Name: Register Number: Class:

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2017
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **12 September 2017, Tuesday**

Additional Materials : Writing Papers (8 sheets) **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Ms Ting Shi Yun

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions.

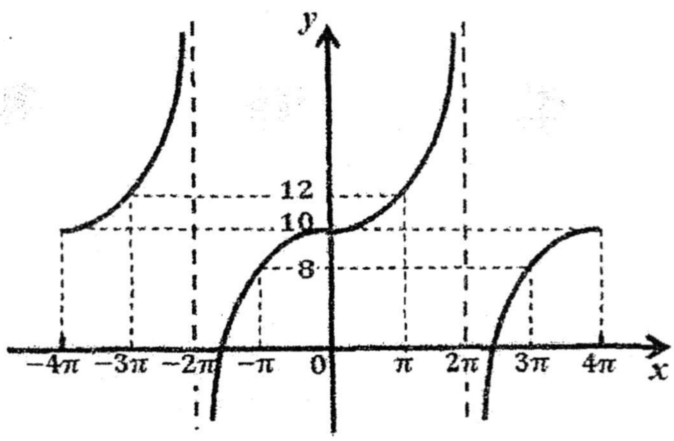
1 (a) Find the range of values of p which satisfy the inequality $px^2 + 8x + p > 6$. [4]

(b) Show that the line $y + qx = q$ will intersect the curve $y = (q+1)x^2 + qx - 1$ at two distinct points for all real values of q, where $q \neq -1$. [3]

2 A right circular cone has a vertical height of $(2\sqrt{3} - \sqrt{2})$ cm and slanted height of l cm. The volume of the cone is $(\sqrt{48} + \sqrt{18})\pi$ cm^3. **Without using a calculator,** show that l^2 can be expressed as $a - b\sqrt{6}$, where a and b are integers. [5]

3 (a) State the values between which the principal value of $\tan^{-1} x$ must lie. [1]

(b)



The figure shows part of the graph $y = a\tan bx + c$.

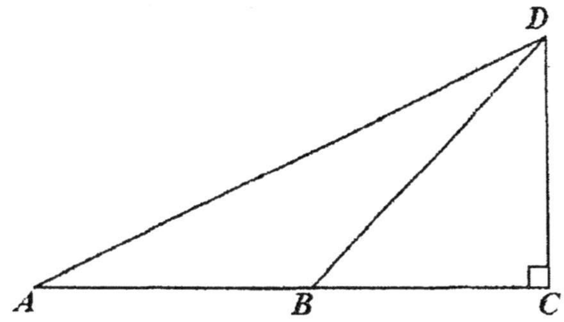
Find the value of each of the constants a, b and c. [3]

4 (i) On the same axes, sketch the curves $y = \sqrt{288x}$ and $y = 3x^3$ for $x > 0$. [2]

(ii) The tangent to the curve $y = 3x^3$ at the point A is parallel to the line passing through **the two** points of intersection of the curves drawn in **(i)**. Find the x–coordinate of A. [4]

5 The diagram shows a right-angled ΔACD such that $\cos \angle ADC = \frac{5}{13}$.

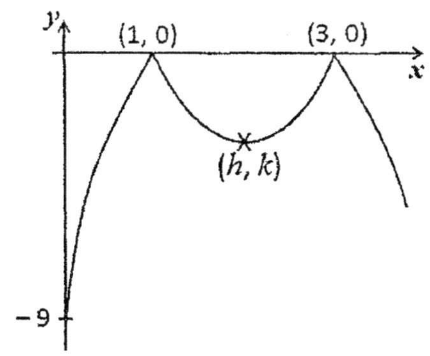
B is a point on the line AC such that $\tan \angle ABD = -1$.



Without finding the values of any angles, show that $\sin \angle ADB = \frac{7\sqrt{2}}{26}$. [4]

6 The diagram shows part of the curve $y = -|a(x-h)^2 + k|$, where $a > 0$.

The curve touches the x–axis at (1, 0) and (3, 0) and has a minimum point at (h, k). The curve also cuts the y–axis at -9.

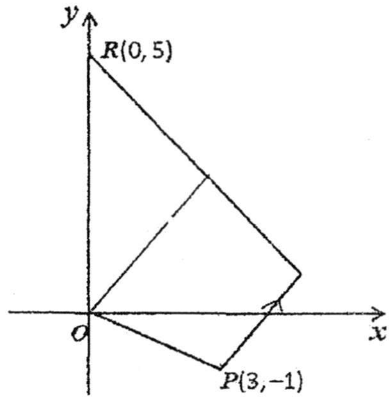


(i) Explain why $h = 2$. [1]

(ii) Determine the value of a and of k. [3]

(iii) State the set of values of m for which the line $y = m$ intersects the curve at four distinct points. [1]

7 The equations of the line PQ and QR are $y = 3x - 10$ and $4y + 3x = 20$ respectively. The coordinates of P and R are $(3, -1)$ and $(0, 5)$ respectively.



(i) Find the coordinates of Q. [2]

(ii) Name the quadrilateral $OPQR$.
Justify your answers with appropriate workings. [3]

(iii) Given that T is a point on PR such that $OPQT$ is a rhombus, find the coordinates of T. [2]

(iv) Find the ratio of the area of ΔOTP to the area of ΔOTR. [2]

8 (a) Calculate the minimum gradient of $y = 2x^3 - 9x^2 - 1$. [4]

(b) Given that $f(x) = e^{-x}(x^2 - 3x + 1)$,

(i) Find the range of values of x for which $f(x)$ is an increasing function. [4]

(ii) Hence, state the coordinates and nature of all the stationary points of $f(x)$. [3]

9 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(x^2+4\right)\right]$. [1]

(ii) Express $\frac{4x+24}{(3x-2)(x^2+4)}$ in partial fractions. [3]

(iii) Hence, find $\int \frac{x+6}{(3x-2)(x^2+4)}\mathrm{d}x$. [3]

10 At 8 a.m., Ship A is 100 km due North of Ship B.

Ship A is sailing due South at 20 km/h.

Ship B is sailing at a bearing of 120° at 10 km/h.

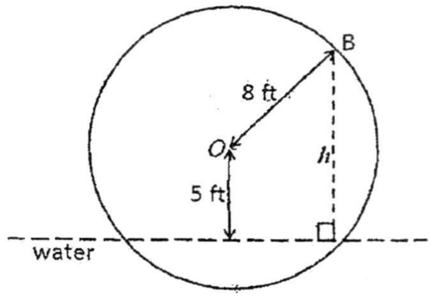
(i) Show that the distance between the two ships after t hours is given by $AB = 10\sqrt{3t^2 - 30t + 100}$. [3]

(ii) At what time is Ship A closest to Ship B? [3]

11 **(i)** Prove $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right) = 4\cos 2\theta$. [4]

(ii) Hence, solve $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right) = 4\sin 2\theta + 2\sqrt{2}$ for $0° \le \theta \le 360°$. [5]

12



A waterwheel rotates 5 revolutions anticlockwise in 1 minute. Tom starts a stopwatch when the bucket B is at its highest height above water level. The radius of the waterwheel is 8 ft and its centre is 5 ft above the water level.

The height of bucket B above water level is given by $h = a\cos bt + c$, where t is the time, in seconds, since Tom started the stopwatch.

(i) Determine the value of each of the constant a, b and c. [3]

(ii) For how long is $h < 0$? [3]

(iii) Explain what does the answer in (ii) mean. [1]

~~~End of paper~~~

Do check your work. All the best!
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2017 Prelim 2 AM P1 Solutions

1a) $px^2+8x+p>6$

$px^2+8x+p-6>0$

$\therefore p>0$ and $(8)^2-4(p)(p-6)<0$

$-4p^2+24p+64<0$

$p^2-6p-16>0$

$(p+2)(p-8)>0$

$\therefore p<-2$ or $p>8$

$\therefore p>8$

1b) $(q+1)x^2+qx-1=-qx+q$

$(q+1)x^2+2qx-1-q=0$

Method 1

Discriminant $=(2q)^2-4(q+1)(-1-q)$

$=(2q)^2+4(q+1)^2$

Since $(2q)^2\geq 0$ and $4(q+1)^2>0$,

$\therefore$ Discriminant >0 (Shown)

Method 2

Discriminant $=(2q)^2-4(q+1)(-1-q)$

$=8q^2+8q+4$

$=8\left(q+\frac{1}{2}\right)^2+4-8\left(\frac{1}{2}\right)^2$

$=8\left(q+\frac{1}{2}\right)^2+2>0$ (Shown)

2) $\frac{1}{3}\pi r^2(2\sqrt{3}-\sqrt{2})=(\sqrt{48}+\sqrt{18})\pi$

$r^2=\dfrac{3(4\sqrt{3}+3\sqrt{2})}{2\sqrt{3}-\sqrt{2}}$

$=\dfrac{3(4\sqrt{3}+3\sqrt{2})}{2\sqrt{3}-\sqrt{2}}\times\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$

$=\dfrac{72+18\sqrt{6}+18+12\sqrt{6}}{12-2}$

$=\dfrac{90+30\sqrt{6}}{10}$

$=9+3\sqrt{6}$

$l^2=9+3\sqrt{6}+(2\sqrt{3}-\sqrt{2})^2$

$=23-\sqrt{6}$

(Do not need to specify $a=23$, $b=1$. Read On!)

3a) $-90^\circ<\tan^{-1}x<90^\circ$

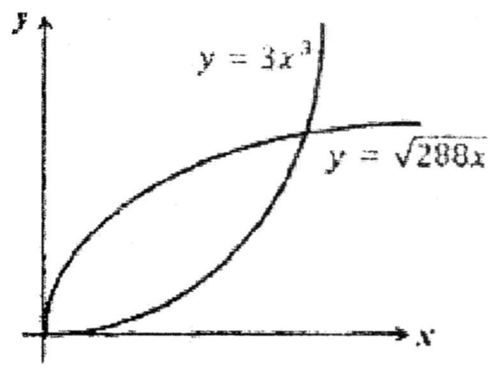
or $-\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2}$

3b) Period $=4\pi$, $b=\frac{1}{4}$

$c=10$

$a=2$

4a) **Must label graphs!**



4b) $\sqrt{288x}=3x^3$

$288x=9x^6$

$9x(32-x^5)=0$

$x=0$ or $x=2$

$y=0$ or $y=24$

$\therefore$ gradient $=\frac{24}{2}=12$

$\frac{dy}{dx}=9x^2$

$9x^2=12$

$x^2=\frac{4}{3}$

$x=\frac{2\sqrt{3}}{3}$ (exact only because it's x-coord!)

5) Method [illegible] (cont'd)

$\sin \angle A$[illegible]

$= \sin(\angle ADC - \angle BDC)$

$= \sin \angle ADC \cos \angle BDC - \cos \angle ADC \sin \angle BDC$

$= \left(\frac{12}{13}\right)\left(\frac{5}{5\sqrt{2}}\right) - \left(\frac{5}{13}\right)\left(\frac{5}{5\sqrt{2}}\right)$

$= \frac{12}{13\sqrt{2}} - \frac{5}{13\sqrt{2}}$

$= \frac{7}{13\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{7\sqrt{2}}{26}$ (Shown)

Method 2 *(The ratio happen to be length)*

$\tan \angle ADB = -\tan \angle DBC = 1$

$\sin \angle ABD = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{\sin \angle ADB}{7} = \frac{\sin \angle ABD}{13}$

$\sin \angle ADB = \frac{7\sqrt{2}}{26}$ (Shown)

Method 3 *(The ratio happen to be length)*

Area of $\Delta ADB = \frac{1}{2} \times 7 \times 5 = 17.5$

$\therefore 17.5 = \frac{1}{2}(13)(5\sqrt{2})\sin \angle ADB$

$\sin \angle ADB = \frac{7\sqrt{2}}{26}$ (Shown)

6i) The curve is symmetrical.

6ii) $(x-1)(x-3) = 0$

$x^2 - 4x + 3 = 0$

$3(x^2 - 4x + 3) = 0$

$3[(x-2)^2 + 3 - 2^2] = 0$

$3(x-2)^2 - 3 = 0$

$a = 3$

$k = -3$

6iii) $-3 < m < 0$

7i) $4(3x - 10) + 3x = 20$

$15x = 60$

$x = 4$

$y = 2$

$Q(4, 2)$

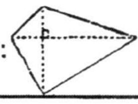
7ii) OR [illegible]

$OQ = \sqrt{[illegible] + (5-2)^2} = 5$

$OP = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

$PQ = \sqrt{(4-3)^2 + (2+1)^2} = \sqrt{10}$

Since $OR = OQ$ and $OP = PQ$, $OPQR$ is a kite.

> Do **not** accept if students **only** find the grad of diagonals and conclude that they are perpendicular.
> Counter example:  which is not a kite!

> Students **can** prove diagonals are perpendicular **AND** one pair of adjacent sides is equal in length.

> Students **can** also show that the midpoint of *OQ* lies on the diagonal *PR*.

7iii) Midpoint of $OQ = (2, 1)$

Let T be (x, y)

$\left(\frac{x+3}{2}, \frac{y-1}{2}\right) = (2, 1)$

$x = 1$ and $y = 3$

$T(1, 3)$

7iv) $TR = \sqrt{1^2 + 2^2} = \sqrt{5}$

$PT = \sqrt{2^2 + 4^2} = 2\sqrt{5}$

area of ΔOTP : area of ΔOTR

$2\sqrt{5} : \sqrt{5}$

$2 : 1$

> Students **can** also use area shoelace method.
> Area of OTP = 5
> Area of OTR = 2.5

8a) **Method 1**

$\frac{dy}{dx} = 6x^2 - 18x \rightarrow \frac{d^2y}{dx^2} = 12x - 18$

When $12x - 18 = 0$, $x = 1.5$

Gradient $= 6(1.5)^2 - 18(1.5) = -13.5$

$\frac{d^3y}{dx^3} = 12 > 0 \quad \therefore$ minimum gradient

Method 2

$\frac{dy}{dx} = 6x^2 - 18x$

$= 6(x - 1.5)^2 - 6(1.5)^2$

$= 6(x - 1.5)^2 - 13.5$

$\therefore$ min gradient $= -13.5$

10ii) $\frac{dAB}{dt} = \frac{10\left(\frac{1}{2}\right)(6t-30)}{\sqrt{3t^2-30t+100}}$

When $\frac{dAB}{dt} = 0,\quad 6t = 30,\quad \therefore t = 5$

Method 1

t	4.9	5	5.1
$\frac{dAB}{dt}$	−	0	+
Sketch	\	—	/

$\therefore AB$ is a minimum.

Method 2

$$\frac{d^2AB}{dt^2} = \frac{30}{\sqrt{3t^2-30t+100}} + \frac{(6t-30)(75-15t)}{\left(3t^2-30t+100\right)^3}$$

When $t = 5$, $\frac{d^2AB}{dt^2} = 6 \quad > 0$

$\therefore AB$ is a minimum.

Time $= 0800 + 5\text{h} = 1300$ or 1pm

11i) $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right)$

$= (2\sin\theta\cos\theta)^2\left(\frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\right)$

$= 4\sin^2\theta\cos^2\theta\left(\frac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta}\right)$

$= 4\,(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= 4\,(\cos 2\theta)(1)$

$= 4\cos 2\theta$

11ii) $4\cos 2\theta - 4\sin 2\theta = 2\sqrt{2}$

$4\cos 2\theta - 4\sin 2\theta = 2\sqrt{2}$

$\cos 2\theta - \sin 2\theta = \frac{\sqrt{2}}{2}$

$\sqrt{2}\cos(2\theta + 45°) = \frac{\sqrt{2}}{2}$

$\cos(2\theta + 45°) = 0.5$

Basic angle $= 60°$

$2\theta + 45° = 60, 300, 420, 660$

$\theta = 7.5°, 127.5°, 187.5°, 307.5°$

12i) Period = 12 seconds

$\frac{2\pi}{b} = 12 \rightarrow b = \frac{\pi}{6}$, $a = 8$, $c = 5$

12ii) $8\cos\frac{\pi}{6}t + 5 = 0$

$\cos\frac{\pi}{6}t = -\frac{5}{8}$

Basic angle = 0.89566

$\frac{\pi}{6}t = 2.2459,\quad 4.03726$

$t = 4.28935,\quad 7.7106$

Duration $= 7.7106 - 4.28935$

$= 3.42125$

=3.42 seconds

12iii) It is the duration of the bucket when it is in the water.
(accept "below water level", "submerged under water")

8bi) $f'(x) = -e^{-x}(x^2 - 3x + 1) + e^{-x}(2x - 3)$

$= e^{-x}(-x^2 + 5x - 4)$

For f(x) to be increasing, $f'(x) > 0$.

Since $e^{-x} > 0$, $-x^2 + 5x - 4 > 0$

$x^2 - 5x + 4 < 0$

$(x-1)(x-4) < 0$

$1 < x < 4$

8bii) When $f'(x) = 0$, $x = 1$ and $x = 4$

$\left(1, -\frac{1}{e}\right)$ min point and $\left(4, \frac{5}{e^4}\right)$ max point

(Exact value of coords only!)

9i) $\frac{d}{dx}\left[\ln(x^2+4)\right] = \frac{2x}{x^2+4}$

9ii) $\frac{4x+24}{(3x-2)(x^2+4)} = \frac{A}{3x-2} + \frac{Bx+C}{x^2+4}$

$4x + 24 = A(x^2+4) + (Bx+C)(3x-2)$

When $x = \frac{2}{3}$, $\frac{80}{3} = \frac{40}{9}A$ → $A = 6$

Comparing coeff of x^2: $0 = A + 3B$ → $B = -2$

Comparing coeff of x^0: $24 = 4A - 2C$ → $C = 0$

$\frac{4x+24}{(3x-2)(x^2+4)} = \frac{6}{3x-2} - \frac{2x}{x^2+4}$

9iii) **Method 1**

$\int \frac{x+6}{(3x-2)(x^2+4)}\,dx$

$= \frac{1}{4}\int \frac{4x+24}{(3x-2)(x^2+4)}\,dx$

$= \frac{1}{4}\int \frac{6}{3x-2} - \frac{2x}{x^2+4}\,dx$

$= \frac{1}{4}\left[\frac{6\ln(3x-2)}{3} - \ln(x^2+4)\right] + c$

$= \frac{1}{2}\ln(3x-2) - \frac{1}{4}\ln(x^2+4) + c$

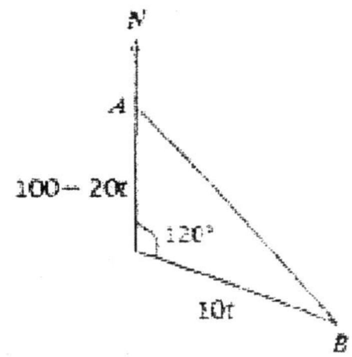
9iii) **Method 2**

$4\int \frac{x+6}{(3x-2)(x^2+4)}\,dx = \int \frac{6}{3x-2} - \frac{2x}{x^2+4}\,dx$

$\int \frac{x+6}{(3x-2)(x^2+4)}\,dx = \frac{1}{4}\left[\frac{6\ln(3x-2)}{3} - \ln(x^2+4)\right] + c$

$= \frac{1}{2}\ln(3x-2) - \frac{1}{4}\ln(x^2+4) + c$

10i) **Method 1**



$AB^2 = (100-20t)^2 + (10t)^2 - 2(100-20t)(10t)\cos 120°$

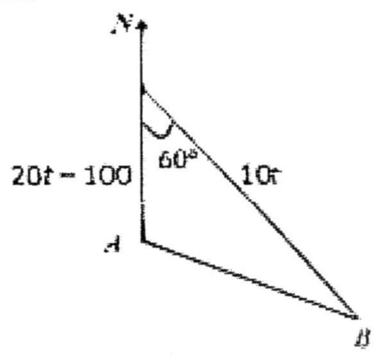
$AB^2 = 10000 - 4000t + 400t^2 + 100t^2 + 1000t - 200t^2$

$AB^2 = 300t^2 - 3000t + 10000$

$AB^2 = 100(3t^2 - 30t + 100)$

$AB = 10\sqrt{3t^2 - 30t + 100}$ (Shown)

Method 2



$AB^2 = (20t-100)^2 + (10t)^2 - 2(20t-100)(10t)\cos 60°$

$AB^2 = 300t^2 - 3000t + 10000$

$AB = 10\sqrt{3t^2 - 30t + 100}$ (Shown)

Answer all the questions.

1 Given that $\tan A = \frac{5}{12}$, $\cos B = -\frac{4}{5}$ and that A and B are in the same quadrant, calculate without the use of calculator, the value of

(i) $\cos(A+B)$, [2]

(ii) $\cos\frac{A}{2}$. [2]

2 (i) Find the equation of a curve with $\frac{dy}{dx} = \frac{6}{(2x-1)^2}$ and which passes through the point $(1, 7)$. [2]

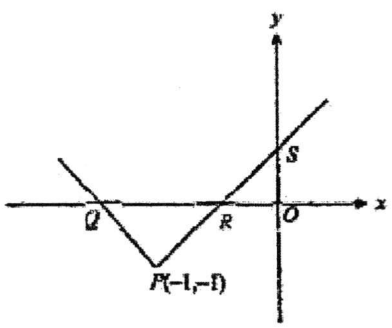
(ii) The curve for which $\frac{dy}{dx} = 2x + k$, where k is a constant, has a turning point at $(2, -9)$.

(a) Show that $k = -4$. [1]

(b) Hence, find the equation of the curve. [2]

3 Given that $\sqrt{a + b\sqrt{3}} = \frac{2\sqrt{3}}{3-\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and b. [5]

4



The diagram shows part of the graph of $y = |kx - 2| - 1$ where $P(-1, -1)$ is the minimum point of the graph.

(i) Show that the value of k is -2. [1]

(ii) Find the coordinates of the points Q, R and S. [3]

(iii) Hence, write down the range of values of x for which y is positive. [1]

5 (i) Write down the principal value, in radians as a multiple of π, of

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$, [1]

(b) $\tan^{-1}(-1)$. [1]

(ii) Solve the equation $2\sin 2x = -1$, for $0 \le x \le 2\pi$. [3]

6 (i) Find the range of values of k for which the line $y = k - 2kx$ does not intersect the curve $y = x^2 + 2$. [4]

(ii) Hence, describe the relationship between the curve $y = x^2 + 2$ and the line $y = 1 - 2x$. [1]

7 Determine the nature of each of the stationary points of the curve $y = \frac{x^2+3}{x-1}$. [6]

8 (i) Prove the identity $\frac{1+\cos A+\cos 2A}{\sin A+\sin 2A} = \cot A$. [3]

(ii) Sketch the graph of $y = 2\sin 2x - 3$ for $-180° \le x \le 180°$. [3]

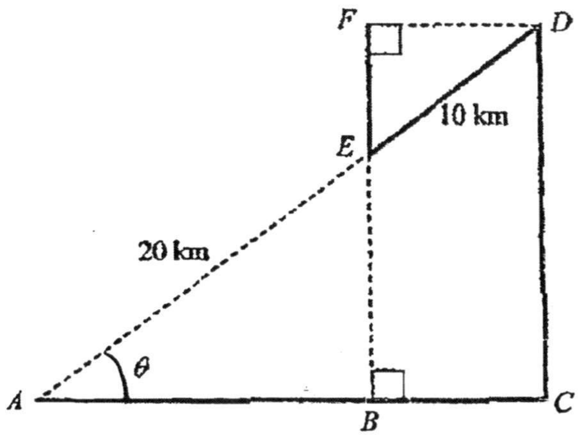
9 (i) Given that $y = 2xe^{\frac{x}{2}}$, show that $\frac{dy}{dx} = xe^{\frac{x}{2}} + 2e^{\frac{x}{2}}$. [3]

(ii) Hence, find the exact value of $\int_0^2 xe^{\frac{x}{2}}\,dx$. [3]

10 (i) Express $\frac{2x^3+5x^2+3}{2x^2+x-1}$ in partial fractions. [5]

(ii) Hence, find $\int \frac{2x^3+5x^2+3}{2x^2+x-1}\,dx$. [2]

11 The diagram shows a proposed route *ABCDEF* travelled by a group of cyclist.



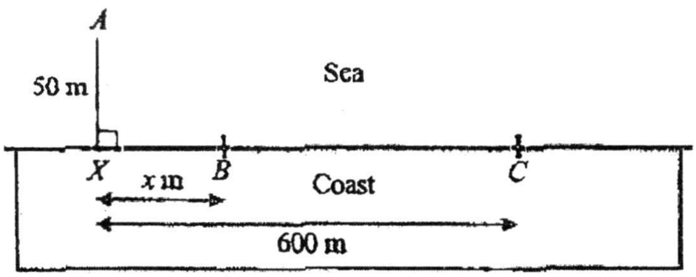
BCDF is a rectangle. *ABC* is a straight line and the line *AD* intersects *BF* at *E*.
$AE = 20$ km, $ED = 10$ km and angle *EAB* is θ, where $0° < \theta < 90°$.
The total distance of the route is S km.

(i) Show that the total distance, S, is given by $S = 10 + 30\cos\theta + 40\sin\theta$. [2]

(ii) Express $30\cos\theta + 40\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Given that the total distance of the route is 60 km, find the value of θ. [2]

12 A man is sitting on a boat at point *A*, 50 metres from a point *X* on a straight coast. He wishes to get as quickly as possible to a point *C* on the coast 600 metres from *X*. From *A*, he rows the boat at 40 m/min to point *B*, and then cycles along the coast at 50 m/min to point *C*.

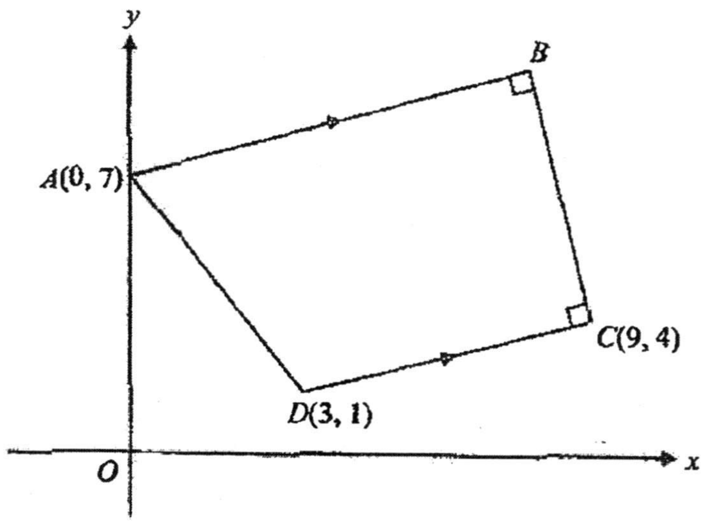


(i) Express the time taken for the man to travel from *A* to *B* and *B* to *C* in terms of *x*, and show that the total time taken, *T* min, for the man to travel from *A* to *C* is given by $T = \dfrac{\sqrt{x^2 + 2500}}{40} - \dfrac{x}{50} + 12$. [3]

(ii) Obtain an expression for $\dfrac{dT}{dx}$. [2]

(iii) Find the distance from *X* where the man should land so that he can get to *C* in the least possible time. (Note: Proof of minimum value is not required) [3]

13



ABCD is a trapezium where *AB* is parallel to *DC*, and angle *ABC* = angle *BCD* = 90°.

The coordinates of *A*, *C* and *D* are (0, 7), (9, 4) and (3, 1) respectively.

(i) Calculate the gradient of *DC*. [1]

(ii) Find the equation of the line *BC*. [2]

(iii) Show that the coordinates of *B* is (6, 10). [3]

(iv) Calculate the area of the trapezium *ABCD*. [2]

(v) If *B* is the midpoint of a line segment *DE*, find the coordinates of *E*. [2]

--------END OF PAPER--------

1 Given that $2^{3(x-1)} = 27^{2-x}$, find the value of 6^x without using a calculator. [3]

2 The expression $ax^3 + 4x^2 + bx - 1$ is exactly divisible by $2x - 1$ and has a remainder of -6 when divided by $x + 1$. Find the value of each of the constants a and of b. [4]

3 The function f is defined, for all values of x, by $f(x) = \frac{x}{x^2 + 9}$. Find the values of x for which f is a decreasing function. [4]

4 Solve the equation $x^3 - 4x^2 - 8x + 8 = 0$, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

5 Solve the equation $5\log_2 y = 4 + \log_y 2$. [5]

6 The curve $y = x^3 - 6x^2 + k$ touches the positive x-axis at point A.

(i) Find the coordinates of A. [2]

(ii) Find the value of k for and the value of $\frac{d^2y}{dx^2}$ at A. [3]

7 The mass, m grams, of a radioactive substance is modelled by an equation in the form

$$m = m_0 e^{-kt},$$

where m_0 and k are constants and t is the time in days after the mass was first recorded. The table below gives values of m and t for the days recorded from 10 to 40 days.

t days	10	20	30	40
m grams	40.3	27.0	18.1	12.2

(i) Plot ln m against t for the given data and draw straight line graph. [2]

(ii) Estimate the value of m_0 and k. [3]

(iii) Assuming that the radioactive substance decayed at a constant rate, estimate the number of grams of substance at 5 days. [2]

8 (i) Find, in ascending powers of x, the first 3 terms in the expansion of $(2-3x)^5$. [3]

The first 3 terms in the expansion of $(a+bx)(2-3x)^5$ in ascending powers of x are $32-176x+cx^2$.

(ii) Find the value of a, of b and of c. [5]

9 (i) Using $\sin 3x = \sin(2x+x)$, show that $\sin 3x$ may be expressed as $\sin x(4\cos^2 x - 1)$. [3]

(ii) Find all the values between $0°$ and $360°$ for which $3\sin 3x = 16\sin x\cos x$. [5]

10 The roots of the quadratic equation $x^2+5x+2=0$ are α and β. Find

(i) the value of $\alpha^3+\beta^3$, [5]

(ii) a quadratic equation with roots $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$. [3]

11 The equation of a curve is $y = x^3 + 6x^2 - 15x + k$, where k is a constant.

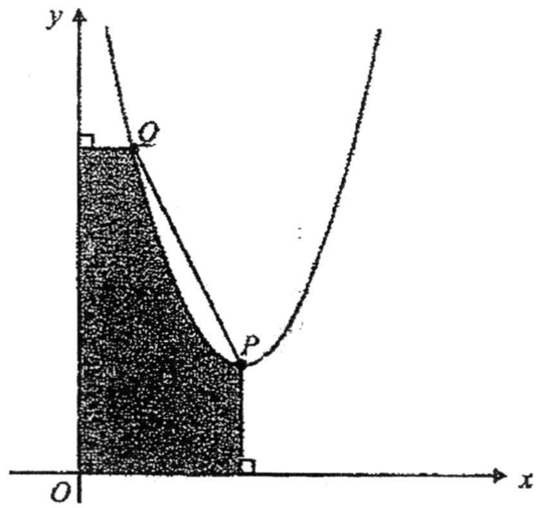
(i) Find the set of values of x for which y is a decreasing function, [4]

(ii) Find the positive value of k for which the x-axis is a tangent to the curve. [3]

The variables of x and y increase in such a way that, when $x = 2$, y is increasing at a rate of 0.5 units per second.

(iii) Find the rate of change of x, when $x = 2$. [2]

12



The diagram shows part of the graph of $y = x^2 - 6x + 11$ passing through the points P and Q. The curve has a minimum point at P.

(i) Find the coordinates of P. [3]

The gradient of the line PQ is -2.

(ii) Show that the coordinates of Q is $(1, 6)$. [4]

(iii) Showing all your working, find the total area of the shaded region. [3]

13 (i) Given that $u = 2^x$, express $2^{2x} = 2^{x+3} - 7$ as an equation in u. [3]

(ii) Hence, find the values of x for which $2^{2x} = 2^{x+3} - 7$, giving your answer, where appropriate, to 1 decimal place. [4]

(iii) Explain why the equation $2^{2x} = 2^{x+3} - k$ has no solution if $k > 16$. [3]

14 The equation of a circle, C_1, with centre A, is $x^2 + y^2 - 12x - 6y + 35 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

(ii) Show that the point $P(3, 4)$ lies on C_1. [1]

(iii) Find the equation of the tangent to C_1 at P. [3]

A second circle, C_2, has a diameter EF. The point E has coordinates $(-2, 2)$ and the equation of the tangent to C_2 at F is $4y = -3x - 48$.

(iv) Find the equation of the diameter EF and hence the coordinates of F. [4]

(v) Find the coordinates of its centre and radius of C_2. [3]

——End of Paper——

Sec 4ESA Add Mathematics
Prelim 1, 2017 (Solutions)
Paper 1

1(i) Quadrant 3

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(-\frac{5}{13}\right)\left(-\frac{3}{5}\right)$ M1

$= \frac{33}{65}$ A1

(ii) $\cos\frac{A}{2} = -\sqrt{\frac{\cos A + 1}{2}}$

$= -\sqrt{\frac{\left(-\frac{12}{13}\right)+1}{2}}$ M1

$= -\frac{1}{\sqrt{26}}$ A1

2(i) $\frac{dy}{dx} = 6(2x-1)^{-2}$

$y = \frac{6(2x-1)^{-1}}{(-1)(2)} + c$

$= \frac{-3}{2x-1} + c$ M1

Subst (1, 7) into equation,

$7 = \frac{-3}{2(1)-1} + c$

$c = 10$

Equation is $y = \frac{-3}{2x-1} + 10$ A1

(ii)(a) $\frac{dy}{dx} = 2x + k$

At $x = 2$, $\frac{dy}{dx} = 0$

$0 = 2(2) + k$

$k = -4$ B1

(b) $\frac{dy}{dx} = 2x - 4$

$y = \frac{2x^2}{2} - 4x + c$

Subst (2, −9) into equation,

$-9 = (2)^2 - 4(2) + c$

$-5 = c$

Equation is $y = x^2 - 4x - 5$ A1

3. $\sqrt{a+b\sqrt{3}} = \frac{2\sqrt{3}}{3-\sqrt{3}} \frac{\times(3+\sqrt{3})}{\times(3+\sqrt{3})}$ M1

$\sqrt{a+b\sqrt{3}} = \frac{6\sqrt{3}+2(3)}{9-3}$

$\sqrt{a+b\sqrt{3}} = \sqrt{3}+1$ M1

$a+b\sqrt{3} = (\sqrt{3}+1)^2$

$a+b\sqrt{3} = 3+2\sqrt{3}+1$ M1

$a+b\sqrt{3} = 4+2\sqrt{3}$

$a = 4,\ b = 2$ A1, A1

4(i) $y = |kx-2| - 1$

Subst (−1, −1)

$-1 = |-k-2| - 1$

$0 = |-k-2|$

$0 = -k-2$

$k = -2$ B1

(ii) $y = |-2x-2| - 1$

Let $x = 0$,

$y = |-2| - 1 = 1$ $S(0, 1)$ B1

Let $y = 0$,

$0 = |-2x-2| - 1$

$1 = |-2x-2|$

$-2x-2 = 1$ or $-2x-2 = -1$

$x = -1.5$ $x = -0.5$

$Q(-1.5, 0)$ $R(-0.5, 0)$ B1, B1

(iii) $x < -1.5$ or $x > -0.5$ B1

5(i)(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ B1

(b) $\tan^{-1}(-1) = -\frac{\pi}{4}$ B1

(ii) $2\sin 2x = -1$

$\sin 2x = \frac{-1}{2}$

(Q3, Q4)

Basic angle $= \sin^{-1}\frac{1}{2} = 0.52360$ (or $\frac{\pi}{6}$) M1

New range: $0 \le x \le 4\pi$

$Q3 = \pi + 0.52360$, $Q4 = 2\pi - 0.52360$ M1

$= 3.6652$ $= 5.7596$

$2x = 3.6652, 5.7596, 9.9484, 12.0428$

$x = 1.83, 2.88, 4.97, 6.02$ A1

6(i) $y = k - 2kx$ ---(1)

$y = x^2 + 2$ ---(2)

Subst (1) into (2)

$k - 2kx = x^2 + 2$

$x^2 + 2kx + (2 - k) = 0$ M1

$b^2 - 4ac < 0$

$(2k)^2 - 4(1)(2-k) < 0$ M1

$4k^2 + 4k - 8 < 0$

$k^2 + k - 2 < 0$ M1

$(k-1)(k+2) < 0$

$-2 < k < 1$ A1

(ii) $y = 1 - 2x$ $y = k - 2kx$

Comparing, $k = 1$

$b^2 - 4ac = k^2 + k - 2$

$= (1)^2 + (1) - 2$

$= 0$

Hence, $y = 1 - 2x$ is a tangent to the curve $y = x^2 + 2$. B1

7. $y=\dfrac{x^2+3}{x-1}$

$\dfrac{dy}{dx}=\dfrac{(x-1)(2x)-(x^2+3)(1)}{(x-1)^2}$ M1

(Quotient Rule)

$=\dfrac{x^2-2x-3}{(x-1)^2}$

Let $\dfrac{dy}{dx}=0$

$\dfrac{x^2-2x-3}{(x-1)^2}=0$ M1

$(x-3)(x+1)=0$

$x=3$ or -1 M1

$y=6$ or -2

$\dfrac{dy}{dx}=\dfrac{x^2-2x-3}{(x-1)^2}$

$\dfrac{d^2y}{dx^2}=\dfrac{(x-1)^2(2x-2)-(x^2-2x-3)\,2(x-1)}{(x-1)^4}$ (Or 1st derivative test)

$=\dfrac{(x-1)[(x-1)(2x-2)-(x^2-2x-3)\,2]}{(x-1)^4}$

$=\dfrac{2x^2-2x-2x+2-2x^2+4x+6}{(x-1)^3}$

$=\dfrac{8}{(x-1)^3}$ M1

At $x=3$,

$\dfrac{d^2y}{dx^2}=1>0 \rightarrow (3,6)$ is a minimum point A1

At $x=-1$,

$\dfrac{d^2y}{dx^2}=-1<0 \rightarrow (-1,-2)$ is a maximum point A1

8(i) $\dfrac{1+\cos A+\cos 2A}{\sin A+\sin 2A}=\cot A$

LHS $=\dfrac{1+\cos A+\cos 2A}{\sin A+\sin 2A}$

$=\dfrac{1+\cos A+(2\cos^2 A-1)}{\sin A+(2\sin A\cos A)}$ M1

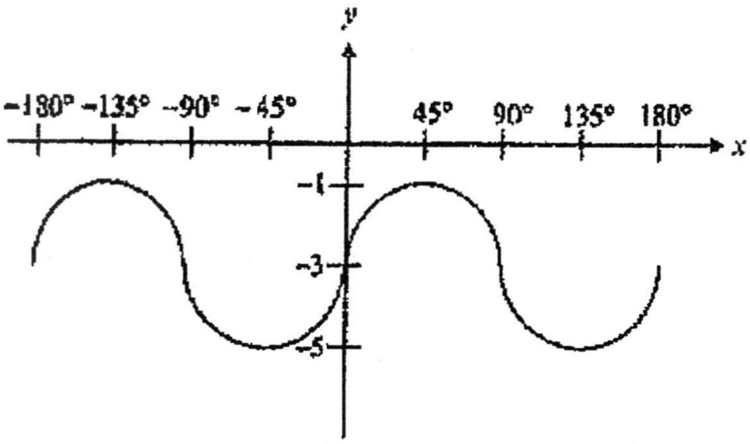
$=\dfrac{\cos A+2\cos^2 A}{\sin A+2\sin A\cos A}$

$=\dfrac{\cos A(1+2\cos A)}{\sin A(1+2\cos A)}$ M1

$=\dfrac{\cos A}{\sin A}$

$=\cot A=\text{RHS}$ A1

(ii)



B1 – intervals
B1 – shape of sine graph
B1 – position on y-axis

9(i) $y = 2xe^{\frac{x}{2}}$

Let $u = 2x$ $\quad v = e^{\frac{x}{2}}$

$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = \frac{1}{2}e^{\frac{x}{2}}$ M1

$\frac{dy}{dx} = (2x)\left(\frac{1}{2}e^{\frac{x}{2}}\right) + \left(e^{\frac{x}{2}}\right)(2)$ M1

$= xe^{\frac{x}{2}} + 2e^{\frac{x}{2}}$ A1

(ii) From (i),

$\int_0^2 xe^{\frac{x}{2}} + 2e^{\frac{x}{2}} \, dx = \left[2xe^{\frac{x}{2}}\right]_0^2$ M1

$\int_0^2 xe^{\frac{x}{2}} \, dx + \int_0^2 2e^{\frac{x}{2}} \, dx = \left[2xe^{\frac{x}{2}}\right]_0^2$

$\int_0^2 xe^{\frac{x}{2}} \, dx = \left[2xe^{\frac{x}{2}}\right]_0^2 - \int_0^2 2e^{\frac{x}{2}} \, dx$

$= \left[2xe^{\frac{x}{2}}\right]_0^2 - \left[\frac{2e^{\frac{x}{2}}}{\left(\frac{1}{2}\right)}\right]_0^2$ M1

$= \left[2(2)e^{\frac{2}{2}} - 2(0)e^{\frac{0}{2}}\right] - \left[4e^{\frac{2}{2}} - 4e^{\frac{0}{2}}\right]$

$= 4e - (4e - 4) = 4$ A1

$$\begin{array}{r}
x+2 \\
2x^2+x-1 \overline{\smash{\big)}\, 2x^3+5x^2+0x+3} \\
\underline{2x^3+x^2-x} \\
4x^2+x+3 \\
\underline{4x^2+2x-2} \\
-x+5
\end{array}$$

M1

$$\frac{-x+5}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$$ M1

$-x+5 = A(x+1) + B(2x-1)$

Let $x = -1$ M1

$-(-1)+5 = B(-2-1) \rightarrow B = -2$ A1

Let $x = 0.5$

$-0.5+5 = A(1.5) \rightarrow A = 3$ A1

$$\frac{2x^3+5x^2+3}{2x^2+x-1} = x+2+\frac{3}{2x-1}-\frac{2}{x+1}$$

$$\int \frac{2x^3+5x^2+3}{2x^2+x-1}\,dx = \int x+2+\frac{3}{2x-1}-\frac{2}{x+1}\,dx$$

$$= \frac{x^2}{2}+2x+\frac{3\ln(2x-1)}{2}-2\ln(x+1)+c$$ B2

11(i) $\sin\theta = \frac{BE}{20} \rightarrow BE = 20\sin\theta$

$\cos\theta = \frac{AB}{20} \rightarrow AB = 20\cos\theta$

$\sin\theta = \frac{EF}{10} \rightarrow EF = 10\sin\theta$

$\cos\theta = \frac{FD}{10} \rightarrow FD = 10\cos\theta$ M1

$S = AB + BC + CD + DE + EF$
$= 20\cos\theta + 10\cos\theta + (20\sin\theta + 10\sin\theta) + 10 + 10\sin\theta$
$= 10 + 30\cos\theta + 40\sin\theta$ A1

(ii) $30\cos\theta + 40\sin\theta = R\cos(\theta - \alpha)$

$30\cos\theta + 40\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

By comparison,

$30 = R\cos\alpha$ ---- (1) and $40 = R\sin\alpha$ ---- (2)

$(1)^2 + (2)^2$, $30^2 + 40^2 = R^2(\cos^2\alpha + \sin^2\alpha)$ M1

$2500 = R^2$

$R = 50$ A1

$\frac{(2)}{(1)}$, $\tan\alpha = \frac{40}{30}$ M1

$\alpha = \tan^{-1}\frac{4}{3} = 53.1301°$ A1

Thus, $30\cos\theta + 40\sin\theta = 50\cos(\theta - 53.1°)$

(iii) $10 + 30\cos\theta + 40\sin\theta = 60$
$50\cos(\theta - 53.1301°) = 50$
$\cos(\theta - 53.1301°) = 1$ M1
$\theta - 53.1301° = 0°$ (Q1, reject Q4)
$\theta = 53.1°$ A1

12(i) Distance from A to $B = \sqrt{x^2 + 50^2}$

Time taken from A to $B = \dfrac{\sqrt{x^2 + 2500}}{40}$ min — M1

Time taken from B to $C = \dfrac{600 - x}{50}$ min — M1

Total time, $T = \dfrac{\sqrt{x^2 + 2500}}{40} + \dfrac{600 - x}{50}$

$= \dfrac{\sqrt{x^2 + 2500}}{40} - \dfrac{x}{50} + 12$ min — A1

(ii) $T = \dfrac{(x^2 + 2500)^{\frac{1}{2}}}{40} - \dfrac{x}{50} + 12$

$\dfrac{dT}{dx} = \dfrac{\frac{1}{2}(x^2 + 2500)^{-\frac{1}{2}}(2x)}{40} - \dfrac{1}{50}$

$= \dfrac{x}{40\sqrt{x^2 + 2500}} - \dfrac{1}{50}$ — B2

(iii) Let $\dfrac{dT}{dx} = 0$

$\dfrac{x}{40\sqrt{x^2 + 2500}} - \dfrac{1}{50} = 0$ — M1

$\dfrac{x}{40\sqrt{x^2 + 2500}} = \dfrac{1}{50}$

$50x = 40\sqrt{x^2 + 2500}$

$2500x^2 = 1600(x^2 + 2500)$ — M1

$900x^2 = 4000000$

$x^2 = \dfrac{40000}{9}$

$x = 66.7$ m or $x = -66.7$ m (reject) — A1

13(i) Gradient of $DC = \frac{4-1}{9-3} = \frac{1}{2}$ B1

(ii) Gradient of $BC = -2$ M1

$y = -2x + c$

Subst (9, 4) into equation

$4 = -2(9) + c$

$c = 22$

Equation of BC is $y = -2x + 22$ A1

(iii) Gradient of $AB = \frac{1}{2}$

$y = \frac{1}{2}x + c$

Subst (0, 7) into equation

$7 = \frac{1}{2}(0) + c$

$c = 7$

Equation of AB is $y = \frac{1}{2}x + 7$ ---(1) M1

$y = -2x + 22$ ---(2)

Subst (1) into (2),

$\frac{1}{2}x + 7 = -2x + 22$ M1

$\frac{5}{2}x = 15$

$x = 6$

$y = 10$ $B(6, 10)$ A1

(iv) Area $= \frac{1}{2}\begin{vmatrix} 0 & 3 & 9 & 6 & 0 \\ 7 & 1 & 4 & 10 & 7 \end{vmatrix}$

$= \frac{1}{2}[0 + 12 + 90 + 42 - 21 - 9 - 24 - 0]$ M1

$= 45 \text{ units}^2$ A1

(v) $(6, 10) = \left(\frac{x+3}{2}, \frac{y+1}{2}\right)$

Comparing,

$x =$ M1, A1

4E5N AMath Prelim 1 Paper 2, 2017
Answer Scheme

Qn	Answer
1	$2^{3(x-1)} = 27^{2-x}$ $\frac{2^{3x}}{2^3} = \frac{3^6}{3^{3x}}$ $6^{3x} = 5832$ $6^x = 18$
2	$f(\frac{1}{2}) = ax^3 + 4x^2 + bx - 1 = 0$ $a(\frac{1}{2})^3 + 4(\frac{1}{2})^2 + b(\frac{1}{2}) - 1 = 0$ $a = -4b$ ------(1) $f(-1) = ax^3 + 4x^2 + bx - 1 = 0$ $a(-1)^3 + 4(-1)^2 + b(-1) - 1 = 0$ $a + b = 9$ ---------(2) Sub (1) into (2) $-4b + b = 9$ $b = -3$ $\therefore a = -12$
3	$f(x) = \frac{x}{x^2+9}$ $f'(x) = \frac{(x^2+9)(1) - (x)(2x)}{(x^2+9)^2}$ $= \frac{x^2 + 9 - 2x^2}{(x^2+9)^2}$ $= \frac{9 - x^2}{(x^2+9)^2}$ Since $(x^2+9)^2 > 0$, and the function f is a decreasing function, $9 - x^2 < 0$ $x^2 - 9 > 0$ $(x+3)(x-3) > 0$ $x < -3$ or $x > 3$

4	$x^3-4x^2-8x+8=0$ By trial and error, $(x+2)$ is a factor $(x+2)(x^2-6x+4)=0$ $x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$ $x=\dfrac{6\pm\sqrt{36-16}}{2}$ $x=2$ or $x=\dfrac{6\pm\sqrt{20}}{2}$ $x=\dfrac{6\pm2\sqrt{5}}{2}$ $x=3\pm\sqrt{5}$
5	$5\log_2 y=4+\log_y 2$ $5\log_2 y-\log_y 2-4=0$ $5\log_2 y-\dfrac{1}{\log_2 y}-4=0$ $5(\log_2 y)^2-4\log_2 y-1=0$ $(\log_2 y-1)(5\log_2 y+1)=0$ $\log_2 y=1$ *or* $\log_2 y=-\dfrac{1}{5}$ $y=2$ $\quad$ $y=0.871$
6i	Since the curve touches the x-axis, point A is a stationery point $\dfrac{dy}{dx}=0$ $y=x^3-6x^2+k$ $\dfrac{dy}{dx}=3x^2-12x$ $3x^2-12x=0$ $3x(x-4)=0$ $x=0$ *or* $x=4$ Coordinates of A is (4, 0)
6ii	At point A(4, 0) $y=x^3-6x^2+k$ $0=4^3-6(4)^2+k$ $k=32$ $\dfrac{dy}{dx}=3x^2-12x$ At $x=4$,

$$\frac{d^2y}{dx^2} = 6(4) - 12$$
$$= 12$$

7i

t	10	20	30	40
$\ln m$	3.7	3.3	2.9	2.5

Refer to graph attached behind

7ii

$$-k = \frac{-3.3 - 2.9}{20 - 30}$$
$$k = 0.04(\pm 0.01)$$

$$\ln m_0 = 4.1$$
$$m_0 = 60.3$$

7iii

$$\ln m = 3.9$$
$$m = 49.4$$

8i

$$(2-3x)^5$$
$$= 2^5 + \binom{5}{1}(2)^4(-3x) + \binom{5}{2}(2)^3(-3x)^2 + ...$$
$$= 32 - 240x + 720x^2 + ...$$

8ii

$$(a+bx)(2-3x)^5$$
$$= 32a - 240ax + 720ax^2 + 32bx - 240bx^2 + ...$$
$$= 32a + (32b - 240a)x + (720a - 240b)x^2 + ...$$

By comparing terms,

$$32a = 32$$
$$a = 1$$

$$-240a + 32b = -176$$
$$b = 2$$

$$720a - 240b = c$$
$$c = 240$$

9i

$$\sin(2x + x)$$
$$= \sin 2x \cos x + \cos 2x \sin x$$
$$= 2\sin x \cos^2 x + \cos 2x \sin x$$
$$= 2\sin x \cos^2 x + (2\cos^2 x - 1)\sin x$$
$$= \sin x(4\cos^2 x - 1) \text{ (shown)}$$

9ii	$3\sin 3x = 16\sin x\cos x$ $3\sin x(4\cos^2 x - 1) - 16\sin x\cos x = 0$ $\sin x(12\cos^2 x - 3 - 16\cos x) = 0$ $\sin x = 0$ $x = 0°, 180°$ $12\cos^2 x - 16\cos x - 3 = 0$ $(2\cos x - 3)(6\cos x + 1) = 0$ $\cos x = 1.5(\textit{reject})$ *or* $\cos x = -\frac{1}{6}$ $x = 180 - 80.41, \quad 180 + 80.41$ $x = 99.6°, 260.4°$
10i	$x^2 + 5x + 2 = 0$ Sum of roots: $\alpha + \beta = -5$ Product of roots: $\alpha\beta = 2$ $\alpha^3 + \beta^3$ $= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= (\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta]$ $= (-5)[(-5)^2 - 3(2)]$ $= -95$
10ii	Sum of roots: $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ $= \frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}$ $= \frac{-95}{(2)^2}$ $= \frac{-95}{4}$ Product of roots: $\frac{\alpha}{\beta^2} \times \frac{\beta}{\alpha^2}$ $= \frac{1}{\alpha\beta}$ $= \frac{1}{2}$

	Quadratic equation: $x^2 - \left(-\frac{95}{4}\right)x + \frac{1}{2} = 0$ $x^2 + \frac{95}{4}x + \frac{1}{2} = 0$
11i	$y = x^3 + 6x^2 - 15x + k$ $\frac{dy}{dx} = 3x^2 + 12x - 15$ $3x^2 + 12x - 15 < 0$ $x^2 + 4x - 5 < 0$ $(x+5)(x-1) < 0$ $-5 < x < 1$
11ii	$\frac{dy}{dx} = 3x^2 + 12x - 15 = 0$ $x^2 + 4x - 5 = 0$ $(x+5)(x-1) < 0$ $x = -5 \, or \, x = 1$ When $x = 1$, $0 = 1^3 + 6(1)^2 - 15(1) + k$ $k = 8$ When $x = -5$, $0 = (-5)^3 + 6(-5)^2 - 15(-5) + k$ $k = -100 (reject)$
11iii	$\frac{dy}{dx} = 3x^2 + 12x - 15$ $\frac{dy}{dx} = 3(2)^2 + 12(2) - 15$ $= 21$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $0.5 = 21 \times \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{1}{42}$
12i	$y = x^2 - 6x + 11$ $\frac{dy}{dx} = 2x - 6 = 0$ $x = 3$ $y = 3^2 - 6(3) + 11$ $y = 2$ Coordinates of P is (3, 2)

12ii	Let coordinates of Q be $(a, a^2 - 6a + 11)$. Gradient PQ = -2 $\frac{a^2 - 6a + 11 - 2}{a - 3} = -2$ $a^2 - 6a + 9 = -2a + 6$ $a^2 - 4a + 3 = 0$ $(a-1)(a-3) = 0$ $a = 1$ or $a = 3$(reject as this is the value of x for point P) y – coordinate of Q $1^2 - 6(1) + 11 = 6$ $Q(1,6)$
12iii	Total area of shaded region $= (1 \times 6) + \int_1^3 x^2 - 6x + 11 \, dx$ $= 6 + \left[\frac{x^3}{3} - 3x^2 + 11x \right]_1^3$ $= 6 + \left[(9 - 27 + 33) - (\frac{1}{3} - 3 + 11) \right]$ $= 12\frac{2}{3}$
13i	$2^{2x} = 2^{x+3} - 7$ $(2^x)^2 - 8.2^x + 7 = 0$ $u^2 - 8u + 7 = 0$
13ii	$u^2 - 8u + 7 = 0$ $(u-7)(u-1) = 0$ $u = 7$ or $u = 1$ $2^x = 7$ or $2^x = 2^0$ $x = \frac{\ln 7}{\ln 2}$ or $x = 0$ $x = 2.8$(1dp)
13iii	$u^2 - 8u + k = 0$ $(-8)^2 - 4(1)(k) < 0$ $64 - 4k < 0$ $k > 16$ Therefore, the equation has no solution if $k > 16$.
14i	$x^2 + y^2 - 12x - 6y + 35 = 0$ [illegible] $6^2 + 3^2$ Centre, $A = (6,3)$ radius $= \sqrt{10}$

14ii	$x^2+y^2-12x-6y+35$ $=3^2+4^2-12(3)-6(4)+35$ $=0$ Therefore the point P lies on C_1.
14iii	Gradient AP $=\frac{4-3}{3-6}$ $=-\frac{1}{3}$ Gradient of tangent: 3 Equation of tangent is $y=mx+c$ $3=3(4)+c$ $c=-9$ $y=3x-9$
14iv	$4y=-3x-48$ $y=\frac{-3x-48}{4}$ $Gradient=-\frac{3}{4}$ Gradient of diameter EF(tangent) $=\frac{4}{3}$ $y=\frac{4x}{3}+c$ $2=\frac{4(-2)}{3}+c$ $c=\frac{14}{3}$ $y=\frac{4x}{3}+\frac{14}{3}$ $\frac{-3x-48}{4}=\frac{4x}{3}+\frac{14}{3}$ $-9x-144=16x+56$ $x=-8$ $\therefore y=\frac{4(-8)}{3}+\frac{14}{3}$ $y=-6$ F
14v	C $=\left(\frac{-2+(-8)}{2},\frac{2+(-6)}{2}\right)$ $=(-5,-2)$

	Radius $= \sqrt{(-8+5)^2 + (-6+2)^2}$ $= 5$
	Total