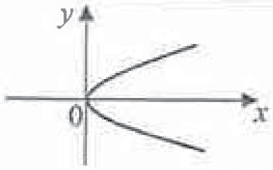
2019 PRELIMINARY EXAMINATION
SECONDARY 4E5N
AMATH PAPER 1 – MARK SCHEME

1		Let $\frac{10x^2-7x+10}{(3x-2)(x^2+2)}=\frac{A}{3x-2}+\frac{Bx+C}{x^2+2}$ $10x^2-7x+10=A(x^2+2)+(Bx+C)(3x-2)$ Sub $x=\frac{2}{3}$ to get $A=4$ Sub $x=0$ to get $C=-1$ Sub $x=1$ (or any other value) to get $B=2$ $\frac{10x^2-7x+10}{(3x-2)(x^2+2)}=\frac{4}{3x-2}+\frac{2x-1}{x^2+2}$
		TOTAL: 5m
2	(i)	
	(ii)	Equate both equations to reduce to one variable $2y^2-3y-5=0$ $(2y-5)(y+1)=0$ Solve for x and y The points of intersection are $\left(\frac{25}{12},\frac{5}{2}\right)$ and $\left(\frac{1}{3},-1\right)$
		TOTAL: 5m
3	(i)	Coordinates of two points on the straight line are $\left(1,\frac{1}{2}\right)$ and $(2,-1)$ Gradient of line $=\frac{\frac{1}{2}-(-1)}{1-2}=-\frac{3}{2}$ Equation of curve is $\frac{xy-(-1)}{\sqrt{x}-2}=-\frac{3}{2}$ $y=\frac{4-3\sqrt{x}}{2x}$
	(ii)	$y=-\frac{1}{4}$
		TOTAL: 5m

4	(i)	$\frac{\cos 2x-\cos 4x}{2\sin^2 x}=\frac{\cos 2x-(2\cos^2 2x-1)}{2\sin^2 x}$ $=\frac{(1+2\cos 2x)(1-\cos 2x)}{2\sin^2 x}$ $=\frac{(1+2\cos 2x)(1-1+2\sin^2 x)}{2\sin^2 x}$ $=1+2\cos 2x$ (shown)	
	(ii)	$1+2\cos 2x=2$ $\cos 2x=\frac{1}{2}$ $2x=60^\circ, 300^\circ, 420^\circ, 660^\circ$ $x=30^\circ, 150^\circ, 210^\circ, 330^\circ$	
			TOTAL: 6m
5	(i)	$\alpha^2\beta^2=\frac{9}{4}$ $\alpha<0<\beta \quad \Rightarrow \quad \alpha\beta<0$ Hence $\alpha\beta=-\sqrt{\frac{9}{4}}=-\frac{3}{2}$ (shown) $\alpha^2+\beta^2=\frac{37}{4} \Rightarrow (\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2=\frac{25}{4}$ $\beta<\lvert\alpha\rvert$ and $\alpha<0<\beta \Rightarrow \alpha+\beta<0$ Hence $\alpha+\beta=-\sqrt{\frac{25}{4}}=-\frac{5}{2}$	
	(ii)	SOR: $\frac{\alpha}{\alpha+\beta}+\frac{\beta}{\alpha+\beta}=\frac{\alpha+\beta}{\alpha+\beta}=1$ POR: $\frac{\alpha}{\alpha+\beta}\times\frac{\beta}{\alpha+\beta}=\frac{\alpha\beta}{(\alpha+\beta)^2}=\frac{-\frac{3}{2}}{\left(-\frac{5}{2}\right)^2}=-\frac{6}{25}$ The equation is $x^2-x-\frac{6}{25}=0$ (or $25x^2-25x-6=0$)	
			TOTAL: 6m

6		$\frac{dy}{dx}=\int\left[1-\frac{4}{(2x+5)^2}\right]dx = x+\frac{2}{2x+5}+c$ Sub $x=-2$, $\frac{dy}{dx}=0$ to get $c=0$, so $\frac{dy}{dx}=x+\frac{2}{2x+5}$ $y=\int\left[x+\frac{2}{2x+5}\right]dx=\frac{1}{2}x^2+\ln(2x+5)+k$ Sub $x=-2$, $y=5$ to get $k=3$ $y=\frac{1}{2}x^2+\ln(2x+5)+3$	
			TOTAL: 5m
7	(i)	$\frac{1}{3}(AB)^2(1+2\sqrt{3})=6\sqrt{3}-8$ $AB^2=\frac{18\sqrt{3}-24}{1+2\sqrt{3}}\times\frac{1-2\sqrt{3}}{1-2\sqrt{3}}$ $=\frac{18\sqrt{3}-108-24+48\sqrt{3}}{1-12}$ $=\frac{66\sqrt{3}-132}{-11}=12-6\sqrt{3}$ (shown)	
	(ii)	Let M be midpoint of AC. By Pythagoras Theorem, $AC^2=AB^2+BC^2=2AB^2$, so $AM^2=\left(\frac{1}{2}AC\right)^2=\frac{1}{2}AB^2=6-3\sqrt{3}$ $VA^2=AM^2+VM^2$ $=(6-3\sqrt{3})+(1+2\sqrt{3})^2=19+\sqrt{3}$	
			TOTAL: 7m
8	(i)	$\angle BDC=\angle CBD$ (base angles in isos $\triangle$) $=\angle TCD$ (alternate segment theorem) By the alternate-angle property, BD is parallel to CT.	
	(ii)	$\angle TCD=\angle TAC$ (alternate segment theorem) $\angle CTD=\angle ATC$ (common angle) Hence ΔTCD is similar to ΔTAC (AA-test) $\frac{CT}{AT}=\frac{DT}{CT}\Rightarrow CT^2=AT\times DT$ (shown)	
			TOTAL: 7m

9	(i)	When $y=4$, $x=2+\frac{1}{(4-1)^2}=\frac{19}{9}$ Gradient of line L $=\frac{4-2}{\frac{19}{9}-0}=\frac{18}{19}$ Equation of L is $y=\frac{18}{19}x+2$
	(ii)	Area $=\int_2^4\left[2+\frac{1}{(y-1)^2}\right]dy-\frac{1}{2}(2)\left(\frac{19}{9}\right)$ $=\left[2y-\frac{1}{y-1}\right]_2^4-\frac{19}{9}$ $=\left[\left(8-\frac{1}{3}\right)-(4-1)\right]-\frac{19}{9}=\frac{23}{9}$ sq units
		TOTAL: 7m
10	(i)	$p=1000$ Sub $t=8$, $N=1492$ and $p=1000$ (found value) $1492=1000e^{8k}$ $8k=\ln\left(\frac{1492}{1000}\right)$ $k=0.05001\approx 0.05$
	(ii)	Sub $t=24$ $N=1000e^{0.05(24)}=3320.1\approx 3320$
	(iii)	$1000e^{0.05t}\geq 20000$ $t\geq\frac{\ln\left(\frac{20000}{1000}\right)}{0.05}$ $t\geq 59.9$ hours $=2$ days 11.9 hours On Wednesday 2100 (or 9pm)
		TOTAL: 8m
11	(i)	$(x+3)^2+(y-8)^2=49$ Centre $P=(-3,8)$, Radius $=7$
	(ii)	Gradient of $PM=\frac{8-12}{-3-(-1)}=2$ Gradient of chord $AB=-\frac{1}{2}$ (AB perpendicular to PM) Equation of chord AB is

		$\frac{y-12}{x-(-1)}=-\frac{1}{2}$ $2y=-x+23$
	(iii)	Note that P, M and Q lie on a straight line. Case 1: M is between P and Q $x_Q=x_M+2(x_M-x_P)=-1+2(-1-(-3))=3$ $y_Q=y_M+2(y_M-y_P)=12+2(12-8)=20$ So coordinate of Q is $(3,20)$ (shown) Case 2: P is the midpoint of Q and M $x_P=\frac{x_Q+x_M}{2} \Rightarrow x_Q=2x_P-x_M=2(-3)-(-1)=-5$ $y_P=\frac{y_Q+y_M}{2} \Rightarrow y_Q=2y_P-y_M=2(8)-(12)=4$ So coordinate of Q is $(-5,4)$.
		TOTAL: 9m
12	(i)	Using cosine rule, $QT^2=4^2+4^2-2(4)(4)\cos(\pi-2x)$ $=32+32\cos 2x$ $=32+32(2\cos^2 x-1)$ $=64\cos^2 x$ $QT=\sqrt{64\cos^2 x}=8\cos x$
	(ii)	$A=\frac{1}{2}(4)(4)\sin(\pi-2x)+3(8\cos x)$ $=8\sin 2x+24\cos x$ (shown)
	(iii)	$\frac{\mathrm{d}A}{\mathrm{d}x}=16\cos 2x-24\sin x=0$ $16(1-2\sin^2 x)-24\sin x=0$ $4\sin^2 x+3\sin x-2=0$ $\sin x=0.4253$ or -1.175 (rejected) For stationary point, $x=0.4392$, $A=27.8799\approx 27.9\text{ cm}^2$ $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=-32\sin 2x-24\cos x$ When $x=0.4392$, $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=-46.35<0$, so $A=27.9\text{ cm}^2$ is a maximum area.
		TOTAL: 10m

Marking Scheme

1 A curve has the equation $y=(ax-3)\ln x$, where $x>0$, $x\neq\frac{3}{a}$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $x+5y-4=0$. Find the value of a. [7]

$(ax-3)\ln x=0$	M1
$\ln x=0$	
$x=1$	M1
$x+5y-4=0$	
$y=-\frac{1}{5}x+\frac{4}{5}$	
$m_{line}=-\frac{1}{5}$	M1
$m_{\perp}=5$	M1
$y=(ax-3)\ln x$	
$\frac{dy}{dx}=\frac{(ax-3)}{x}+a\ln x$	M1
@$x=1$, $m_{tan}=\frac{a-3}{1}+a\ln 1$	M1
$=a-3$	
$\therefore\ a-3=5$	
$a=8$	A1

2a Differentiate the following with respect to x,
(i) $\ln(\cos 2x)$ **[2]**

$y=\ln(\cos 2x)$	
$\frac{dy}{dx}=\frac{1}{\cos 2x}\cdot -\sin 2x\cdot 2$	M1
$=-2\tan 2x$	A1

(ii) $\frac{x}{2}\tan 2x$ **[2]**

$y=\frac{x}{2}\tan 2x$	
$\frac{dy}{dx}=\frac{x}{2}\cdot\sec^2 2x\cdot 2+\tan 2x\cdot\frac{1}{2}$	M1
$=x\sec^2 2x+\frac{1}{2}\tan 2x$	A1

b Using your results from part (a) find $\int 2x\sec^2 2x\,dx$. **[4]**

$\int x\sec^2 2x+\frac{1}{2}\tan 2x\,dx=\frac{x}{2}\tan 2x$	M1
$\int x\sec^2 2x\,dx=\frac{x}{2}\tan 2x-\frac{1}{2}\int\tan 2x\,dx$	M1
$2\int x\sec^2 2x\,dx=x\tan 2x-\int\tan 2x\,dx$	M1
$\int 2x\sec^2 2x\,dx=x\tan 2x+\frac{1}{2}\ln\cos 2x+c$	A1

3 (i) Given that the constant term in the binomial expansion of $\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$ is 60, find the value of the positive constant k. [4]

$T_{r+1}=\left(\frac{2}{x}\right)^{6-r} C_r^6\left(\frac{x^2}{k}\right)^r$	M1
$\rightarrow x^{-6+r}\times x^{2r}$	M1
$\therefore 3r-6=0$	
$r=2$	M1
$T_3=\left(\frac{2}{x}\right)^4 C_2^6\left(\frac{x^2}{k}\right)^2$	
$\frac{240}{k^2}=60$	
$k=2,\quad -2(NA)$	A1

(ii) Using the value of k found in part (i), find the term independent of x in the expression $\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$. [4]

$\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$	
$=\left(1+x^3\right)\left[\left(\frac{2}{x}\right)^6+\left(\frac{2}{x}\right)^5 C_1^6\left(-\frac{x^2}{2}\right)^1+\left(\frac{2}{x}\right)^4 C_2^6\left(-\frac{x^2}{2}\right)^2+\ldots\right]$	M2
$=\left(1+x^3\right)\left[\ldots-6\left(\frac{2^{5-1}}{x^{5-2}}\right)+15\left(\frac{2^{4-2}}{x^{4-4}}\right)+\ldots\right]$	M1
$=-96+60$	
$=-36$	A1

4a A particle moves along the curve $y = 3x^2 - 2x + 5$. At the point P, the x-coordinate of the particle is increasing at a rate of 0.002 units/sec and the y-coordinate is increasing at 0.02 units/sec. Find the coordinates of P. [4]

$y = 3x^2 - 2x + 5$	
$\frac{dy}{dx} = 6x - 2$	M1
$\frac{dx}{dt} = 0.002\ u/s$ $\quad \frac{dy}{dt} = 0.02\ u/s$	Both seen M1
$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	
$6x - 2 = \frac{0.02}{0.002}$	M1
$x = 2$	A1
$y = 3(2)^2 - 2(2) + 5$	
$= 13$	
$P(2, 13)$	

b The equation of a curve is $y = x^3 + 5x^2 - 8x + k$, where k is a constant. Find the set of values of x for which y is decreasing. [4]

$y = x^3 + 5x^2 - 8x + k$	
$\frac{dy}{dx} = 3x^2 + 10x - 8$	M1
For decreasing function, $\frac{dy}{dx} < 0$	
$3x^2 + 10x - 8 < 0$	M1
$(3x - 2)(x + 4) < 0$	M1
$-4 < x < \frac{2}{3}$	A1

5 (i) Show that $\frac{d}{dx}\left(\frac{\ln 2x}{x^3}\right)=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$ **[4]**

$\frac{d}{dx}\left(\frac{\ln 2x}{x^3}\right)=\frac{x^3\cdot\frac{1}{2x}\cdot 2-\ln 2x\cdot(3x^2)}{(x^3)^2}$	M1, M1
$=\frac{x^2}{x^6}-\frac{3x^2\ln 2x}{x^6}$	M1
$=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$	A1

(ii) Hence, integrate $\frac{\ln 2x}{x^4}$ with respect to x. **[3]**

$\int\frac{1}{x^4}-\frac{3\ln 2x}{x^4}\,dx=\frac{\ln 2x}{x^3}$	M1
$\int\frac{3\ln 2x}{x^4}\,dx=\int\frac{1}{x^4}\,dx-\frac{\ln 2x}{x^3}$	M1
$3\int\frac{\ln 2x}{x^4}\,dx=\frac{x^{-3}}{-3}-\frac{\ln 2x}{x^3}+c$	
$\int\frac{\ln 2x}{x^4}\,dx=-\frac{1}{9x^3}-\frac{\ln 2x}{3x^3}+c$	A1

(iii) Given that the curve $y=f(x)$ passes through the point $\left(1,\frac{8}{9}\right)$ and is such that $f'(x)=\frac{\ln 2x}{x^4}$, find f(x). **[2]**

$f(x)=\int\frac{\ln 2x}{x^4}\,dx$	
$y=-\frac{1}{9x^3}[1+3\ln 2x]+c$	M1
$\frac{8}{9}=-\frac{1}{9}[1+3\ln 1]+c$	
$c=1$	A1
$f(x)=-\frac{1}{9x^3}[1+3\ln 2x]+1$	

6 Mr Tan drives his car along a straight road. As he passes a point A he applies the brake and his car slows down, coming to a rest at point B.
For the journey from A to B, the distance, s meters, of the car from A, t seconds after passing A, is given by

$$s = 600\left(1 - e^{-\frac{t}{6}}\right) - 12t$$

(i) Find an expression, in terms of t, for the velocity of the car during the journey from A to B. **[2]**

$s = 600 - 600e^{-\frac{t}{6}} - 12t$	
$\frac{ds}{dt} = -600 \cdot e^{-\frac{t}{6}} \cdot \left(-\frac{1}{6}\right) - 12$	M1
$v = 100e^{-\frac{t}{6}} - 12$	A1

(ii) Find the velocity of the car at A. **[1]**

$v = 100e^{-\frac{t}{6}} - 12$	
$= 100 - 12$	
$= 88\ m/s$	B1

(iii) Find the time taken for the journey from A to B. **[3]**

$0 = 100e^{-\frac{t}{6}} - 12$	M1
$100e^{-\frac{t}{6}} = 12$	
$-\frac{t}{6} = \ln\left(\frac{12}{100}\right)$	M1
$t = 12.72\ s$	A1

(iv) Find the average speed of the car for the journey from A to B. **[3]**

Ave speed $= \frac{\text{tot dist}}{\text{tot time}}$	
$= \frac{600\left(1 - e^{-\frac{12.72}{6}}\right) - 12(12.72)}{12.72}$	M1 (num) M1 (den)
$= 29.5\ m/s$	A1

7 Solve each of the following equations.

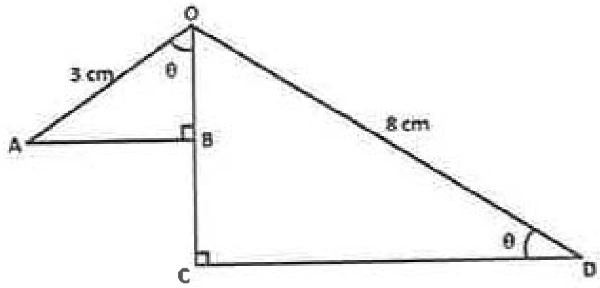
(i) $e^{2\ln x} + \ln e^{2x} = 8$ **[5]**

$e^{2\ln x} + \ln e^{2x} = 8$	
$e^{\ln x^2} + 2x\ln e = 8$	M1, M1
$x^2 + 2x - 8 = 0$	M1
$(x+4)(x-2) = 0$	
$x = 2, \quad or\ x = -4\ (NA)$	A1, A1

(ii) $\log_5 50 + 4\log_{25} x - \log_5(2x+4) = 2$ **[5]**

$\log_5 50 + 4\log_{25} x - \log_5(2x+4) = 2$	
$\log_5 25 \times 2 + \frac{4\log_5 x}{\log_5 25} - \log_5(2x+4) = 2$	M1, M1
$\log_5 5^2 + \log_5 2 + \frac{4\log_5 x}{\log_5 5^2} - \log_5 2(x+2) = 2$	
$2 + \log_5 2 + \frac{4\log_5 x}{2} - [\log_5 2 + \log_5(x+2)] = 2$	M1
$2 + \log_5 2 + 2\log_5 x - \log_5 2 - \log_5(x+2) = 2$	
$\log_5(x+2) = 2\log_5 x$	
$x + 2 = x^2$	
$x^2 - x - 2 = 0$	M1
$(x-2)(x+1) = 0$	
$x = 2, \quad or\ x = -1\ (NA)$	A1

8 In the diagram, triangles OAB and OCD are right-angled triangles.



Angle AOB = angle $ODC = \theta$., $OA = 3$ cm and $OD = 8$ cm.

(i) Show that the length of $AB + CD = 3\sin\theta + 8\cos\theta$. [1]

$AB + CD = 3\sin\theta + 8\cos\theta$	B1

(ii) Express $3\sin\theta + 8\cos\theta$ in the form $R\sin(\theta+\alpha)$ where $R > 0$ and α is acute. [4]

$3\sin\theta + 8\cos\theta = R\sin(\theta+\alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
$R\cos\alpha = 3$ $R\sin\alpha = 8$	M1 both
$\tan\alpha = \frac{8}{3}$	
$\alpha = 69.44^\circ$	M1
$R = \sqrt{73}$	M1
$3\sin\theta + 8\cos\theta = \sqrt{73}\sin(\theta + 69.44^\circ)$	A1

(iii) Find the maximum length of $AB + CD$ and the corresponding value of θ. [3]

Max = $\sqrt{73}$ *or* 8.544	B1
$\sin(\theta + 69.44^\circ) = 1$ $\theta = 90^\circ - 69.44^\circ$	M1
$\theta = 20.56^\circ$	A1

(iv) Find the value of θ, if B is the midpoint of OC. [2]

$2OB = OC$ $2(3\cos\theta) = 8\sin\theta$	M1
$\tan\theta = \frac{6}{8}$ $\theta = 36.87^\circ$	A1

9 The function f is defined by $f(x) = 4\cos 2x - 3$.

(i) State the amplitude of f. **[1]**

Amplitude = 4	B1

(ii) State the period of f in terms of π. **[1]**

$\text{Period} = \frac{2\pi}{2}$ $= \pi$	B1

The equation of a curve is $y = 4\cos 2x - 3$ for $0 \le x \le \pi$.

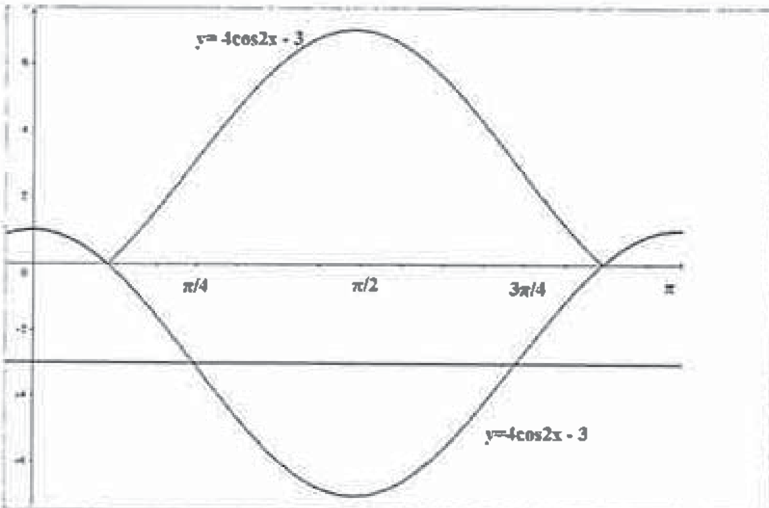
(iii) Find the minimum value of the curve. **[1]**

$\text{Min} = -4 - 3$ $= -7$	B1

(iv) Find the x-coordinates of the points where the curve meets the x-axis. **[3]**

$4\cos 2x - 3 = 0 \quad 0 \le x \le \pi$	M1
$\cos 2x = \frac{3}{4} \quad 0 \le 2x \le 2\pi$	M1
$2x = 0.7227, \quad 5.560$ $x = 0.3614, \quad 2.780$	A1

(v) Sketch the graph of $y = \lvert 3\cos 2x - 4 \rvert$ for $0 \le x \le \pi$. **[3]**



Shape B1
Correct keypoints:
x-axis B1
y-axis B1

(vi) Hence, find the range of values of c, for which $\lvert 3\cos 2x - 4 \rvert = c$ has exactly two solutions only. **[1]**

$1 < c < 7$	B1

10 The diagram shows a triangle ABC with vertices at $A(0, 3)$, $B(8, 12)$ and $C(k, 13)$.

(i) Given that $AB = BC$, find the value of k. **[4]**

$AB^2 = BC^2$	
$(k-8)^2+(13-12)^2=(8-0)^2+(12-3)^2$	M1
$(k-8)^2=64+81-1$	
$(k-8)^2-144=0$	M1
$(k-8+12)(k-8-12)=0$	
$(k+4)(k-20)=0$	M1
$k=20, \quad k=-4(NA)$	A1

A line is drawn from B to meet the x-axis at D such that $AD = CD$.

(ii) Name the quadrilateral $ABCD$. **[1]**

Kite	B1

(iii) Find the equation of BD and the coordinates of D. **[4]**

Property of Kite △. Diagonals intersect at 90°	
$m_{AC}=\dfrac{13-3}{20-0}$	M1
$=\dfrac{1}{2}$	
$m_{BD}=-2$	M1
$12=-2(8)+c$	
$c=28$	
$y=-2x+28$	A1
$0=-2x+28$	
$x=14$	
$D(14,0)$	A1

(iv) Find the area of the triangle ABC. **[2]**

$A=\dfrac{1}{2}\begin{vmatrix}0 & 8 & 20 & 0\\ 3 & 12 & 13 & 3\end{vmatrix}$	M1
$=\dfrac{1}{2}[(264)-(104)]$	
$=80\ units^2$	A1

11a (i) Find the range of values of x for which $x^2-8x+15\geq 0$ **[2]**

$x^2-8x+15\geq 0$ $(x-5)(x-3)\geq 0$	M1
$x\leq 3 \quad or\ x\geq 5$	A1

(ii) Hence, find the range of values of x for which $(x+2)^2-8x-1<0$ **[3]**

$(x+2)^2-8(x+2)+16-1<0$ $(x+2)^2-8(x+2)+15<0$	M1
$[(x+2)-5][(x+2)-3]<0$	M1
$(x-3)(x-1)<0$ $1<x<3$	A1

b Show that $my=x^2-4(x-1)$ meets the curve $y=x^2-3x+2$ at two distinct points for all real values of m, except $m=0$ and $m=1$. **[5]**

$my=x^2-4(x-1)$ $y=\dfrac{x^2-4x+4}{m}$	M1
$y=x^2-3x+2$ $\dfrac{x^2-4x+4}{m}=x^2-3x+2$	M1
$x^2-4x+4=mx^2-3mx+2m$ $(m-1)x^2+(4-3m)x+(2m-4)=0$	M1
$b^2-4ac=(4-3m)^2-4(m-1)\cdot 2(m-2)$ $=16-24m+9m^2-8(m^2-3m+2)$ $=16-24m+9m^2-8m^2+24m-16$ $=m^2$	M1
$m^2>0$ $\therefore b^2-4ac>0$ $\therefore$ 2 distinct roots	A1