	Class	Register Number
Name MARKING SCHEME		

4047/01 **15/S4PR2/AM/1**

ADDITIONAL MATHEMATICS **PAPER 1**

Wednesday **5 August 2015** **2 hours**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of 5 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

55

1 Find the range of the values of x which satisfy both inequalities
$0 < x^2 - 4x$ and $x^2 - 4x \leq 3x + 10$. [4]

2 Solve

(i) $\dfrac{3^{2-x}}{9^x} = \dfrac{1}{\sqrt{27^x}}$, [2]

(ii) $3e^x - e = 2e^{2-x}$. [3]

3 (i) Find the coefficient of the term in x in the expansion of $\left(x^2 - \dfrac{1}{2x^3}\right)^8$. [3]

(ii) The coefficient of x^2 in the expansion $(5-3x)(1+5x)^n$ is 1785. Find the value of n. [4]

4 The gradient to a curve is given by $\dfrac{dy}{dx} = (kx+3)^2$, where k is a non-zero constant. The equation of the tangent to the curve at the point $(1, 2)$ is $9x - y - 5 = 0$. Find the

(i) value of k, [2]

(ii) equation of the curve. [2]

5 Sketch the graph of $y = -|x+1| + 2$ for $-4 \leq x \leq 2$. [3]

(i) State the range of values of p for which the equation $-|x+1| = p - 2$ has at least 1 solutions for $-4 \leq x \leq 2$. [1]

(ii) Using your graph, state the number of solutions for $-|x+1| + 2 = x + 3$. [1]

6 (i) Find the exact value of x in the equation $\sqrt{112}x + 5 = \sqrt{7}x + 19$. [4]

(ii) A cuboid with a square base of length $\sqrt{3} + 1$ cm, has a volume of $\left(5\sqrt{2}\right)^2 - 8\sqrt{3}$ cm^3. Find the height of the cuboid in the form $a + b\sqrt{3}$. [4]

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(i) Find $\frac{dy}{dx}$. [2]

(ii) Hence show that $\int_0^{\ln 2} 4xe^{4x}\,dx = 16\ln 2 - 3\frac{3}{4}$. [4]

(iii) Find the range of values of x for which the function $y = xe^{4x}$ is decreasing. [2]

8 AB is a chord of the circle $x^2 + y^2 - 8x - 2y - 3 = 0$ and $M\left(\frac{4}{5}, 2\frac{2}{5}\right)$ is the midpoint of chord AB. Find the

(i) radius and the coordinates of the centre of the circle, [2]

(ii) equation of chord AB. [3]

If P is a variable point on the circle, find the

(iii) maximum area of triangle ABP. [4]

9 The function f is defined, for $0 \le x \le 2\pi$, by $f(x) = 2\cos ax + b$, where a and b are integers. The minimum value of f is -1 and the period of f is $\frac{4\pi}{3}$.

(i) State the amplitude of f. [1]

(ii) State the values of a and of b. [1]

(iii) Using the values of a and b found in part (ii),

(a) solve $f(x) = 0$ for $0 \le x \le 2\pi$, leaving your answers in terms of π, [4]

(b) sketch the graph of $f(x) = 2\cos ax + b$ for $0 \le x \le 2\pi$. [3]

10 A particle moves in a straight line such that t seconds after leaving a fixed point O, the velocity v m/s, is given by $v = 3t^2 - t - 10$. Find the

(i) initial acceleration of the particle, [2]

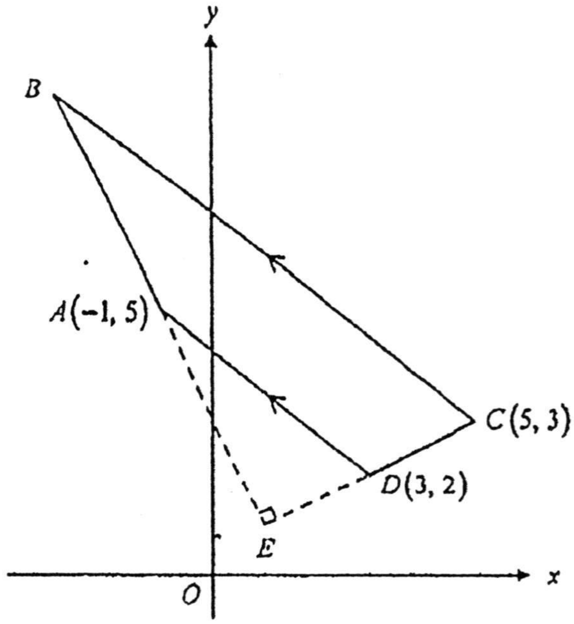
(ii) minimum velocity of the particle, [2]

(iii) total distance travelled by the particle in the first 3 seconds, [4]

(iv) average speed of the particle during the first 3 seconds. [2]

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows the trapezium *ABCD* in which *BC* is parallel to *AD* while *BA* produced is perpendicular to *CD* produce at point *E*. The point *A* is $(-1, 5)$, *C* is $(5, 3)$ and *D* is $(3, 2)$.



(i) Show that the coordinates of *B* are $(-3, 9)$. [6]

(ii) Find the area of trapezium *ABCD*. [2]

(iii) Given that $\frac{\text{area of } \Delta AED}{\text{area of } \Delta BEC} = \frac{1}{4}$, find the coordinates of *E*. [3]

End of Paper

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Answer Key

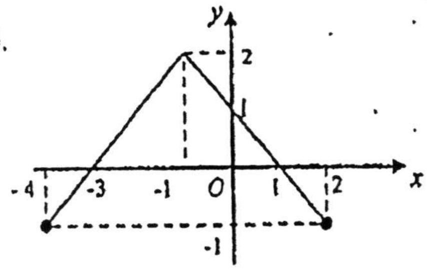
1. $-1.22 \le x < 0$ or $4 < x \le 8.22$

2(i) $1\frac{1}{3}$ 2(ii) $x = 1$

3(i) Coefficient of $x = -7$ 3(ii) $n = 6$

4(i) $k = -6$ 4(ii) $y = \frac{1}{2} - \frac{3(1-2x)^3}{2}$

5.



5(i) $-1 \le p \le 2$

5(ii) There are infinite number of solutions.

6(i) $x = \frac{2\sqrt{7}}{3}$ 6(ii) height $= 62 - 33\sqrt{3}$

7(i) $\frac{dy}{dx} = e^{4x}(4x+1)$ 7(iii) $x < -\frac{1}{4}$

8(i) centre of circle is $(4, 1)$

(ii) radius $= 4.47$ units

(iii) Max area $= 22.2$ units^2

9(i) Amplitude $= 2$, (ii) $a = 1.5$, $b = 1$

(iii) (a) $x = \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}$

10(i) -1 m/s^2

(ii) min velocity $= -10\frac{1}{12}$ m/s

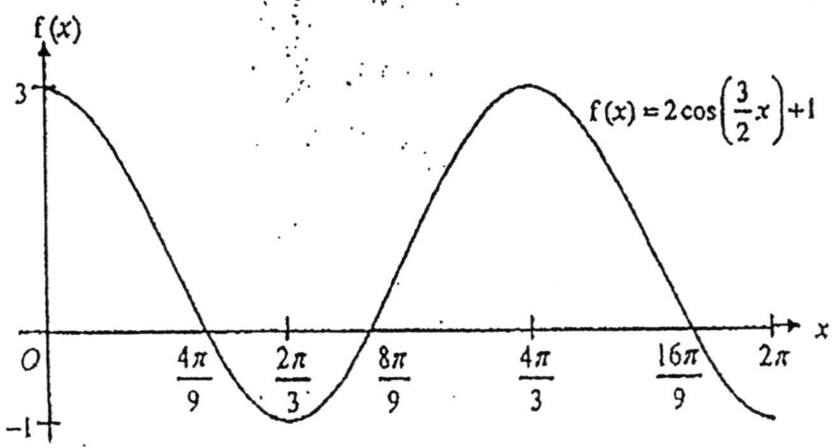
(iii) Total Distance $= 20.5$ m

(iv) Ave Speed $= 6\frac{5}{6}$ or 6.83 m/s

11(ii) Area $= 15$ units^2

(iii) $E(1,1)$

9(iii)(b)



E4

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1 Find the range of the values of x which satisfy both inequalities $0 < x^2 - 4x$ and $x^2 - 4x \leq 3x + 10$. [4]

$0 < x^2 - 4x$ and $x^2 - 4x \leq 3x + 10$

$x^2 - 4x > 0$ $\quad$ $x^2 - 7x - 10 \leq 0$

$x(x-4) > 0$ $\quad$ for $x^2 - 7x - 10 = 0$

$x < 0$ or $x > 4$ $\quad$ $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-10)}}{2(1)}$

$= -1.22$ or 8.22

$\therefore x^2 - 7x - 10 \leq 0$

$-1.22 \leq x \leq 8.22$

Hence the solution is $-1.22 \leq x < 0$ or $4 < x \leq 8.22$

2 Solve

(i) $\dfrac{3^{2-x}}{9^x} = \dfrac{1}{\sqrt{27^x}}$, [2]

$$\frac{3^{2-x}}{9^x} = \frac{1}{\sqrt{27^x}}$$

$$\frac{3^{2-x}}{3^{2x}} = \frac{1}{3^{\frac{3x}{2}}}$$

$$2 - x - 2x = -\frac{3x}{2}$$

$$2 = \frac{3}{2}x$$

$$x = \frac{4}{3} = 1\frac{1}{3}$$

(ii) $3e^x - e = 2e^{2-x}$. [3]

$$3e^x - e = 2e^{2-x}$$

$$3e^x - e = \frac{2e^2}{e^x}$$

$$3(e^x)^2 - e \cdot e^x - 2e^2 = 0$$

$$(e^x - e)(3e^x + 2e) = 0$$

$e^x = e$ or $3e^x = -2e$

$x = 1$ $\quad$ (NA)

3 (i) Find the coefficient of the term in x in the expansion of $\left(x^2-\frac{1}{2x^3}\right)^8$. [3]

For $\left(x^2-\frac{1}{2x^3}\right)^8$,

$$T_{r+1}=\binom{8}{r}(x^2)^{8-r}\left(-\frac{1}{2x^3}\right)^r$$

$$=\binom{8}{r}\left(\frac{-1}{2}\right)^r x^{16-2r}x^{-3r}$$

$$=\binom{8}{r}\left(\frac{-1}{2}\right)^r x^{16-5r}$$

For term in x, $16-5r=1$

$5r=15$

$r=3$

Coefficient of $x=\binom{8}{3}\left(\frac{-1}{2}\right)^3$

$=-7$

(ii) The coefficient of x^2 in the expansion $(5-3x)(1+5x)^n$ is 1785. Find the value of n. [4]

$(5-3x)(1+5x)^n$

$=(5-3x)\left(1+\binom{n}{1}(5x)+\binom{n}{2}(5x)^2+\ldots\right)$

$=(5-3x)\left(1+5nx+\frac{n(n-1)}{2}\times 25x^2+\ldots\right)$

coefficient of x^2 in the above expansion $=1785$

$125\times\frac{n(n-1)}{2}-3(5n)=1785$

$125n(n-1)-30n=3570$

$125n^2-125n-30n-3570=0$

$125n^2-155n-3570=0$

$25n^2-31n-714=0$

$(n-6)(25n+119)=0$

$n-6=0$ or $25n+119=0$

$n=6$ or $n=-\frac{119}{25}$ (N.A)

4 The gradient to a curve is given by $\frac{dy}{dx}=(kx+3)^2$, where k is a non-zero constant. The equation of the tangent to the curve at the point $(1, 2)$ is $9x-y-5=0$. Find the

(i) value of k, [2]

$9x-y-5=0$
$y=9x-5$
Gradient of tangent $=9$
At $(1,2)$, $\frac{dy}{dx}=9$
$(k+3)^2=9$
$k+3=3$ or $k+3=-3$
$k=0$ (N.A) or $k=-6$

(ii) equation of the curve. [2]

Equation of curve is,

$$y=\int(-6x+3)^2\,dx$$
$$=\frac{(-6x+3)^3}{3(-6)}+c$$
$$=\frac{(3-6x)^3}{-18}+c$$

At $(1,2)$,

$$2=\frac{(3-6)^3}{-18}+c$$
$$c=\frac{1}{2}$$

$\therefore$ equation of curve is,

$$y=\frac{(3-6x)^3}{-18}+\frac{1}{2}$$
$$y=\frac{1}{2}-\frac{3(1-2x)^3}{2}$$

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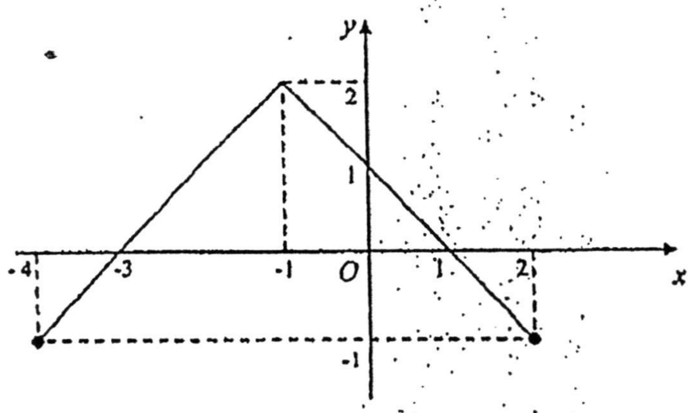
5 Sketch the graph of $y=-|x+1|+2$ for $-4 \leq x \leq 2$. [3]



(i) State the range of values of p for which the equation $-|x+1|=p-2$ has at least 1 solutions for $-4 \leq x \leq 2$. [1]

$-1 \leq p \leq 2$

(ii) Using your graph, state the number of solutions for $-|x+1|+2=x+3$ [1]

There are infinite number of solutions.

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6 (i) Find the exact value of x in the equation $\sqrt{112}x+5=\sqrt{7}x+19$. [4]

$$\sqrt{112}x+5=\sqrt{7}x+19$$
$$4\sqrt{7}x-\sqrt{7}x=14$$
$$3\sqrt{7}x=14$$
$$x=\frac{14}{3\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$$
$$=\frac{14\sqrt{7}}{21}$$
$$=\frac{2\sqrt{7}}{3}$$

$$\sqrt{112}x+5=\sqrt{7}x+19$$
$$\sqrt{112}x-\sqrt{7}x=14$$
$$\left(\sqrt{112}-\sqrt{7}\right)x=14$$
$$x=\frac{14}{\sqrt{112}-\sqrt{7}}\times\frac{\sqrt{112}+\sqrt{7}}{\sqrt{112}+\sqrt{7}}$$
$$=\frac{14\cdot4\sqrt{7}+14\sqrt{7}}{112-7}$$
$$=\frac{56\sqrt{7}+14\sqrt{7}}{105}$$
$$=\frac{70\sqrt{7}}{105}$$
$$=\frac{2\sqrt{7}}{3}$$

(ii) A cuboid with a square base of length $\sqrt{3}+1$ cm, has a volume of $\left(5\sqrt{2}\right)^2-8\sqrt{3}$ cm². Find the height of the cuboid in the form $a+b\sqrt{3}$. [4]

$$\text{Height}=\frac{\left(5\sqrt{2}\right)^2-8\sqrt{3}}{\left(\sqrt{3}+1\right)^2}$$
$$=\frac{25(2)-8\sqrt{3}}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$
$$=\frac{200-100\sqrt{3}-32\sqrt{3}+48}{16-12}$$
$$=\frac{248-132\sqrt{3}}{4}$$
$$=62-33\sqrt{3}$$

7 A curve has the equation $y = xe^{4x}$.

(i) Find $\frac{dy}{dx}$. [2]

$y = xe^{4x}$

$\frac{dy}{dx} = x4e^{4x} + e^{4x}(1)$

$= e^{4x}(4x+1)$

(ii) Hence show that $\int_0^{\ln 2} 4xe^{4x}dx = 16\ln 2 - 3\frac{3}{4}$. [4]

$\int_0^{\ln 2} e^{4x}(4x+1)\, dx = \left[xe^{4x}\right]_0^{\ln 2}$

$\int_0^{\ln 2} 4xe^{4x} + e^{4x}\, dx = \ln 2 \times e^{4\ln 2} - 0$

$\int_0^{\ln 2} 4xe^{4x}\, dx + \int_0^{\ln 2} e^{4x}\, dx = \ln 2 \times e^{\ln 16}$

$\int_0^{\ln 2} 4xe^{4x}\, dx = \ln 2 \times 16 - \int_0^{\ln 2} e^{4x}\, dx$

$= 16\ln 2 - \int_0^{\ln 2} e^{4x}\, dx$

$= 16\ln 2 - \left[\frac{e^{4x}}{4}\right]_0^{\ln 2}$

$= 16\ln 2 - \frac{1}{4}\left(e^{4\ln 2} - e^0\right)$

$= 16\ln 2 - \frac{1}{4}\left(e^{\ln 16} - 1\right)$

$= 16\ln 2 - \frac{1}{4}(16-1)$

$= 16\ln 2 - 3\frac{3}{4}$

(iii) Find the range of values of x for which the function $y = xe^{4x}$ is decreasing. [2]

For y to be decreasing,

$\frac{dy}{dx} < 0$

$e^{4x}(4x+1) < 0$

Since $e^{4x} > 0$ for all values of x,

then $4x + 1 < 0$

$x < -\frac{1}{4}$

8 AB is a chord of the circle $x^2+y^2-8x-2y-3=0$ and $M\left(\frac{4}{5},2\frac{2}{5}\right)$ is the midpoint of chord AB. Find the

(i) radius and the coordinates of the centre of the circle, [2]

$x^2+y^2-8x-2y-3=0$

$x^2+y^2+2(-4)x+2(-1)y+(-3)=0$

$\therefore\ g=-4,\quad f=-1,\quad c=-3$

Hence centre of circle is $(4,1)$

radius of circle is $\sqrt{g^2+f^2-c}$

$=\sqrt{(-4)^2+(-1)^2-(-3)}$

$=\sqrt{20}$

$=4.47$ units (3 sf)

(ii) equation of chord AB. [3]

Let the centre of circle be C.

$\therefore$ gradient of $CM=\dfrac{2\frac{2}{5}-1}{\frac{4}{5}-4}$

$=\dfrac{-7}{16}$

$\therefore$ gradient of chord $AB=\dfrac{16}{7}$

Hence equation of chord AB is,

$y-2\frac{2}{5}=\frac{16}{7}\left(x-\frac{4}{5}\right)$

$=\frac{16}{7}x-\frac{64}{35}$

$\therefore\ y=\frac{16}{7}x+\frac{4}{7}$

If P is a variable point on the circle, find the

(iii) maximum area of triangle ABP. [4]

Length of $CM=\sqrt{\left(4-\frac{4}{5}\right)^2+\left(1-2\frac{2}{5}\right)^2}$

$=\sqrt{12\frac{1}{5}}$

Length of $BM=\sqrt{BC^2-CM^2}$

$=\sqrt{20-12\frac{1}{5}}$

$=\sqrt{7\frac{4}{5}}$

Length of chord $AB=2\times BM$

$=2\times\sqrt{7\frac{4}{5}}$

Area of ΔABP is maximum when P, C & M are collinear and PM is $\perp AB$.

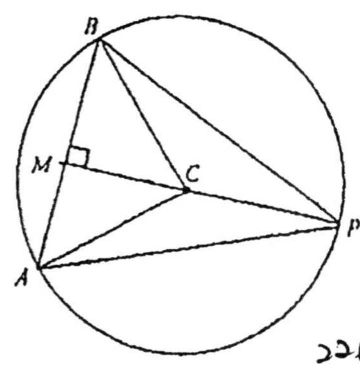
$\therefore$ maximum area of ΔABP

$=\frac{1}{2}\times AB\times PM$

$=\frac{1}{2}\times AB\times(CM+CP)$

$=\frac{1}{2}\times\left(2\times\sqrt{7\frac{4}{5}}\right)\times\left(\sqrt{12\frac{1}{5}}+\sqrt{20}\right)$

$=22.2$ units^2 (3 sf)



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9 The function f is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 2\cos ax + b$, where a and b are integers. The minimum value of f is -1 and the period of f is $\frac{4\pi}{3}$.

(i) State the amplitude of f. [1]

Amplitude $= 2$

(ii) State the values of a and of b. [1]

$a = 2\pi \div \frac{4\pi}{3} = 1.5$

$b = 1$

(iii) Using the values of a and b found in part (ii),

(a) solve $f(x) = 0$ for $0 \leq x \leq 2\pi$, leaving your answers in terms of π, [4]

$$2\cos\left(\frac{3}{2}x\right) + 1 = 0$$

$$\cos\left(\frac{3}{2}x\right) = -\frac{1}{2}$$

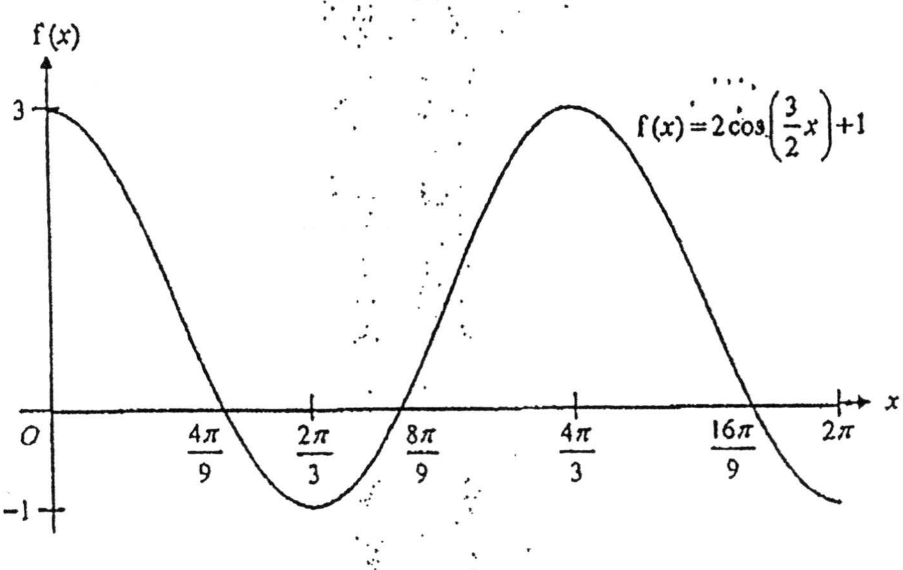
$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\frac{3}{2}x = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3},\ 2\pi + \left(\pi - \frac{\pi}{3}\right)$$

$$\frac{3}{2}x = \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{8\pi}{3}$$

$$x = \frac{4\pi}{9},\ \frac{8\pi}{9},\ \frac{16\pi}{9}$$

(b) sketch the graph of $f(x) = 2\cos ax + b$ for $0 \leq x \leq 2\pi$. [3]



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(i) initial acceleration of the particle, [2]

$$v = 3t^2 - t - 10$$
$$a = \frac{dv}{dt}$$
$$= 6t - 1$$

Initial acceleration $= 6(0) - 1$

$= -1$ m/s^2

(ii) minimum velocity of the particle, [2]

Minimum velocity of the particle occurs when $a = 0$

$$6t - 1 = 0$$
$$t = \frac{1}{6}$$

$\therefore$ minimum velocity of the particle,

$$= 3\left(\frac{1}{6}\right)^2 - \left(\frac{1}{6}\right) - 10 = -10\frac{1}{12} \text{ m/s}$$

(iii) total distance travelled by the particle in the first 3 seconds, [4]

$$s = \int 3t^2 - t - 10 \, dt$$
$$= t^3 - \frac{1}{2}t^2 - 10t + c$$

At $t = 0$, $s = 0$, $\therefore c = 0$

Hence, $s = t^3 - \frac{1}{2}t^2 - 10t$

When $v = 0$,

$$3t^2 - t - 10 = 0$$
$$(3t + 5)(t - 2) = 0$$
$$3t + 5 = 0 \quad \text{or} \quad t - 2 = 0$$
$$t = \frac{-5}{3} \ (N.A) \qquad t = 2$$

At $t = 2$,

$$s = 2^3 - \frac{1}{2}(2)^2 - 10(2) = -14$$

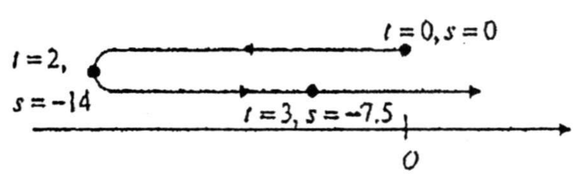
At $t = 3$,

$$s = 3^3 - \frac{1}{2}(3)^2 - 10(3) = -7\frac{1}{2}$$

$\therefore$ total distance travelled in the first 3 seconds

$= 14 + (14 - 7.5)$

$= 20.5$ m



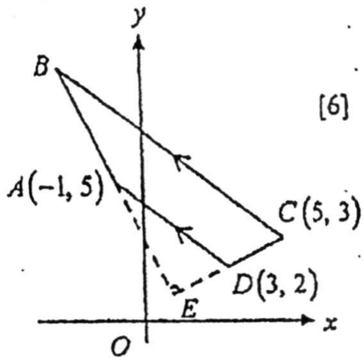
(iv) the average speed of the particle during the first 3 seconds. [2]

Average speed of the particle during the first 3 seconds

$$= \frac{\text{total distance}}{\text{total time}} = \frac{20.5}{3}$$
$$= 6\frac{5}{6} \text{ or } 6.83 \text{ m/s}$$

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows the trapezium $ABCD$ in which BC is parallel to AD while BA produced is perpendicular to CD produce at point E. The point A is $(-1, 5)$, C is $(5, 3)$ and D is $(3, 2)$.



(i) Show that the coordinates of B are $(-3, 9)$. [6]

Gradient of BC = Gradient of AD

$$= \frac{5-2}{-1-3}$$

$$= -\frac{3}{4}$$

Sub $(5, 3)$ into $y = -\frac{3}{4}x + c$, $\quad 3 = -\frac{3}{4}(5) + c$

$$c = 6\frac{3}{4}$$

Equation of BC is $y = -\frac{3}{4}x + 6\frac{3}{4}$

Gradient of $CD = \frac{3-2}{5-3}$

$$= \frac{1}{2}$$

Gradient of $BA = -2$

Sub $(-1, 5)$ into $y = -2x + d$, $\quad 5 = -2(-1) + d$

$$d = 3$$

Equation of BA is $y = -2x + 3$

$$-\frac{3}{4}x + 6\frac{3}{4} = -2x + 3$$

$$-3x + 27 = -8x + 12$$

$$5x = -15$$

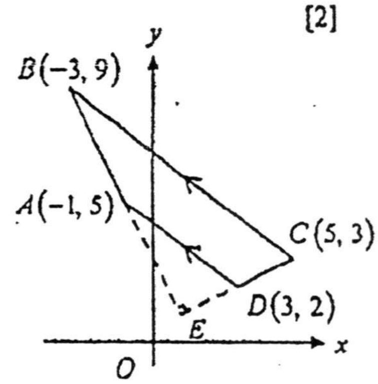
$$x = -3$$

when $x = -3$, $\quad y = -2(-3) + 3$

$$= 9$$

$\therefore B(-3, 9)$

11 (ii) Find the area of trapezium $ABCD$. [2]



Area of trapezium $ABCD$

$$= \frac{1}{2}\begin{vmatrix} 5 & -3 & -1 & 3 & 5 \\ 3 & 9 & 5 & 2 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left[(45-15-2+9)-(-9-9+15+10)\right]$$

$$= \frac{1}{2}(37-7)$$

$$= 15 \text{ units}^2$$

(iii) Given that $\dfrac{\text{area of } \Delta AED}{\text{area of } \Delta BEC} = \dfrac{1}{4}$, find the coordinates of E. [3]

Since ΔAED and ΔBEC are similar,

$$\frac{ED}{EC} = \frac{EA}{EB} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$\therefore D$ and E are the midpoints of EC and EB respectively.

Let $E(m, n)$,

$$\left(\frac{5+m}{2}, \frac{3+n}{2}\right) = (3, 2)$$

$$\frac{5+m}{2} = 3 \qquad \frac{3+n}{2} = 2$$

$$m = 1 \qquad n = 1$$

$\therefore E(1, 1)$ ⟵ A1

or

$$\left(\frac{-3+m}{2}, \frac{9+n}{2}\right) = (-1, 5)$$

Sub $(3, 2)$ into $y = \frac{1}{2}x + f$,

$$2 = \frac{1}{2}(3) + f$$

$$f = \frac{1}{2}$$

Equation of CD is $y = \frac{1}{2}x + \frac{1}{2}$

Equation of BA is $y = -2x + 3$

$$\frac{1}{2}x + \frac{1}{2} = -2x + 3$$

$$x + 1 = -4x + 6$$

$$5x = 5$$

$$x = 1$$

when $x = 1$, $y = -2(1) + 3 = 1$

$\therefore E(1, 1)$

End of Paper

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	Class	Register Number
Name ______________________		

4047/02 **15/S4PR2/AM/2**

ADDITIONAL MATHEMATICS PAPER 2

Tuesday 11 August 2015 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Materials: Answer Paper
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 7 printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

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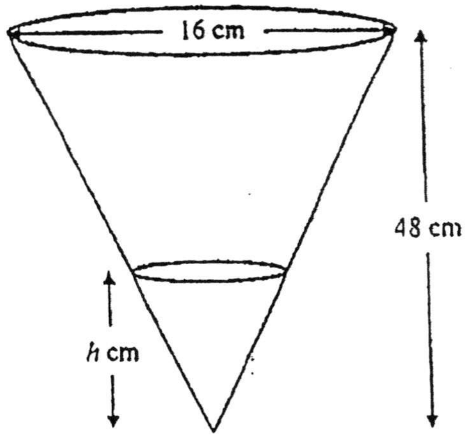
1



Water from a tank in the shape of an inverted cone flows out at the rate of $5\text{ cm}^3/\text{min}$. The height of the cone is 48 cm and the base diameter is 16 cm. After t minutes the water level is h cm.

(i) Show that the volume of water in the tank, $V\text{ cm}^3$, at time t is given by $V = \frac{\pi h^3}{108}$. [2]

(ii) Find the rate of change of the water level when $h = 6$. [3]

(iii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

2 The displacement, y mm, of a mass fixed on a vertical spring can be described by the simple harmonic motion equation, $y = A\sin(\omega t)$, where A and ω are constants and t is the time in seconds after the mass is displaced from its equilibrium position, 0 mm.

Given that the maximum displacement of the mass is 20 mm and that the mass first returns to its equilibrium position after 0.25 seconds.

(i) State the positive value of A. [1]

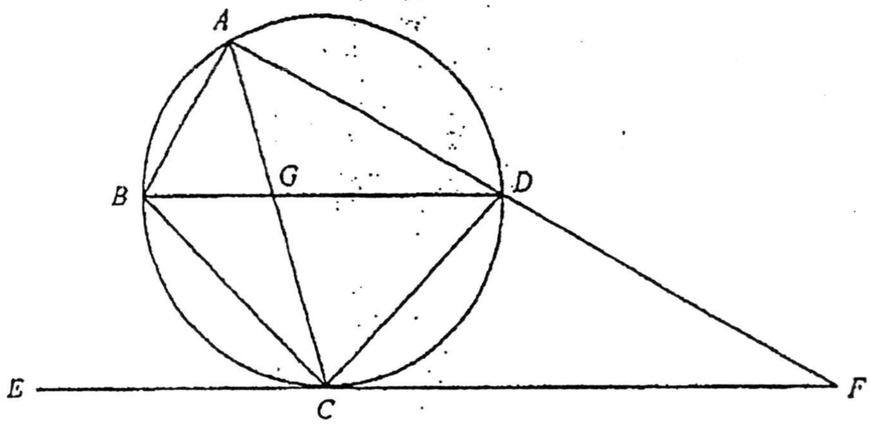
(ii) Show that the value of ω is 4π radians per second. [2]

(iii) Find the exact value of t when the mass first reach a position 10 mm below its equilibrium position. [3]

3 (i) Given that $f(x) = 2x^3 + ax^2 + bx - 30$ has a factor $(x+3)$ and leaves a remainder of -28 when divided by $(x-1)$. Find the values of a and of b and solve $f(x) = 0$. [6]

(ii) Hence solve $2(y+1)^3 + a(y+1)^2 + by + b - 30 = 0$. [2]

4



The diagram shows points A, B, C and D lying on a circle. The chords BD and AC intersect at G. EF is a tangent to the circle at C. AD is produced meet the tangent at F and $\angle ABC = \angle BGC$.

Prove that

(i) BD is parallel to EF, [2]

(ii) triangle CFD and triangle AFC are similar, [2]

(iii) $FC^2 - FD^2 = FD \times DA$. [3]

5 (i) Express $\dfrac{3x^2+10x}{(x+2)(x^2-4)}$ in partial fractions. [5]

(ii) Using your answer from (i), find $\displaystyle\int \frac{3x^2+10x}{(x+2)(x^2-4)}\,dx$ and hence show that

$$\int_3^4 \frac{3x^2+10x}{(x+2)(x^2-4)}\,dx = \ln\left(\frac{24}{5}\right)+\frac{1}{15}.$$ [4]

6 (a) The quadratic equation $3x^2-2x+4=0$ has roots $3\alpha+\beta$ and $\alpha+3\beta$.

(i) Show that the values of $\alpha+\beta=\dfrac{1}{6}$ and $\alpha\beta=\dfrac{5}{16}$. [4]

(ii) If the roots of the equation $gx^2-hx-1=0$ where g and h are constants, are α and β, find the value of g and of h. [2]

(b) Find the range of values of k for which $(k+3)x^2+kx+1$ is always positive for all real values of x. [4]

7 **Answer the whole of this question on a sheet of graph paper.**

The table shows experimental values of two variables, x and y.

x	0.4	0.6	0.8	1.0	1.2
y	2.22	2.13	1.97	1.73	1.37

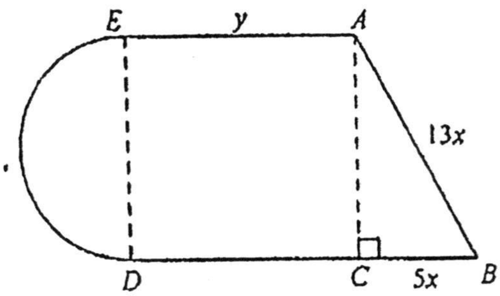
It is known that x and y are related by the equation $y^2 = (ax+1)x - b$, where a and b are constants.

(i) On graph paper, plot $(y^2 - x)$ against x^2, using a scale of 2 cm to represent 0.2 unit on the x^2 - axis and 4 cm to represent 1 unit on the $(y^2 - x)$ - axis. Draw a straight line graph to represent the equation $y^2 = (ax+1)x - b$. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $(a-2) = \frac{1+b}{x^2}$. [2]

8 A piece of wire 160 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc whose diameter is given by the length DE, and a right-angled triangle ABC on the opposite ends of a rectangle of length y cm. The length of BC and AB are $5x$ cm and $13x$ cm respectively.



(i) Express y in terms of x. [2]

(ii) Show that the area enclosed, A cm^2, is given by $A = 960x - 6(3\pi + 13)x^2$. [2]

(iii) Determine the value of x for which A has a stationary value. [3]

(iv) Find the stationary value of A and determine if it is a maximum or a minimum value. [3]

9 (a) (i) Prove that $\cos A = \dfrac{\cos 2A}{\cos A} + \tan A \sin A$. [3]

(ii) Solve, for $0° \le A \le 360°$, $\cos A - \tan A \sin A = -1$. [5]

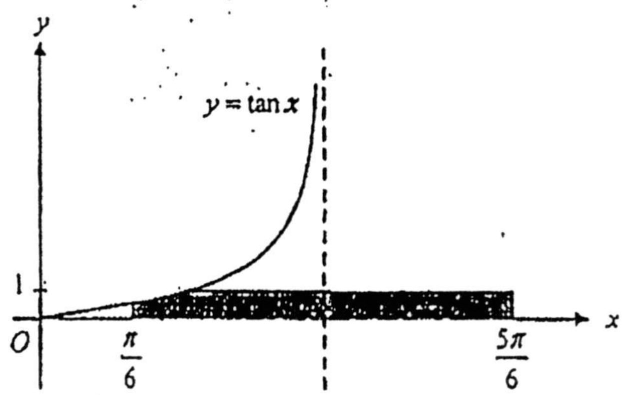
(b) Given $\cos\theta = -\dfrac{4}{5}$ and θ is in the third quadrant. Without using a calculator, find the value of $\cos\dfrac{\theta}{2}$. [3]

10 (a) Solve the equation $\log_2 \dfrac{1}{2} = \log_2 x - \log_4 (9x-2)$. [3]

(b) Given that $\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$, express y in terms of x. [3]

(c) (i) Differentiate $\ln \cos x$. [1]

(ii) State the principal value of $\tan^{-1} 1$, giving your answer as a multiple of π. [1]



The diagram shows part of the graph $y = \tan x$. The shaded region is bounded by the curve, the x - axis, lines $x = \dfrac{\pi}{6}$, $x = \dfrac{5\pi}{6}$ and $y = 1$.

(iii) Using your results from (i) and (ii), or otherwise, find the area of the shaded region. [4]

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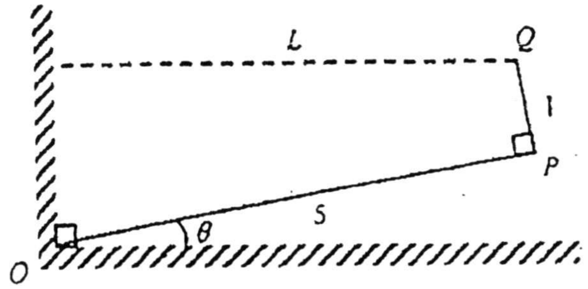

A L-shaped structure, OPQ, can be rotated about O. OP and PQ measures 5 m and 1 m respectively. OP makes an acute angle, θ, with the ground. Given that L m is the shortest distance from Q to the wall,

(i) show that $L = 5\cos\theta - \sin\theta$, [2]

(ii) express L in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, [4]

(iii) state the minimum value of L and find the corresponding value of θ, [3]

(iv) find the value of θ when $L = 3$, [2]

(v) explain why the maximum value of L is not R. [1]

End of Paper

Answer Key

1(ii) $\frac{dh}{dt}=-1.59$ cm / min

(iii) As t increases, h decreases.
Since $\frac{dh}{dt}=\frac{-180}{\pi h^2}$, $\frac{dh}{dt}$ is inversely proportional to h^2,
hence rate of change of water level increases when h decreases.

Or
$\frac{dh}{dt}=\frac{-180}{\pi h^2}$ $\therefore \frac{d^2h}{dt^2}=\frac{360}{\pi h^3}$
Since $h^3>0$ for all positive h, then $\frac{d^2h}{dt^2}>0$.
Hence $\frac{dh}{dt}$ is an increasing function.

2(i) $A=20$ (iii) $t=\frac{7}{24}$

3(i) $a=7$, $b=-7$
$x=-3$ or $x=2$ or $x=-2.5$

(ii) $y=-4$ or $y=1$ or $y=-3.5$

4 Plane Geometry

5 (i) $\frac{2}{x-2}+\frac{1}{x+2}+\frac{2}{(x+2)^2}$

6a(ii) $g=-3\frac{1}{5}$ $h=-\frac{8}{15}$
6b $-2<k<6$

7(ii) $a=-3.00$ $b=-5$
7(iii) Draw $y^2-x=2x^2+1$
$x=\pm 0.894$

8(i) $y=80-3(\pi+3)x$
8(iii) $x=3.57$
8(iv) Stationary value of $A=1710$
A is maximum

9a(ii) $A=60°, 180°, 300°$
9(b) $\cos\frac{\theta}{2}=-\frac{\sqrt{10}}{10}$

10(a) $x=\frac{1}{4}$ or $x=2$

(b) $y=\frac{(x+3)^3}{64}$

(c) (i) $-\tan x$
(ii) Principal value of $\tan^{-1}1=\frac{\pi}{4}$
(iii) Area = 2.04 units^2

11(ii) $L=5.10\cos(\theta+11.3°)$

(iii) Min $L=0$ when $\theta=78.7°$

(iv) $\theta=42.7°$

(v) If $L=R$ then $\theta<0°$.
Since $0°\le\theta<90°$, $\therefore$ maximum $L\ne R$
[Since $\theta\ge 0°$, maximum L occurs when $\theta=0°$, maximum $L=5$.]

	Class	Register Number
Name SOLUTION		

4047/02 **15/S4PR2/AM/2**

ADDITIONAL MATHEMATICS **PAPER 2**

Tuesday 11 August 2015 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper
Graph paper (1 sheet)

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[Turn over

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

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1 Water from a tank in the shape of an inverted cone flows out at the rate of $5 \text{ cm}^3/\text{min}$. The height of the cone is 48 cm and the base diameter is 16 cm. After t minutes the water level is h cm.

(i) Show that the volume of water in the tank, $V \text{ cm}^3$, at time t is given by $V = \frac{\pi h^3}{108}$. [2]

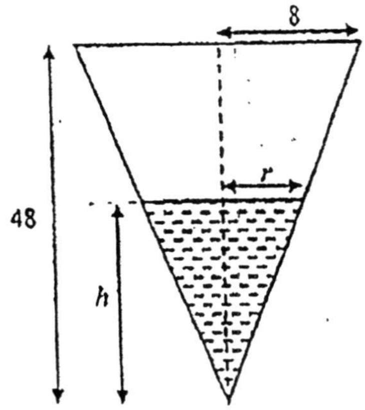
Using similar triangles,

$\frac{r}{8} = \frac{h}{48} \quad \therefore r = \frac{h}{6}$

Volume of water, $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{6}\right)^2 h$

$\therefore V = \frac{\pi h^3}{108}$



(ii) Find the rate of change of the water level when $h = 6$. [3]

$\frac{dV}{dh} = \frac{\pi h^2}{36}$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$-5 = \frac{\pi h^2}{36} \times \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{-180}{\pi h^2}$

At $h = 6$, $\frac{dh}{dt} = \frac{-180}{\pi 6^2}$

$= -1.59 \text{ cm}^3/\text{min}$ (3sf)

(iii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

As t increases, h decreases. Since $\frac{dh}{dt} = \frac{-180}{\pi h^2}$, $\frac{dh}{dt}$ is inversely proportional to h^2, hence rate of change of water level increases when h decreases.

2 The displacement, y mm, of a mass fixed on a vertical spring can be described by the simple harmonic motion equation, $y = A\sin(\omega t)$, where A and ω are constants and t is the time in seconds after the mass is displaced from its equilibrium position, 0 mm.

Given that the maximum displacement of the mass is 20 mm and that the mass first returns to its equilibrium position after 0.25 seconds.

(i) State the positive value of A. [1]

$A = 20$

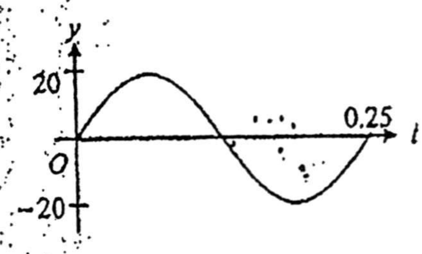
(ii) Show that the value of ω is 4π radians per second. [2]

$0 = 20\sin\omega(0.25)$

$\sin\frac{1}{4}\omega = 0$

$\frac{1}{4}\omega = 0, \pi$

$\omega = 0$ (rej), 4π



(iii) Find the exact value of t when the mass first reach a position 10 mm below its equilibrium position. [3]

$-10 = 20\sin 4\pi t$

$\sin 4\pi t = -\frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$4\pi t = \pi + \frac{\pi}{6}$

$t = \frac{7\pi}{6} \times \frac{1}{4\pi}$

$= \frac{7}{24}$ s

3 (i) Given that $f(x)=2x^3+ax^2+bx-30$ has a factor $(x+3)$ and leaves a remainder of -28 when divided by $(x-1)$. Find the values of a and of b and solve $f(x)=0$. [6]

$$f(-3)=2(-3)^3+a(-3)^2+b(-3)-30=0$$
$$-54+9a-3b-30=0$$
$$3a-b=28 \quad \cdots(1)$$

$$f(1)=2(1)^3+a(1)+b-30=-28$$
$$a+b=0 \quad \cdots(2)$$

$$(1)+(2):\quad 4a=28$$
$$a=7$$
$$b=-7$$

$$
\begin{array}{r}
2x^2+x-10 \\
x+3\overline{)2x^3+7x^2-7x-30} \\
\underline{2x^3+6x^2\qquad\qquad} \\
x^2-7x\qquad \\
\underline{x^2+3x\qquad} \\
-10x-30 \\
\underline{-10x-30} \\
0
\end{array}
$$

$$f(x)=0$$
$$2x^3+7x^2-7x-30=0$$
$$(x+3)(2x^2+x-10)=0$$
$$(x+3)(x-2)(2x+5)=0$$
$$x=-3 \quad \text{or} \quad x=2 \quad \text{or} \quad x=-2.5$$

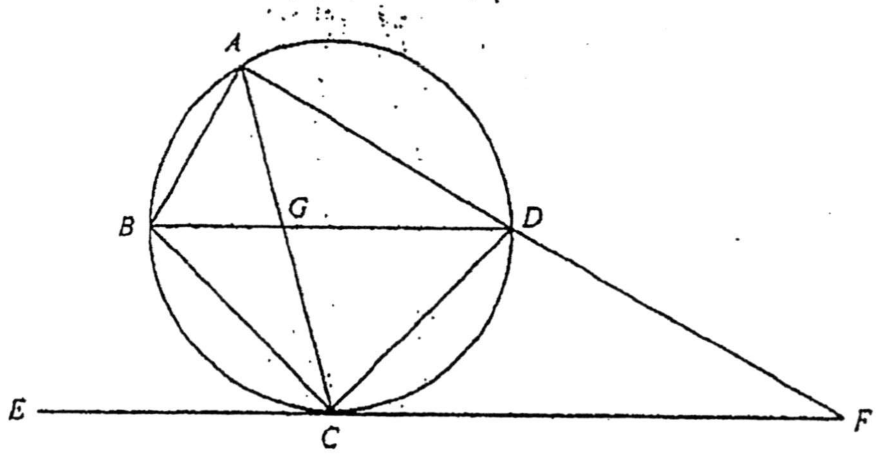
(ii) Hence solve $2(y+1)^3+a(y+1)^2+by+b-30=0$. [2]

$$2(y+1)^3+a(y+1)^2+by+b-30=0$$
$$2(y+1)^3+a(y+1)^2+b(y+1)-30=0$$

Let $x=y+1$,

$$y+1=-3 \quad \text{or} \quad y+1=2 \quad \text{or} \quad y+1=-2.5$$
$$y=-4 \qquad\qquad y=1 \qquad\qquad y=-3.5$$

4



The diagram shows points A, B, C and D lying on a circle. The chords BD and AC intersect at G. EF is a tangent to the circle at C. AD is produced meet the tangent at F and $\angle ABC = \angle BGC$.

Prove that

(i) BD is parallel to EF, [2]

$\angle ACF = \angle ABC$ ($\angle$s in alternate segment)

$\angle ABC = \angle BGC$ (Given)

$\therefore \angle BGC = \angle ACF$

By the angle property of alternate angles, BD is parallel to EF.

(ii) triangle CFD and triangle AFC are similar, [2]

$\angle CFD = \angle AFC$ (Common $\angle$)

$\angle DCF = \angle CAF$ ($\angle$s in alternate segment)

Hence ΔCFD is similar to ΔAFC.

(iii) $FC^2 - FD^2 = FD \times DA$. [3]

Since ΔCFD is similar to ΔAFC,

$$\left.\begin{aligned} &\frac{FD}{FC} = \frac{CF}{AF} \\ &FC^2 = FD \times AF \end{aligned}\right\}$$

$$= FD \times (FD + DA)$$

$$= FD^2 + FD \times DA$$

$\therefore FC^2 - FD^2 = FD \times DA$ (Proven)

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6 (ii) If the roots of the equation $gx^2 - hx - 1 = 0$ where g and h are constants, are α and β, find the value of g and of h. [2]

$\alpha + \beta = \frac{h}{g}$

$\frac{1}{6} = \frac{h}{g}$

$\therefore h = \frac{g}{6}$ ....(1)

$\alpha\beta = \frac{-1}{g}$

$\frac{5}{16} = \frac{-1}{g}$

$g = \frac{-16}{5}$

$= -3\frac{1}{5}$

Sub $g = -3\frac{1}{5}$ into (1)

$\therefore h = -3\frac{1}{5} \div 6$

$= \frac{-8}{15}$

Alternative solution:

$x^2 - \frac{1}{6}x + \frac{5}{16} = 0$

$-\frac{16}{5}x^2 + \frac{8}{15}x - 1 = 0$

$gx^2 - hx - 1 = 0$

$\therefore g = \frac{-16}{5}$

$= -3\frac{1}{5}$

$\therefore -h = \frac{8}{15}$

$h = -\frac{8}{15}$

(b) Find the range of values of k for which $(k+3)x^2 + kx + 1$ is always positive for all real values of x. [4]

$(k+3)x^2 + kx + 1 > 0$

Since the expression is always positive,

$k + 3 > 0$ and $b^2 - 4ac < 0$

$k > -3$

$k^2 - 4(k+3)(1) < 0$

$k^2 - 4k - 12 < 0$

$(k-6)(k+2) < 0$

$\therefore -2 < k < 6$

Hence $k > -3$ and $-2 < k < 6$

$\therefore$ the solution is $-2 < k < 6$

7 Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables, x and y.

x	0.4	0.6	0.8	1.0	1.2
y	2.22	2.13	1.97	1.73	1.37

It is known that x and y are related by the equation $y^2=(ax+1)x-b$, where a and b are constants.

(i) On graph paper, plot (y^2-x) against x^2, using a scale of 2 cm to represent 0.2 unit on the x^2 axis and 4 cm to represent 1 unit on the (y^2-x) axis. Draw a straight line graph to represent the equation $y^2=(ax+1)x-b$. [3]

x^2	0.160	0.360	0.640	1.00	1.44
y^2-x	4.53	3.94	3.08	1.99	0.677

(ii) Use your graph to estimate the value of a and of b. [4]

$y^2=(ax+1)x-b$

$y^2=ax^2+x-b$

$y^2-x=ax^2-b$

Gradient $=a$

Gradient $=\dfrac{5-3.5}{0-0.5}$

$\therefore a=-3.00$ (3sf).

(y^2-x)–intercept $=-b$

$-b=5$

$\therefore b=-5$

(iii) By drawing a suitable straight line on your graph, solve the equation $(a-2)=\dfrac{1+b}{x^2}$. [2]

$(a-2)=\dfrac{1+b}{x^2}$

$ax^2-2x^2=1+b$

$ax^2-b=2x^2+1$

Draw $y^2-x=2x^2+1$

From graph, $x^2=0.8$

$x=\pm\ 0.894$ (3sf)

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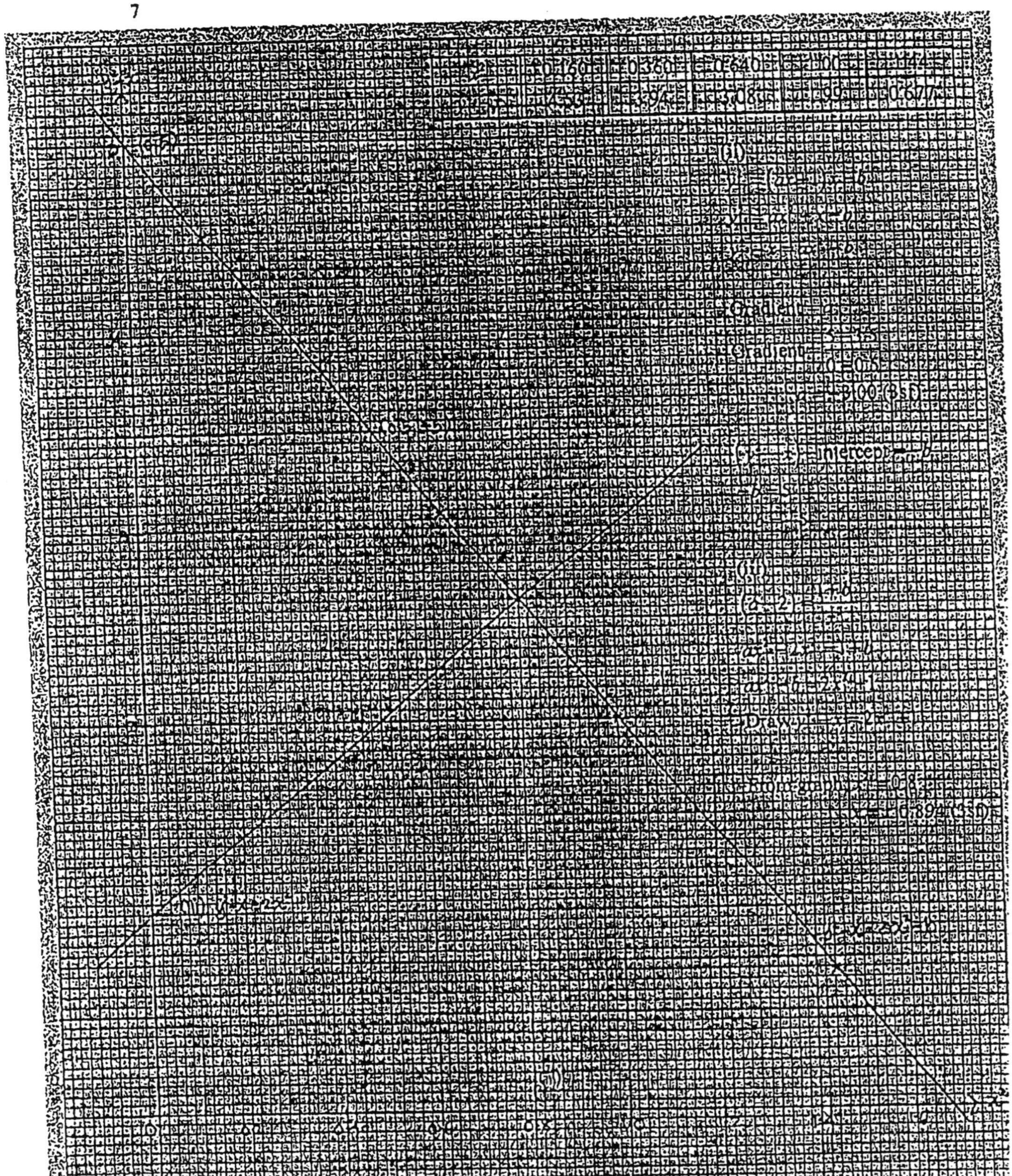
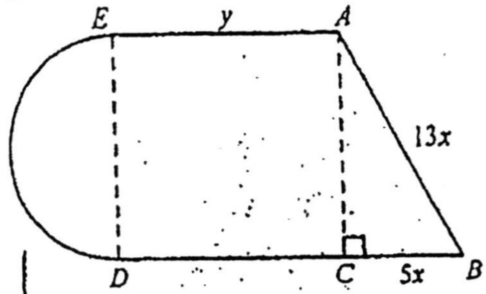


8 A piece of wire 160 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc whose diameter is given by the length DE, and a right-angled triangle ABC on the opposite ends of a rectangle of length y cm. The length of BC and AB are $5x$ cm and $13x$ cm respectively.



(i) Express y in terms of x. [2]

$AC=\sqrt{(13x)^2-(5x)^2}=12x$

Perimeter of figure = Length of wire

$2y+\frac{\pi 12x}{2}+13x+5x=160$

$2y+6\pi x+18x=160$

$y+3\pi x+9x=80$

$\therefore\ y=80-3(\pi+3)x$

(ii) Show that the area enclosed, A cm^2, is given by $A=960x-6(3\pi+13)x^2$. [2]

$$\begin{aligned}A&=\frac{1}{2}\pi(6x)^2+y(12x)+\frac{1}{2}(5x)(12x)\\&=18\pi x^2+30x^2+12x[80-3(\pi+3)x]\\&=(18\pi+30)x^2+960x-36(\pi+3)x^2\\&=[18\pi+30-36(\pi+3)]x^2+960x\\&=(18\pi+30-36\pi-108)x^2+960x\\&=(-18\pi-78)x^2+960x\\&=960x-6(3\pi+13)x^2\end{aligned}$$

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8 (iii) Determine the value of x for which A has a stationary value. [3]

$$A = 960x - 6(3\pi + 13)x^2$$

$$\frac{dA}{dx} = 960 - 12(3\pi + 13)x$$

For stationary value of A,

$$\frac{dA}{dx} = 0$$

$$960 - 12(3\pi + 13)x = 0$$

$$x = \frac{960}{12(3\pi + 13)}$$

$$\approx 3.567$$

$$= 3.57 \text{ (3 sf)}$$

(iv) Find the stationary value of A and determine if it is a maximum or a minimum value. [3]

$$\text{Stationary value of } A = 960(3.567) - 6(3\pi + 13)(3.567)^2$$

$$\approx 1712.39$$

$$= 1710 \text{ (3 sf)}$$

$$\frac{d^2A}{dx^2} = -12(3\pi + 13)$$

since $\frac{d^2A}{dx^2} < 0$, A is a maximum value.

---

9 (a) (i) Prove that $\cos A = \frac{\cos 2A}{\cos A} + \tan A \sin A$. [3]

$$\text{RHS} = \frac{\cos 2A}{\cos A} + \tan A \sin A$$

$$= \frac{2\cos^2 A - 1}{\cos A} + \frac{\sin A}{\cos A} \cdot \sin A$$

$$= \frac{2\cos^2 A - 1 + \sin^2 A}{\cos A}$$

$$= \frac{2\cos^2 A - 1 + 1 - \cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

$$= \text{LHS}$$

9 (a) (ii) Solve, for $0° \le A \le 360°$, $\cos A - \tan A \sin A = -1$. [5]

$\cos A - \tan A \sin A = -1$

$\frac{\cos 2A}{\cos A} = -1$

$2\cos^2 A - 1 = -\cos A$

$2\cos^2 A + \cos A - 1 = 0$

$(\cos A + 1)(2\cos A - 1) = 0$

$\cos A = -1$ or $\cos A = \frac{1}{2}$

$A = 180°$ $\quad$ $\alpha = 60°$

$A = 60°, 360° - 60°$

$= 60°, 300°$

(b) Given $\cos\theta = -\frac{4}{5}$ and θ is in the third quadrant. Without using a calculator, find the value of $\cos\frac{\theta}{2}$. [3]

$\cos\theta = -\frac{4}{5}$ $\quad$ $180° < \theta < 270°$

$2\cos^2\frac{\theta}{2} - 1 = -\frac{4}{5}$ $\quad$ $90° < \frac{\theta}{2} < 135°$

$\cos^2\frac{\theta}{2} = \frac{1}{10}$

$\cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{10}}$

$= \pm\frac{\sqrt{10}}{10}$

Since $90° < \frac{\theta}{2} < 135°$, $\cos\frac{\theta}{2} = -\frac{\sqrt{10}}{10}$

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10 (a) Solve the equation $\log_2 \frac{1}{2} = \log_2 x - \log_4 (9x-2)$. [3]

$$\log_2 \frac{1}{2} = \log_2 x - \log_4 (9x-2)$$

$$\log_4 (9x-2) = \log_2 x - \log_2 \frac{1}{2}$$

$$\frac{\log_2 (9x-2)}{\log_2 4} = \log_2 \left(x \div \frac{1}{2}\right)$$

$$\frac{\log_2 (9x-2)}{2} = \log_2 2x$$

$$\log_2 (9x-2) = 2\log_2 2x$$

$$\log_2 (9x-2) = \log_2 (2x)^2$$

$$\therefore 9x-2 = 4x^2$$

$$4x^2 - 9x + 2 = 0$$

$$(4x-1)(x-2) = 0$$

$$4x-1=0 \quad \text{or} \quad x-2=0$$

$$x = \frac{1}{4} \quad \text{or} \quad x = 2$$

(b) Given that $\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$, express y in terms of x. [3]

$$\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$$

$$\log_2 (x+3) - \log_2 y \times \frac{1}{\log_2 8} = 2$$

$$\log_2 (x+3) - \frac{\log_2 y}{3} = 2$$

$$3\log_2 (x+3) - \log_2 y = 6$$

$$\log_2 (x+3)^3 - \log_2 y = 6$$

$$\log_2 \frac{(x+3)^3}{y} = 6$$

$$\frac{(x+3)^3}{y} = 2^6$$

$$64y = (x+3)^3$$

$$y = \frac{(x+3)^3}{64}$$

$$\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$$

$$\log_2 (x+3) - \log_2 y \times \frac{\log_2 2}{\log_2 8} = 2$$

$$\log_2 (x+3) - \frac{\log_2 y}{3} = 2$$

$$\log_2 (x+3) - \log_2 \sqrt[3]{y} = 2$$

$$\log_2 \frac{(x+3)}{\sqrt[3]{y}} = 2$$

$$\frac{(x+3)}{\sqrt[3]{y}} = 2^2$$

$$\frac{(x+3)^3}{y} = 4^3$$

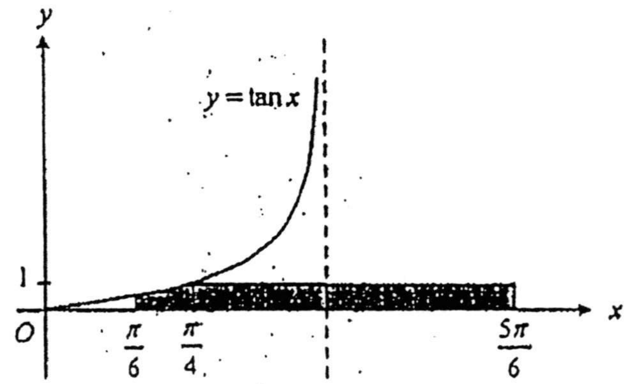
$$y = \frac{(x+3)^3}{64}$$

10 (c) (i) Differentiate $\ln \cos x$. [1]

$$\frac{d}{dx}\ln \cos x = \frac{-\sin x}{\cos x}$$
$$= -\tan x$$

(ii) State the principal value of $\tan^{-1} 1$, giving your answer as a multiple of π. [1]

Principal value of $\tan^{-1} 1 = \frac{\pi}{4}$



The diagram shows part of the graph $y = \tan x$. The shaded region is bounded by the curve, the x axis, lines $x = \frac{5\pi}{6}$ and $y = 1$.

(iii) Using your results from (i) and (ii), or otherwise, find the area of the shaded region. [4]

$$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \tan x \, dx + \left(\frac{5\pi}{6} - \frac{\pi}{4}\right)$$
$$= -\left[\ln \cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \frac{7\pi}{12}$$
$$= -\left[\ln \cos\frac{\pi}{4} - \ln \cos\frac{\pi}{6}\right] + \frac{7\pi}{12}$$
$$= -\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2} + \frac{7\pi}{12}$$
$$= \frac{1}{2}\ln\frac{3}{2} + \frac{7\pi}{12}$$
$$= 2.04 \text{ units}^2 \text{ (3sf)}$$

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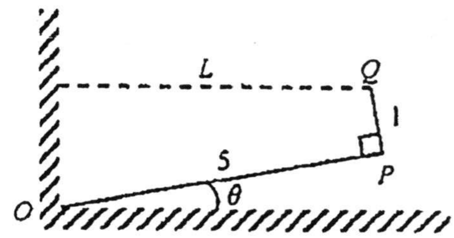
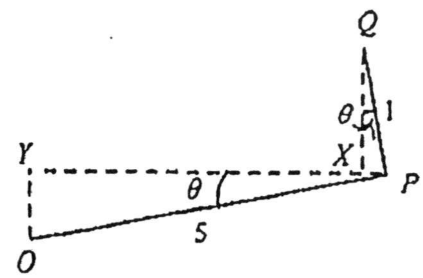


A L-shaped structure, OPQ, can be rotated about O. OP and PQ measures 5 m and 1 m respectively. OP makes an acute angle, θ, with the ground. Given that L m is the shortest distance from Q to the wall,

(i) show that $L = 5\cos\theta - \sin\theta$, [2]



$$\cos\theta = \frac{PY}{5}$$
$$PY = 5\cos\theta$$

$$\sin\theta = \frac{PX}{1}$$
$$PX = \sin\theta$$

$$\therefore L = PY - PX$$
$$= 5\cos\theta - \sin\theta$$

(ii) express L in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, [4]

$$L = 5\cos\theta - \sin\theta$$
$$= R\cos(\theta + \alpha)$$
$$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

$R\cos\alpha = 5 \quad \cdots(1) \qquad R\sin\alpha = 1 \quad \cdots(2)$

$(1)^2 + (2)^2: \quad R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 1^2$
$$R = \sqrt{26}$$
$$= 5.10 \text{ (3sf)}$$

$\frac{(2)}{(1)}: \quad \frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{5}$

$$\tan\alpha = \frac{1}{5}$$
$$\alpha = \tan^{-1}\frac{1}{5}$$
$$= 11.31° \text{ (2dp)}$$

$\therefore L = 5.10\cos(\theta + 11.3°)$ (3sf, 1dp)

11 (iii) state the minimum value of L and find the corresponding value of θ, [3]

Minimum $L=0$, when $\cos(\theta+11.31°)=0$

$\theta+11.31°=90°$

$\theta=78.7°$ (1dp)

(iv) find the value of θ when $L=3$, [2]

$3=\sqrt{26}\cos(\theta+11.31°)$

$\cos(\theta+11.31°)=\frac{3}{\sqrt{26}}$

$\alpha=\cos^{-1}\left(\frac{3}{\sqrt{26}}\right)=53.96°$

$\theta+11.31°=53.96°$

$\theta=42.7°$ (1dp)

(v) explain why the maximum value of L is not R. [1]

If $L=R$ then $\theta<0°$. Since $0°\leq\theta<90°$, $\therefore$ maximum $L\neq R$

[Since $\theta\geq0°$, maximum L occurs when $\theta=0°$, maximum $L=5$.]

End of Paper

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END