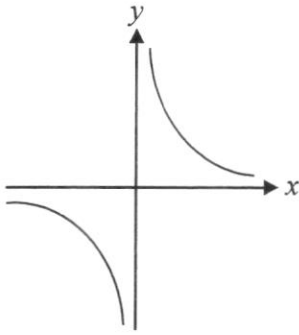


Figure 1

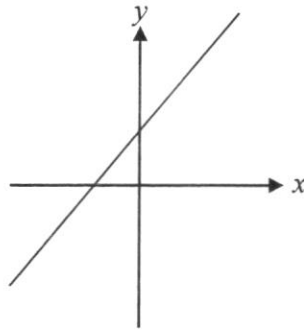


Figure 2

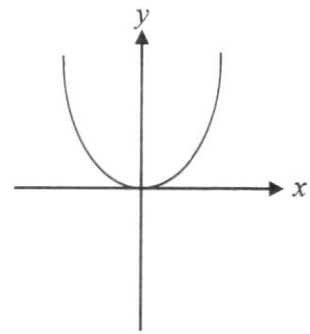


Figure 3

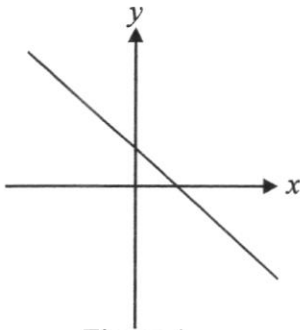


Figure 4

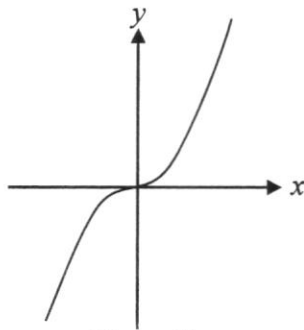


Figure 5

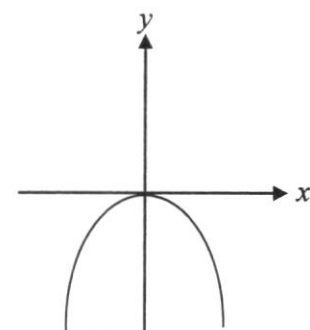


Figure 6

Which of the graphs shown above could be the graph of

(a)  $y = x^3$ ,

Answer Figure ..... [1]

(b)  $y = -x^2$ ,

Answer Figure ..... [1]

(c)  $y = \frac{1}{x}$  ?

Answer Figure ..... [1]

- 4 (a) Solve the inequality  $5 - 3x \leq 1 - 2x < 3x + 7$ .

*Answer* ..... [3]

- (b) Hence, write down the smallest prime number that satisfies  $5 - 3x \leq 1 - 2x < 3x + 7$ .

*Answer* ..... [1]

- 5 In 2020, there were 5.626 million people living in Country A.  
9% of the population was elderly.  
The number of youths was twice the number of elderly.  
By expressing your answers in standard form, correct to 2 significant figures, calculate

- (a) the number of elderly,

*Answer* ..... [1]

- (b) the number of youths.

*Answer* ..... [1]

6 Solve the following equations.

(a)  $3^4 \times 3^q = \frac{1}{27}$

Answer  $q = \dots\dots\dots$  [2]

(b)  $8^r \div 4 = 1$

Answer  $r = \dots\dots\dots$  [2]

7 Given that  $-5 \leq x \leq 4$  and  $-3 \leq y \leq 2$ , find

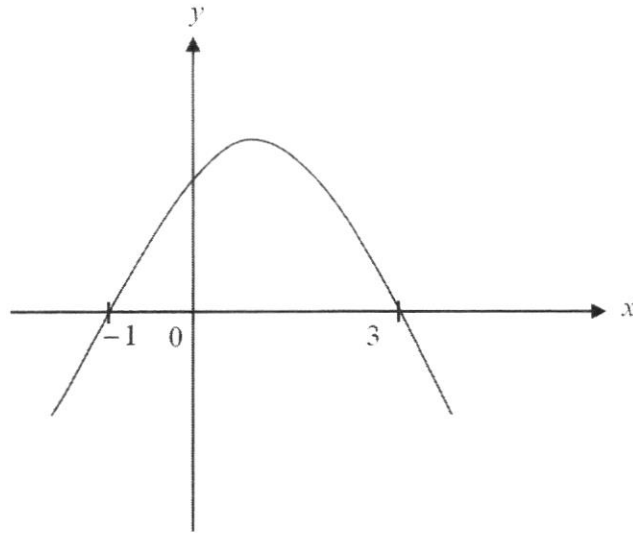
(a) the smallest possible value of  $xy$ ,

Answer  $\dots\dots\dots$  [1]

(b) the greatest possible value of  $x^2 - y^2$ .

Answer  $\dots\dots\dots$  [1]

- 8 The figure below shows a quadratic curve.



- (a) Find the equation of the curve in the form  $y = ax^2 + bx + c$ .

*Answer* ..... [2]

- (b) State the equation of the line of symmetry for the curve.

*Answer* ..... [1]

- (c) Find the coordinates of the maximum point.

*Answer* (....., .....) [1]

- 9** Two bottles are geometrically similar.  
The ratio of the surface areas of the bottles is 9 : 16.
- (a)** Write down the ratio of the length of the smaller bottle to the length of the larger bottle.

*Answer* ..... : ..... [1]

- (b)** The volume of the larger bottle is  $1440 \text{ cm}^3$ .  
Calculate the volume of the smaller bottle.

*Answer* .....  $\text{cm}^3$  [2]

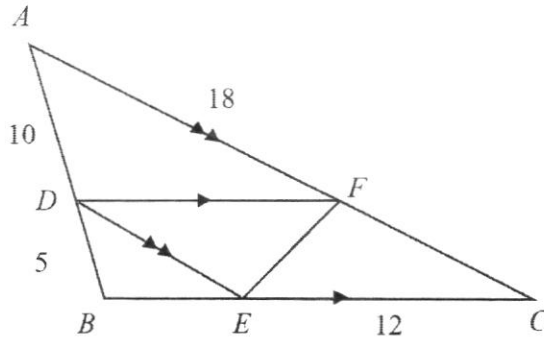
- 10 (a)** Express  $x^2 + 8x - 20$  in the form  $(x + h)^2 - k$ .

*Answer* ..... [2]

- (b)** Hence, solve the equation  $x^2 + 8x - 20 = 0$ .

*Answer*  $x = \dots\dots\dots$  or  $x = \dots\dots\dots$  [2]

- 11 In the diagram,  $DF$  is parallel to  $BC$  and  $AC$  is parallel to  $DE$ .  
 Given that  $DB = 5$  cm,  $AD = 10$  cm,  $EC = 12$  cm and  $AF = 18$  cm.



- (a) Name a triangle that is similar to triangle  $ADF$ .  
 Prove that the two triangles are similar

*Answer*

[3]

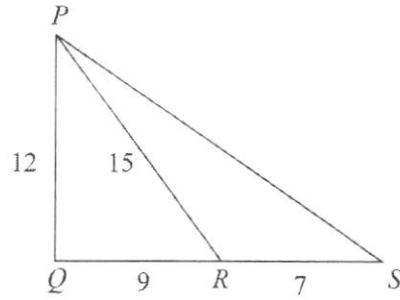
- (b) Given that triangle  $CEF$  and triangle  $DFE$  are congruent, calculate the length of  $BE$ .

*Answer* ..... cm [3]

- (c) Find the ratio of  $\frac{\text{area of triangle } ADF}{\text{area of parallelogram } DBCF}$ , express your answer as a fraction in its simplest form.

*Answer* ..... [2]

- 12 The diagram below shows a figure, where  $PQ = 12$  cm,  $QR = 9$  cm,  $PR = 15$  cm and  $RS = 7$  cm.



- (a) Explain why angle  $PQR = 90^\circ$ .

*Answer*

[2]

- (b) Expressing your answer as a fraction in the simplest form, find

(i)  $\tan \angle QPS$ ,

*Answer* ..... [1]

(ii)  $\cos \angle PRS$ ,

*Answer* ..... [1]

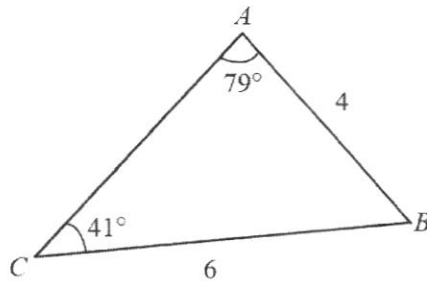
(iii)  $\sin \angle PSQ$ .

*Answer* ..... [2]

- 13 10 workers take 90 days to complete building a bus stop.  
 The manager claims that by hiring 5 more workers, the completion date can be brought forward by 30 days.  
 Explain with calculations whether the manager is correct in his claim.  
 State an assumption made.

*Answer* .....  
 .....[3]

- 14 The diagram shows a triangle with  $AB = 4$  cm,  $BC = 6$  cm,  $\angle BAC = 79^\circ$  and  $\angle ACB = 41^\circ$ .



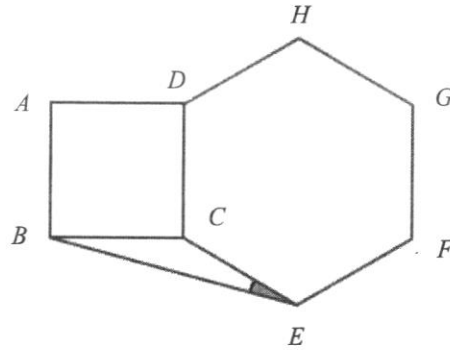
- (a) Calculate the area of triangle  $ABC$ .

*Answer* .....  $\text{cm}^2$  [2]

- (b) Hence, or otherwise, find the shortest distance from  $C$  to  $AB$ .

*Answer* ..... cm [2]

- 15 The diagram below shows a square  $ABCD$  and a regular hexagon  $CEFGHD$ .



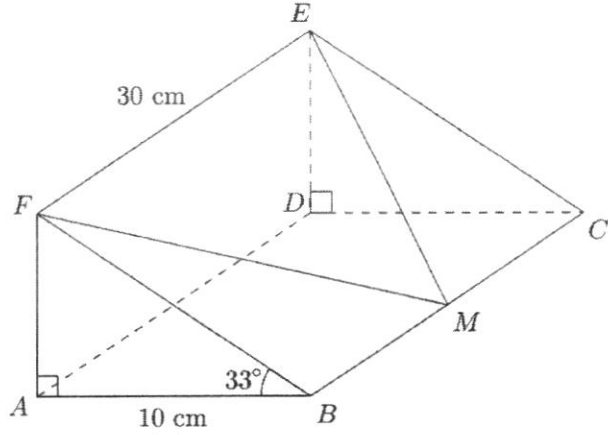
Calculate angle  $BEC$ .

*Answer* ..... [3]

- 16 Zen invests \$3000 in GRC finance company that pays 2.8% compound interest per annum compounded half-yearly.  
Find the total amount at the end of 6 years, giving your answer correct to the **nearest dollar**.

*Answer* \$ ..... [2]

- 17 In the diagram,  $ABCDEF$  is a triangular prism, and  $M$  is the midpoint of  $BC$ . It is also given that  $AB = 10$  cm,  $EF = 30$  cm, and  $\angle ABF = 33^\circ$ .



- (a) Find the length of  $BF$ .

Answer  $BF = \dots\dots\dots$  cm [2]

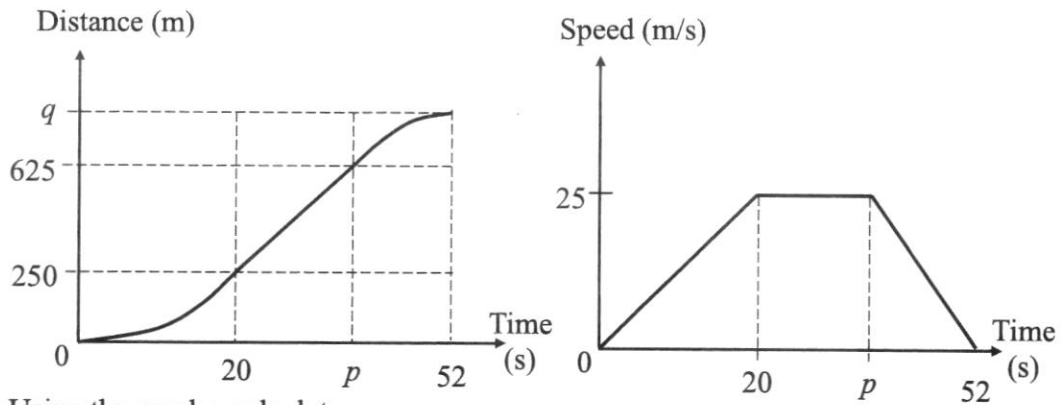
- (b) Find the length of  $FM$ .

Answer  $FM = \dots\dots\dots$  cm [2]

- (c) Find  $\angle FMA$ .

Answer  $\angle FMA = \dots\dots\dots$  [2]

18 The diagrams below show the distance-time and speed-time graphs of a van travelling from Town A to Town B.



Using the graphs, calculate

(a) the acceleration during the first 20 seconds,

Answer .....  $\text{m/s}^2$  [1]

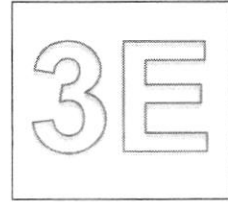
(b) value of  $p$ ,

Answer  $p =$  ..... [2]

(c) the value of  $q$ .

Answer  $q =$  ..... [2]

**End of Paper**



## 2022 End of Year Examination

### Secondary Three Express

**MATHEMATICS**  
4052/02

11 October 2022  
1 hour 45 minutes  
1130 – 1315

Name: \_\_\_\_\_ (     )     Class: \_\_\_\_\_

#### READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all the work you hand in.  
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The number of marks is given in brackets [ ] at the end of each question or part question.  
The total of the marks for this paper is 70.

| FOR MARKER'S USE |               |           |
|------------------|---------------|-----------|
|                  | Marks Awarded | Max Marks |
| Total            |               | <b>70</b> |

- 1 Here are the first four terms of a sequence.

$$T_1 = 1^2 + 1 = 2$$

$$T_2 = 3^2 + 2 = 11$$

$$T_3 = 5^2 + 3 = 28$$

$$T_4 = 7^2 + 4 = 53$$

- (a) Write down the fifth term of the sequence.

*Answer*  $T_5 = \dots\dots\dots$  [1]

- (b)  $T_n$  is the  $n$ th term of the sequence.

Show that  $T_n = 4n^2 - 3n + 1$ .

*Answer*

[3]

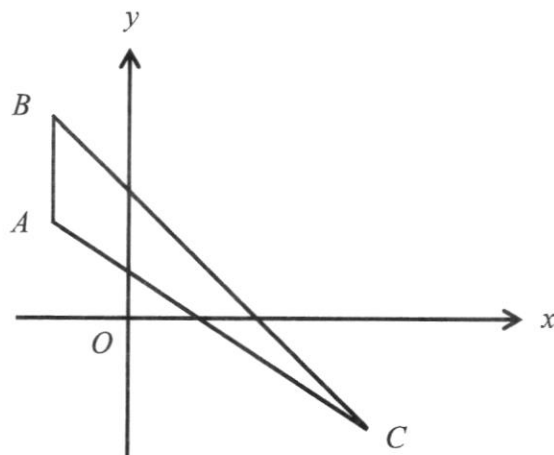
- (c) The difference,  $D$ , between two consecutive terms of the sequence is  $T_{n+1} - T_n$ .  
Find an expression, in terms of  $n$ , for  $D$ .

*Answer*  $D = \dots\dots\dots$  [3]

- (d) Explain why the difference between two consecutive terms of the sequence is always odd.

..... [1]

2



The diagram shows a triangle  $ABC$  with vertices  $A(-2, 4)$  and  $C(10, -4)$ .  
 $AB$  is parallel to the  $y$ -axis.

- (a) Calculate the length of the line segment  $AC$ .

*Answer* ..... units [2]

- (b) Given that the gradient of the line  $BC = -1$ .

Find

- (i) the coordinates of  $B$ ,

*Answer*  $B$  (....., .....) [2]

(ii) the equation of the line  $BC$ .

*Answer* ..... [2]

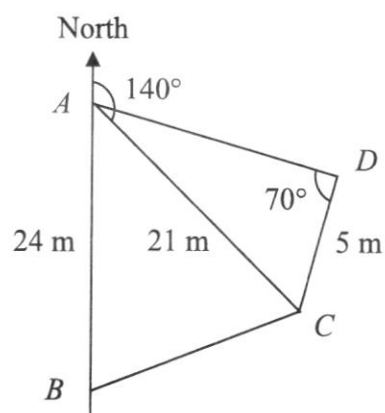
(c) Calculate the area of the triangle  $ABC$ .

*Answer* ..... units<sup>2</sup> [2]

(d) Find the coordinates of point  $D$ , given that  $ABCD$  is a parallelogram.

*Answer*  $D$  (....., .....) [1]

- 3  $ABCD$  represents a plot of land.



$A$  is 24 m due north of  $B$ .

$AC = 21$  m,  $CD = 5$  m and angle  $ADC = 70^\circ$ .

The bearing of  $C$  from  $A$  is  $140^\circ$ .

- (a) Calculate

- (i)  $BC$ ,

Answer ..... m [2]

- (ii) the bearing of  $D$  from  $A$ .

Answer ..... [3]

- (b)  $E$  is a point at the top of a tree at point  $C$ .  
The angle of elevation of  $E$  from  $D$  is  $78^\circ$ .

Calculate

- (i) the height of the tree,

*Answer* ..... m [2]

- (ii) the greatest possible angle of elevation of  $E$  from a point on  $AB$ .

*Answer* ..... [2]

4 Aida and Bernard took part in a marathon race.

They each ran 70 km.

(a) Aida ran at an average speed of  $x$  km/h.

Write down an expression, in terms of  $x$ , for the number of hours she took.

*Answer* ..... h [1]

(b) Bernard ran at an average speed which was 0.5 km/h more than Aida's speed.

Find an expression, in terms of  $x$ , for the number of hours he took.

*Answer* ..... h [1]

(c) The difference between their timings was 50 minutes.

Write down an equation in  $x$  to represent this information, and show that it reduces to  $2x^2 + x - 84 = 0$ .

*Answer*

[3]

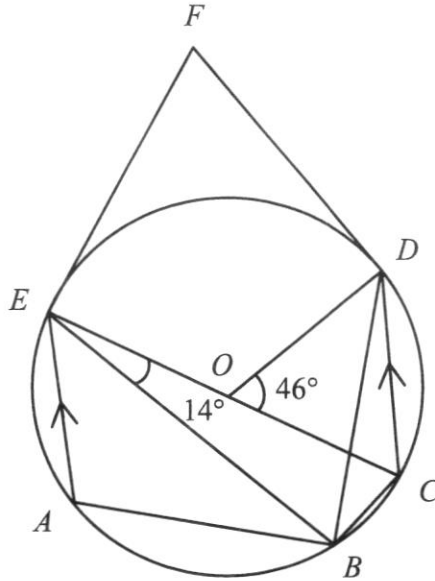
- (d) Solve the equation  $2x^2 + x - 84 = 0$ , giving your solutions correct to 1 decimal place.

*Answer*  $x = \dots\dots\dots$  or  $\dots\dots\dots$  [4]

- (e) Hence calculate the time that Aida took to complete the race.  
Give your answer in hours and minutes, correct to the nearest minute.

*Answer*  $\dots\dots\dots$  h  $\dots\dots\dots$  min [2]

5



The diagram shows a circle  $ABCDE$ , with centre  $O$ .  
 $AE$  is parallel to  $CD$ .  
 Angle  $COD = 46^\circ$  and angle  $CEB = 14^\circ$ .  
 $EF$  and  $DF$  are tangents to the circle at  $E$  and  $D$  respectively.

- (a) Find angle  $DBE$ .  
 Give a reason for each step of your working.

*Answer* Angle  $DBE = \dots\dots\dots$  [2]

- (b) Find angle  $AEB$ .  
 Give a reason for each step of your working.

*Answer* Angle  $AEB = \dots\dots\dots$  [2]

- (c) Find angle  $BAE$ .

Give a reason for each step of your working.

*Answer* Angle  $BAE = \dots\dots\dots$  [2]

- (d) Find angle  $EFO$ .

Give a reason for each step of your working.

*Answer* Angle  $EFO = \dots\dots\dots$  [2]

- (e) A point  $G$  lies on the same side of  $DE$  as  $B$  such that angle  $DGE = 90^\circ$ .

State whether point  $G$  lies inside, outside or on the circle.

Explain your answer.

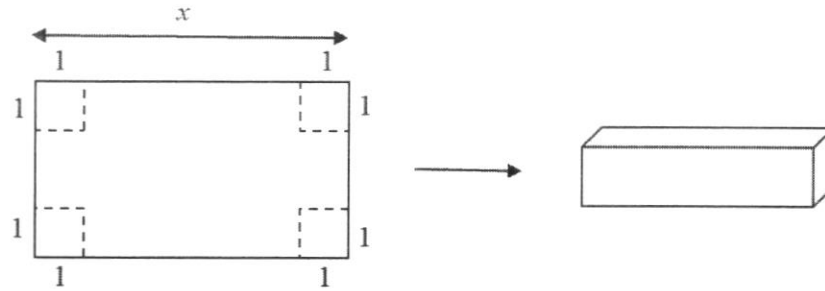
$\dots\dots\dots$  [1]

- (f) The radius of the circle is 5 cm.

Calculate the length of the major arc  $EABCD$ .

*Answer*  $\dots\dots\dots$  cm [2]

- 6 A rectangular sheet of paper has length  $x$  cm and an area of  $36 \text{ cm}^2$ .  
Four identical squares of side 1 cm are cut from the four corners of the paper and the paper is folded to make an open box.



- (a) Show that the volume of the open box,  $y \text{ cm}^3$ , is given by  $y = 2(x-2)\left(\frac{18}{x}-1\right)$ .

*Answer*

[2]

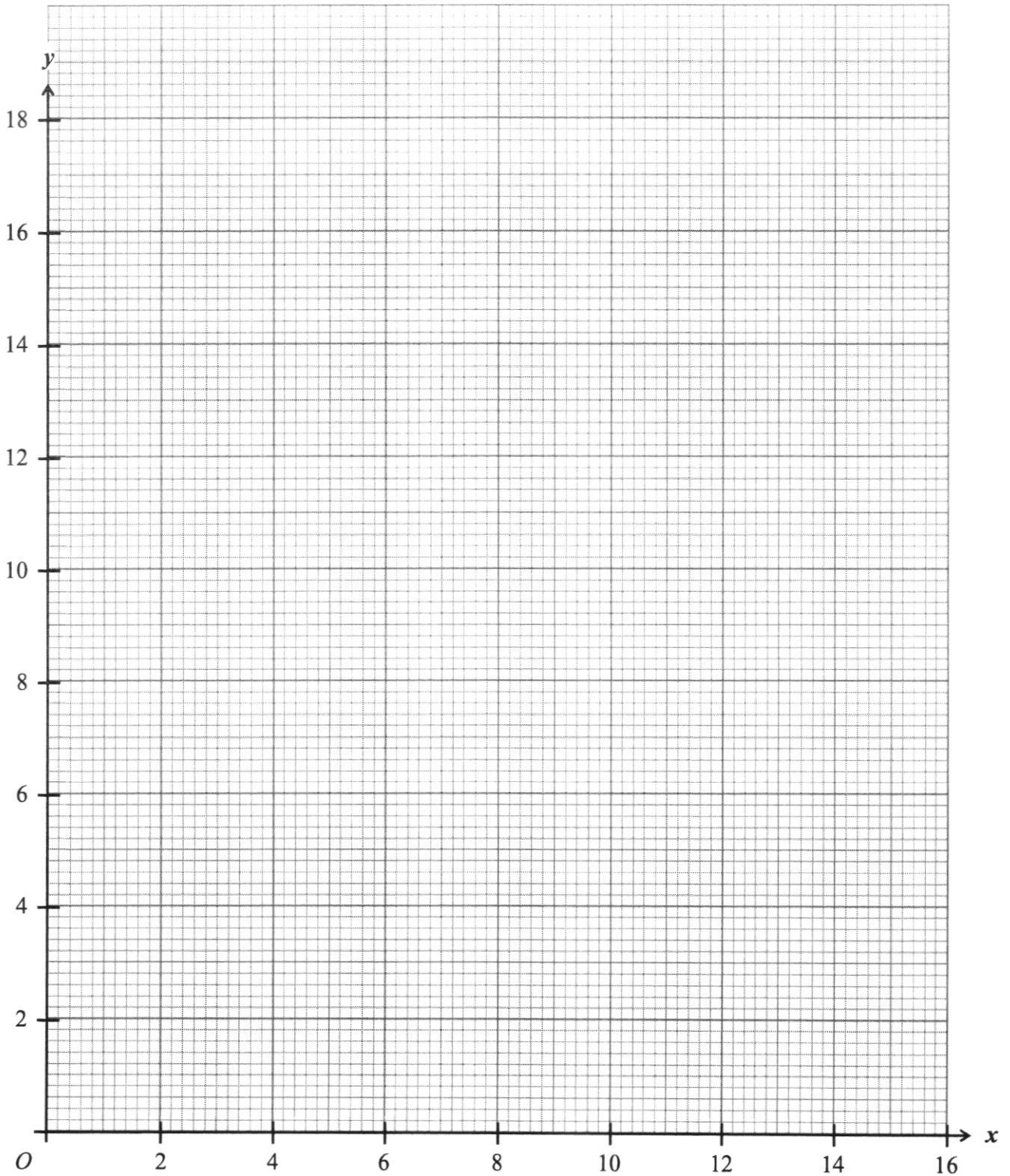
- (b) Complete the table of values for  $y = 2(x-2)\left(\frac{18}{x}-1\right)$ .

Values are given to 1 decimal place where appropriate.

|     |   |    |    |      |      |    |    |    |
|-----|---|----|----|------|------|----|----|----|
| $x$ | 2 | 3  | 4  | 5    | 7    | 9  | 12 | 16 |
| $y$ | 0 | 10 | 14 | 15.6 | 15.7 | 14 | 10 |    |

[1]

(c) On the grid, draw the graph of  $y = 2(x-2)\left(\frac{18}{x}-1\right)$  for  $0 \leq x \leq 16$ .



[2]

- (d) Use your graph to estimate the range of values of  $x$  for which the volume of the box is more than  $7 \text{ cm}^3$ .

*Answer* ..... [1]

- (e) Explain how the graph shows that the volume cannot be equal to  $18 \text{ cm}^3$ .

..... [1]

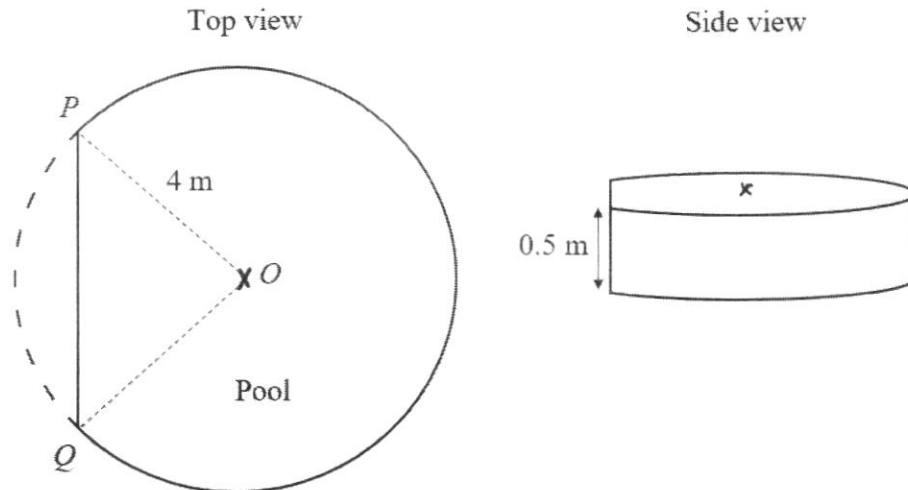
- (f) By drawing a tangent, find the gradient of the curve at the point  $x = 5$ .

*Answer* ..... [2]

- (g) By drawing a suitable straight line on your graph, find the solutions of the equation  $2x^2 + 5x = 4(x - 2)(18 - x)$ .

*Answer*  $x =$  ..... or ..... [3]

7



The diagrams show a kids' pool in a refurbished hotel.

The cross-section of the pool is a major segment of a circle, centre  $O$  and radius 4 m. The depth of the pool is 0.5 m.

- (a) The area of the minor sector  $OPQ$  is  $11.2 \text{ m}^2$ .
- (i) Find angle  $POQ$  in radians.

*Answer* Angle  $POQ = \dots\dots\dots$  rad [1]

- (ii) Show that the capacity of the pool is  $23.47 \text{ m}^3$ , correct to 2 decimal places.

*Answer*

[3]

- (b) For safety reasons, a flight of stairs made from  $1.05 \text{ m}^3$  cement was placed along  $PQ$  inside the pool.

Mr Tan, the manager, needs to fill the pool with water, up to 95% capacity.

Company GRC provides water-filling service for swimming pools.

|                    |                               |
|--------------------|-------------------------------|
| Water pump         | 8 gallons per minute          |
| Cost of water pump | \$17 per 100 gallons of water |
| Labour cost        | \$30 per hour                 |

$1 \text{ gallon} = 3.785 \text{ litres}$

$1 \text{ litre} = 0.001 \text{ m}^3$

Mr Tan thinks that the pool can be filled in 10 hours, within a budget of \$1200.

Is Mr Tan correct?

Justify your answer.

*Answer*

.....  
 ..... [6]

**End of Paper**

**2022 SA2 3E Mathematics Paper 1 Marking Scheme**

|    |   |   |  |
|----|---|---|--|
| 1. | $2x - y = 2 \quad \text{---(1)}$ $x + 3y = 15 \quad \text{---(2)}$ $(2) \times 2 \quad 2x + 6y = 30 \quad \text{---(3)}$ $(1) - (3) \quad -7y = -28$ $y = 4$ <p>Sub <math>y = 4</math> into equation (2)</p> $x = 15 - 12$ $x = 3$ $x = 3, y = 4$ | <p>[M1] Reduce to linear equation with one variable. Substitution Method is also accepted.</p> <p>[A2]</p>  |  |
| 2. | (a)   | $(3a^3)^2 = 9a^6$   | [B1]   |
|    | (b)   | $6pq \times 4p^3q = 24p^4q^2$   | [B1]   |
|    | (c)   | $\sqrt[3]{a^4} \div a^{\frac{1}{3}} \times a^0$ $= a^{\frac{4}{3}} \div a^{\frac{1}{3}} \times a^0$ $= a$   | <p>[M1]</p> <p>[A1]</p>  |
| 3. | (a)   | Figure 5  | [B1]   |
|    | (b)   | Figure 6  | [B1]   |
|    | (c)   | Figure 1  | [B1]   |
| 4. | (a)   | $5 - 3x \leq 1 - 2x < 3x + 7$ $5 - 3x \leq 1 - 2x \qquad 1 - 2x < 3x + 7$ $-x \leq 1 - 5 \qquad -2x - 3x < 7 - 1$ $-x \leq -4 \qquad \text{and} \qquad -5x < 6$ $x \geq 4 \qquad x > -\frac{6}{5}$ <p>Solution: <math>x \geq 4</math></p> | <p>[M1]</p> <p>[M1]</p> <p>For solving inequalities.</p> <p>[A1]</p> |
|    | (b)   | 5   | [B1]   |

|    |     |   |                        |
|----|-----|---|------------------------|
| 5. | (a) | $\text{number of elderly} = \frac{9}{100} \times 5.626 \times 10^6$ $\approx 5.063 \times 10^5$ $\approx 5.1 \times 10^5$ | [B1]                   |
|    | (b) | $\text{number of youths} = 2 \times 5.063 \times 10^5$ $\approx 1.0 \times 10^6$  | [B1]                   |
| 6. | (a) | $3^4 \times 3^q = \frac{1}{27}$ $3^{4+q} = \frac{1}{3^3}$ $3^{4+q} = 3^{-3}$ $4+q = -3$ $q = -7$                          | [M1] Same base<br>[A1] |
|    | (b) | $8^r \div 4 = 1$ $2^{3r} \div 2^2 = 2^0$ $2^{3r-2} = 2^0$ $3r-2 = 0$ $r = \frac{2}{3}$                                    | [M1] Same base<br>[A1] |
| 7. | (a) | $xy = 4(-3)$ $= -12$  | [B1]                   |
|    | (b) | $x^2 - y^2 = (-5)^2 - 0^2$ $= 25$   | [B1]                   |
| 8. | (a) | $y = -(x+1)(x-3)$ $= -(x^2 - 3x + x - 3)$ $= -(x^2 - 2x - 3)$ $= -x^2 + 2x + 3$   | [M1]<br>[A1]           |
|    | (b) | $x = 1$   | [B1]                   |
|    | (c) | $(1, 4)$  | [B1]                   |
| 9. | (a) | $3 : 4$   | [B1]                   |

|     |     |  |  |
|-----|-----|--|--|
|     | (b) | $\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3$ $= \frac{27}{64}$ <p>Volume of smaller bottle = <math>\frac{27}{64} \times 1440</math></p> $= 607.5 \text{ cm}^3$   | [M1]<br><br>[A1]   |
| 10. | (a) | $x^2 + 8x - 20$ $= (x+4)^2 - 16 - 20$ $= (x+4)^2 - 36$   | [B1]<br>[B1]   |
|     | (b) | $x^2 + 8x - 20 = 0$ $(x+4)^2 - 36 = 0$ $(x+4)^2 = 36$ $x+4 = \pm 6$ $x = 2 \text{ or } -10$  | [M1]<br>[A1]   |
| 11. | (a) | <p><math>\triangle ABC</math> is similar to <math>\triangle ADF</math>.</p> <p><math>\angle BAC = \angle DAF</math> (common <math>\angle</math>s)</p> <p><math>\angle ABC = \angle ADF</math> (corresponding <math>\angle</math>s)</p> <p>By AA similarity test, <math>\triangle ABC</math> is similar to <math>\triangle ADF</math>.</p> <p>or</p> <p><math>\triangle DBE</math> is similar to <math>\triangle ADF</math>.</p> <p><math>\angle DBE = \angle ADF</math> (corresponding <math>\angle</math>s)</p> <p><math>\angle DEB = \angle AFD</math> (corresponding <math>\angle</math>s)</p> <p>By AA similarity test, <math>\triangle DBE</math> is similar to <math>\triangle ADF</math>.</p> | [B1]<br>[B1]<br>with angle properties stated<br><br>[A1]<br>conclude with AA similarity test or AA test. |
|     | (b) | $DF = EC$ $= 12 \text{ cm}$ $\frac{BC}{12} = \frac{15}{10}$ $BC = \frac{15}{10} \times 12$ $= 18 \text{ cm}$   | [M1] Ratio<br><br>[M1] Finding   |

|     |     |   |   |
|-----|-----|---|---|
|     |     | $BE = 18 - 12$ $= 6 \text{ cm}$ <p>or</p> $\frac{BE}{12} = \frac{5}{10}$ $BE = \frac{5}{10} \times 12$ $= 6 \text{ cm}$   | $BC$<br>[A1]<br><br>[M1]<br><br>[M1]<br><br>[A1]  |
|     | (c) | $\frac{\text{area of triangle } ADF}{\text{area of triangle } ABC} = \left(\frac{10}{15}\right)^2$ $= \frac{4}{9}$ $\frac{\text{area of triangle } ADF}{\text{area of } DBCF} = \frac{4}{9-4}$ $= \frac{4}{5}$                            | [M1]<br>Mark is not awarded if students attempt to find the areas as there is no vertical height given. Hence the areas calculated will not be correct.<br><br>[A1] |
| 12. | (a) | $PR^2 = 15^2$ $= 225$ $PQ^2 + QR^2 = 12^2 + 9^2$ $= 225$ <p>Since <math>PR^2 = PQ^2 + QR^2</math>, by converse of Pythagoras' Theorem, triangle <math>PQR</math> is a right-angled triangle. Thus <math>\angle PQR = 90^\circ</math>.</p> | [M1]<br>Mark is not awarded if students write $15^2 = 12^2 + 9^2$<br><br>[A1] Students  |

|     |   |  | must state the theorem used.  |              |
|-----|---|--|---|--------------|
|     | (b)   | (i)  | $\tan \angle QPS = \frac{16}{12}$ $= \frac{4}{3}$   | [B1]         |
|     |   | (ii)   | $\cos \angle PRS = -\cos \angle PRQ$ $= -\frac{9}{15}$ $= -\frac{3}{5}$                       | [B1]         |
|     |   | (iii)  | $PS = \sqrt{12^2 + 16^2}$ $= 20 \text{ cm}$ $\sin \angle PSQ = \frac{12}{20}$ $= \frac{3}{5}$ | [M1]<br>[A1] |
| 13. | 10 workers → 90 days<br>1 worker → 900 days<br>15 workers → 60 days   |  | [M1]  |              |
|     | Yes, the manager is correct in his claim as 15 workers will only require 60 days to complete building the bus stop. |  | [A1]  |              |
|     | The assumption made is that all workers work at the same rate.  |  | [B1]  |              |
| 14. | (a)   | $\angle ABC = 180^\circ - 79^\circ - 41^\circ$ $= 60^\circ$<br>$\text{area of } \triangle ABC = \frac{1}{2} \times 4 \times 6 \times \sin 60^\circ$ $\approx 10.392$ $= 10.4 \text{ cm}^2$ | [M1]<br>[A1]  |              |
|     | (b)   | Let the shortest distance from $C$ to $AB$ be $d$ .<br><br>$\frac{1}{2} \times 4 \times d = 10.392$ $d \approx 5.20$   | [M1]<br>[A1]  |              |

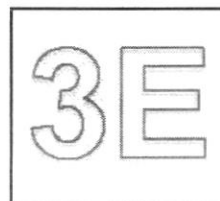
|     |  |   |
|-----|--|---|
| 15. | $1 \text{ exterior } \angle = \frac{360^\circ}{6}$ $= 60^\circ$<br>$\angle BCE = 180^\circ - (90^\circ - 60^\circ)$ $= 150^\circ$<br>$\angle BEC = \frac{180^\circ - 150^\circ}{2}$ $= 15^\circ$ | <br>[M1]<br><br>[M1]<br><br>[A1]                                    |
| 16. | $\text{Total amount} = 3000 \left( 1 + \frac{2.8 \div 2}{100} \right)^{6 \times 2}$ $\approx \$3545$   | <br>[M1]<br><br>[A1]  |
| 17. | <p>(a)</p> $\cos 33^\circ = \frac{10}{BF}$ $BF = \frac{10}{\cos 33^\circ}$ $\approx 11.924$ $\approx 11.9 \text{ cm}$  | <br>[M1]<br>awarded if<br>students use<br>sine rule too<br><br>[A1] |
|     | <p>(b)</p> $BM = 30 \div 2$ $= 15 \text{ cm}$<br>$FM = \sqrt{11.924^2 + 15^2}$ $\approx 19.162$ $\approx 19.2 \text{ cm}$  | <br>[M1]<br><br><br>[A1]  |



|  |  |  |
|--|--|--|
|  | <p>(c) Total distance</p> $= \frac{1}{2} \times [52 + (35 - 20)] \times 25$ $= 837.5 \text{ m}$<br>$q = 837.5$ | <p>[M1]</p><br><br><br><br><br><br><br><br><br><br><p>[A1]</p> |
|--|--|--|



CANBERRA SECONDARY SCHOOL



2022 End of Year Examination

Secondary Three Express

**MATHEMATICS**

4052/02

11 October 2022  
1 hour 45 minutes  
1130 – 1315

Name: \_\_\_\_\_ () **MARKING SCHEME** ( ) Class: \_\_\_\_\_

**READ THESE INSTRUCTIONS FIRST**

Write your full name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 70.

| FOR MARKER'S USE |               |           |
|------------------|---------------|-----------|
|                  | Marks Awarded | Max Marks |
| Total            |               | 70        |

This question paper consists of **16** printed pages including the cover page.

- 1 Here are the first four terms of a sequence.

$$T_1 = 1^2 + 1 = 2$$

$$T_2 = 3^2 + 2 = 11$$

$$T_3 = 5^2 + 3 = 28$$

$$T_4 = 7^2 + 4 = 53$$

- (a) Write down the fifth term of the sequence.

$$9^2 + 5 = 86 \quad [\text{B1}]$$

$$\text{Answer } T_5 = \dots\dots\dots [1]$$

- (b)  $T_n$  is the  $n$ th term of the sequence.

Show that  $T_n = 4n^2 - 3n + 1$ .

Answer

$$T_n = (2n-1)^2 + n \quad [\text{M1}] \text{ form the formula}$$

$$= 4n^2 - 4n + 1 + n \quad [\text{M1}] \text{ expand}$$

$$= 4n^2 - 3n + 1 \quad (\text{Shown}) \quad [\text{A1}]$$

[3]

- (c) The difference,  $D$ , between two consecutive terms of the sequence is  $T_{n+1} - T_n$ .

Find an expression, in terms of  $n$ , for  $D$ .

$$D = T_{n+1} - T_n$$

$$= [4(n+1)^2 - 3(n+1) + 1] - (4n^2 - 3n + 1) \quad [\text{M1}] \text{ substitute } T_n \text{ and } T_{n+1}$$

$$= 4(n^2 + 2n + 1) - 3n - 3 + 1 - 4n^2 + 3n - 1 \quad [\text{M1}] \text{ expand}$$

$$= 4n^2 + 8n + 4 - 3 - 4n^2$$

$$= 8n + 1 \quad [\text{A1}]$$

$$8n + 1$$

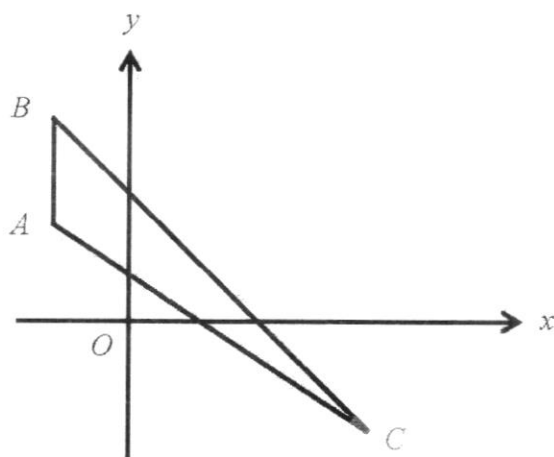
$$\text{Answer } D = \dots\dots\dots [3]$$

- (d) Explain why the difference between two consecutive terms of the sequence is always odd.

$$D = 8n + 1 = 2(4n) + 1 \Rightarrow 8n \text{ is always a multiple of } 2 \Rightarrow D \text{ always odd}$$

..... [1]

2



The diagram shows a triangle  $ABC$  with vertices  $A(-2, 4)$  and  $C(10, -4)$ .  
 $AB$  is parallel to the  $y$ -axis.

- (a) Calculate the length of the line segment  $AC$ .

$$AC = \sqrt{(-2-10)^2 + [4-(-4)]^2} \quad [\text{M1}]$$

$$= \sqrt{208}$$

$$= 14.4 \text{ units (3 s. f.)} \quad [\text{A1}]$$

14.4

Answer ..... units [2]

- (b) Given that the gradient of the line  $BC = -1$ .

Find

- (i) the coordinates of  $B$ ,

Let the coordinates of  $B$  be  $(-2, b)$ .

$$\frac{b-(-4)}{-2-10} = -1 \quad [\text{M1}]$$

$$b+4=12$$

$$b=8$$

$$B = (-2, 8) \quad [\text{A1}]$$

-2                      8

Answer  $B$  (....., .....) [2]

(ii) the equation of the line  $BC$ .

$$y = mx + c$$

$$-4 = (-1)(10) + c \quad [M1]$$

$$c = 6$$

Equation of  $BC$  is  $y = -x + 6$       [A1] reject  $y = -1x + 6$

$y = -x + 6$   
 Answer ..... [2]

(c) Calculate the area of the triangle  $ABC$ .

$$\text{Area of triangle} = \frac{1}{2} \times 4 \times 12 \quad [M1]$$

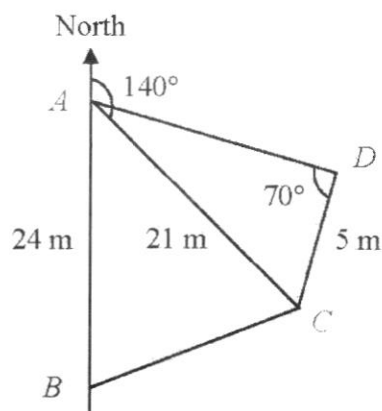
$$= 24 \text{ units}^2 \quad [A1]$$

24  
 Answer ..... units<sup>2</sup> [2]

(d) Find the coordinates of point  $D$ , given that  $ABCD$  is a parallelogram.

10                  -8  
 Answer  $D$  (....., ..... ) [1]

- 3  $ABCD$  represents a plot of land.



$A$  is 24 m due north of  $B$ .

$AC = 21$  m,  $CD = 5$  m and angle  $ADC = 70^\circ$ .

The bearing of  $C$  from  $A$  is  $140^\circ$ .

- (a) Calculate

- (i)  $BC$ ,

Using cosine rule, for  $BC > 0$ .

$$BC = \sqrt{24^2 + 21^2 - 2(24)(21)\cos(180^\circ - 140^\circ)} \quad [\text{M1}]$$

$$= 15.647 \quad (5 \text{ s. f.})$$

$$= 15.6 \text{ m} \quad (3 \text{ s. f.}) \quad [\text{A1}]$$

15.6

Answer ..... m [2]

- (ii) the bearing of  $D$  from  $A$ .

$$\text{Using sine rule, } \frac{\sin \angle CAD}{5} = \frac{\sin 70^\circ}{21} \quad [\text{M1}]$$

$$\angle CAD = \sin^{-1}\left(\frac{5 \sin 70^\circ}{21}\right)$$

$$= 12.929^\circ \quad (3 \text{ d. p.}) \quad [\text{M1}]$$

$$\text{Bearing of } D \text{ from } A = 140^\circ - 12.929^\circ$$

$$= 127.1^\circ \quad (1 \text{ d. p.}) \quad [\text{A1}]$$

127.1°

Answer ..... [3]

- (b)  $E$  is a point at the top of a tree at point  $C$ .  
The angle of elevation of  $E$  from  $D$  is  $78^\circ$ .

Calculate

- (i) the height of the tree,

$$\tan 78^\circ = \frac{h}{5} \quad [\text{M1}]$$

$$h = 5 \tan 78^\circ$$

$$= 23.523 \quad (5 \text{ s. f.})$$

$$= 23.5 \text{ m} \quad (3 \text{ s. f.}) \quad [\text{A1}]$$

23.5

Answer ..... m [2]

- (ii) the greatest possible angle of elevation of  $E$  from a point on  $AB$ .

$$\text{Shortest distance from } C \text{ to } AB = 21 \sin(180^\circ - 140^\circ)$$

$$= 13.499 \quad (5 \text{ s. f.}) \quad [\text{M1}]$$

$$\text{Greatest possible angle of elevation} = \tan^{-1}\left(\frac{23.523}{13.499}\right)$$

$$= 60.2^\circ \quad (1 \text{ d. p.}) \quad [\text{A1}]$$

60.2°

Answer ..... [2]

- 4 Aida and Bernard took part in a marathon race.

They each ran 70 km.

- (a) Aida ran at an average speed of  $x$  km/h.

Write down an expression, in terms of  $x$ , for the number of hours she took.

$$\frac{70}{x} \quad [\text{B1}]$$

Answer ..... h [1]

- (b) Bernard ran at an average speed which was 0.5 km/h more than Aida's speed.

Find an expression, in terms of  $x$ , for the number of hours he took.

$$\frac{70}{x+0.5}$$

$$\frac{140}{2x+1} \quad [\text{B1}]$$

Answer ..... h [1]

- (c) The difference between their timings was 50 minutes.

Write down an equation in  $x$  to represent this information, and show that it reduces to  $2x^2 + x - 84 = 0$ .

Answer

$$\frac{70}{x} - \frac{140}{2x+1} = \frac{5}{6}$$

[M1] forming equation

$$70(6)(2x+1) - 140(6)(x) = 5(x)(2x+1)$$

[M1] removing denominator

$$840x + 420 - 840x = 10x^2 + 5x$$

$$0 = 10x^2 + 5x - 420$$

$$2x^2 + x - 84 = 0 \quad (\text{Shown}) \quad [\text{A1}]$$

[3]

- (d) Solve the equation  $2x^2 + x - 84 = 0$ , giving your solutions correct to 1 decimal place.

$$2x^2 + x - 84 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-84)}}{2(2)} \quad \text{[M1] applying quadratic formula}$$

$$= \frac{-1 \pm \sqrt{673}}{4} \quad \text{[M1]}$$

$$= -6.736 \text{ (3 d. p.) } \text{ or } 6.236 \text{ (3 d. p.)}$$

$$= -6.7 \text{ (1 d. p.) } \text{ or } 6.2 \text{ (1 d. p.)} \quad \text{[A2] correct to 1 d. p.}$$

$$\text{Answer } x = \overset{6.7}{\dots\dots\dots} \text{ or } \overset{6.2}{\dots\dots\dots} \quad \text{[4]}$$

- (e) Hence calculate the time that Aida took to complete the race.

Give your answer in hours and minutes, correct to the nearest minute.

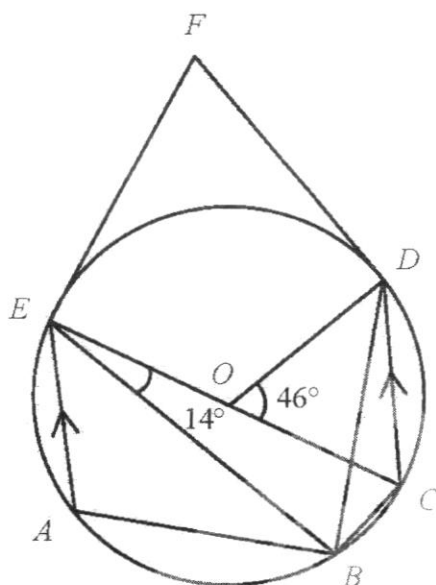
$$\text{Aida's time taken} = \frac{70}{6.236} \quad \text{[M1]}$$

$$= 11.225 \text{ h}$$

$$= 11 \text{ h } 14 \text{ min (to the nearest minute)} \quad \text{[A1]}$$

$$\text{Answer } \overset{11}{\dots\dots\dots} \text{ h } \overset{14}{\dots\dots\dots} \text{ min [2]}$$

5



The diagram shows a circle  $ABCDE$ , with centre  $O$ .  
 $AE$  is parallel to  $CD$ .  
 Angle  $COD = 46^\circ$  and angle  $CEB = 14^\circ$ .  
 $EF$  and  $DF$  are tangents to the circle at  $E$  and  $D$  respectively.

(a) Find angle  $DBE$ .

Give a reason for each step of your working.

$$\begin{aligned} \angle DOE &= 180^\circ - 46^\circ && \text{(adjacent angles on a straight line)} && \text{[M1]} \\ &= 134^\circ \end{aligned}$$

$$\begin{aligned} \angle DBE &= \frac{134^\circ}{2} && \text{(angle at centre is twice angle at circumference)} \\ &= 67^\circ && \text{[A1] with correct reason} \end{aligned}$$

\* one reason one mark

67°

Answer Angle  $DBE = \dots\dots\dots$  [2]

(b) Find angle  $AEB$ .

Give a reason for each step of your working.

$$\angle DCE = \angle DBE = 67^\circ \quad \text{(angles in the same segment)} \quad \text{[M1]}$$

$$\angle AEC = \angle DCE = 67^\circ \quad \text{(alternate angles, } AE \parallel CD)$$

$$\begin{aligned} \angle AEB &= 67^\circ - 14^\circ \\ &= 53^\circ && \text{[A1] with correct reason} \end{aligned}$$

\* one reason one mark

53°

Answer Angle  $AEB = \dots\dots\dots$  [2]

- (c) Find angle
- $BAE$
- .

Give a reason for each step of your working.

$$\angle CBE = 90^\circ \quad (\text{angle in semicircle})$$

$$\begin{aligned} \angle BCE &= 180^\circ - 90^\circ - 14^\circ && (\text{angle sum of triangle}) \quad [\text{M1}] \\ &= 76^\circ \end{aligned}$$

$$\begin{aligned} \angle BAE &= 180^\circ - 76^\circ && (\text{opposite angles in cyclic quadrilateral}) \\ &= 104^\circ && [\text{A1}] \text{ with correct reason} \end{aligned}$$

\* one reason one mark

104°

Answer Angle  $BAE = \dots\dots\dots$  [2]

- (d) Find angle
- $EFO$
- .

Give a reason for each step of your working.

$$\begin{aligned} \angle FOE &= \frac{134^\circ}{2} && (\text{tangent from external point}) \\ &= 67^\circ \end{aligned}$$

$$\angle OEF = 90^\circ \quad (\text{angle between tangent and radius}) \quad [\text{M1}]$$

$$\begin{aligned} \angle EFO &= 180^\circ - 90^\circ - 67^\circ && (\text{angle sum of triangle}) \\ &= 23^\circ && [\text{A1}] \text{ with correct reason} \end{aligned}$$

\* one reason one mark

23°

Answer Angle  $EFO = \dots\dots\dots$  [2]

- (e) A point
- $G$
- lies on the same side of
- $DE$
- as
- $B$
- such that angle
- $DGE = 90^\circ$
- .

State whether point  $G$  lies inside, outside or on the circle.  
Explain your answer.G lies inside the circle. Angle  $DGE = 90^\circ > 67^\circ = \text{angle } DBE$ .

..... [1]

- (f) The radius of the circle is 5 cm.

Calculate the length of the major arc  $EABCD$ .

$$\text{Arc length} = \frac{180^\circ + 46^\circ}{360^\circ} \times 2\pi(5) \quad [\text{M1}]$$

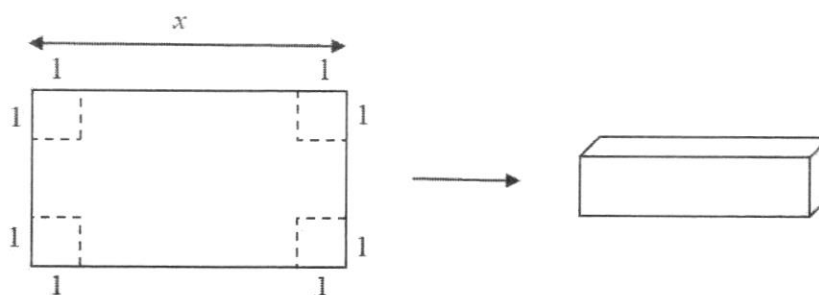
$$= 19.7 \text{ cm (3 s. f.)} \quad [\text{A1}]$$

19.7

Answer ..... cm [2]

- 6 A rectangular sheet of paper has length  $x$  cm and an area of  $36 \text{ cm}^2$ .

Four identical squares of side 1 cm are cut from the four corners of the paper and the paper is folded to make an open box.



- (a) Show that the volume of the open box,  $y \text{ cm}^3$ , is given by  $y = 2(x-2)\left(\frac{18}{x}-1\right)$ .

*Answer*

$$y = (x-1-1)\left(\frac{36}{x}-1-1\right)(1) \quad [\text{M1}]$$

$$= (x-2)\left(\frac{36}{x}-2\right)$$

$$= 2(x-2)\left(\frac{18}{x}-1\right) \quad (\text{Shown}) \quad [\text{A1}]$$

[2]

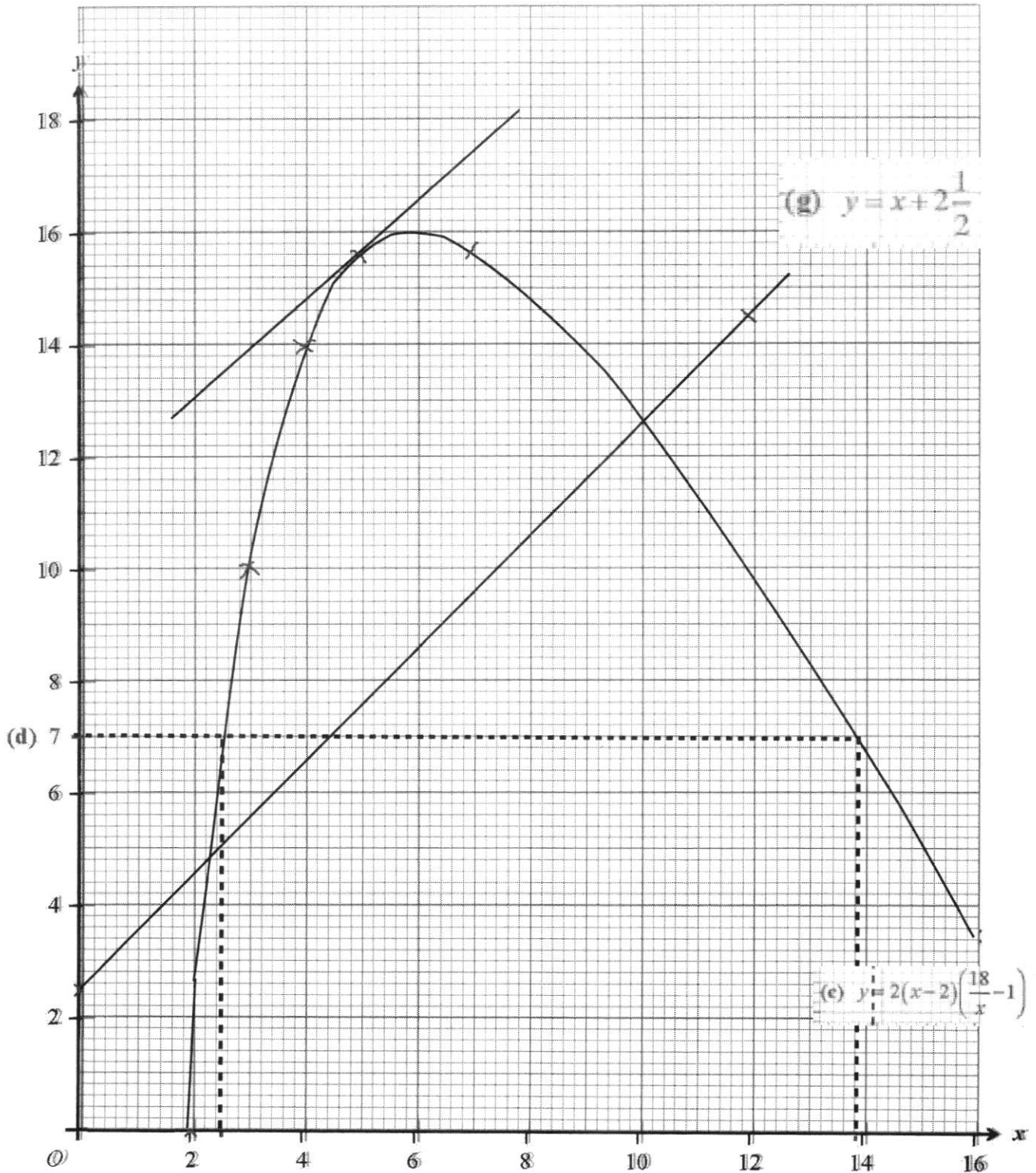
- (b) Complete the table of values for  $y = 2(x-2)\left(\frac{18}{x}-1\right)$ .

Values are given to 1 decimal place where appropriate.

|     |   |    |    |      |      |    |    |            |
|-----|---|----|----|------|------|----|----|------------|
| $x$ | 2 | 3  | 4  | 5    | 7    | 9  | 12 | 16         |
| $y$ | 0 | 10 | 14 | 15.6 | 15.7 | 14 | 10 | <u>3.5</u> |

[1]

(c) On the grid, draw the graph of  $y = 2(x-2)\left(\frac{18}{x}-1\right)$  for  $0 \leq x \leq 16$ .



Correct plotting of all 8 points [G1]  
 Smooth curve passing through all 8 points [G1]

[2]

- (d) Use your graph to estimate the range of values of  $x$  for which the volume of the box is more than  $7 \text{ cm}^3$ .

(accept  $\pm 0.1$ )

$2.6 < x < 13.9$  [B1]

Answer ..... [1]

- (e) Explain how the graph shows that the volume cannot be equal to  $18 \text{ cm}^3$ .  
The maximum volume is  $16 \text{ cm}^3$ , which is less than  $18 \text{ cm}^3$ .

..... [1]

- (f) By drawing a tangent, find the gradient of the curve at the point  $x = 5$ .

$$m = \frac{17.4 - 13}{7 - 2}$$

[M1] gradient working with tangent drawn

$$= 0.88 \left( = \frac{22}{25} \right)$$
 [A1]

0.88

Answer ..... [2]

- (g) By drawing a suitable straight line on your graph, find the solutions of the equation  $2x^2 + 5x = 4(x-2)(18-x)$ .

$$2x^2 + 5x = 4(x-2)(18-x)$$

$$\frac{2x^2 + 5x}{2x} = \frac{4(x-2)(18-x)}{2x}$$

$$x + 2\frac{1}{2} = 2(x-2)\left(\frac{18}{x} - 1\right)$$

$$x + 2\frac{1}{2} = y$$

[M1] correct equation of straight line

Plot the line  $y = x + 2\frac{1}{2}$ .

|     |                |                |                 |
|-----|----------------|----------------|-----------------|
| $x$ | 0              | 6              | 12              |
| $y$ | $2\frac{1}{2}$ | $8\frac{1}{2}$ | $14\frac{1}{2}$ |

[M1] correct (labelled) straight line drawn

From the graph,

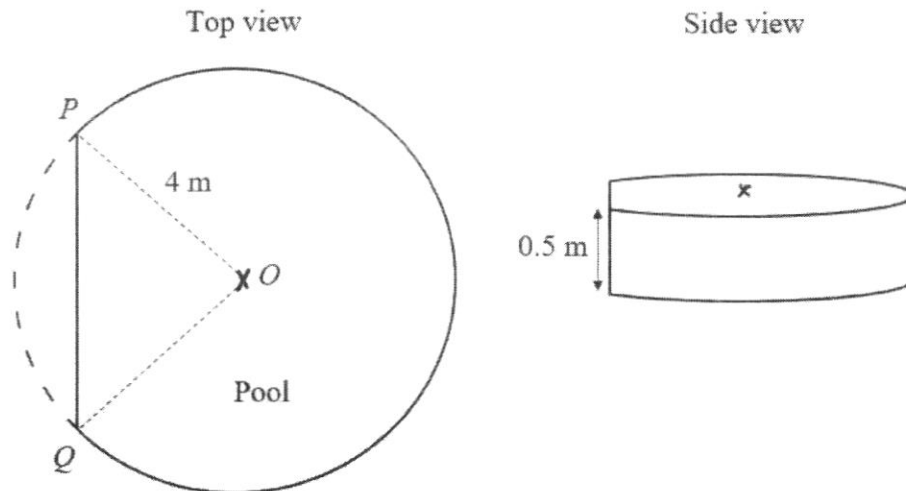
$$x = 2.4 (\pm 0.1) \text{ or } x = 10.1 (\pm 0.1)$$
 [A1]

2.4

10.1

Answer  $x = \dots$  or  $\dots$  [3]

7



The diagrams show a kids' pool in a refurbished hotel.

The cross-section of the pool is a major segment of a circle, centre  $O$  and radius 4 m. The depth of the pool is 0.5 m.

(a) The area of the minor sector  $OPQ$  is  $11.2 \text{ m}^2$ .

(i) Find angle  $POQ$  in radians.

$$\frac{1}{2}(4)^2 \theta = 11.2$$

$$\theta = 1.4$$

1.4 [B1]

Answer Angle  $POQ = \dots\dots\dots$  rad [1]

(ii) Show that the capacity of the pool is  $23.47 \text{ m}^3$ , correct to 2 decimal places.

Answer

$$\text{Area of triangle } POQ = \frac{1}{2}(4)^2 \sin 1.4 \quad [\text{M1}]$$

$$= 7.8836 \text{ m}^2 \quad (4 \text{ d. p.})$$

$$\text{Area of major sector } POQ = \frac{1}{2}(4)^2 (2\pi - 1.4) \quad [\text{M1}]$$

$$= 39.0655 \text{ m}^2 \quad (4 \text{ d. p.})$$

$$\text{Volume} = (7.8836 + 39.0655) \times 0.5$$

$$= 23.47 \text{ m}^3 \quad (2 \text{ d. p.}) \quad (\text{Shown}) \quad [\text{A1}]$$

[3]

- (b) For safety reasons, a flight of stairs made from  $1.05 \text{ m}^3$  cement was placed along  $PQ$  inside the pool.

Mr Tan, the manager, needs to fill the pool with water, up to 95% capacity.

Company GRC provides water-filling service for swimming pools.

|                    |                               |
|--------------------|-------------------------------|
| Water pump         | 8 gallons per minute          |
| Cost of water pump | \$17 per 100 gallons of water |
| Labour cost        | \$30 per hour                 |

$1 \text{ gallon} = 3.785 \text{ litres}$

$1 \text{ litre} = 0.001 \text{ m}^3$

Mr Tan thinks that the pool can be filled in 10 hours, within a budget of \$1200.

Is Mr Tan correct?

Justify your answer.

*Answer*

$$\text{Capacity to be filled} = \frac{95}{100}(23.47 - 1.05)$$

$$= 21.299 \text{ m}^3 \quad \text{[M1] vol in m}^3$$

$$= 21299 \text{ litres}$$

$$= 5627.213 \text{ gallons (3 d. p.)} \quad \text{[M1] vol in gallons}$$

$$\text{Cost of water pump} = \$17 \times 56$$

$$= \$952 \text{ (2 d. p.)} \quad \text{[M1] cost of pump}$$

$$\text{Time taken} = \frac{5627.213}{8}$$

$$= 703.402 \text{ min (3 d. p.)} \quad \text{[M1] time in min}$$

$$= 11.72 \text{ h (2 d. p.)}$$

$$= 12 \text{ h (nearest hour)}$$

$$\text{Total cost} = \$952 + \$30 \times 12 \quad \text{[M1] labour cost}$$

$$= \$1312$$

.....  
Mr Tan is **incorrect** as it will take about **12 h** at a cost of **\$1312**. [A1]

..... [6]

**End of Paper**