YUYING SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal (Academic)

NAME			
CLASS		**REG. NO**	

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **25 August 2016**

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Begin each question on a new page.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
Total	**80**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

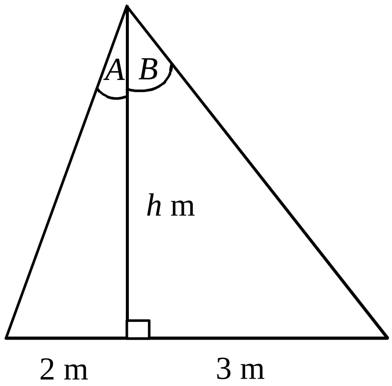
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer **all** the questions.

1 The perimeter of a rectangle has a constant value of 40 cm. One side, of length x cm, is increasing at a rate 0.5 cms^{-1}. Find the rate at which the area is increasing at the instant when $x = 3$. [4]

2



The diagram above shows a triangle of height h m. The angles A and B are such that $A + B = \dfrac{\pi}{4}$ radians. By using the expansion of tan $(A + B)$, or otherwise, find the value of h. [5]

3 The function f is defined, for $0 \le x \le 3\pi$, by $f(x) = 5\cos Ax + B$, where A and B are constants.

(a) Given that the period of f(x) is 4π, state the value of A. [1]

(b) Given that the least value of f(x) is –2, state the value of B. [1]

(c) Sketch the graph of $y = f(x)$ for $0 \le x \le 3\pi$. [2]

(d) Find the range of values of p such that $y = p$ will intersect $y = f(x)$ at exactly 2 points. [1]

4 **(i)** Find $\dfrac{d}{dx}\left(x^2 \ln x - x\right)$. [2]

(ii) Hence find $\int x \ln x \; dx$. [3]

5 Given that the roots of $7x^2 - 10x + 8 = 0$ are $\dfrac{2}{\alpha}$ and $\dfrac{2}{\beta}$, find a quadratic equation with integer coefficients whose roots are α and β. [6]

[Turn over

6 A curve has the equation $y = 8x + \frac{1}{2x^2}$.

(i) Find the coordinates of the turning point on the curve. [4]

(ii) Justify whether this turning point is a minimum or a maximum point. [2]

7 **(i)** Write down and simplify, in terms of x and n, the term containing x^3 in the expansion of $\left(1 - \frac{x}{n}\right)^n$ by the binomial theorem. [3]

(ii) If this term equals $\frac{7}{8}$ when $x = -2$, and n is a positive integer, calculate the value of n. [3]

8 **(i)** Express $2\sin 2\theta(\operatorname{cosec}\theta - \cot\theta)$ as a quadratic expression in cos θ. [3]

(ii) Use your answer to part **(i)** to find, for $0 \le \theta \le 2\pi$, the **exact** solutions of the equation

$$2\sin 2\theta(\operatorname{cosec}\theta - \cot\theta) + 3 = 0.$$ [3]

(iii) Explain why one of the solutions in part **(ii)** is not possible. [1]

9 A positive whole number has two digits which can be expressed as $10x + y$ where $x > y$. When the two digits are reversed, a new number is formed. The difference between the squares of the two numbers is 1584. The sum of the two numbers is 44 times the difference between the digits of the original number. Find the two numbers. [5]

10 Variables x and y are related by the equation $y = px + \frac{q}{x}$, where p and q are constants. By plotting values of xy against x^2, a straight line graph that passes through the points (1, 9), (5, 1) and (4, r) is obtained. Calculate the values of p, q and r. [7]

11 **Without using a calculator,** find

(a) the values of x such that $(8^x)^x = 4(32^x)$, [4]

(b) the values of rational numbers a and b such that $\frac{a + b\sqrt{5}}{2 + 3\sqrt{5}} = \frac{2 + 3\sqrt{5}}{3 + \sqrt{5}}$. [4]

12 The point $A(6, 5)$ lies on the circle $x^2 + y^2 - 4x + ky + 13 = 0$, where k is a constant.

(a) Find

(i) the value of k, [2]

(ii) the centre and radius of the circle. [3]

(b) The line $y = 2x + h$, where h is a constant, is a tangent to the circle. Find the possible values of h, giving your answers in the simplest surd form. [3]

13 A particle moves in a straight line so that, at time t seconds after leaving a fixed point O, its velocity v ms^{-1} is given by $v = \frac{27}{(2t+1)^2} - 3$. Find

(a) the value of t for which the particle is at instantaneous rest. [2]

(b) the initial acceleration of the particle. [2]

(c) the total distance travelled by the particle in the first three seconds. [4]

~~~ End of Paper ~~~
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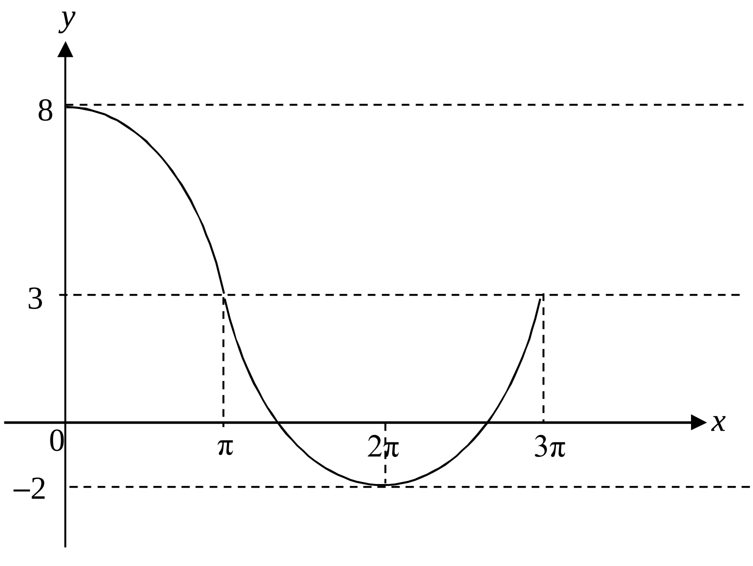
Marking Scheme
Sec 4E5N-A. Maths P1-SA2-2016

1	$A = x(20 - x) = 20x - x^2$ $\frac{dA}{dx} = 20 - 2x$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt} = (20 - 2x)(0.5) = 7\ \text{cm}^2\text{s}^{-1}$ when $x = 3$
2	$\tan A = \frac{2}{h}$, $\tan B = \frac{3}{h}$ $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\tan\left(\frac{\pi}{4}\right) = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \frac{2}{h}\left(\frac{3}{h}\right)}$ $1 - \frac{2}{h}\left(\frac{3}{h}\right) = \frac{2}{h} + \frac{3}{h}$ $1 - \frac{6}{h^2} = \frac{5}{h}$ $h^2 - 5h - 6 = 0$ $(h - 6)(h + 1) = 0$ $h = 6$ or $h = -1$ (rej)
3 (a)	$\frac{1}{2}$
3 (b)	3
3 (c)	
3 (d)	$-2 < p \leq 3$

4 (i)	$\frac{d}{dx}\left(x^2 \ln x - x\right) = x^2\left(\frac{1}{x}\right) + 2x\ln x - 1 = x + 2x\ln x - 1$
4 (ii)	$\int x + 2x\ln x \quad -1 \;\, dx = x^2 \ln x - x + c$ $\int x \;\, dx + \int 2x\ln x \;\, dx - \int 1 \;\, dx = x^2 \ln x - x + c$ $\int 2x\ln x \;\, dx = x^2 \ln x - x - \frac{x^2}{2} + x + c$ $\int x\ln x \;\, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$
5	$7x^2 - 10x + 8 = 0$ $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{10}{7}$ and $\frac{2}{\alpha} \times \frac{2}{\beta} = \frac{8}{7}$ $\frac{2(\alpha + \beta)}{\alpha\beta} = \frac{10}{7}$ $\frac{2(\alpha + \beta)}{\frac{7}{2}} = \frac{10}{7}$ $\alpha + \beta = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$ $\therefore x^2 - \frac{5}{2}x + \frac{7}{2} = 0$ $2x^2 - 5x + 7 = 0$
6 (i)	$y = 8x + \frac{1}{2x^2}$ $\frac{dy}{dx} = 8 - \frac{1}{x^3}$ $8 - \frac{1}{x^3} = 0$ $x^3 = \frac{1}{8}$ $x = \frac{1}{2}$ $\therefore \left(\frac{1}{2}, 6\right)$
6 (ii)	$\frac{d^2y}{dx^2} = \frac{3}{x^4} > 0$ when $x = \frac{1}{2}$ $\therefore \left(\frac{1}{2}, 6\right)$ is a min. pt.

[Turn over

7(i)	$T_{r+1} = \binom{n}{r} 1^{n-r} \left(-\frac{x}{n}\right)^r = \binom{n}{r}\left(-\frac{1}{n}\right)^r x^r$ When $r = 3$, $T_4 = \binom{n}{3}\left(-\frac{1}{n}\right)^3 x^3 = -\frac{(n-1)(n-2)x^3}{6n^2}$
7(ii)	$-\frac{(n-1)(n-2)}{6n^2}(-2)^3 = \frac{7}{8}$ $32(n-1)(n-2) = 21n^2$ $32n^2 - 96n + 64 - 21n^2 = 0$ $11n^2 - 96n + 64 = 0$ $(11n-8)(n-8) = 0$ $n = 8$ or $n = \frac{8}{11}$ (rej)
8 (i)	$2\sin 2\theta(\cos ec\theta - \cot\theta)$ $= 2(2\sin\theta\cos\theta)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$ $= 4\sin\theta\cos\theta\left(\frac{1-\cos\theta}{\sin\theta}\right)$ $= 4\cos\theta - 4\cos^2\theta$
8 (ii)	$2\sin 2\theta(\cos ec\theta - \cot\theta) + 3 = 0$ $4\cos\theta - 4\cos^2\theta + 3 = 0$ $4\cos^2\theta - 4\cos\theta - 3 = 0$ $(2\cos\theta + 1)(2\cos\theta - 3) = 0$ $\cos\theta = -\frac{1}{2}$ or $\cos\theta = \frac{3}{2}$ $\alpha = \frac{\pi}{3}$ (no sol.) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$
8 (iii)	There's no solution for $\cos\theta = \frac{3}{2}$ because max. value of $\cos\theta$ is 1.

9	Let the positive number by $10x + y$. $(10x+y)^2-(10y+x)^2=1584$ $99x^2-99y^2=1584$ $x^2-y^2=16 \quad \rightarrow (1)$ $(10x+y)+(10y+x)=44(x-y)$ $y=\frac{3}{5}x \quad \rightarrow (2)$ $x^2-\frac{9}{25}x^2=16$ $x^2=25$ $x=5$ or $x=-5$ (rej.) $\therefore y=3$ The two numbers are 53 and 35.
10	$y=px+\frac{q}{x}$ $xy=px^2+q$ $at(1,9),\ 9=p+q---(1)$ $at(5,1),\ 1=5p+q---(2)$ $at(4,r),\ r=4p+q---(3)$ $(2)-(1): 4p=-8$ $\therefore p=-2$ $q=9+2=11$ $r=-8+11=3$
11 (a)	$(8^x)^x=4(32^x)$ $2^{3x^2}=2^{2+5x}$ $3x^2-5x-2=0$ $(3x+1)(x-2)=0$ $x=-\frac{1}{3}$ or $x=2$
11 (b)	$\frac{a+b\sqrt{5}}{2+3\sqrt{5}}=\frac{2+3\sqrt{5}}{3+\sqrt{5}}$ $3a+5b+(a+3b)\sqrt{5}=4+45+12\sqrt{5}$ $3a+5b=49 \rightarrow (1)$ $a+3b=12 \rightarrow (2)$ Solve (1) & (2) simultaneously: $a=\frac{87}{4},\ b=-\frac{13}{4}$

[Turn over

12 (a)(i)	at $A(6,5)$, $6^2+5^2-4(6)+k(5)+13=0$ $5k=-50$ $k=-10$
12 (a)(ii)	$x^2+y^2-4x-10y+13=0$ $(x-2)^2+(y-5)^2=-13+4+25$ $(x-2)^2+(y-5)^2=16$ $centre=(2,5)$ $radius=\sqrt{16}=4units$
12 (b)	$x^2+y^2-4x-10y+13=0 \quad \rightarrow (1)$ $y=2x+h \quad \rightarrow (2)$ Sub (2) into (1), $x^2+(2x+h)^2-4x-10(2x+h)+13=0$ $5x^2+(4h-24)x+(h^2-10h+13)=0$ $b^2-4ac=0$, $(4h-24)^2-4(5)(h^2-10h+13)=0$ $h^2-2h-79=0$ $h=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-79)}}{2(1)}$ $=1+4\sqrt{5}$ or $1-4\sqrt{5}$
13 (a)	$\dfrac{27}{(2t+1)^2}-3=0$ $(2t+1)^2=9$ $2t+1=3$ or $2t+1=-3$ $t=1$ or $t=-2$ (rej)
13 (b)	$a=-\dfrac{108}{(2t+1)^3}$ When t = 0, $a=-108\text{ms}^{-2}$
13 (c)	$s=-\dfrac{27}{2(2t+1)}-3t+c$ When t = 0, s = 0, $c=\dfrac{27}{2}$ $\therefore s=-\dfrac{27}{2(2t+1)}-3t+\dfrac{27}{2}$ When t = 1, s = 6 When t = 3, $s=\dfrac{18}{7}$ Dist travelled = $6+6-\dfrac{18}{7}=\dfrac{66}{7}=9\dfrac{3}{7}$ m

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NAME			
CLASS		**REG. NO**	

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **26 August 2016**

2 hours 30 minutes

Additional Materials: Answer Paper

Setters: Mr Tai Kay Seng
Mr Tan Boon Kah

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Begin each question on a new page.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Total	/ **100**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae of Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

Answer **all** the questions.

1 The equation of a curve is given by $y = x \sin^2 x$, for $0 < x < \frac{\pi}{2}$.

(i) Find an expression for $\frac{dy}{dx}$. [3]

(ii) Calculate the **exact** gradient of the normal to the curve at the point where $x = \frac{\pi}{4}$. [2]

(iii) Explain why the value of y is always increasing for $0 < x < \frac{\pi}{2}$. [2]

2 It is given that $\sin A = -\frac{5}{13}$ and $\cos B = \frac{7}{25}$. If angles A and B are both **reflex** angles in **different** quadrants, find, without the use of a calculator, the exact value of

(i) $\cos(B - A)$, [3]

(ii) $\cos\frac{B}{2}$. [3]

3 **(i)** Express $\frac{3x^2 + 4x - 12}{x^2 - 4}$ in the form of $A + \frac{B}{x+2} + \frac{C}{x-2}$, where A, B and C are constants to be determined. [4]

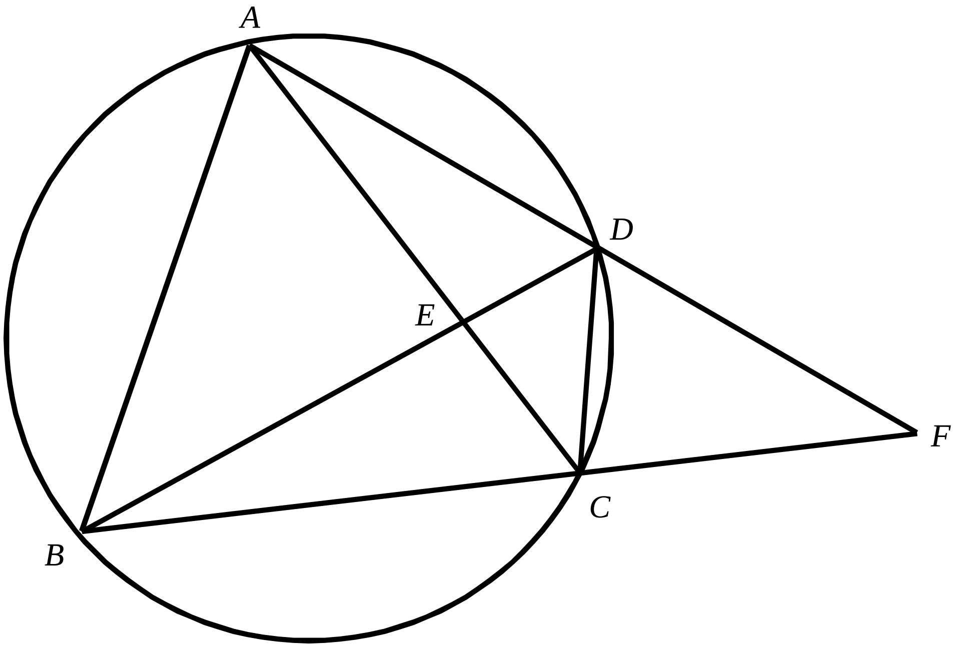
(ii) Hence, find the value of $\int_3^4 \frac{3x^2 + 4x - 12}{x^2 - 4}\,dx$, giving your answer correct to three significant figures. [4]

4 **(i)** The gradient function of a curve is given by $\frac{dy}{dx} = Ae^{1-x} + 5x$, where A is a real constant. The gradient of the tangent at the point $(0, -7e)$ is $9e$. Find the value of A and hence, find the equation of the curve. [4]

(ii) Hence, by using the value of A found in **(i)**, evaluate $\int_2^3 y\,dx$. [4]

[Turn over

5 In the diagram, *ABCDA* is a circle, and *D* and *C* are points on *AF* and *BF* respectively. *E* is the point of intersection of lines *AC* and *BD*.



Prove that

(i) angle *BDF* = angle *ACF*, [2]

(ii) triangle *BDF* is similar to triangle *ACF*, [3]

(iii) $(FD)^2 - (FC)^2 = (FC)(CB) - (FD)(DA)$. [3]

6 **(a)** Solve $\frac{3}{e^y} = 4 - e^y$. [3]

(b) **(i)** Find the value of x for which $\log_3(8-x) + \log_3 x = 2\log_9 15$. [3]

(ii) Given that $\log_x a = 3$ and $\log_x b = 4$, find the value of $\log_{ab} x$. [3]

7 The polynomial $3x^3 + ax^2 + bx + 8$, where a and b are constants, has a factor of $3x - 2$ but leaves a remainder of 16 when divided by $x - 2$.

(i) Find the value of a and of b. [6]

(ii) Using the values of a and b found in part **(i)**, find the remainder when $3x^3 + ax^2 + bx + 8$ is divided by $x^2 + 2x - 1$. [3]

8 **(a)** Find the value(s) of k for which the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$. [4]

(b) **(i)** Sketch the graph of $y = 5 - |2x + 3|$ for $-5 \le x \le 3$, showing clearly the coordinates of the points where the graph meets the coordinate axes and the coordinates of the vertex. [3]

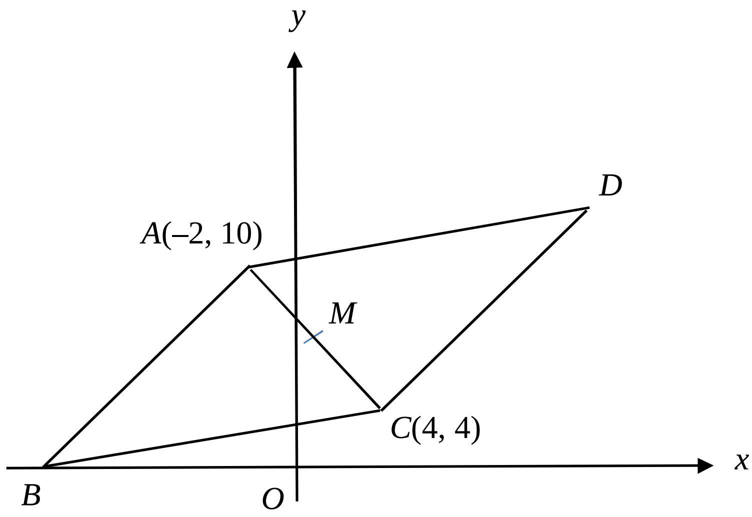
(ii) On the same axes in **(i)**, a line $y = mx + c$ is drawn. In each of the following cases, by justifying your answer, state the number of solutions of the equation $5 - |2x + 3| = mx + c$ when

(a) $m = -2$ and $c < 2$, [2]

(b) $m = \frac{1}{2}$ and $c = 2$. [2]

[Turn over

9



The diagram shows a **rhombus** *ABCD* where *A*(–2, 10) and *C*(4, 4) are opposite corners. The mid-point of *AC* is *M* and *B* lies on the *x*-axis.

(a) Find

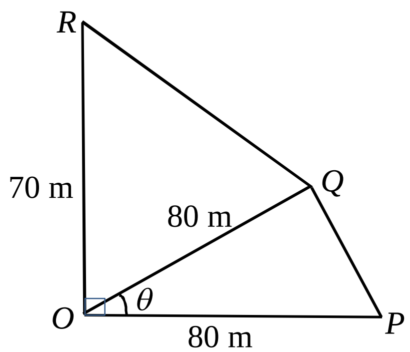
(i) the coordinates of *M* and of *B*, [3]

(ii) the equation of *BD*, [3]

(iii) the area of *ABCD*. [2]

(b) Justify if angle *ABC* is a right angle. [3]

10



The figure shows two triangular plots of land *OPQ* and *OQR*. It is given that $OP = OQ = 80$ m, $OR = 70$ m, angle *POR* is a right angle and angle $POQ = \theta$.

(i) Show that the total area of the two plots of land, *A* m^2 is given by

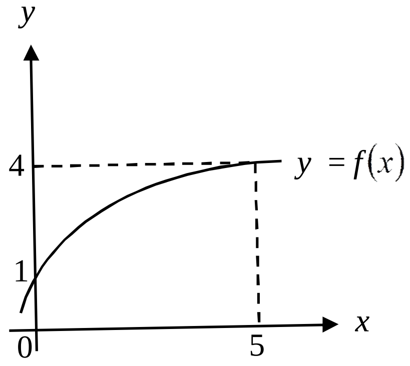
$$A = 3200\sin\theta + 2800\cos\theta.$$ [2]

(ii) Express *A* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^0 < \theta < 90^0$. [4]

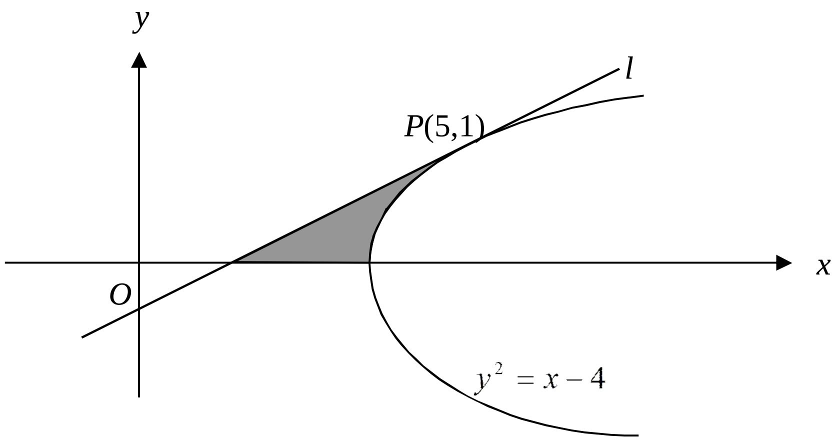
(iii) Find the greatest possible value of *A* and the value of θ at which this occurs. [3]

(iv) Find the value of θ when $A = 4000$ m^2. [2]

11



(a) The diagram above shows part of the curve $y = f(x)$. The curve cuts the y-axis at (0, 1). Explain why $\int_0^5 f(x)dx > 12.5$. [2]



(b) In the above diagram, the line l is the tangent to the curve $y^2 = x - 4$ at the point $P(5,1)$. Q is the point where the line l cuts the x-axis.

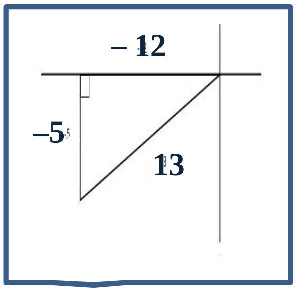
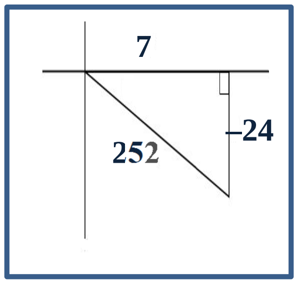
(i) Find the equation of the tangent l and the coordinates of Q. [5]

(ii) Calculate the area of the shaded region enclosed by the curve, the tangent l and the x-axis. [5]

~~~ End Of Paper ~~~

SOLUTION TO 2016 PRELIM 4EXP/5NA A. MATHS P2

Qn	Solution	Marks	Remarks
1i	$y = x\sin^2 x$ $\frac{dy}{dx}$ $= (1)\sin^2 x + (x)(2\sin x)(\cos x)$ $= \sin^2 x + x\sin 2x$	**M2** **A1**	
1ii	When $x = \frac{\pi}{4}$, Gradient of tangent $= \sin^2\left(\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right)\sin 2\left(\frac{\pi}{4}\right)$ $= \frac{1}{2} + \left(\frac{\pi}{4}\right)$ $= \left(\frac{2+\pi}{4}\right)$ Therefore, gradient of normal $= \left(-\frac{4}{2+\pi}\right)$	**M1** **A1**	
1iii	Since $0 < x < \left(\frac{\pi}{2}\right)$ (acute angle) $\Rightarrow \sin x > 0$ $\Rightarrow \sin^2 x > 0$ $Also, \sin 2x > 0$ $\Rightarrow x\sin 2x > 0$ $Therefore, \frac{dy}{dx} > 0$ Therefore, y is increasing for $0 < x < \left(\frac{\pi}{2}\right)$.	**M1** **B1**	

2i	A is in the 3rd quadrant, B is in the 4th quadrant.   $\cos(B-A)$ $=\cos B\cos A+\sin B\sin A$ $=\left(\frac{7}{25}\right)\left(-\frac{12}{13}\right)+\left(-\frac{24}{25}\right)\left(-\frac{5}{13}\right)$ $=\frac{36}{325}$	**M2** **A1**	
2ii	Using : $2\cos^2\left(\frac{B}{2}\right)-1=\cos B$ $\cos\left(\frac{B}{2}\right)=\pm\sqrt{\frac{\cos(B)+1}{2}}$ $=\pm\sqrt{\frac{\frac{7}{25}+1}{2}}$ $=\pm\frac{4}{5}$ If $270^{\circ}<B<360^{\circ}$, then $135^{\circ}<\frac{B}{2}<180^{\circ}$, So $\frac{B}{2}$ is in the 2nd quadrant where only the sine ratio is positive. Therefore, $\cos\left(\frac{B}{2}\right)=-\frac{4}{5}$	**M1** **M1** **A1**	

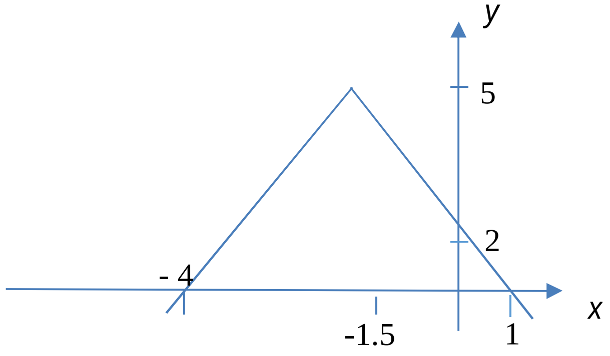
3i	Long Division : $x^2-4\,\overline{)\,3x^2+4x-12}$ quotient 3 $-(3x^2-12)$ $4x$ So, $\frac{3x^2+4x-12}{x^2-4} \equiv 3+\frac{4x}{(x+2)(x-2)}$ $\equiv 3+\frac{B}{x+2}+\frac{C}{x-2}$ Consider $4x = B(x-2) + C(x+2)$ Sub. $x = 2$, $8 = 4C$ so $C = 2$. Sub. $x = -2$, $-8 = -4B$ so $B = 2$. Therefore, the partial fractions are $3+\frac{2}{x+2}+\frac{2}{x-2}$ where $A = 3$, $B = 2$, $C = 2$.	**M1** **M1** **A1** **A1**	
3ii	$\int_3^4 \frac{3x^2+4x-12}{x^2-4}dx$ $=\int_3^4 3+\frac{2}{x+2}+\frac{2}{x-2}dx$ $=[3x+2\ln(x+2)+2\ln(x-2)]_3^4$ $=[12+2\ln 6+2\ln 2]-[9+2\ln 5+2\ln 1]$ $=4.75(3sf)$	**M2** **M1** **A1**	

4i	$\frac{dy}{dx} = Ae^{1-x} + 5x$ $When\ x = 0, \frac{dy}{dx} = 9e,$ $\Rightarrow 9e = Ae^{1-0} + 5(0)$ $\Rightarrow 9e = Ae$ $\Rightarrow A = 9$ $y \quad = \int 9e^{1-x} + 5x\ dx$ $\quad = -9e^{1-x} + \frac{5x^2}{2} + C$ $When\ x = 0, y = -7e,$ $\Rightarrow -7e = -9e^{1-0} + (0) + C$ $\Rightarrow 2e = C$ Thus, equation of curve is $y = -9e^{1-x} + \frac{5x^2}{2} + 2e$	**M1** **A1** **M1** **A1**	
4ii	$\int_2^3 -9e^{1-x} + \frac{5x^2}{2} + 2e\ dx$ $= [9e^{1-x} + \frac{5x^3}{6} + 2ex]_2^3$ $= [9e^{-2} + \frac{5(3)^3}{6} + 6e] - [9e^{-1} + \frac{5(2)^3}{6} + 4e]$ $= 19.177...$ $= 19.2(3sf)$	**M2** **M1** **A1**	**-1 for each wrong integral**

5i	Let $\angle FAC = x = \angle DBF$ ($\angle$s in same seg.) Let $\angle ADE = y = \angle BCE$ ($\angle$s in same seg.) $\Rightarrow \angle BDF = 180^\circ - y = \angle ACF$ ($\angle$s on a str line)	**M1** **A1**	
5ii	$\angle DBF = \angle CAF = x$ (A) $\angle BDF = \angle ACF = 180^\circ - y$ (A) $\Rightarrow$By AA similarity, ΔBDF is similar to ΔACF.	**M1** **M1** **A1**	
5iii	$\dfrac{FD}{FC} = \dfrac{FB}{FA}$ $\Rightarrow (FD)(FA) = (FC)(FB)$ $\Rightarrow (FD)(FD + DA) = (FC)(FC + CB)$ $\Rightarrow (FD)^2 + (FD)(DA) = (FC)^2 + (FC)(CB)$ $\Rightarrow (FD)^2 - (FC)^2 = (FC)(CB) - (FD)(DA)$	**M1** **M1** **A1**	

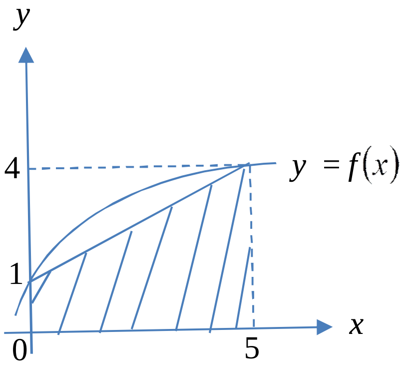
6(a)	$\frac{3}{e^y}=4-e^y$ $3=4e^y-e^{2y}$ $let \quad u=e^y$ $3=4u-u^2$ $u^2-4u+3=0$ $(u-3)(u-1)=0$ $u=1 \quad or \quad 3$ $y=0 \quad or \quad 1.10$	**M1** **M1** **A1**	
6(b)(i)	$\log_3(8-x)+\log_3 x=2\log_9 15$ $\log_3 x(8-x)=2\left(\frac{\log_3 15}{\log_3 9}\right)$ $\log_3 x(8-x)=\log_3 15$ $x^2-8x+15=0$ $(x-3)(x-5)=0$ $x=3 \quad or \quad 5$	**M1** **M1** **A1**	
6(b)(ii)	$\log_x a=3 \Rightarrow a=x^3$ $\log_x b=4 \Rightarrow b=x^4$ $ab=x^7$ $\lg ab=\lg x^7$ $\lg ab=7\lg x$ $\frac{\lg x}{\lg ab}=\frac{1}{7}$ $\log_{ab} x=\frac{1}{7}$	**M1** **M1** **A1**	

7(i)	$3\left(\frac{2}{3}\right)^3+a\left(\frac{2}{3}\right)^2+b\left(\frac{2}{3}\right)+8=0$ $\frac{8}{9}+\frac{4}{9}a+\frac{2}{3}b+8=0$ $8+4a+6b+72=0$ $2a+3b=-40\rightarrow(1)$ $3(2)^3+a(2)^2+b(2)+8=16$ $24+4a+2b+8=16$ $2a+b=-8\rightarrow(2)$ $(1)-(2)$ $b=-16$ $Sub \quad b=-16 \quad \text{int}\, o(2)$ $a=4$	**M1** **M1** **M1** **M1** **A1** **A1**	
7(ii)	Using long division, Quotient $= 3x-2$ Remainder $= -9x+6$	**M2** **A1**	

8(a)	$y = kx + 6$ --------(1) $2x^2 - xy = 3$ ------(2) Sub (1) into (2) $2x^2 - x(kx+6) = 3$ $2x^2 - kx^2 - 6x - 3 = 0$ $(2-k)x^2 - 6x - 3 = 0$ $(-6)^2 - 4(2-k)(-3) = 0$ $36 + 24 - 12k = 0$ $k = 5$	**M1** **M1** **M1** **A1**	
8(b)(i)	 	**A3**	1 mark – for the coordinates of vertex. 1 mark – for the points on the axes. 1 mark for the correct graph.
8(b)(ii)(a) **8(b)(ii)(b)**	$y = -2x + c$ and c < 2 This line is parallel to the line on the right of the vertex whose gradient is –2. It will intersect the line on the left of the vertex at one point. Hence only 1 solution. $y = \frac{1}{2}x + 2$ The gradient of the line on the left is $\frac{5-0}{-\frac{3}{2}+4} = 2$ Thus, this line is less steep as compared to the line on the left and will intersect both arms on at one point each. Hence, there will be 2 solutions.	**M1** **A1** **M1** **A1**	

9(a)(i)	$M=\left(\frac{-2+4}{2},\frac{10+4}{2}\right)=(1,7)$ Let $B(x,0)$ Length BA = Length BC $(x+2)^2+(0-10)^2=(x-4)^2+(0-4)^2$ $x^2+4x+104=x^2-8x+32$ $12x=-72$ $x=-6$ B (-6, 0)	**A1** **M1** **A1**	
9(a)(ii)	Eqn of BD = Eqn of BM Gradient of BM = $\frac{0-7}{-6-1}=1$ $y = x + \text{C}$ Sub (-6, 0) into eqn C = 6 Eqn of BD : $y = x + 6$	 **M1** **M1** **A1**	
9(a) (iii)	Area of $ABCD$ = 2 x Area of ΔABC $=2\times\frac{1}{2}\begin{vmatrix}-2 & -6 & 4 & -2\\10 & 0 & 4 & 10\end{vmatrix}$ $=(0-24+40)-(-60+0-8)$ $=84 \quad units^2$	 **M1** **A1**	
9(b)	$\frac{1}{2}(AB)(BC)\sin\angle ABC=42$ $\frac{1}{2}(\sqrt{116})(\sqrt{116})\sin\angle ABC=42$ $\angle ABC=\sin^{-1}\left(\frac{42\times 2}{116}\right)=46.4^0$ Therefore angle ABC is not a right angle.	**M1** **M1** **A1**	Alternatively, $m_{AB}=\frac{10-0}{-2+6}=\frac{5}{2}$ $m_{BC}=\frac{4-0}{4+6}=\frac{2}{5}$ $m_{AB}\times m_{BC}=1\neq -1.$ So angle ABC is not a right angle.

10(i)	$A=\frac{1}{2}(80)(80)\sin\theta+\frac{1}{2}(70)(80)\sin(90^0-\theta)$ $=3200\sin\theta+2800\cos\theta$	**M1** **A1**	
10(ii)	$A=\sqrt{3200^2+2800^2}\cos\left(\theta-\tan^{-1}\left(\frac{3200}{2800}\right)\right)$ $=4252.0583\cos(\theta-48.814^0)$ $=4250\cos(\theta-48.8^0)$	**M2** **M1** **A1**	
10(iii)	$\cos(\theta-48.8^0)=1$ $(\theta-48.8^0)=0$ $\theta=48.8^0$ $A=4250$	**M1** **A1** **A1**	
10(iv)	$4252.0583\cos(\theta-48.814^0)=4000$ $\theta=\cos^{-1}\left(\frac{4000}{4252.0583}\right)+48.814^0$ $=68.6^0$	**M1** **A1**	

11(a)	 Area of the shaded trapezium $=\frac{1}{2}(1+4)(5)=12.5 \quad units^2$ Since the area under the curve is greater than the area of the trapezium, hence $\int_0^5 f(x)dx > 12.5$.	 **M1** **A1**	
11(b)(i)	$y=(x-4)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}(x-4)^{-\frac{1}{2}}$ Sub $x = 5$ $\frac{dy}{dx}=\frac{1}{2}(5-4)^{-\frac{1}{2}}=\frac{1}{2}$ $\frac{y-1}{x-5}=\frac{1}{2}$ $y=\frac{1}{2}x-\frac{3}{2}$ Sub $y = 0$ into eqn $x = 3$ Q (3, 0)	 **M1** **M1** **M1** **M1** **A1**	

11(b)(ii)	Sub y = 0 into eqn of curve $0=(x-4)^{\frac{1}{2}}$ $x=4$ Shaded area $=\frac{1}{2}(5-3)(1)-\int_4^5 (x-4)^{\frac{1}{2}}\,dx$ $=1-\left[\frac{2}{3}(x-4)\right]_4^5$ $=1-\frac{2}{3}$ $=\frac{1}{3}\quad units^2$	**M1** **M2** **M1** **A1**	