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Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS

4047/1

15 September 2015

2 hour

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **All** questions.

Attempt Question **1 to 8** in **Answer Booklet 1A**
Question **9 to 13** in **Answer Booklet 1B**.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This document consists of **5** printed pages, including this cover page.

43

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2}ab \sin C$$

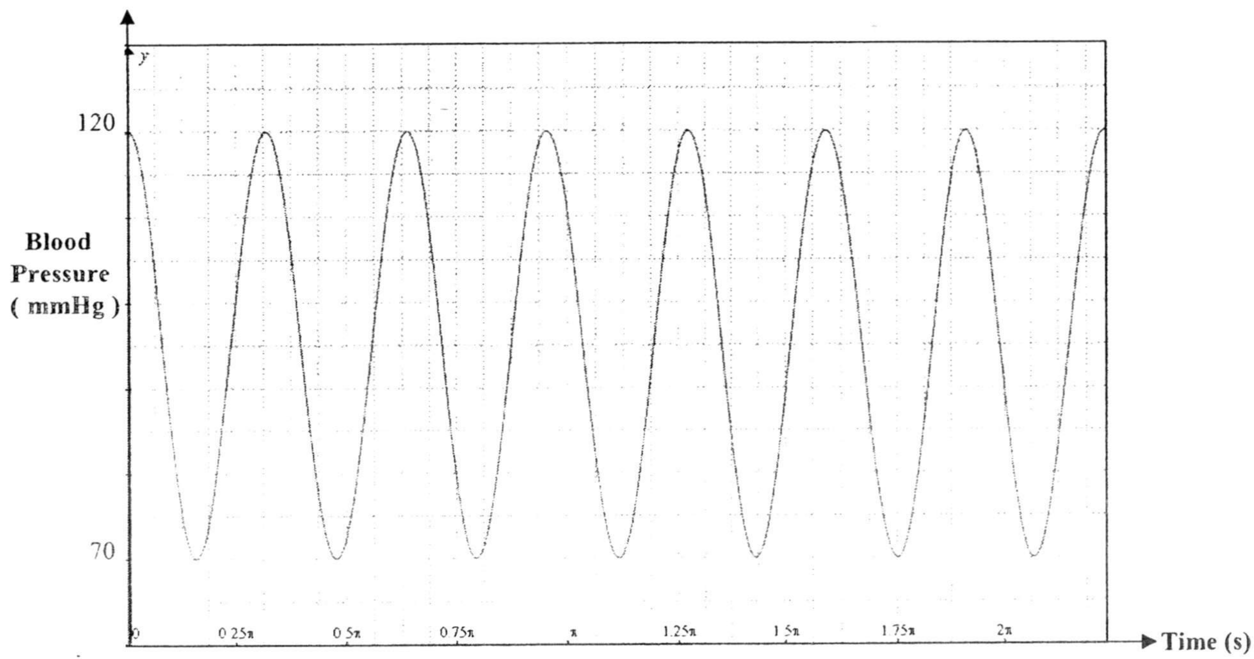
1 Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x **without using a calculator**. [3]

2 Express $\dfrac{4x+7}{x^2+6x+9}$ in partial fractions. [4]

3 Given that θ is acute and that $\sin\theta = \dfrac{1}{\sqrt{3}}$, express, **without using a calculator**,

$\dfrac{1}{\cos\theta - \sin\theta}$ in the form $\sqrt{a}+\sqrt{b}$ where a and b are integers. [5]

4



The diagram shows a part of the curve of a person's blood pressure, which is modelled using

$y = a\cos bt + c$

where t is time in seconds and y is the blood pressure measured in mm(of mercury).

The length of the same person's heartbeat is the time between two consecutive peaks on the curve.

Given that the person's heartbeat is 60 beats per minute,

(a) Write down the amplitude of y. [1]

(b) Explain why the period of the function is 1 second. [1]

(c) Write down the value of

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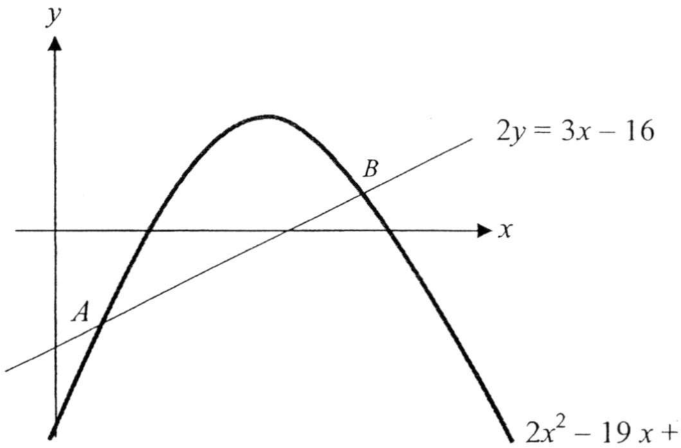
5



The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points A and B. Given that A lies below the x-axis and that the point P lies on AB such that $AP : PB = 3 : 1$, find the co-ordinates of P. [6]

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(i) Find the gradient of the curve when $x = \frac{\pi}{6}$. [4]

(ii) Given that x is decreasing at a constant rate of $2\sqrt{3}$ units per second, find the rate of change of y when $x = \frac{\pi}{6}$. [2]

7 **(i)** Given that $y = x^2\sqrt{2x-1}$, show that $\frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}}$. [2]

(ii) Hence evaluate $\int_1^5 \frac{5x^2 - 2x + 1}{\sqrt{2x-1}}\,dx$. [4]

8 Find the coordinates of the stationary point on the curve $y = 2x^3 - 6x^2 + 6x - 11$ and determine the nature of the stationary point. [7]

9 **(a)** Show that the roots of the equation $6x^2 + 5(m-1) = 3(x+m)$ are real if $m < 2\frac{11}{16}$. [3]

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10 A particle P moves in a straight line so that t seconds after leaving a fixed point O, its velocity v ms^{-1} is given by $v=(2t-3)^2-9$.

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(i) Find the coordinates of the centre and radius of C. [3]

(ii) Give a reason why the y-axis is a tangent to C. [1]

The circle C crosses the x-axis at the point $P(1\ ,\ 0)$.

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It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T=Ke^{-qt}$, where q is a constant.
Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	72.8	60.2	45.2	34.0	28.1

(i) On graph paper, plot $\ln T$ against t and draw a straight line graph. [3]

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

~ End Of Paper ~

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SOLUTION:

1	$2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$
	$4(2^{2x}) \times \frac{5^x}{5} = 2^{3x} \times 5^{2x}$
	$\frac{(2^{3x})(5^{2x})}{(2^{2x})(5^x)} = \frac{4}{5}$
	$(2^x)(5^x) = \frac{4}{5}$
	$10^x = \frac{4}{5}$

2 Express $\frac{4x+7}{x^2+6x+9}$ in partial fractions. [4]

SOLUTION:

2	$\frac{4x+7}{x^2+6x+9}$
	$\frac{4x+7}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$
	$4x+7 = A(x+3)+B$
	Let $x=-3$, $B=-5$ Let $x=0$, $7=3A-5$ $A=4$
	$\frac{4x+7}{x^2+6x+9} = \frac{4}{x+3} - \frac{5}{(x+3)^2}$

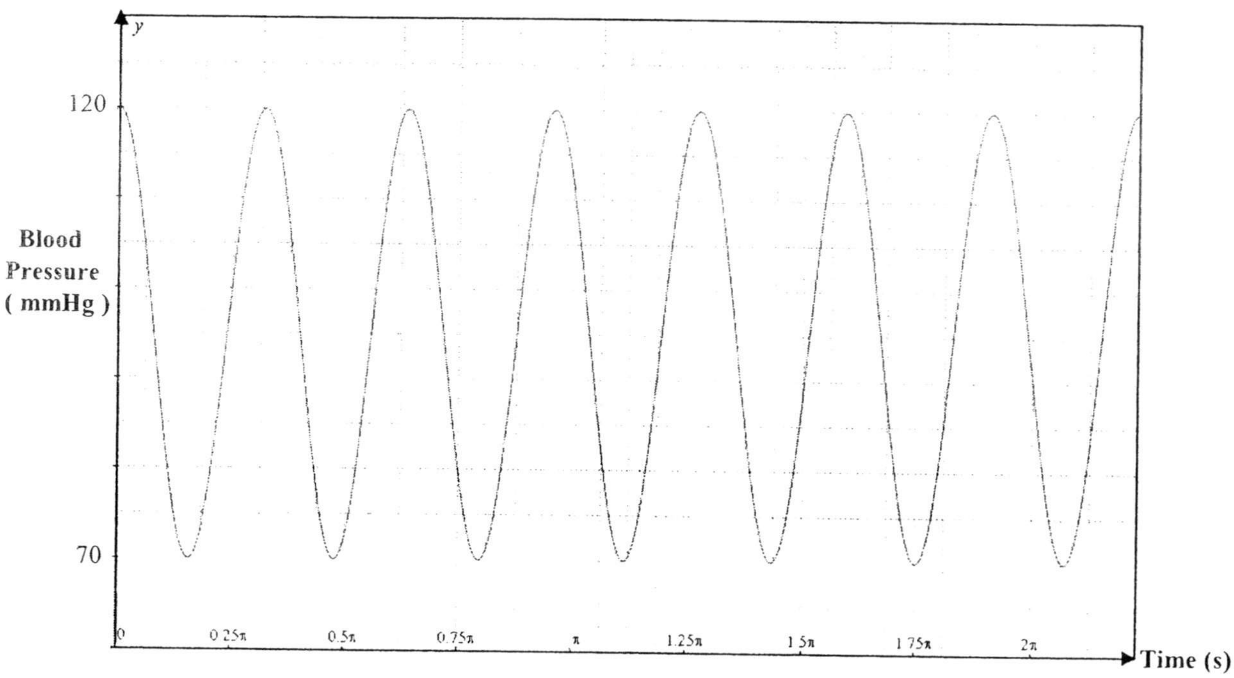
47

3 Given that θ is acute and that $\sin\theta = \frac{1}{\sqrt{3}}$, express, **without using a calculator**, $\frac{1}{\cos\theta - \sin\theta}$ in the form $\sqrt{a}+\sqrt{b}$ where a and b are integers. [5]

SOLUTION:

3.	$1^2 + x^2 = \left(\sqrt{3}\right)^2$
	$x = \sqrt{2}$
	$\therefore \cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$
	$\frac{1}{\cos\theta - \sin\theta} = \frac{1}{\frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{6}+\sqrt{3}}{2-1}$
	$= \sqrt{6}+\sqrt{3}$

4



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where t is time in seconds and y is the blood pressure measured in mm (of mercury).

The length of the same person's heartbeat is the time between two consecutive peaks on the curve.

Given that the person's heartbeat is 60 beats per minute,

(a) Write down the amplitude of y. [1]

(b) Explain why the period of the function is 1 second. [1]

(c) Write down the value of

(i) a,

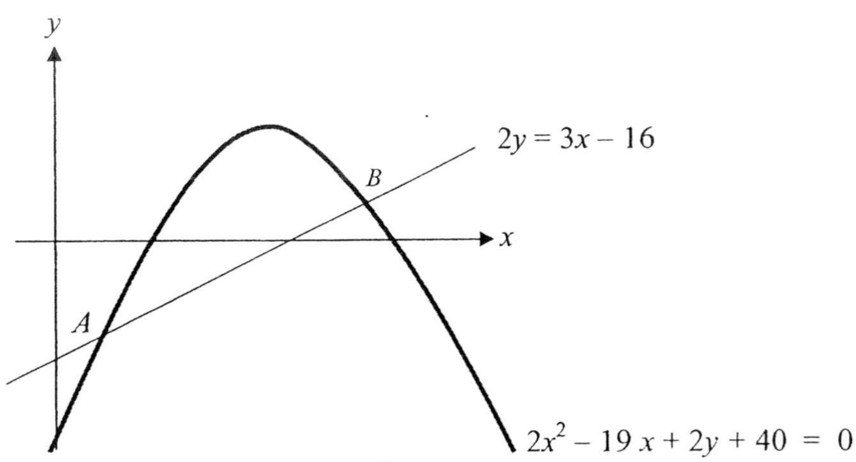
(ii) b

(iii) c. [3]

SOLUTION

4	$y = a\cos bt + c$
(a)	amplitude of $y = 25$
(b)	60 beats/cycles per 60 seconds. Therefore 1 cycle takes 1 second
(c)	$a = 25$ $b = \frac{2\pi}{1} = 2\pi$ $c = 95$

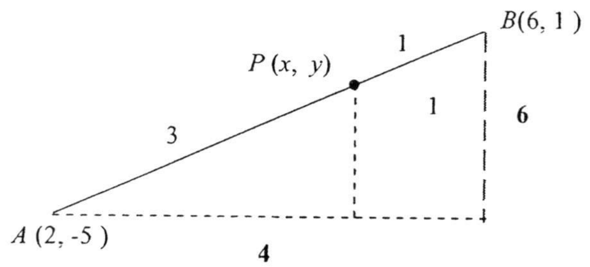
5



The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points A and B. Given that A lies below the x-axis and that the point P lies on AB such that $AP : PB = 3 : 1$, find the co-ordinates of P. [6]

48

SOLUTION

5	$2y = 3x - 16$ into $2x^2 - 19x + 2y + 40 = 0$
	$2x^2 - 19x + 3x - 16 + 40 = 0$ $2x^2 - 16x + 24 = 0$ $(x-2)(x-6) = 0$ $x = 2, y = -5 \quad A(2,-5)$ $x = 6, y = 1 \quad B(6,1)$
	 $\therefore P\left(2 + \frac{3}{4} of 4, -5 + \frac{3}{4} of 6\right)$ $\therefore P\left(2+3, -5+4\frac{1}{2}\right) \Rightarrow P\left(5, -\frac{1}{2}\right)$

6 A curve has the equation $y = \sin x - 3\cos 2x$.

(i) Find the gradient of the curve when $x = \frac{\pi}{6}$. [4]

(ii) Given that x is decreasing at a constant rate of $2\sqrt{3}$ units per second, find the rate of change of y when $x = \frac{\pi}{6}$. [2]

SOLUTION

6	$y = \sin x - 3\cos 2x$
(i)	$\frac{dy}{dx} = \cos x + 6\sin 2x$ At $x = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + 6\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{7\sqrt{3}}{2}$ *or* 6.06
(ii)	At $x = \frac{\pi}{6}$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= \frac{7\sqrt{3}}{2} \times \left(-2\sqrt{3}\right)$ $= -21$ units /s **OR** y is decreasing at 21 units /s

7 **(i)** Given that $y = x^2\sqrt{2x-1}$, show that $\frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}}$. [2]

(ii) Hence evaluate $\int_1^5 \frac{5x^2-2x+1}{\sqrt{2x-1}}\,dx$. [4]

SOLUTION

7	$y = x^2\sqrt{2x-1}$
(i)	$\frac{dy}{dx} = \left(\sqrt{2x-1}\right)(2x) + x^2\left(\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)\right)$ $= (2x-1)^{-\frac{1}{2}}(x)\left[2(2x-1)+x\right]$ $= \frac{x(5x-2)}{\sqrt{2x-1}}$ (Shown)
(ii)	$\int \frac{x(5x-2)}{\sqrt{2x-1}}\,dx = x^2\sqrt{2x-1}$ $\int_1^5 \frac{5x^2-2x+1}{\sqrt{2x-1}}\,dx = \left[x^2\sqrt{2x-1}\right]_1^5 + \int_1^5 \frac{1}{\sqrt{2x-1}}\,dx$ $= \left[x^2\sqrt{2x-1}\right]_1^5 + \left[\frac{\left(\sqrt{2x-1}\right)(2)}{2}\right]_1^5$ $= \left[x^2\sqrt{2x-1}\right]_1^5 + \left[\sqrt{2x-1}\right]_1^5$ $= 74 + 2 = 76$

49

8 Find the coordinates of the stationary point on the curve $y = 2x^3 - 6x^2 + 6x - 11$ and determine the nature of the stationary point. [7]

SOLUTION

8	$y = 2x^3 - 6x^2 + 6x - 11$
	$\frac{dy}{dx} = 6x^2 - 12x + 6$ At turning point, $6x^2 - 12x + 6 = 0 \Rightarrow (x-1)^2 = 0$ $x = 1$ x: 1^-, 1, 1^+ $\frac{dy}{dx}$: +ve, 0, +ve Shape: /, --, / At $x = 1$, $y = -9$ $\therefore$ Point of inflexion at $(1, -9)$

9 **(a)** Show that the roots of the equation $6x^2 + 5(m-1) = 3(x+m)$ are real if $m < 2\frac{11}{16}$. [3]

(b) Find the range of values of k for which $(k+3)x^2 + 4x + k$ is always negative for all real values of x. [4]

SOLUTION

9	$6x^2 + 5(m-1) = 3(x+m)$
(a)	$6x^2 - 3x + 2m - 5 = 0$ $b^2 - 4ac = 9 - 4(6)(2m-5) = 129 - 48m$ For real roots, $129 - 48m > 0$ $48m < 129$ $m < 2\frac{11}{16}$.
(b)	For function to be negative, $b^2 - 4ac < 0$ and $(k+3) < 0$ $16 - 4k(k+3) < 0$ $-4k^2 - 12k + 16 < 0$ $-4(k+4)(k-1) < 0$ $k < -4$ or $k > 1$ (reject)

10 A particle P moves in a straight line so that t seconds after leaving a fixed point O, its velocity v ms^{-1} is given by $v=(2t-3)^2-9$.

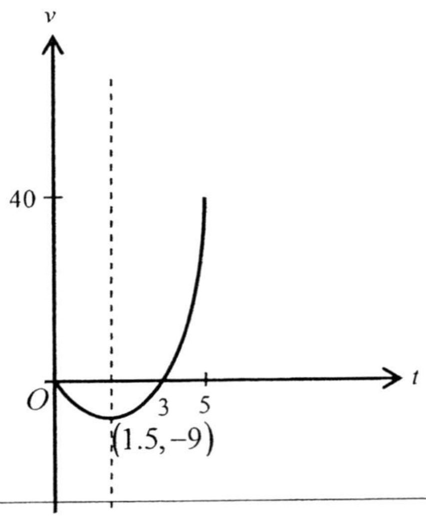
(a) Sketch the v-t graph of the particle P for $0 \le t \le 5$. [2]

(b) Hence or otherwise,

(i) find the range of values of t for which the acceleration of P is less than 4 m/s^2. [2]

(ii) find the distance travelled by P in the first 5 seconds. [3]

SOLUTION

10	$v=(2t-3)^2-9.$
(a)	
b(i)	$\frac{dv}{dt}=4(2t-3)$ $\frac{dv}{dt}<4 \quad \Rightarrow 4(2t-3)<4 \quad \Rightarrow t<2 \qquad \therefore 0 \le t<2$
(ii)	Distance traveled $=\left|\int_0^3 (2t-3)^2-9\,dt\right|+\int_3^5 (2t-3)^2-9\,dt$ $=\left|\left[\frac{(2t-3)^3}{6}-9t\right]_0^3\right|+\left|\left[\frac{(2t-3)^3}{6}-9t\right]_3^5\right|$ $=18+16\frac{2}{3}+18$ $=52\frac{2}{3}$ m

50

11 In the expansion of $\left(x^2 - \frac{k}{2x}\right)^6$, where k is a positive constant, the term independent of x is 15.

(i) Show that $k = 2$. [4]

(ii) With this value of k, find the coefficient of x^4 in the expansion of

$\left(x^2 - \frac{k}{2x}\right)^6 (8x+1)$. [3]

SOLUTION

7	$\left(x^2 - \frac{k}{2x}\right)^6$
(i)	${}^6C_4 (x^2)^2 \left(-\frac{k}{2x}\right)^4 = 15$ $\frac{k^4}{2^4} \times 15 = 15$ $k^4 = 2^4$ $k = 2$ OR General Term $= {}^6C_r (x^2)^{6-r} \left(-\frac{k}{2x}\right)^r = {}^6C_r \left(-\frac{k}{2}\right)^r (x^2)^{6-r} (x)^{-r}$ Independent of x : $(x^2)^{6-r}(x)^{-r} = x^0$ Therefore $12 - 3r = 0$, $r = 4$ Term $= {}^6C_4 \left(-\frac{k}{2}\right)^4 = 15$ $15 \times \left(-\frac{k}{2}\right)^4 = 15$ $k^4 = 2^4$ $k = 2$
(ii)	$(\ldots - 20x^3 + \ldots)(8x+1)$ x^4 term $= -160x^3$ $\therefore$ Coefficient of $x^4 = -160$

12 A circle, C, has equation $x^2+y^2-10x+6y+9=0$.

(i) Find the coordinates of the centre and radius of C. [3]

(ii) Give a reason why the y-axis is a tangent to C. [1]

The circle C crosses the x-axis at the point $P(1\ ,0)$.

(iii) Show that the equation of the tangent to the circle C at P is $3y-4x=-4$. [3]

(iv) Find the coordinates of the point where the circle C crosses the x-axis again. [1]

<u>SOLUTION</u>

12	$x^2+y^2-10x+6y+9=0$
(i)	$x^2-10x+5^2-5^2+y^2+6y+9=0$ $(x-5)^2+(y+3)^2=25$ So, centre is $(5\ ,-3)$ and radius is 5
(ii)	Since radius is 5, leftmost x-coordinate of circle C is $5-5=0$ Hence, the y-axis is a tangent to C.
(iii)	$\text{grad}_{P.centre}=\dfrac{0+3}{1-5}=-\dfrac{3}{4}$ Equation of tangent is $y=\dfrac{4}{3}x+c$ At $P(1\ ,0)$, $0=\dfrac{4}{3}+c$ $c=-\dfrac{4}{3}$ $y=\dfrac{4}{3}x-\dfrac{4}{3}$ *or* $3y-4x=-4$
(iv)	$x^2-10x+5^2-5^2+y^2+6y+9=0$ sub $y=0$, $x^2-10x+9=0$ $(x-1)(x-9)=0$ Coordinates $=(9,\ 0)$

51

13 In a Science experiment, a container of liquid was heated to a temperature of K °C.

It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T = Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	72.8	60.2	45.2	34.0	28.1

(i) On graph paper, plot $\ln T$ against t and draw a straight line graph. [3]

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

SOLUTION

13	$T = Ke^{-qt}$
(i)	Labelling of axes of graph correct plots straight line almost passing all points
(ii)	$T = Ke^{-qt}$ $\ln T = -qt + \ln K$
	$-q = \dfrac{4.2 - 3.72}{3 - 8} = -0.096$ $q = 0.096$
	$\ln K = 4.48$ $K = e^{4.48} \approx 88.2$
(iii)	$T = 88.2e^{-0.096(5)}$ $\approx 54.6°C$ Alternatively from graph, $t = 5, \ln T = 4$ $T = e^4 \approx 54.6°C$

CATHOLIC HIGH SCHOOL, SINGAPORE

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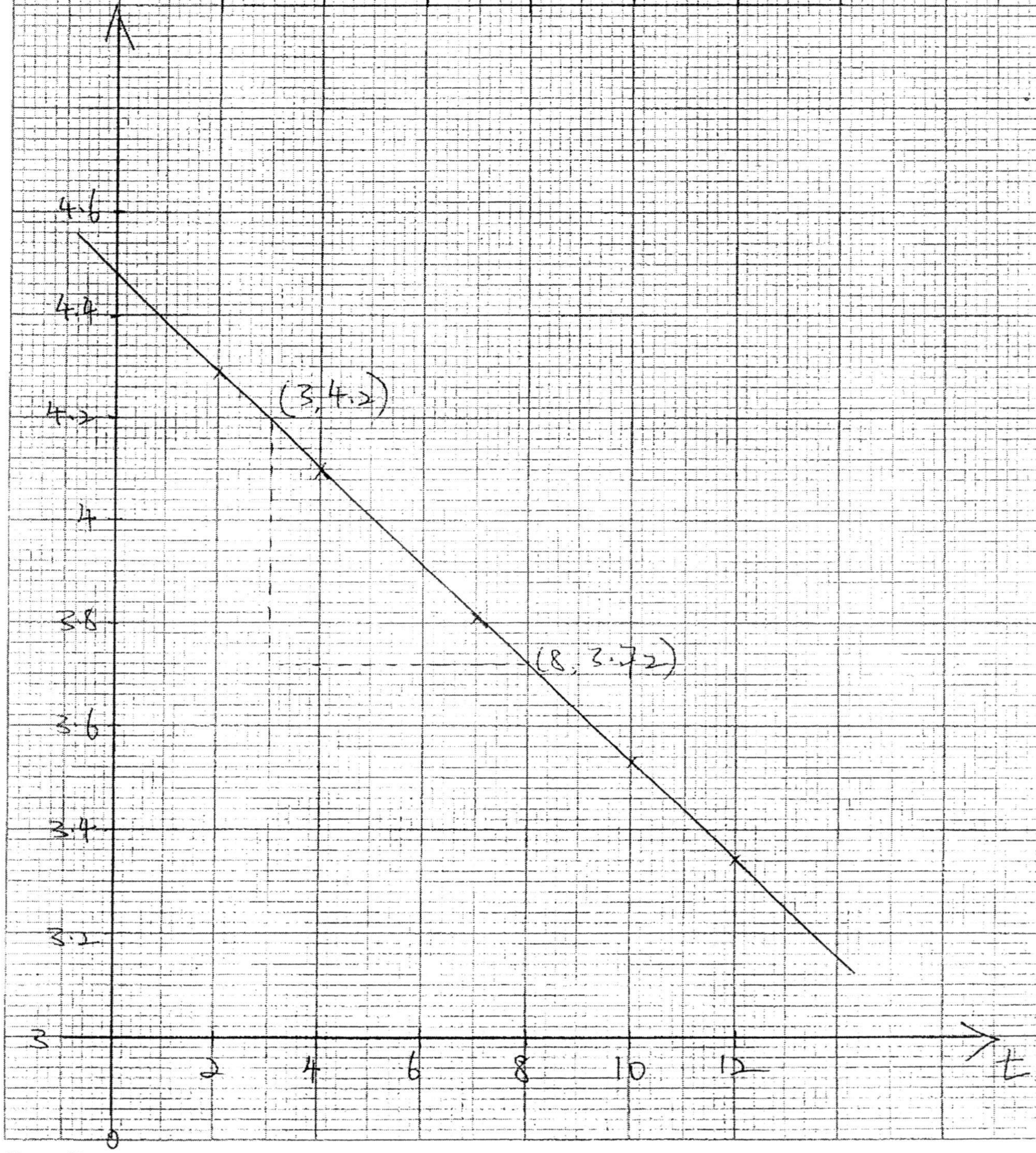
Subject ______________ Class __________ Date __________

$\ln T$	4.29	4.10	3.81	3.53	3.34
t	2	4	7	10	12



20 cm x 24 cm

52

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

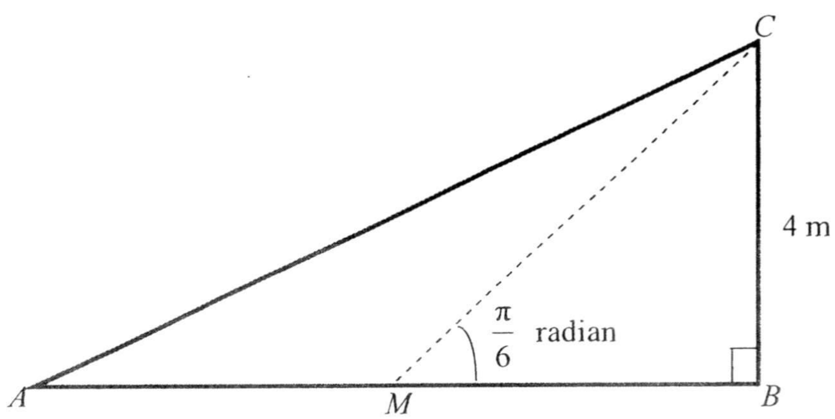
$$\Delta = \tfrac{1}{2} ab \sin C$$

1 The roots of the quadratic equation $4x^2 - 33x + 16 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, given that $\alpha + \beta > 0$ and $\alpha\beta > 0$. [6]

2 **(a)** Solve the equation $\sin^2 y + 2\cos 2y = 2\cos y$ for $0^\circ \le y \le 360^\circ$. [3]

(b) Prove that $\dfrac{\cos(A+B)+\cos(A-B)}{\sin(A+B)-\sin(A-B)} = \cot B$. [4]

3



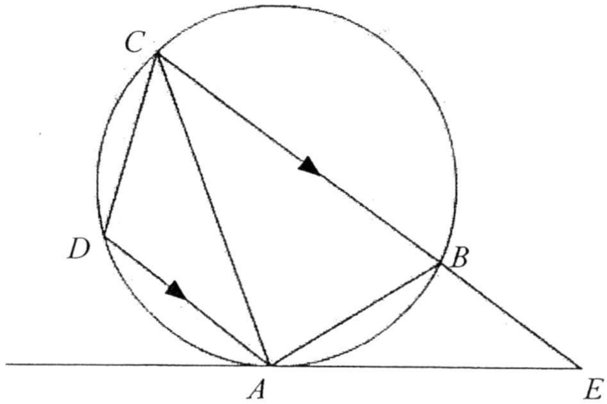
The diagram shows a triangle ABC in which angle CMB is $\frac{\pi}{6}$ radians, angle B is a right angle, M is the mid-point of AB and the length of CB is 4 m.

Without using a calculator, find the value of the integer k such that

$$\angle ACM = \sin^{-1}\left(\frac{\sqrt{k}}{26}\right).$$ [6]

54 **[TURN OVER**

4



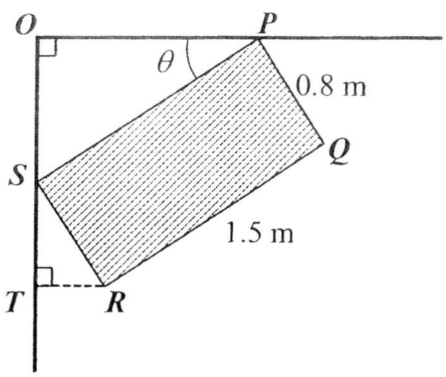
The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point E lies on the extended line CB such that AE is a tangent to the circle. CE and AD are parallel lines.

(i) Explain why angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [2]

(iii) Given that $AB = BE$, explain why the line AC bisects the angle BCD. [2]

5



The diagram shows the plan of a rectangular desk, $PQRS$, in a corner of a room. Given that the desk has length 1.5 m and width 0.8 m, and that $\angle POS = \angle STR = 90^\circ$ and $\angle OPS = \theta$.

(i) Show the length of OT, L can be expressed as $L = 1.5\sin\theta + 0.8\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta+\alpha)$ where $0^\circ < \alpha < 90^\circ$ and $R > 0$. [3]

Hence, find the value of θ for which

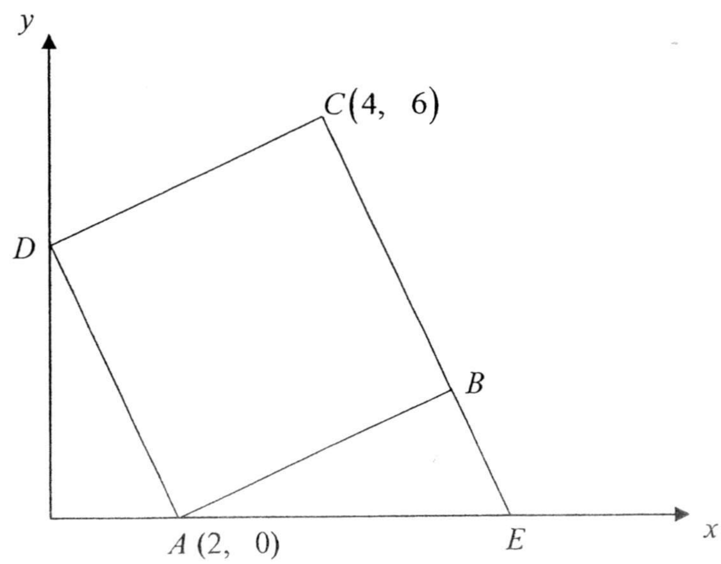
(iii) L has a maximum length, [2]

(iv) $L = 1.2$ m. [2]

6 **(a)** Simplify $\dfrac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}}$. [4]

(b) Solve the equation $5^{x+1} = 8 + 4\left(5^{-x}\right)$. [5]

7



The diagram shows a rhombus *ABCD* with vertices *A* and *C* at the points (2 , 0) and (4 , 6) respectively. *D* lies on the *y*-axis and the line *CB* produced intersects the *x*-axis at *E*.

(i) Show that the *y*-coordinate of *D* is 4. [3]

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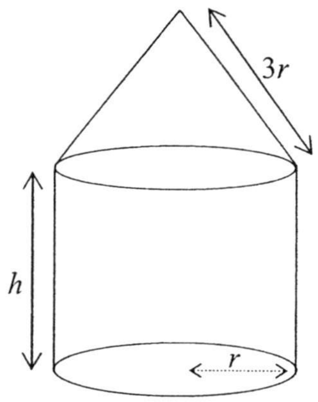
(iv) Calculate the area of the quadrilateral *AECD*. [2]

55 [TURN OVER

8



The diagram shows a solid body which consists of a cone fixed to the top of a right circular cylinder of radius r cm and height h cm. The slant edge of the cone is $3r$ cm.

(i) Given that the volume of the cylinder is 108π cm^3, express h in terms of r. [1]

(ii) Show that the total surface area, A cm^2 of the solid is given by $A = 4\pi\left(\frac{54}{r} + r^2\right)$. [3]

(iii) Given that r and h can vary,

(a) find the value of r for which A has a stationary value, [3]

(b) determine whether this stationary value is a maximum or minimum. [2]

9 **(i)** Find the range of values of m for which the curve $y = (x-1)(x-4)$ and the line $y = mx + 3$ do not intersect. [3]

(ii) Sketch the graph of $y = |(x-1)(x-4)|$, showing the coordinates of the turning point and the points where the curve meets the x-axis. [3]

(iii) Find the number of solutions of the equation $|(x-1)(x-4)| = -x + 1$. [2]

10 **(a)** **Without using a calculator**, show that $\frac{\log_2 5 \times \log_5 4}{\log_{25} 5} = 4$. [3]

(b) Given that $y = \ln\sqrt{\frac{2x}{x+4}}$, $x > 0$ and $x < -4$,

(i) find $\frac{dy}{dx}$. [4]

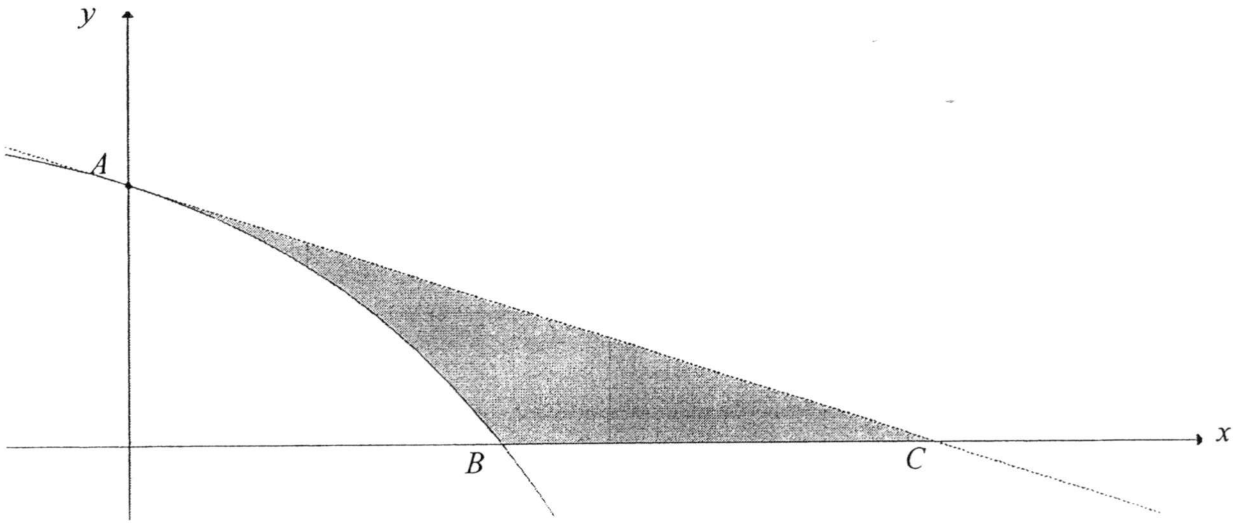
(ii) Hence show that y has no stationary value. [2]

11 The polynomial $P(x)=2x^3+ax^2+bx+8$, where a and b are constants, leaves a remainder of 10 when divided by $2x-1$. Given that $x+2$ is a factor of $P(x)$,

(i) find the value of a and of b. [5]

(ii) Explain why the equation $P(x)=0$ has only 1 real root. Hence find this root. [4]

12 The diagram shows part of the curve $y=4-e^{\frac{1}{2}x}$ which cuts the axes at A and at B.



(i) Find the coordinates of A and of B. [4]

The tangent to the curve at A meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region. [4]

~ End of Paper ~

56

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS

4047/2

16 September 2015
2 hour 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **All** questions.

Attempt Questions 1 to 4 in **Answer Booklet A**, Questions 5 to 8 in **Answer Booklet B** and Question 9 to 12 in **Answer Booklet C.**

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

This document consists of **6** printed pages, including this cover page.

57

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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Formulae for ΔABC

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1 The roots of the quadratic equation $4x^2 - 33x + 16 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, given that $\alpha + \beta > 0$ and $\alpha\beta > 0$. [6]

SOLUTION

1	$4x^2 - 33x + 16 = 0$
	$\alpha^2\beta^2 = 4$ $\alpha\beta = 2$ or -2(reject) $\alpha^2 + \beta^2 = \frac{33}{4}$ $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ $= \frac{33}{4} + 4 = \frac{49}{4}$ $\alpha + \beta = \frac{7}{2}$ or $-\frac{7}{2}$(reject bec $\alpha+\beta>0$) $\therefore$ Equation: $x^2 - \frac{7}{2}x + 2 = 0 \Rightarrow 2x^2 - 7x + 4 = 0$

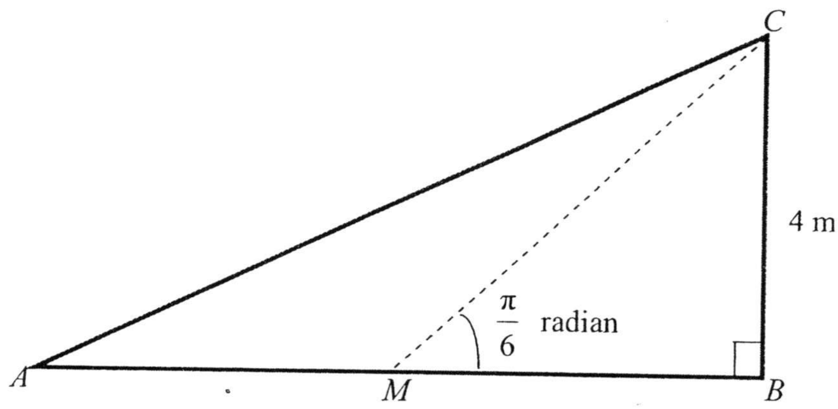
2 **(a)** Solve the equation $\sin^2 y + 2\cos 2y = 2\cos y$ for $0^\circ \le y \le 360^\circ$. [3]

(b) Prove that $\frac{\cos(A+B) + \cos(A-B)}{\sin(A+B) - \sin(A-B)} = \cot B$. [4]

SOLUTION:

2	$\sin^2 y + 2\cos 2y = 2\cos y$
(a)	$1 - \cos^2 y + 4\cos^2 y - 2 - 2\cos y = 0$ $3\cos^2 y - 2\cos y - 1 = 0$ $(3\cos y + 1)(\cos y - 1) = 0$
	$\cos y = -\frac{1}{3}$ or $\cos y = 1$ Basic Angle $= 70.53^\circ$ $y = 0^\circ, 360^\circ$ $y = 109.5^\circ, 250.5^\circ$
(b)	LHS $= \frac{\cos(A+B) + \cos(A-B)}{\sin(A+B) - \sin(A-B)}$ $= \frac{\cos A\cos B + SinA\sin B + \cos A\cos B - \sin A\sin B}{\sin A\cos B + \cos A\sin B - \sin A\cos B + \cos A\sin B}$
	$= \frac{2\cos A\cos B}{2\cos A\sin B}$
	$= \frac{\cos B}{\sin B}$
	$= \cot B$

3



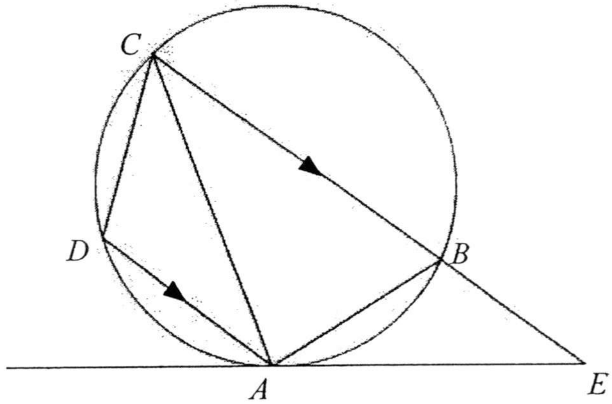
The diagram shows a triangle ABC in which angle CMB is $\frac{\pi}{6}$ radians, angle B is a right angle, M is the mid-point of AB and the length of CB is 4 m.

Without using a calculator, find the value of the integer k such that $\angle ACM = \sin^{-1}\left(\frac{\sqrt{k}}{26}\right)$. [6]

SOLUTION

3	$\tan\frac{\pi}{6} = \frac{4}{MB}$ $AM = MB = \frac{4}{\tan\frac{\pi}{6}} = 4\sqrt{3}$
	$AC = \sqrt{(8\sqrt{3})^2 + 4^2} = 4\sqrt{13}$
	$\frac{\sin\angle ACM}{4\sqrt{3}} = \frac{\sin\frac{5\pi}{6}}{4\sqrt{13}}$ $\sin\angle ACM = \frac{\frac{1}{2}}{4\sqrt{13}} \times 4\sqrt{3} = \frac{\sqrt{3}}{2\sqrt{13}}$ $= \frac{\sqrt{3}}{2\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$ $\angle ACM = \sin^{-1}\left(\frac{\sqrt{39}}{26}\right)$
	Therefore $k = 39$

4



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(i) Explain why angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [2]

(iii) Given that $AB = BE$, explain why the line AC bisects the angle BCD. [2]

SOLUTION:

4	
(i)	$\angle BAE = \angle ACB$ (tangent chord theorem) $= \angle CAD$ (alternate angles)
(ii)	In triangles BAE and DAC, $\angle BAE = \angle CAD$ (part (i)) $\angle CDA = 180° - \angle ABC$ (opposite angles of cyclic quadrilateral) $= \angle ABE$ (angles on straight line) $\angle ACD = \angle AEB$ (angle sum of triangle) Hence, triangles BAE and DAC are similar.
	$AB = BE$, implying that triangles BAE and DAC are similar isosceles triangles. So, $\angle ACD = \angle CAD$ $= \angle BCA$ (alternate angles) Hence, the line AC bisects the angle BCD.

59

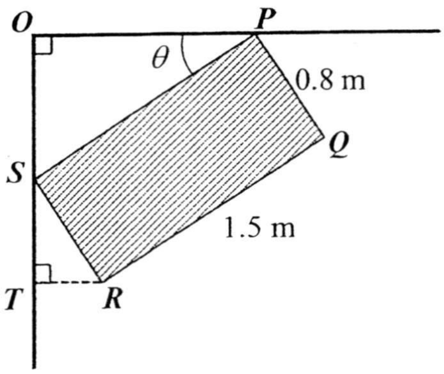
5 **(a)** Simplify $\dfrac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}}$. [4]

(b) Solve the equation $5^{x+1} = 8 + 4\left(5^{-x}\right)$. [5]

SOLUTION:

5 (a)	$\dfrac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}}$
	$= \dfrac{2^{4(x+1)} + 48(2^{4x})}{2^{x+3} \times 2^{3(x+2)}}$ $= \dfrac{2^{4x+4} + 48(2^{4x})}{2^{4x+9}}$
	$= \dfrac{2^{4x}(2^4 + 48)}{2^{4x}(2^9)}$
	$= \dfrac{2^6}{2^9} = \dfrac{1}{2^3}$
	$= \dfrac{1}{8}$
(b)	$5^{x+1} = 8 + 4\left(5^{-x}\right)$
	$5\left(5^x\right) = 8 + 4\left(5^x\right)^{-1}$
	Let $u = 5^x$ $5u = 8 + \dfrac{4}{u}$ $5u^2 = 8u + 4$
	$\left(5u+2\right)\left(u-2\right) = 0$ $u = -\dfrac{2}{5}$ (rejected) or $u = 2$
	$5^x = 2$ $\therefore x = 0.4306 \approx 0.431$ (3 s.f.)

6



The diagram shows the plan of a rectangular desk, *PQRS*, in a corner of a room.

Given that the desk has length 1.5 m and width 0.8 m, and that $\angle POS = \angle STR = 90^\circ$ and $\angle OPS = \theta$.

(i) Show the length of OT, L can be expressed as $L = 1.5\sin\theta + 0.8\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta+\alpha)$ where $0^\circ < \alpha < 90^\circ$ and $R > 0$. [3]

Hence, find the value of θ for which

(iii) L has a maximum length, [2]

(iv) $L = 1.2$ m. [2]

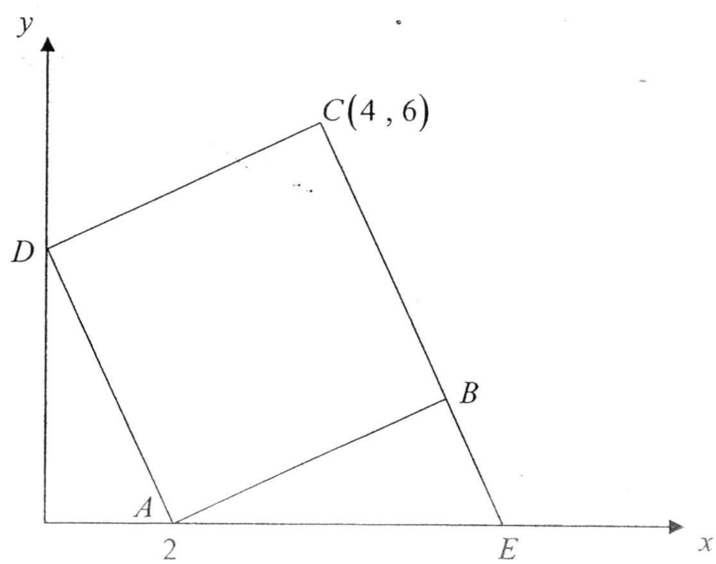
<u>SOLUTION:</u>

6	
(i)	$\angle TSR = \theta$, $\cos\theta = \dfrac{ST}{0.8} \Rightarrow ST = 0.8\cos\theta$ $\sin\theta = \dfrac{OS}{1.5} \Rightarrow OS = 1.5\sin\theta$ $OT = OS + ST$ $L = 1.5\sin\theta + 0.8\cos\theta$
(ii)	$L = 1.5\sin\theta + 0.8\cos\theta = R\sin(\theta+\alpha)$ where $R = \sqrt{1.5^2 + 0.8^2} = 1.7$ $\tan\alpha = \dfrac{0.8}{1.5}, \Rightarrow \alpha = 28.07^\circ$ $\therefore\ L = 1.7\sin(\theta + 28.07^\circ)$
(iii)	L has maximum length when $\sin(\theta + 28.07) = 1$ $\theta + 28.07^\circ = 90^\circ$ $\theta = 61.9^\circ$ (1 dp)

60

(iv)	$1.7\sin\left(\theta+28.07^{\circ}\right)=1.2$ $\sin\left(\theta+28.07^{\circ}\right)=\frac{1.2}{1.7}$ Basic Angle $=44.90^{\circ}$ $\theta+28.07^{\circ}=44.9^{\circ}$ $\theta=16.8^{\circ}$ (1 dp)

7



The diagram shows a rhombus $ABCD$ with vertices A and C at the points (2 , 0) and (4 , 6) respectively. D lies on the y-axis and the line BC produced intersects the x-axis at E.

(i) Show that the y-coordinate of D is 4. [3]

(ii) Explain why the rhombus $ABCD$ is also a square. [2]

(iii) Find the coordinates of E. [2]

(iv) Calculate the area of the quadrilateral $AECD$. [2]

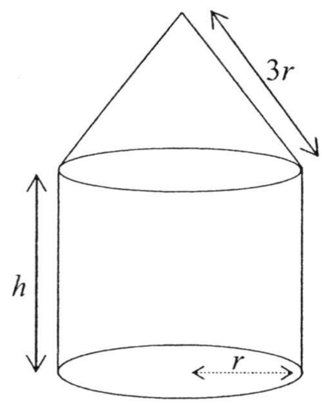
SOLUTION:

7	
(i)	Midpoint of $AC=\left(\frac{2+4}{2},\frac{0+6}{2}\right)$ $=(3,\ 3)$
	$m_{AC}=\frac{6-0}{4-2}=3$

	Equation of perpendicular bisector of AC is $y=-\frac{1}{3}x+c$
	At (3 , 3), $3=-\frac{1}{3}(3)+c$ $c=4$ $\therefore y$-coordinate of D is 4.
(ii)	$\text{grad}_{AD}=\frac{0-4}{2-0}=-2$ $\text{grad}_{CD}=\frac{6-4}{4-0}=\frac{1}{2}$ $-2\times\frac{1}{2}=-1 \Rightarrow$ AD and CD are perpendicular, hence $ABCD$ is a square.
(iii)	Equation of BC is $y=-2x+c$ At (4 , 6), $6=-2(4)+c$ $c=14$ $\therefore y=-2x+14$ Along x-axis, $y=0$. $0=-2x+14$ $x=7$ $E(7\,,0)$
(iv)	$\text{Area}=\frac{1}{2}\begin{vmatrix}2 & 7 & 4 & 0 & 2\\ 0 & 0 & 6 & 4 & 0\end{vmatrix}$ $=\frac{1}{2}[58-8]$ $=25$

61

8



The diagram shows a solid body which consists of a cone fixed to the top of a right circular cylinder of radius r cm and height h cm. The slant edge of the cone is $3r$ cm.

(i) Given that the volume of the cylinder is 108π cm^3, express h in terms of r. [1]

(ii) Show that the total surface area, A cm^2 of the solid is given by $A = 4\pi\left(\frac{54}{r} + r^2\right)$. [3]

(iii) Given that r and h can vary,

(a) find the value of r for which A has a stationary value, [3]

(b) determine whether this stationary value is a maximum or minimum. [2]

SOLUTION:

8	
(i)	$\pi r^2 h = 108\pi$ $h = \frac{108}{r^2}$
(ii)	Total surface area = area of cylinder + area of cone $= 2\pi rh + \pi r^2 + \pi rl$ $= 2\pi rh + \pi r^2 + 3\pi r^2$ $= 2\pi r\left(\frac{108}{r^2}\right) + 4\pi r^2$ $= 4\pi\left(\frac{54}{r} + r^2\right)$ (shown)
(iii)	$\frac{dA}{dr} = \frac{-216\pi}{r^2} + 8\pi r$

(a)	$\frac{-216\pi}{r^2}+8\pi r=0$ $\frac{216\pi}{r^2}=8\pi r$ $216=8r^3$ $r=3$
	Sub $r=3$ into $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$, $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=\frac{432\pi}{r^3}+8\pi$ $=\frac{432\pi}{(3)^3}+8\pi$ $=24\pi$ Since $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ is positive, A is a minimum. (shown)

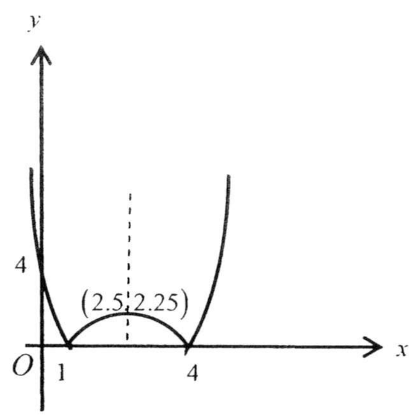
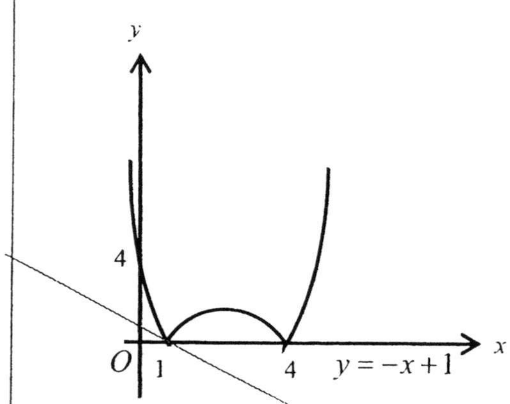
62

9 (i) Find the range of values of m for which the curve $y=(x-1)(x-4)$ and the line $y=mx+3$ do not intersect. [3]

(ii) Sketch the graph of $y=|(x-1)(x-4)|$, showing the coordinates of the turning point and the point where the curve meets the x-axis. [3]

(iii) Find the number of solutions of the equation $|(x-1)(x-4)|=-x+1$. [2]

SOLUTION:

9	$y=(x-1)(x-4)$
(i)	$(x-1)(x-4)=mx+3$ $x^2-5x+4-mx-3=0$ $x^2-(m+5)x+1=0$ $b^2-4ac<0$ $(m+5)^2-4<0$ $(m+7)(m+3)<0$ $\therefore\quad -7<m<-3$
(ii)	
(iii)	
	(a) 1 solution

10 **(a)** **Without using a calculator**, show that $\dfrac{\log_2 5 \times \log_5 4}{\log_{25} 5} = 4$. [3]

(b) Given that $y = \ln\sqrt{\dfrac{2x}{x+4}}$, $x > 0$ and $x < -4$,

(i) find $\dfrac{dy}{dx}$. [4]

(ii) Hence show that y has no stationary value. [2]

SOLUTION:

10	$\dfrac{\log_2 5 \times \log_5 4}{\log_{25} 5}$
(a)	$\dfrac{\log_2 5 \times \dfrac{\log_2 4}{\log_2 5}}{\dfrac{\log_2 5}{\log_2 25}} = \log_2 4 \div \dfrac{\log_2 5}{\log_2 25}$
	$2\log_2 2 \div \dfrac{\log_2 5}{2\log_2 5}$
	$2 \div \dfrac{1}{2} = 2 \times 2 = 4$
(b) **(i)**	$y = \ln\sqrt{\dfrac{2x}{x+4}}$ $= \ln\left(\dfrac{2x}{x+4}\right)^{\frac{1}{2}} = \dfrac{1}{2}\ln\left(\dfrac{2x}{x+4}\right)$ $= \dfrac{1}{2}\left[\ln 2x - \ln(x+4)\right]$
	$\dfrac{dy}{dx} = \dfrac{1}{2}\left[\dfrac{2}{2x} - \dfrac{1}{x+4}\right]$
	$= \dfrac{1}{2}\left[\dfrac{(x+4)-x}{x(x+4)}\right] = \dfrac{2}{x(x+4)}$
(ii)	$\dfrac{2}{x(x+4)} \neq 0$ $\text{since } \dfrac{dy}{dx} \neq 0 \Rightarrow$ there is no stationary value

63

11 The polynomial $P(x) = 2x^3 + ax^2 + bx + 8$, where a and b are constants, leaves a remainder of 10 when divided by $2x - 1$. Given that $x + 2$ is a factor of $P(x)$,

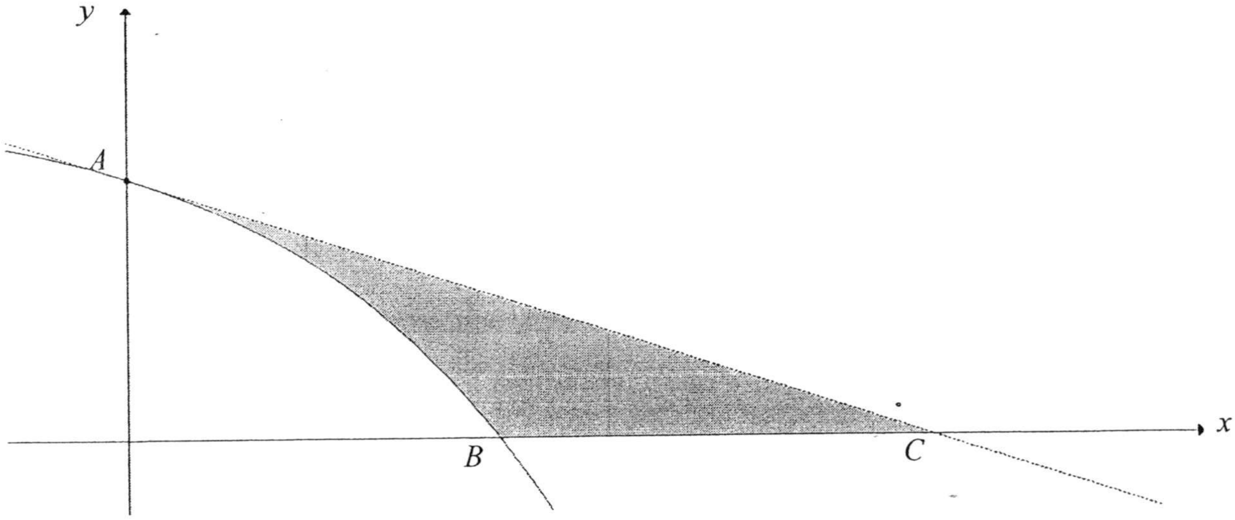
(i) find the value of a and of b. [5]

(ii) Explain why the equation $P(x) = 0$ has only 1 real root. Hence find this root. [4]

SOLUTION:

11	$P(x) = 2x^3 + ax^2 + bx + 8$
(i)	$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 8 = 10$ $\frac{1}{4}a + \frac{1}{2}b = \frac{7}{4}$ $a + 2b = 7$
	$P(-2) = 2(-2)^3 + a(-2)^2 + b(-2) + 8 = 0$ $4a - 2b = 8$ $2a - b = 4$
	$2(7 - 2b) - b = 4 \Rightarrow -5b = -10$
	$b = 2$, $a = 7 - 2(2) = 3$
(ii)	$P(x) = 2x^3 + 3x^2 + 2x + 8$ $= (x + 2)(2x^2 + bx + 4)$ term in x^2 : $3x^2 = bx^2 + 4x^2$, $b = -1$ $P(x) = 2x^3 + 3x^2 + 2x + 8$ $= (x + 2)(2x^2 - x + 4)$
	for $2x^2 - x + 4$, $b^2 - 4ac = 1 - 4(2)(4)$ $= -31 < 0$
	Hence, the equation $2x^2 - x + 4 = 0$ has no roots. So $P(x) = 0$ has only 1 real root.
	The root is $x + 2 = 0$ ie $x = -2$

12 The diagram shows part of the curve $y = 4 - e^{\frac{1}{2}x}$ which cuts the axes at A and at B.



(i) Find the coordinates of A and of B. [4]

The tangent to the curve at A meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region [4]

SOLUTION:

12	$y = 4 - e^{\frac{1}{2}x}$
(i)	When $x = 0$, $y = 4 - e^{\frac{1}{2}(0)} = 3 \Rightarrow A(0, 3)$ When $y = 0$, $0 = 4 - e^{\frac{1}{2}x}$ $e^{\frac{1}{2}x} = 4$ $\frac{1}{2}x = \ln 4$ $x = 2\ln 4$ or $4\ln 2 \Rightarrow B(2\ln 4, 0)$ or $B(4\ln 2, 0)$
(ii)	$\frac{dy}{dx} = -\frac{1}{2}e^{\frac{1}{2}x}$ $= -\frac{1}{2}e^{\frac{1}{2}(0)}$ $= -\frac{1}{2}$

64

	Equation of tangent: $y=-\frac{1}{2}x+3$
	When $y=0$, $0=-\frac{1}{2}x+3$ $x=6 \Rightarrow C(6,0)$
(iii)	Shaded area $=\frac{1}{2}\times 6\times 3-\int_0^{4\ln 2} 4-e^{\frac{1}{2}x}\,dx$ $=9-\left[4x-2e^{\frac{1}{2}x}\right]_0^{4\ln 2}$ $=9-\left[4(4\ln 2)-2e^{\frac{1}{2}(4\ln 2)}-(-2)\right]$ $=3.9096$ ≈ 3.91 units^2

~ **End of Paper** ~