Name | Reg. No | Class

OWER SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL

4E5N

ADDITIONAL MATHEMATICS **4047/01**

[80 marks]

SEMESTER ONE EXAMINATION

13 May 2019

2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers on the writing paper provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use

Brand / Model of Calculator

This question paper consists of **7** printed pages, including the cover page.

Setter: Ms Shen Sirui

Vetter: Mr Nara

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

1 **(i)** On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

2 A particle moves along the curve $y = e^{2x}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the y-coordinate of the particle at the instant when the x-coordinate of the particle is increasing at 0.01 units per second.

[4]

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where k is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of k. [5]

4 **(a)** Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of 6^x . [3]

(b) The side of an equilateral triangle is $6(\sqrt{3} - 1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a + b\sqrt{c})$ cm^2, where a, b and c are integers. [4]

5 Find the range of values of x for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]

6 A curve has the equation $y = (2x - 3)^2 - 1$.

(i) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(ii) Sketch the graph of $y = |(2x - 3)^2 - 1|$. [3]

(iii) Using your graph, state the range of values of k for which $|(2x - 3)^2 - 1| = k$ has 4 solutions. [1]

7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.

Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

8 Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. [5]

9 **(a)** Given that the first two non-zero terms in the expansion, in ascending powers of x, of $(1 + bx)(1 + ax)^6$ are 1 and $-\frac{21}{4}x^2$ and that $a > 0$, find the value of a and of b.

[5]

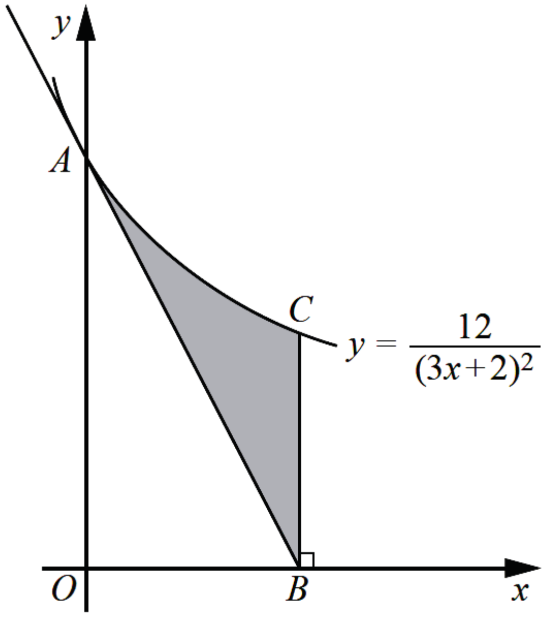
(b) Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^9$. [3]

10 The equation of a curve is $y = \frac{x^2}{2x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

11



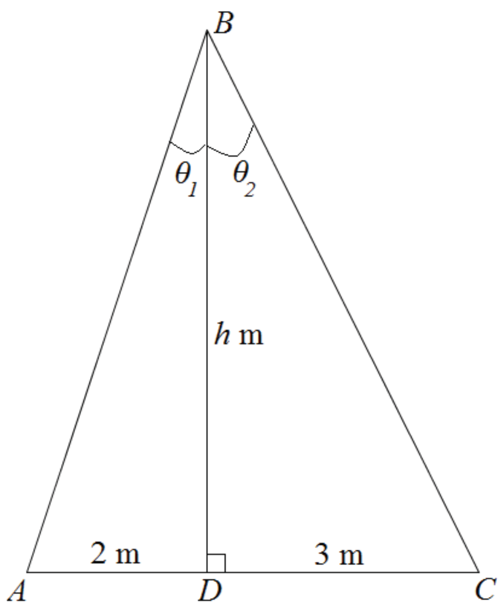
The diagram shows part of the curve $y = \frac{12}{(3x+2)^2}$ meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, [4]

(ii) the area of the shaded region. [5]

12 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of π . [1]

(b) The diagram below shows triangle *ABC* where *AD* = 2 m, *DC* = 3 m and *BD* = h m. *BD* is perpendicular to *AC* and $\theta_1 + \theta_2 = 45°$.



By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of h. [5]

13 The Ultraviolet Index describes the level of solar radiation on the earth's surface. The Ultraviolet Index, U, measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours, $0 \le t \le 20$, from the lowest value of Ultraviolet Index and q is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]

END OF PAPER

Name	Reg. No	Class

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4E/5N

ADDITIONAL MATHEMATICS

4047/02

Paper 2 [100 marks]

SEMESTER ONE EXAMINATION

May 2019

2 hours 30 minutes

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES

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Write your name, register number and class on all the work you hand in.
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Omission of essential working will result in loss of marks.
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INFORMATION FOR CANDIDATES

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The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Brand / Model of Calculator

For Examiner's Use	
Total	/ 100

This question paper consists of **15** printed pages.

Setter: Mr. Gabriel Cheow
Vetter: Mr. Narayanan

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

For Examiner's Use

1 The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

(ii) Find a quadratic equation whose roots are $\dfrac{1}{\alpha^2 + \beta}$ and $\dfrac{1}{\alpha + \beta^2}$. [4]

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2 The function $f(x) = 6x^3 + ax^2 + bx - 12$, where a and b are constants, is exactly divisible by $x + 2$ and leaves a remainder of 5 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

(ii) By showing your working clearly, factorise $f(x)$. [3]

(iii) Hence, solve the equation $6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$ [4]

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3 **(i)** Express $\dfrac{2x+16}{(x^2+4)(2x-1)}$ in partial fractions. [5]

(ii) Differentiate $\ln(x^2+4)$ with respect to x. [2]

(iii) Hence, using your results in **(i)** and **(ii)**, find $\int \dfrac{x+8}{(x^2+4)(2x-1)}\,dx$. [4]

For Examiner's Use

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4 Prove the following identities.

(a) $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$ [3]

$LHS = (\sec x - \tan x)(cosec\ x + 1)$

(b) $\dfrac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$ [3]

For Examiner's Use

For Examiner's Use

5 The lines $y=8$ and $4x+3y=30$ are tangent to a circle C at the points $(-1,8)$ and $(3,6)$ respectively.

(i) Show that the equation of C is $x^2+y^2+2x-6y-15=0$. [5]

(ii) Explain whether or not the x-axis is tangent to C. [3]

(iii) The points Q and R also lie on the circle, and the length of the chord QR is 2 units. Calculate the shortest distance from the center of C to the chord QR. [2]

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6 The table shows experimental values of two variables x and y, which are known to be connected by the equation $yx^n = A$, where n and A are constants.

x	1.0	1.5	2.0	2.5	3.0
y	22.0	13.0	8.9	6.9	5.3

(i) Plot lg y against lg x and draw a straight line graph. [3]

For Examiner's Use

For Examiner's Use

(ii) Use your graph to estimate the value of A and of n. [4]

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x which satisfies the equation $x^{n+2} = A$. [2]

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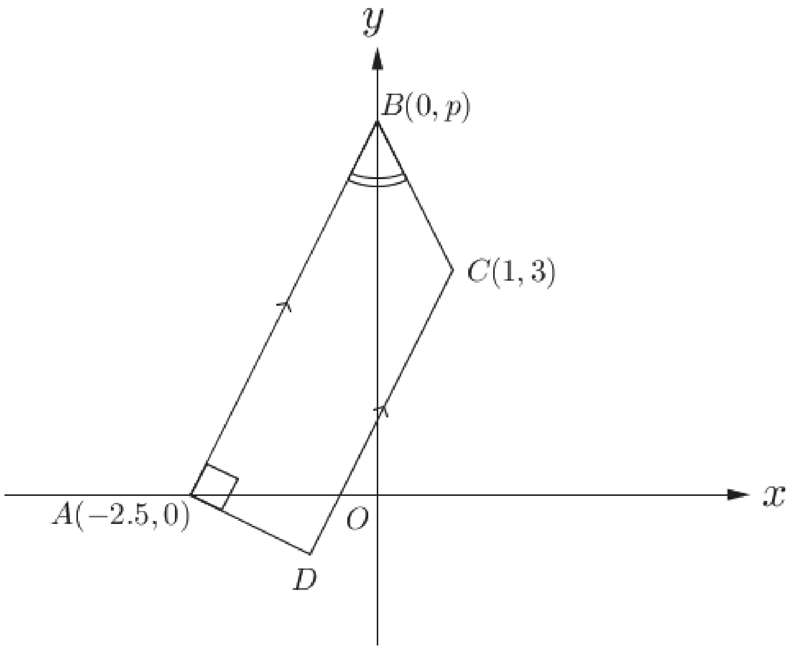
For Examiner's Use

7 The diagram shows a trapezium with vertices $A(-2.5, 0)$, $B(0, p)$, $C(1, 3)$ and D. The sides AB and DC are parallel and the angle DAB is 90°.
Angle ABO is equal to angle CBO.



(i) Express the gradients of the lines AB and CB in terms of p and hence, or otherwise, show that $p = 5$. [3]

For Examiner's Use

For Examiner's Use

(ii) Find the coordinates of *D*. [4]

(iii) Find the area of the trapezium *ABCD*. [2]

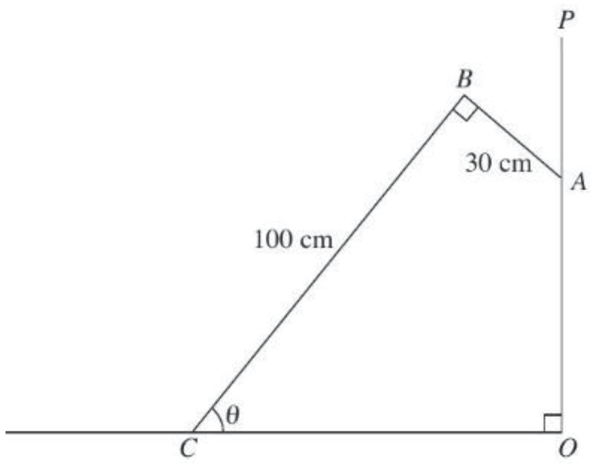
For Examiner's Use

8 **(a)** Solve the equation $3\log_x 3 = 8 - 4\log_3 x$. [5]

(b) It is given that $\log_a x = p$ and $\log_a y = q$.
Express $\log_y ax^2y^3$ in terms of p and q. [3]

For Examiner's Use

9 The figure shows a stage prop ABC used by a member of the theatre, leaning against a vertical wall OP. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let D be foot of B on OC, let E be foot of A on BD.

(ii) Express OC in terms of $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle. [3]

(iii) State the maximum value of OC and the corresponding value of θ. [2]

(iv) Find the value of θ for which $OC = 80$ cm. [3]

For Examiner's Use

10 Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians,

(i) state the period of y. [1]

Given that the maximum and minimum values of y are 3 and – 5 respectively, find

(ii) the amplitude of y, [1]

[2]

Using the values of a and b found in part **(iii)**,

(iv) sketch the graph of $y = a + b\cos 4x$ for $0 \le x \le \pi$. [3]

(v) On the same set of axes, sketch the graph of $y = |4\sin 3x|$, and hence state the number of solutions of $a + b\cos 4x = |4\sin 3x|$. [3]

For Examiner's Use

For Examiner's Use

11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by h cm and its total surface area is 312 cm^2. The volume of the cuboid is V cm^3.

(i) Express h in terms of x. [2]

(ii) Show that $V = \frac{36}{5}x(26 - x^2)$. [2]

(iii) Find the maximum volume of the cuboid as x varies, giving your answer to the nearest cm^3. [5]

– End of Paper –