(i) **Without using a calculator**, find the exact value of 12^x, given that $3^{2(x+1)} - 4^{1-2x} = 0$. [3]

(ii) Solve the equation $2e^{2x} = 13e^x - 15$. [4]

(i) Find the range of values of x for which $x(10-x) > 24$. [2]

(ii) Find the range of values of c for which $x(10-x) < c^2$. [4]

(i) Sketch, on the same diagram, the graphs of $y = x^{-\frac{1}{2}}$ and $y^2 = 4x$ for $x > 0$. [2]

(ii) Find the coordinates of the point of intersection of the graphs. [3]

(i) Find the exact value of $\sin 165°$. [3]

(ii) Hence, show that $\cot^2 165°$ can be expressed in the form $a + b\sqrt{3}$ where a and b are integers. [4]

iven that the term independent of x in the expansion of $(3+5x^2)\left(1-\frac{1}{2x}\right)^n$ is 38, where n is a onstant, find

(a) the value of n, [4]

(b) the coefficient of $\frac{1}{x}$. [3]

The population, P, of a certain species of frogs is given by

$$P = Ae^{-kt},$$

here A and k are constants and t is the time in years from 1 January 2000.

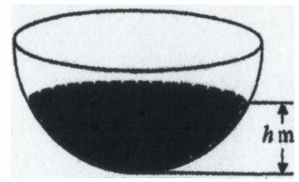
Over a period of 10 years from 1 January 2000 to 1 January 2010, P decreased from 90 000 to 40 000.

Find

(i) the value of A and of k, [3]

(ii) the year in which the population will be reduced by 70% as compared to the year 2000. [2]

7. A hemispherical container is shown in the diagram below.



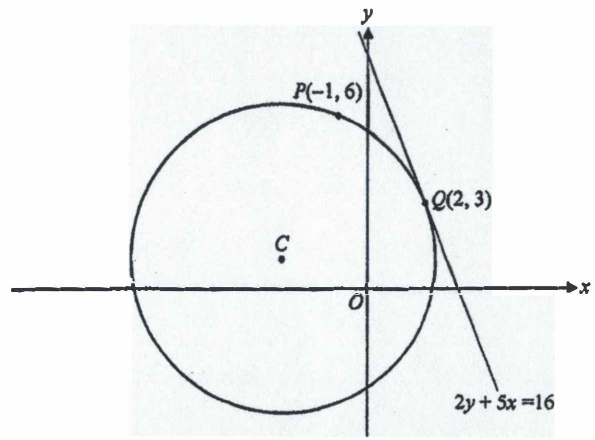
When the depth of the water is h m, the volume, V m^3, is given by

$$V = \frac{1}{12}\pi h(3-4h^2), \text{ where } 0 \le h \le 0.5.$$

(i) Find the value of h for which $\frac{dV}{dt} = \frac{5\pi}{36}\frac{dh}{dt}$. [3]

(ii) If water is flowing into the bowl at a constant rate of $\frac{\pi}{800}$ m^{3}s^{-1}, find the rate of change of h when $h = 0.25$. [2]

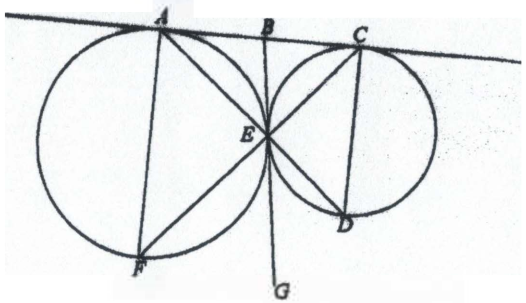
8. A circle, centre C, passes through the point $P(-1, 6)$ and touches the line $2y + 5x = 16$ at the point $Q(2, 3)$.



(i) Find the equation of the perpendicular bisector of PQ. Hence find the equation of the circle. [6]

(ii) Find the coordinates of R such that $CPQR$ is a parallelogram. [2]

…e diagram, the two circles touch at E. ABC and BEG are common tangents to the two circles. …and CE are produced to D and F respectively.

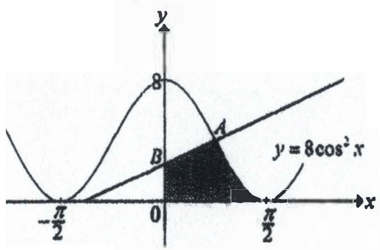


Prove that AF is parallel to CD. [4]

) Prove that AC is a diameter of a circle which passes through A, E and C. [4]

) Show that $\frac{d}{dx}(2x+\sin 2x)=4\cos^2 x$. [2]

)



The diagram shows part of the graph of $y=8\cos^2 x$. The normal to the curve at A, where $x=\frac{\pi}{4}$, meets the y-axis at B.

(i) Show that the y-coordinates of B is $\frac{128-\pi}{32}$. [4]

(ii) Determine the area of the shaded region bounded by the curve, the line AB, the x-axis and the y-axis. [5]

11. It is given that $f(x)=x^2-8x+9$ for $2\le x\le 7$.

(i) Find the value of a and of b for which $f(x)=(x-a)^2+b$. [2]

(ii) Find the stationary point of the graph $y=|f(x)|$ and determine its nature. [2]

(iii) Sketch the graph of $y=|f(x)|$. [3]

(iv) Find the range of values of x for which $|f(x)|>6$. [2]

(v) Determine the number of solutions of the equation $|f(x)|=mx+c$ in each of the following cases, when

(a) $m=1$ and $c=2$, [1]

(b) $m=-\frac{1}{2}$ and $c=4$. [1]

End of Paper 1

Find a quadratic equation for which the sum of roots is $\frac{1}{2}$ and the sum of the cubes of the roots is $\frac{13}{8}$. [5]

(a) Variables x and y are connected by the equation $\log_3 y = a\log_3 x + b$, where a and b are constants. Using experimental values of x and y, a graph was drawn in which $\log_3 y$ was plotted on the vertical axis against $\log_3 x$ on the horizontal axis. The straight line which was obtained passed through the points (1, 3) and (−1, 5).

(i) Find the value of a and of b. [3]

(ii) Show that x and y can be expressed in the form $y = kx^n$, where k and n are constants to be found. [3]

(b) Given that $\log_8 x^3 = \log_{\sqrt{2}} u$, express u in terms of x. [3]

(i) Show that $\frac{\sin 2x + 1 - \cos 2x}{\sin 2x - 1 + \cos 2x} = \frac{1+\tan x}{1-\tan x}$. [3]

(ii) Hence, solve for $-3 < x < 2$, the equation $\frac{\sin 2x + 1 - \cos 2x}{\sin 2x - 1 + \cos 2x} = 6\tan x$. [5]

(a) Find the value of m, where $m > 0$, for which $2x^2 + x + m$ is a factor of $4x^3 + 5x - 3$. [3]

(b) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 3 and the roots of the equation $f(x) = 0$ are −2, 3 and k. Given that $f(x)$ has a remainder of 42 when divided by $(x+1)$, find

(i) the value of k, [3]

(ii) the remainder when $f(x)$ is divided by x. [2]

(i) Express $\frac{-2x-6}{(x+1)(x^2-3)}$ in partial fractions. [4]

(ii) Differentiate $\ln(x^2-3)$. [1]

(iii) Given that $\int_2^3 \frac{-3x-9}{(x+1)(x^2-3)}\,dx = \frac{9}{2}\ln a$, using the results in parts (i) and (ii), find the value of a. [4]

6. A device is used to simulate the breathing patterns of a certain mammal's lungs. The volume, V litres, of air in the lungs of this mammal, t seconds after the beginning of one breath can be modelled by

$$V = 0.45 - 0.4\cos(kt),\ \ 0 \le t \le 4.$$

The time for one breath is 4 seconds.

(i) Explain why this model suggests that the maximum capacity of the lungs is 0.85 litres. [1]

(ii) Show that the value of k is $\frac{\pi}{2}$. [2]

(iii) Find the length of time for which the lungs contain at least 0.5 litres of air. [3]

(iv) Sketch the graph of $V = 0.45 - 0.4\cos(kt)$, $0 \le t \le 4$. [2]

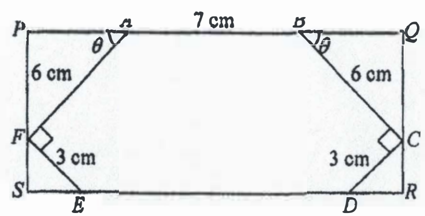
7. A curve has the equation $y = 4x^3e^{2x}$. It has a stationary point at $\left(p, \frac{q}{e^3}\right)$ where $p < 0$.

(i) Find the exact value of p and of q. [5]

(ii) By considering the sign of $\frac{dy}{dx}$, determine the coordinates and the nature of the other stationary point. [2]

(iii) Find the range of values of x for which $y = 4x^3e^{2x}$ is a decreasing function. [2]

8. In the diagram, $PQRS$ is a rectangle. $ABCDEF$ is a hexagon with angle AFE = angle BCD = 90°. AB = 7 cm, BC = AF = 6 cm, CD = EF = 3 cm and angle QBC = angle PAF = θ, where $0° \le \theta \le 90°$.

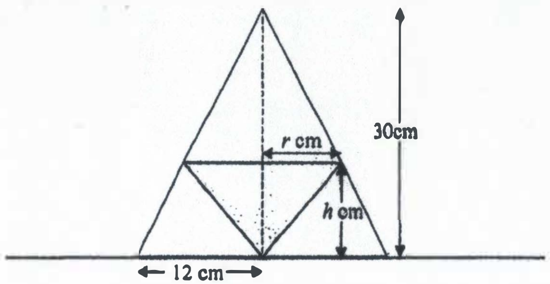


(i) Show that the perimeter, L cm, of $ABCDEF$ is given by $L = 32 + 12\cos\theta - 6\sin\theta$. [3]

(ii) Express L in the form $k + R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Find the value of θ, if $L = 35$. [3]

9. The diagram shows the cross-section of a hollow cone of height 30 cm and base radius of 12 cm and an inverted cone of radius r cm and height h cm. Both stand on a horizontal surface with the inverted cone inside the hollow cone. The upper circular edge of the inverted cone is in contact with the hollow cone.



(i) Express h in terms of r and hence show that the volume, V cm³ of the inverted cone is given by

$$V = \pi\left(10r^2 - \frac{5r^3}{6}\right).$$ [4]

Given that r can vary,

(ii) find, in terms of π, the volume of the largest inverted cone which can stand inside the hollow cone, and show that, in this case, the inverted cone occupies $\frac{4}{27}$ of the volume of the hollow cone. [7]

10. The population P, in millions, of a country was recorded in various years and the results are shown in the table below.

Year	2000	2005	2010	2015
P	12.88	14.61	17.38	21.88

It is known that P and t are related by an equation of the form $P = 10 + Ab^t$, where t is the time measured in years from January 1995 and A and b are constants.

(i) Using graph paper, draw a straight line graph of $\lg(P-10)$ against t and use your graph to estimate the value of A and of b.
[7]

Use your graph to estimate

(ii) the population, in millions, in the country in January 1995, [2]

(iii) the year in which the population exceeds 35 million. [2]

11. The velocity, v ms^{-1}, of a particle, P, moving in a straight line is given by $v = 3t^2 + pt + q$, where t is the time in seconds after the start of motion. At $t = 0$, the displacement of the particle from O is 3 m.

Given also that when $t = 2$, the displacement of the particle from O is 23 m and the acceleration of the particle is -6 ms^{-2},

(i) find the value of p and of q, [7]
(ii) explain with clear working whether P will return to its starting point. [5]

End of Paper 2

2016 SCGS Prelim Paper 1

1. (i) $\frac{2}{3}$
   (ii) 0.405, 1.61

2. (i) $4 < x < 6$
   (ii) $c < -5$ or $c > 5$

3. (ii) $\left(\frac{1}{2}, \sqrt{2}\right)$

4. (i) $\frac{\sqrt{6}-\sqrt{2}}{4}$
   (ii) $7 + 4\sqrt{3}$

5. (a) 8
   (b) −47

6. (i) 0.0811
   (ii) 2014

7. (i) $\frac{1}{3}$
   (ii) $\frac{1}{150}$ ms^{-1} or 0.00667 ms^{-1}

8. (i) $(x+3)^2 + (y-1)^2 = 29$
   (ii) (0, −2)

10. (bii) 4.24 units^2

11. (i) $a = 4$, $b = -7$
   (ii) (4, 7), Maximum point
   (iv) $3 < x < 5$
   (va) 2 solutions
   (vb) 3 solutions

2016 SCGS Prelim Paper 2

1. $x^2 - \frac{x}{2} - 1 = 0$

2. (ai) $a = -1$, $b = 4$
   (ii) $y = 81x^{-1}$
   (b) $u = \sqrt{x}$

3. (ii) $x = -2.82, -2.68, 0.322, 0.464$

4. (a) 3
   (bi) $k = \frac{5}{2}$
   (bii) 45

5. (i) $\frac{2}{x+1} - \frac{2x}{x^2-3}$
   (ii) $\frac{2x}{x^2-3}$
   (iii) $\frac{2}{3}$

6. (iii) 1.84 s

7. (i) $p = -\frac{3}{2}$, $q = -\frac{27}{2}$ or -13.5
   (ii) point of inflexion
   (iii) $x < -\frac{3}{2}$

8. (ii) $L = 32 + 6\sqrt{5}\cos(\theta + 26.6°)$
   (iii) 50.5°

9. (ii) $\frac{640\pi}{3}$ cm^3

10. (ii) 11.8
   (iii) 2023

11. (i) $p = -18$, $q = 24$
   (ii) $L = 29 + \sqrt{3362}\cos(\theta - 45°)$