ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2016

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **25 August 2016**

Secondary 4 Express / 5 Normal (Academic) **2 hours**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **7** printed pages

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\dbinom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities $\qquad \sin^2 A + \cos^2 A = 1$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC, $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The function f is defined by

$$f(x) = 3 + \frac{1}{2x-1}, \text{ where } x \neq \frac{1}{2}.$$

Show that f is a decreasing function. [3]

2 Find the range of values of p for which $(p+2)x^2 - 12x + 2(p-1)$ is always negative. [4]

3 The line $y = mx + c$ intersects the curve $y^2 = ax$ at $A(4, 4)$ and $B(1, k)$.

B is a point that lies below the x-axis.

(i) Sketch the curve $y^2 = ax$, indicating point A. [1]

(ii) Find the values of a, m, c and k. [4]

4 Sketch the graph of $y = |x-3| + 2$ for $-3 \leq x \leq 6$. [3]

Find the range of values of c for which $|x-3| - c = x - 2$ has

(i) only 1 solution, [1]

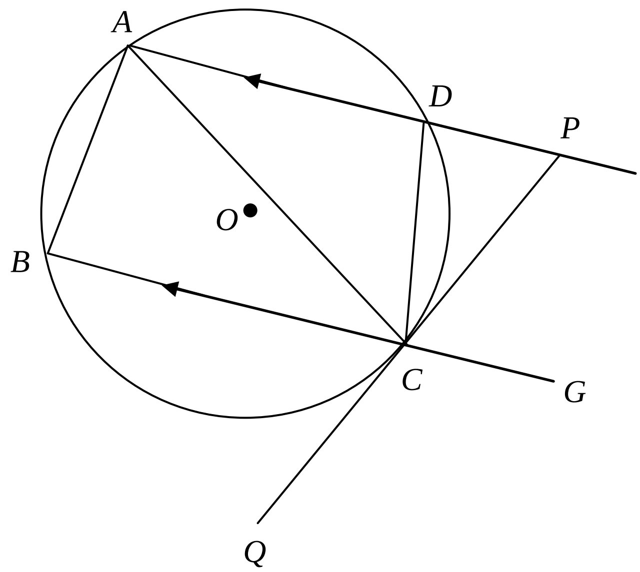
(ii) no solution. [1]

5 Air is pumped into a spherical balloon at a constant rate of 60 cm^3/s.

(i) Find the rate of increase of the radius, at the instant when the radius is 12 cm. [3]

(ii) Hence, find the rate of change of the surface area of the balloon at this instant. [2]

6 In the figure, O is the centre of the circle. PCQ is the tangent to the circle at C and AD is parallel to BC.



(i) Name an angle equal to $\angle BAC$, giving your reason(s) clearly. [1]

(ii) Show that $\angle CPD = \angle BAC$. [2]

(iii) Show that ΔBAC is similar to ΔCPD. [3]

7 Given that $\mathrm{f}(x) = \dfrac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$,

(i) express f(x) in the form $a \sin bx + c$, stating the value of each of the integers a, b and c, [4]

(ii) state the greatest and least values of f(x), [2]

(iii) state the period and amplitude of f(x). [2]

8 The decay of a certain radioactive isotope can be modelled by the exponential equation $N = N_o e^{-at}$ after t weeks, where N represents the amount of radioactive isotope, N_0 and a are constants. A sample of this radioactive isotope has a mass of 100.9 g initially.

(i) After 2 weeks, it is found that the amount of this sample left is 84.6 g. Calculate the value of a. [3]

(ii) What percentage of this sample has decayed after 5 weeks? [2]

(iii) After 9 weeks, the amount of this sample is found to be only 34.6 g. Suggest a reason why this might be so. [2]

9 **(i)** Show that $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$. [3]

(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta - 3\cos\theta = 2$ for $0 < \theta < 360°$. [4]

10 A particle moves in a straight line such that, t seconds after leaving a fixed point O, its velocity, v m s^{-1}, is given by $v = 15 - e^{-3t}$.

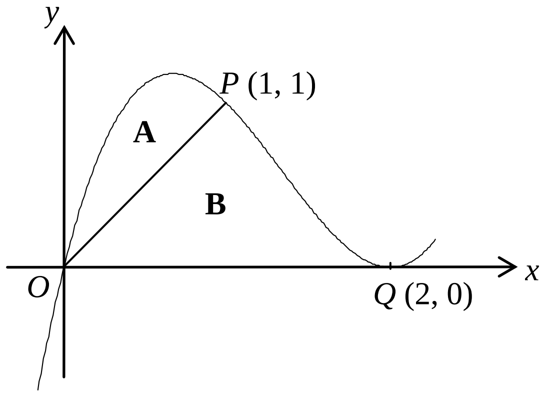
(i) Write down the initial velocity of the particle. [1]

(ii) If t becomes very large, what value will v approach? Explain your answer clearly and its significance. [2]

(iii) Find the acceleration of the particle when t = 3, giving your answer in cm s^{-2} correct to 3 decimal places. [2]

(iv) Find the distance travelled by the particle in the first 4 seconds of its journey, giving your answer correct to 2 decimal places. [2]

11 The diagram above shows part of the curve $y = x(x-2)^2$ which passes through *P* (1, 1) and touches the *x*-axis at *Q* (2, 0).



(i) Find the equation of the tangent at *P* and show that line *OP* is the normal to the curve at *P*. [4]

(ii) Show that the area of the region labelled A is $\frac{5}{12}$ unit^2 and determine the ratio of the area of **A** to the area of **B**. [6]

12 In figure 1, *ABCD* is a square plastic plate of side 4 cm and *PQRS* is a square whose centre coincides with that of *ABCD*. The shaded regions are to be cut off and the remaining plastic is folded to form a right pyramid with base *PQRS*, as shown in figure 2.

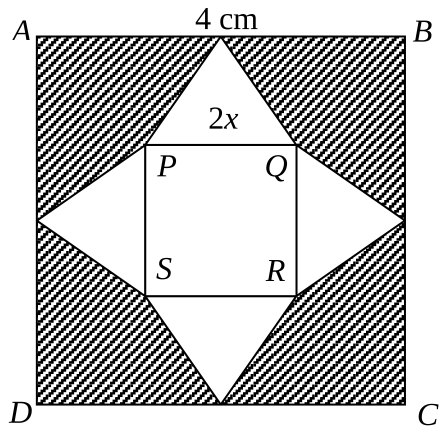


Figure 1

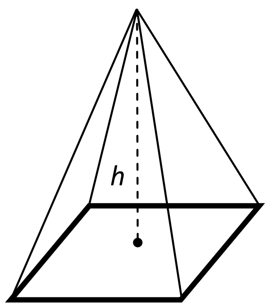


Figure 2

Let $PQ = 2x$ cm and let V be the volume of the pyramid.

(i) Show that the height of the pyramid is $2\sqrt{1-x}$ cm. [2]

(ii) Show that $V = \frac{8}{3}x^2\sqrt{1-x}$ cm^3. [2]

(iii) Find the value of x such that V is maximum. [7]

(iv) Showing your working clearly, explain why the volume of the pyramid will not exceed 0.8 cm^3. [2]

- The End –

Answers

1. $f'(x) = -\frac{2}{(2x-1)^2}$

   $(2x-1)^2 > 0$

   Therefore, $-\frac{2}{(2x-1)^2} < 0$

   Since $f'(x) < 0$, f(x) is a decreasing function.

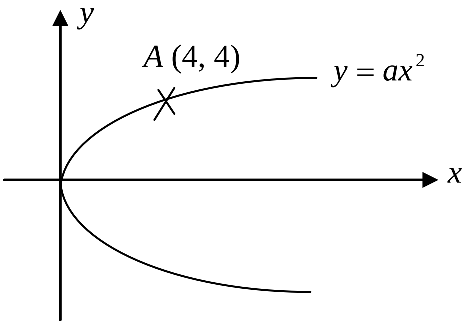
2. $b^2 - 4ac < 0$,

   $p < -5$ or $p > 4$

   But $p + 2 < 0$,

   $\therefore\ p < -5$

3.



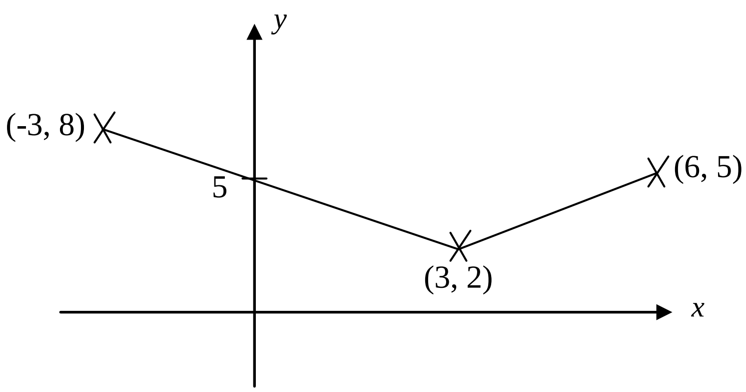
   $a = 4$

   $k = -2$

   $m = 2$

   $c = -4$

4



   (i) $-1 < c \leq 11$

   (ii) $c < -1$ or $c > 11$

5(i) $\frac{dr}{dt} = 0.0332$ cm/s

(ii) $\frac{dA}{dt} = 10.0\,\text{cm}^2/\text{s}$

6(i) $\angle BCQ$.

Alternate Segment Theorem

(ii) from (i),

$\angle BAC = \angle BCQ$.

$\angle BCQ = \angle GCP$ (vert. opp. $\angle$s)

$\therefore \angle CPD = \angle GCP$ (alt. $\angle$s)

$= \angle BAC$

(iii) from (ii),

$\angle BAC = \angle CPD$.

$\angle DCP = \angle DAC$ (alt. seg. thm)

$\angle DAC = \angle BCA$ (alt. $\angle$s)

$\therefore \Delta BAC$ similar to ΔCPD (AA Similarity or 2 pairs of corr. $\angle$s equal)

7. (i) $f(x) = \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$

$= \frac{1}{2}\sin 2x + 1.$

$\therefore a = \frac{1}{2},\ b = 2,\ c = 1$

(ii) greatest = $\frac{3}{2}$, least = $\frac{1}{2}$

(iii) amplitude = $\frac{1}{2}$, period = π or 180°

8(i) $a = 0.0881$

(ii) 35.6%

(iii) Difference = 11.06

Possible reasons:

- Error in data collection
- Due to other external factors that expedited the decay
- Any other logical reasoning with explanation

9(ii) $\theta = 120°$, $180°$, $240°$

10(i) initial velocity = 14m/s

(ii) when t is very large, e^{-3t} becomes insignificant,

$\therefore v$ will approach 15 m/s.

Velocity will approach a maximum speed of 15m/s and held constant at 15m/s

(iii) acceleration = 0.037 cm/s^2

(iv) s = 59.67 m

11(i) Equation of tangent at P: $y = -x + 2$

gradient of $OP \times$ gradient at P = 1×-1

$= -1$

Since gradient of $OP \times$ gradient at P = -1, OP is normal to curve at P.

(ii) 5 : 11

12(ii) $V = \frac{8}{3}x^2\sqrt{1-x}$

(iii) stationary point, $x = \frac{4}{5}$

Use 1st or 2nd derivative test to prove that it is a maximum point.

(iv) When $x = \frac{4}{5}$,

Max $V = 0.763$ cm^3 , therefore, V will never exceed 0.8cm^3

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2016

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **30 August 2016**

Secondary 4 Express / 5 Normal (Academic) **2 hours 30 minutes**

Additional Materials: Answer Paper

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The total number of marks for this paper is 100.

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where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities $\quad \sin^2 A + \cos^2 A = 1$

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC, $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Differentiate $5xe^{2x}$ with respect to x. Hence evaluate $\int_0^1 3xe^{2x}\,dx$, giving answer correct to 2 decimal places. [5]

2 **(i)** Given that $y = \dfrac{\sin 2x}{1+\cos 2x}$, show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{k}{1+\cos 2x}$ and state the value of k. [4]

(ii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{4(1+\cos 2x)}\,dx$. [3]

3 A curve has the equation $y = px - x\ln x$ for $x > 0$ and p is a constant. Find, in terms of p,

(i) the x-coordinate of the point at which the curve crosses the x-axis, [2]

(ii) the value of x, for which the curve has a turning point, [3]

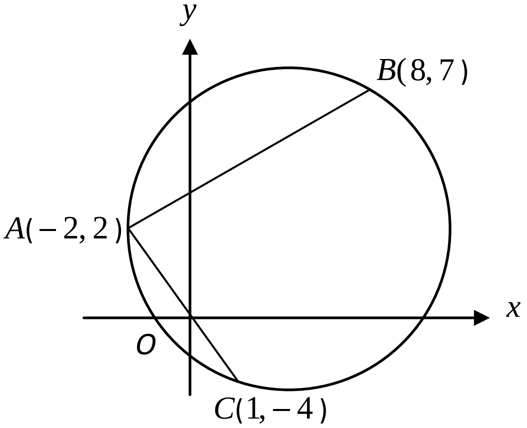
(iii) the coordinates of the turning point and the nature of this point. [3]

4 A curve is such that $\dfrac{dy}{dx} = \dfrac{x^2 - 3}{x^2}$.

(i) Given that the curve passes through the point $P(3, 5)$, find the equation of the curve. [3]

(ii) Find the equation of the tangent at P and determine if this tangent cuts the curve again. [5]

5 In the diagram below, $A(-2, 2)$, $B(8, 7)$ and $C(1, -4)$ are points on a circle.



(i) Find the gradient of *AB* and of *AC*. [2]

(ii) Show that*BC* is a diameter of the circle and hence find the centre of the circle. [4]

(iii) Find the equation of the circle. [2]

6 **(a)** Express $\dfrac{8\sqrt{2}+\sqrt{80}-\sqrt{98}}{\sqrt{18}+2\sqrt{45}-4\sqrt{5}}$ in the form $a+b\sqrt{c}$. [4]

(b) Without using calculators, express the value of $\dfrac{4\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{3}\right)-\sin\left(\frac{\pi}{6}\right)}$ in theform $a\sqrt{3}+b$, where *a* and *b* are integers. [4]

7 **(a)** Express $\dfrac{2x^2 + x + 3}{x^3 + 3x}$ in partial fractions. [4]

(b) A polynomial $\mathrm{P}(x)$ of degree three is exactly divisible by $x^2 - 2$.

Given also that $4\mathrm{P}(-1) = \mathrm{P}(2)$, show that x is a factor of $\mathrm{P}(x)$. [4]

8 The roots of the quadratic equation $2x^2 - 4x + 3 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [2]

(ii) Show that the value of $\alpha^3 + \beta^3$ is -1. [2]

(iii) Find a quadratic equation whose roots are $\dfrac{\alpha}{\beta^2} + 1$ and $\dfrac{\beta}{\alpha^2} + 1$. [5]

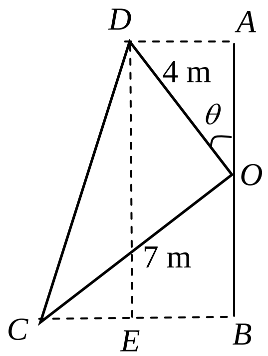
9 **(a)** Find the middle term in the expansion of $\left(x^2 - \dfrac{1}{3x^3}\right)^{10}$. [3]

(b) Write down the first three terms in the expansion, in ascending powers of *x* of $\left(1 - \dfrac{x}{2}\right)^n$, where *a* is a constant and *n* is a positive integer greater than 6. [2]

The first three terms in the expansion, in ascending powers of *x*, of $(2 + ax)\left(1 - \dfrac{x}{2}\right)^n$ are $2 - 6x + 7x^2$.

Find the value of *a* and of *n*. [5]

10 In the diagram, OD = 4 m, angle DOC = angle DAO = angle CBO = 90°, and OC = 7 m. Angle DOA = θ and varies between 0° and 90° . The point E is on the line CB such that DE is parallel to AB.



(i) Show that $AB = 7\sin\theta + 4\cos\theta$. [2]

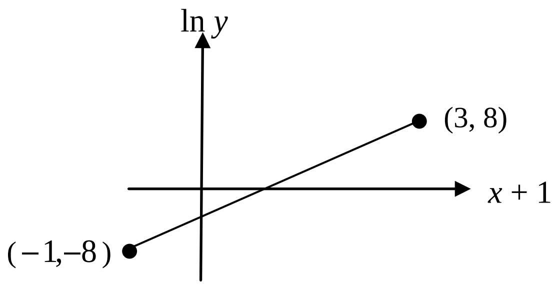
(ii) Express AB in the form $R\sin(\theta+\alpha)$, where R is positive and α is acute. Hence find the value of θ for AB = 7.5 m. [4]

(iii) State which line in the diagram has a length of R and which angle in the diagram has a value of α. [2]

(iv) Show that the area of triangle CDE is $\dfrac{65\sin 2(\theta+\alpha)}{4}$. [3]

(v) Find the maximum value of the area of triangle CDE as θ varies and state the corresponding value of θ. [3]

11 **(a)** The diagram shows a part of a straight line graph obtained by plotting $\ln y$ against $x+1$, together with coordinates of two points on the line. Express *y* in terms of *x*. [4]



(b) At time *t* minutes, the temperature of a liquid, which is left to cool, exceeds room temperature by $T°C$. The table shows the temperature difference at given times. It is known that one value of *T* has been recorded incorrectly.

Time, *t*(min)	5	10	15	20	25
Temperature difference, $T°C$	14.7	8.1	6.5	2.4	1.3

The variables *T* and *t* are related by the equation $T = ke^{at}$, where *k* and *a* are constants.

(i) Plot $\ln T$ against *t* for the given data and draw a straight line graph. [4]

(ii) Use your graph to

(a) identify the abnormal reading and estimate the correct value of *T*, [2]

(b) estimate the value of *k* and of *a*. [3]

(c) explain why the temperature of the liquid will never reach room temperature. [2]

<u>Answer Keys</u>

1 (i) $5(1+2x)e^{2x}$ (ii) 6.26

2 (i) $k = 2$ (ii) $\frac{1}{8}$

3 (i) $x = e^{p}$ (ii) $x = e^{p-1}$ (iii) $\left(e^{p-1}, e^{p-1}\right)$, max

4 (i) $y = x + \frac{3}{x} + 1$ (ii) $y = \frac{2}{3}x + 3$, No

5 (i) $\frac{1}{2}$, -2 (ii) $\left(\frac{9}{2}, \frac{3}{2}\right)$ (iii) $\left(x - \frac{9}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{85}{2}$

6 (a) $17 - 5\sqrt{10}$ (b) $2\sqrt{3} + 6$

7 (a) $\frac{2x^2 + x + 3}{x^3 + 3x} = \frac{1}{x} + \frac{x+1}{x^2+3}$

8 (i) 1 (iii) $x^2 - \frac{14}{9}x + \frac{11}{9} = 0$

9 (a) $-\frac{28}{27x^5}$ (b) $1 - \frac{n}{2}x + \frac{n(n-1)}{8}x^2 + \ldots\ldots$, $n = 7$, $a = 1$

10 (ii) $AB = \sqrt{65}\sin(\theta + 29.7°)$ or $AB = 8.06\sin(\theta + 29.7°)$

(iii) *CD* has a length of *R*, $\angle DCO = \alpha$

(v) $16\frac{1}{4}\,\text{m}^2$, $\theta = 15.3°$

11 (a) $y = e^{4x}$

(b) (i) $\ln y = at + \ln k$

(iia) abnormal reading is 6.5, correct reading is 4.5

(iib) $a \approx -0.12$, $k \approx 27.1$

(c) $T = 0$ at room temperature and $\ln T$ will become undefined. Hence the temperature of the liquid.