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**PEICAI SECONDARY SCHOOL**  
**SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC**  
**PRELIMINARY EXAMINATION 2019**

CANDIDATE  
NAME

CLASS

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REGISTER NUMBER

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**ADDITIONAL MATHEMATICS**

**4047/01**

**27 August 2019**

**2 hours**

Candidates answer on the Question Paper.  
No Additional Materials are required.

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**READ THESE INSTRUCTIONS FIRST**

Write your register number, class and name in the spaces at the top of this page.  
Write in dark blue or black pen.  
You may use an HB pencil for any diagrams or graphs.  
Do not use staples, paper clips, glue or correction fluid.  
**DO NOT WRITE IN ANY BARCODES.**

Answer all the questions.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
The use of an approved scientific calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

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This document consists of **19** printed pages and **1** blank page.

Setter: Mrs Ariel Leong

[Turn over

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*For the quadratic equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$ **2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

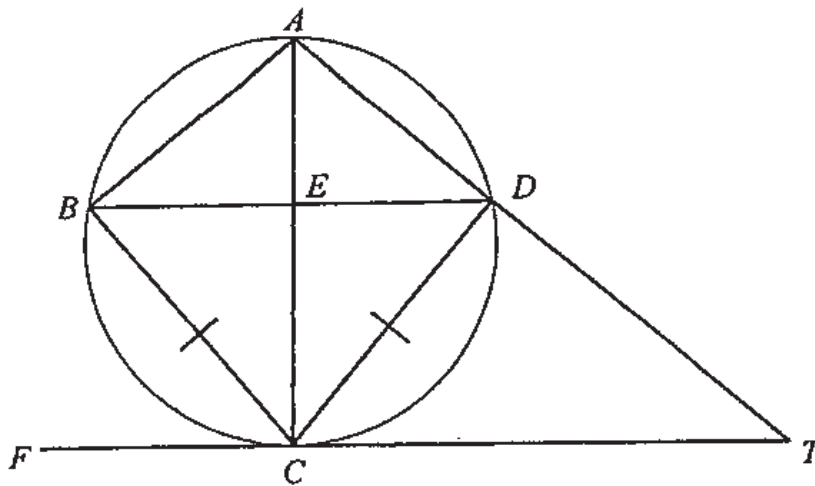
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 Find the range of values of the constant  $p$  for which the line  $y = px - 1$  intersects the curve  $y = 2x^2 - 5x + 1$  at two points. [4]

- 2 Show that  $\frac{36^{x+1} + 15(6^{2x})}{3^{2x+1}}$  is divisible by 17, where  $x$  is a positive integer. [4]

- 3 Find the coordinates of the stationary points of the curve  $y = 4x + \frac{25}{x}$ , and determine the nature of these stationary points. [6]

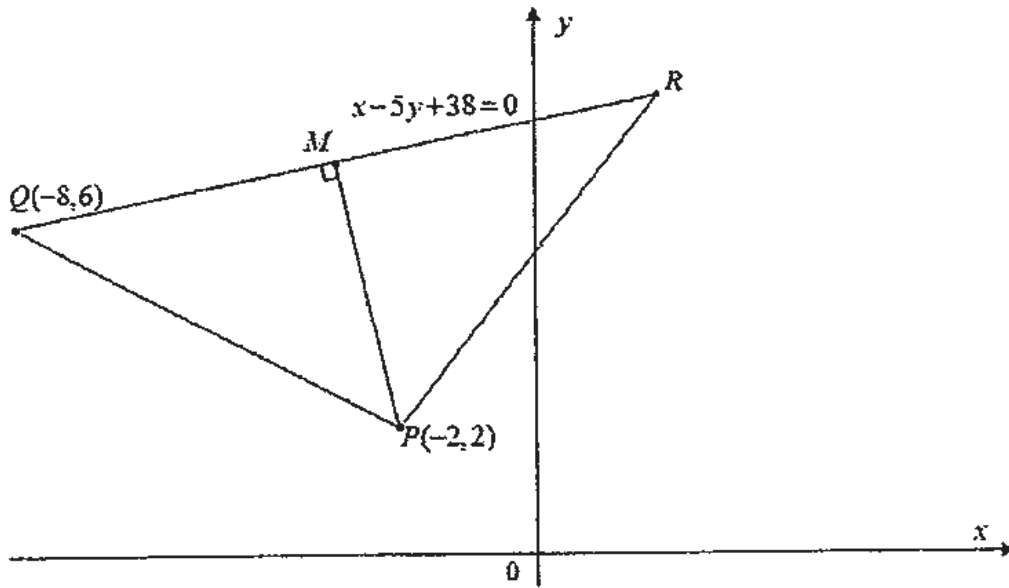


$A, B, C$  and  $D$  are four points on the circle.  $CT$  is the tangent to the circle at point  $C$ . The diagonals of  $BD$  and  $AC$  intersect at point  $E$  and  $BC = CD$ .

(i) Prove that  $BD$  is parallel to  $FT$ . [3]

(ii) Show that  $(CT)^2 = TD \times TA$ . [3]

5

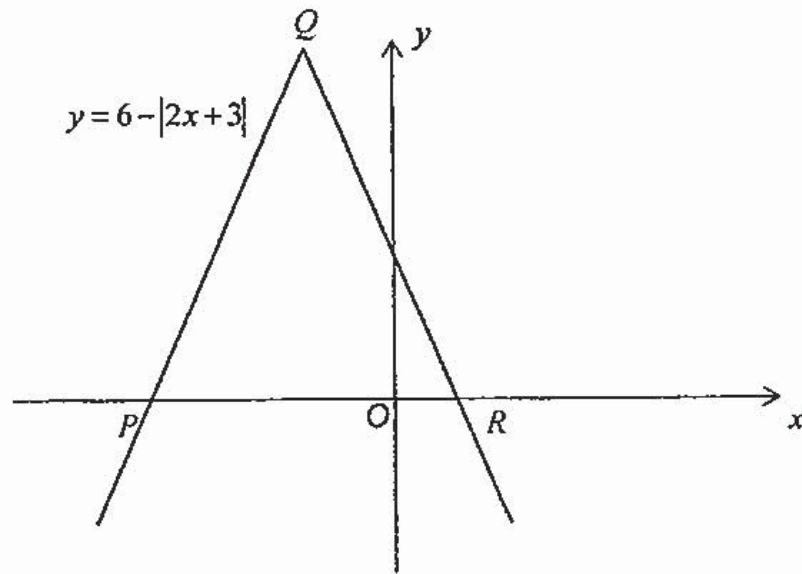


The diagram shows an isosceles triangle  $PQR$  with vertices  $P(-2, 2)$  and  $Q(-8, 6)$ .  $QP = PR$ . The line  $QR$  with equation  $x - 5y + 38 = 0$  passes through  $M$ , the mid-point of  $QR$ .

Find the area of triangle  $PQR$ .

[6]

6



The diagram shows part of the graph  $y = 6 - |2x + 3|$ .

(a) Find the coordinates of  $P$ ,  $Q$  and  $R$ .

[4]

(4)

(b) In each of the following cases determine the number solutions of the equation  $6 - |2x + 3| = mx - 1$ . Justify your answer.

(i)  $m = 2$ , [2]

(ii)  $m = -\frac{1}{2}$ . [2]

7 The roots of the quadratic equation  $2x^2 - 4x + 3 = 0$  are  $\alpha$  and  $\beta$ .

(i) Show that  $\alpha^3 + \beta^3 = -1$ .

[3]

(ii) Find a quadratic equation whose roots are  $\alpha^3 + 1$  and  $\beta^3 + 1$ .

[4]

- 8 Solve the equation  $\sin \theta \tan \theta + 2 \sin \theta = 3 \cos \theta$ , where  $\cos \theta \neq 0$ , for  $0^\circ < \theta < 180^\circ$ .  
[6]

9 A particle moves in a straight line such that, at time  $t$  seconds after leaving a fixed point  $O$ , its velocity  $v$  m/s is given by  $v = t^2 (t - 4)$ . Find

(i) the acceleration of the particle when  $t = 3$ , [2]

(ii) the value of  $t$  when the particle comes to an instantaneous rest, [2]

(iii) the time taken for the particle to return to the point  $O$ , [4]

- (iv) the total distance travelled by the particle in the interval  $t = 0$  to  $t = 5$ . [3]

10 Variables  $x$  and  $y$  are connected by the equation  $y = a^{x+b}$ , where  $a$  and  $b$  are constants.

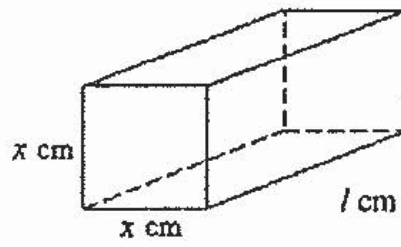
When a graph of  $\lg y$  is plotted against  $x$ , a straight line passing through the points  $(3, 1)$  and  $(6, 4)$  is obtained. Find

(i) the value of  $a$  and of  $b$ ,

[4]

- (ii) the coordinates of the point on the line at which  $\lg y = 2x - 4$  . [3]

11



The diagram shows a rectangular block of ice,  $x$  cm by  $x$  cm by  $l$  cm.  
Volume of the ice is 1 litres.

- (i) Show that the total surface area,  $A$  cm<sup>2</sup>, is given by  $A = 2x^2 + \frac{4000}{x}$ . [2]

- (ii) The block of ice is melting such that the total surface area is changing at a constant rate of  $3 \text{ cm}^2/\text{s}$ . Find the rate of decrease of  $x$  when  $x = 5$ . [4]

- 12 On a particular day, the water level at the beach of Tanjung Rima first reached a maximum of 2 m at 8 am. The lowest water level was forecast to be at 6 pm. The depth of water at the beach may be modelled by the equation

$$h = 0.8 \cos(kt) + c$$

where  $h$  is the water level in metres and  $t$  is the number of hours after 8 am.

- (i) Explain why this model suggests that the minimum water level will be 0.4 m.

[1]

- (ii) Show that  $c = 1.2$ .

[1]

- (iii) Show that the value of  $k$  is  $\frac{\pi}{10}$ .

[2]

19

The corals at the beach of Tanjung Rima are visible at low tide when the water level is less than 0.45 m.

- (iv) Between what times in the daylight hours will corals be visible at Tanjung Rima? [5]



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**ADDITIONAL MATHEMATICS**

Paper 2

**4047/02**

**28 August 2019**

**2 hours 30 minutes**

Candidates answer on the Question Paper.  
No Additional Materials are required.

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At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

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Setter: Mrs Ho Thuk Lan

[Turn over

**Mathematical Formulae****1. ALGEBRA****Quadratic Equation**

For the quadratic equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Binomial expansion**

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

**2. TRIGONOMETRY****Identities**

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

**Formulae for  $\Delta ABC$** 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (i) In the expansion of  $(1+4x)^n$  the coefficient of  $x^2$  is 8 times the coefficient of  $x$ . Show that  $n = 5$ . [4]

(ii) Using your answer to part (i), find, in terms of  $p$ , the coefficient of  $x^2$  in the expansion of  $(1+px+3x^2)(1+4x)^n$ . [1]

(iii) If the coefficient of  $x^2$  in the expansion of  $(1+px+3x^2)(1+4x)^n$  is 263, find the value of the constant  $p$ . [2]

2 (i) Differentiate  $x \cos 3x$  with respect to  $x$ . [3]

(ii) Using your answer to part (i), find  $\int x \sin 3x \, dx$ . [3]

(iii) Hence show that  $\int_0^{\frac{\pi}{3}} x \sin 3x \, dx = \frac{\pi}{9}$ . [2]

3 The equation of a curve is  $\frac{x^2-1}{x^2+1}$ .

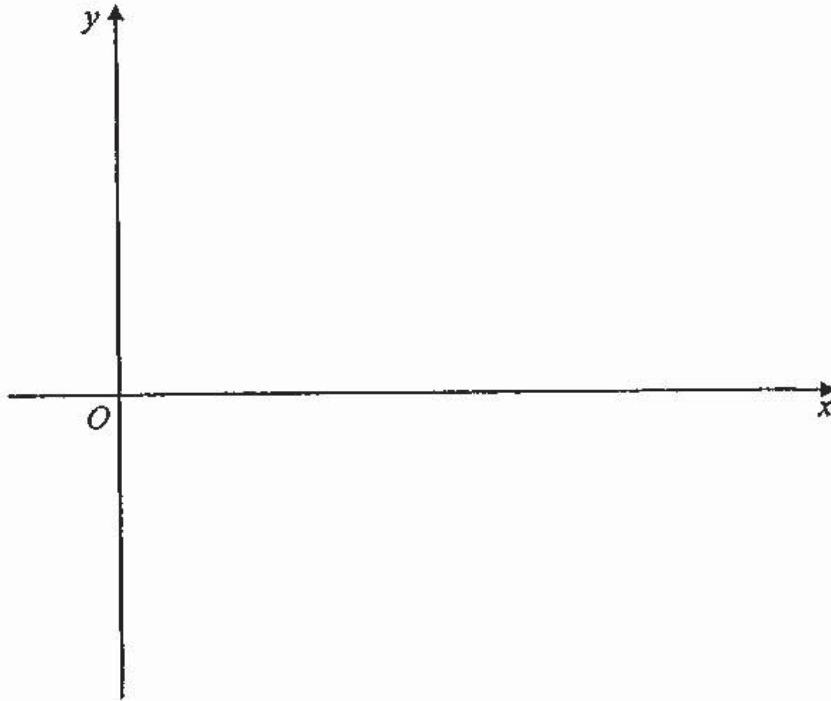
(i) Obtain an expression for  $f'(x)$ . [2]

(ii) Find the equations of the tangent to the curve at the points where the curve meets the  $x$ -axis. [3]

(iii) Show that  $f(x)$  is increasing when  $x > 0$ .

[2]

- 4 (i) Sketch the graph of  $y = \frac{1}{4}x^{\frac{2}{3}}$  for  $x > 0$ . [1]

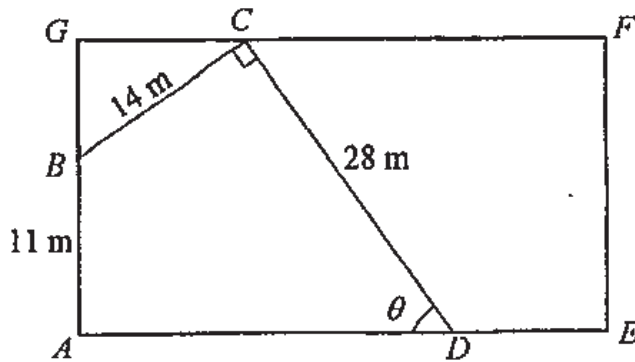


- (ii) On the same diagram sketch the graph of  $y = -\frac{1}{4}x^{\frac{2}{3}}$  for  $x > 0$ . [1]

- (iii) What is the relationship between the graphs of  $y = \frac{1}{4}x^{\frac{2}{3}}$  and  $y = -\frac{1}{4}x^{\frac{2}{3}}$ ? [1]

- (iv) Calculate the coordinates of the point of intersection of  $y = \frac{1}{4}x^{\frac{2}{3}}$  and  $y = 4x^{-\frac{2}{3}}$  for  $x > 0$ . [2]
- (v) On the same diagram sketch the graph of  $y = 4x^{-\frac{2}{3}}$  for  $x > 0$ . [1]
- (vi) Determine, with explanation, whether the tangents to the graphs of  $y = \frac{1}{4}x^{\frac{2}{3}}$  and  $y = 4x^{-\frac{2}{3}}$  for  $x > 0$  at the point of intersection are perpendicular. [3]

5



The diagram shows a rectangular basketball court,  $AEFG$ .

From a point  $A$  on the court, players are to run along the straight paths  $AB$ ,  $BC$ ,  $CD$  and  $DA$ .

The lengths of  $AB$ ,  $BC$  and  $CD$  are 11 m, 14 metres and 28 metres respectively.

Angle  $ADC$  is  $\theta$ , where  $0^\circ < \theta < 90^\circ$ .

The total distance covered by each player is  $T$  metres.

- (i) Show that  $T$  can be expressed as  $p + q \cos \theta + r \sin \theta$  where  $p$ ,  $q$  and  $r$  are constants to be found. [3]

- (ii) Express  $T$  in the form  $p + R \cos(\theta - \alpha)$ , where  $R > 0$  and  $\alpha$  is an acute angle. [4]

Given that the total distance is found to be 78 metres.

(iii) Find the value of  $\theta$ . [2]

Given that the length of  $DE$  is  $(5 + 2\sqrt{5})$  m and the area of triangle  $DEF$  is  $(45 + 42\sqrt{5})$ .

(iv) Find the length of  $EF$  in the form  $a\sqrt{5} - b$ . [2]

6 A circle,  $C_1$ , has centre  $A(4, 2)$  and radius  $\sqrt{13}$ .

(i) Write down the equation of circle,  $C_1$ . [1]

(ii) Determine whether the point  $R(8, 3)$  lies inside or outside of circle,  $C_1$ . [2]

Circle,  $C_1$  intersects the  $x$ -axis at points  $P$  and  $Q$ .

(iii) Find the mid-point of  $PQ$ . [4]

A second circle,  $C_2$ , with centre  $B$  and radius  $\sqrt{18}$  also passes through  $P$  and  $Q$ .

(iv) State the  $x$ -coordinate of  $B$ . [1]

(v) Given that the  $y$ -coordinate of  $B$  is positive, find the centre of circle  $C_2$ . [3]

7 (a) Prove the identity  $\frac{1}{1 + \tan^2 x} = (1 + \sin x)(1 - \sin x)$ . [3]

(b) Find all the angles between  $0^\circ$  and  $360^\circ$  that satisfy the equation  $8 \tan x = 3 \cos x$ . [4]

- (c) Solve the equation  $2 \cos 2y - 5 \cos y = 4$  for  $0 \leq y \leq 2\pi$ , giving your answers in radians. Correct your answers to 2 decimal places. [4]

8 A prism has a volume of  $(6x^2 - 21x + 25)$   $\text{cm}^3$  and a base area of  $(2x^2 - 5x)$   $\text{cm}^2$ .

(i) Find an expression for the height,  $h(x)$ , of the prism. [1]

(ii) Using your answer to part (i), express the height in partial fractions. [5]

(iii) Differentiate  $\ln(2x - 5)$  with respect to  $x$ . [1]

- (iv) Using your answers to part (ii) and (iii), find  $\int_3^4 h(x) dx$  in the form  $a + \ln \frac{b}{c}$  where  $a$ ,  $b$  and  $c$  are integers. [5]

- 9 (a) Given that  $\log_3 p = m$ ,  $\log_{27} q = n$  and  $\frac{p}{q} = 3^r$ , express  $r$  in terms of  $m$  and  $n$ . [3]

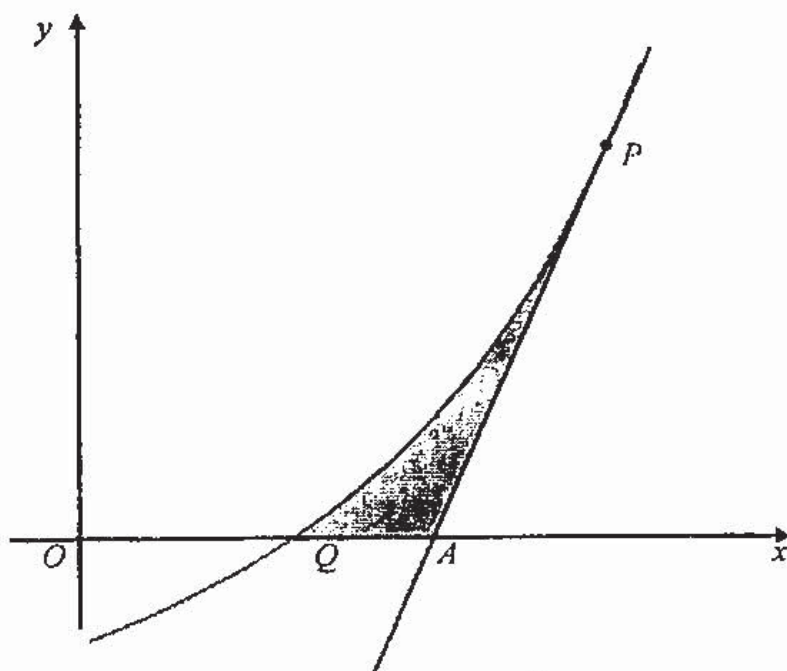
(b) Solve the equation

(i)  $\log_3(3x-7) = \log_3(2x-3) - 2$ , [4]

(ii)  $3\log_5 y = 2 + \log_5 5.$

[5]

10



The diagram shows part of the curve  $y = \frac{9}{(7-x)^2} - 1$ , cutting the  $x$ -axis at  $Q$ . The tangent at the point  $P$  on the curve cuts the  $x$ -axis at  $A$ . Given that the gradient of this tangent is  $\frac{9}{4}$ , calculate

- (i) the coordinates of  $P$ ,

[5]

(ii) the area of the shaded region  $PQA$ .

[7]

**The End**

**[Turn over**

