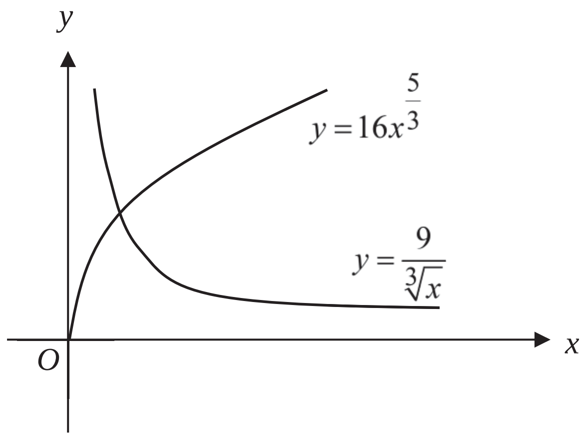
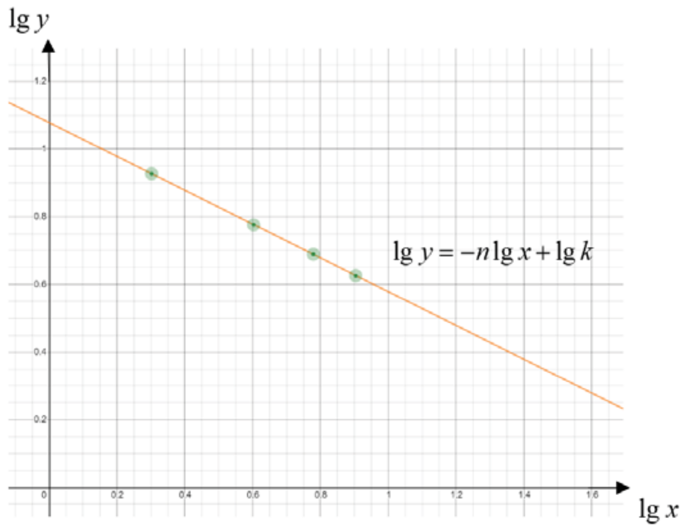
Answer key for AM P1

1	**(i)** $\alpha^2+\beta^2=8$ (ii) $\dfrac{\alpha^2+\beta^2}{\alpha\beta}=16$ (iii) $x^2+20x+37=0$	**2**	(i)The remaining mass after 30 days is 14.2g. (ii) The number of days required is 28 days. (iii) As $t\to\infty$, $30\,e^{-0.025t}\to\infty$, the value of m approaches 0.
3	**(a)** $a=\dfrac{1}{4}$ **(b)** $-\dfrac{3003}{32}$ *or* $-93\dfrac{27}{32}$	**4**	**(i)** $y=\dfrac{9}{2}x+15$ **(ii)** $(-4,-3)$ **(iii)** $y=-\dfrac{2}{9}x+\dfrac{5}{6}$ (iv) 68 units^2 (v) $76.0°$ (1 d.p.)
5	**(ii)** $\dfrac{1}{15}\tan^3 5x+\dfrac{1}{5}\tan 5x+C_2$		
6	**(i)** $\dfrac{15-6x}{\sqrt{(5-4x)^3}}$ **(ii)** $9y+x=28$	**7**	**(i)**  **(ii)** $x=\dfrac{3}{4}$ $(x>0)$
8	**(ii)** For a decreasing function, $\dfrac{dy}{dx}<0$ For $0\le x\le 2\pi$, $\sec^2 x>0,\ x\ne\dfrac{\pi}{2},\dfrac{3\pi}{2}$, $\sin x<0$ for $\pi<x<2\pi$ Hence, y is decreasing for $\pi<x<2\pi,\ x\ne\dfrac{3\pi}{2}$		
9	**(ii)** The area is increasing at 0.0498 units^2/s.		
10	**(ii)** $A\approx\sqrt{3076425}\sin(\theta+41.2°)$ or $A\approx165\sqrt{113}\sin(\theta+41.2°)$ **(iii)** $\theta=3.4°$		
12	$\cot x=-\dfrac{1}{3}$ $\qquad$ $\cot x=2$ $\tan x=-3$ or $\tan x=\dfrac{1}{2}$ $x=-1.25$ $\qquad$ $x=0.464$	**11**	**(i)** $y=\dfrac{x^2}{13x-2}$ (ii)  $k=12.0$ (11.5 − 12.6) $n=0.500$ (0.45 − 0.55)

Answer key for AM paper 2

1

(a) $m > 1$

(b) $a = -1$ and $c = -5$ or

$a = -2$ and $c = -2$ or

$a = -5$ and $c = -1$ or any pairs of values that fulfill the above criteria

2

(a) Since degree of dividend = degree of quotient + degree of divisor,

degree of $Q(x) = 1$

ALTERNATIVE: Must multiply with x^3 to give degree 4 in the polynomial on the left.

(b) $a = -19, \quad b = -6, \quad c = -5$ (c) $\frac{326}{81}$

3

(a) $7x + y - 5$

(b) $x = 2^{\frac{3}{2}}$ or $x = 8^{\frac{1}{2}}$ or $x = 64^{\frac{1}{4}}$

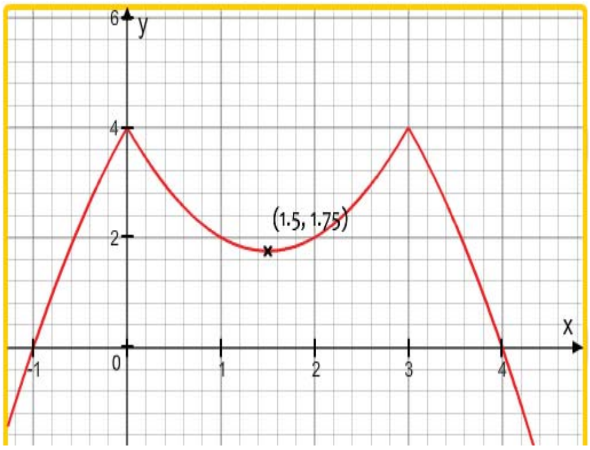
4

(a) $\frac{4}{13}\sqrt{10} - \frac{7}{13}\sqrt{3}$ (b) $x = -\frac{4}{3}$

5

(a)(i) $x = 4$ or $x = -1$

(ii)



(b)

$-\left|3x - x^2\right| = k - 4$

$-\left|3x - x^2\right| + 4 = k$

$y = k$

$y = k$ is a **horizontal line** and when k is lesser than 2.25, it will be **below the turning point** and so it will only **intersect the curve at the two outer arms thereby giving two solutions only.**

(c) $m = 1$

6

(i) $\frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2}$

(ii) $\ln(x-1) - \ln(x+2) - \frac{1}{2(x+2)} + c$

7

$x = \frac{k}{4}$, $\frac{d^2A}{dx^2} = -2 < 0$

Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square.

8

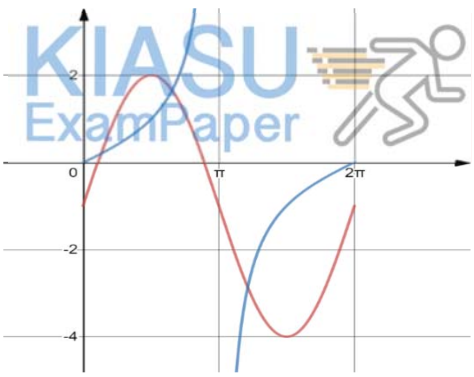
(i) $\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$

(ii) 5.14 units (to 3 s.f.)

9

(a) (ii) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$ rad

(b)(i)



(ii) From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \le x \le 2\pi$

10

(a) $v_B = t + \frac{1}{6}t^2$

(b) $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$

(c) 486 m

(d) 72 m/s

11	(i) $2p\sqrt{1-p^2}$ (ii) $\sqrt{\dfrac{p+1}{2}}$	12	$R(\ln 2, -9)$ $P\left(\ln 2 - \dfrac{9}{2}, 0\right)$ $Q\left(-\dfrac{1}{2}\ln\dfrac{5}{2}, 0\right)$ Area = 13.2 units^2

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020

S4

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **1 September 2020 Tuesday**

2 hours

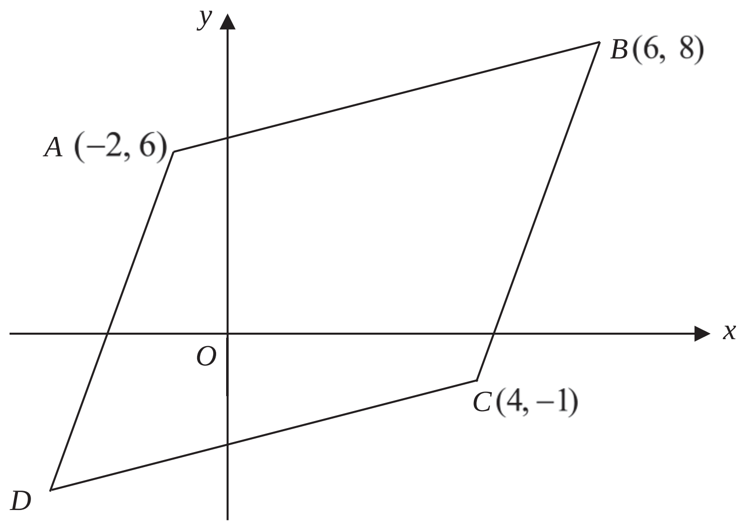
Marking Scheme

1	The roots of the quadratic equation $2x^2-6x+1=0$ are α and β. **(i)** Find the value of $\alpha^2+\beta^2$. [3] **(ii)** Find the value of $\dfrac{\alpha^2+\beta^2}{\alpha\beta}$. [1] **(iii)** Form a quadratic equation with roots $\dfrac{\alpha}{\beta}+2$ and $\dfrac{\beta}{\alpha}+2$. [3]	
1(i)	$\alpha+\beta=-\left(\dfrac{-6}{2}\right)=3$ $\alpha\beta=\dfrac{1}{2}$ $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $\alpha^2+\beta^2=(3)^2-2\left(\dfrac{1}{2}\right)$ $\alpha^2+\beta^2=8$	M1 for both roots M1 A1
(ii)	$\dfrac{\alpha^2+\beta^2}{\alpha\beta}=8\div\dfrac{1}{2}=16$	B1
(iii)	$\dfrac{\alpha}{\beta}+2+\dfrac{\beta}{\alpha}+2=\dfrac{\alpha^2+\beta^2}{\alpha\beta}+4$ $\dfrac{\alpha}{\beta}+2+\dfrac{\beta}{\alpha}+2=16+4=20$ $\left(\dfrac{\alpha}{\beta}+2\right)\left(\dfrac{\beta}{\alpha}+2\right)=1+\dfrac{2\alpha}{\beta}+\dfrac{2\beta}{\alpha}+4$ $\left(\dfrac{\alpha}{\beta}+2\right)\left(\dfrac{\beta}{\alpha}+2\right)=5+\dfrac{2\alpha^2+2\beta^2}{\alpha\beta}$ $\left(\dfrac{\alpha}{\beta}+2\right)\left(\dfrac{\beta}{\alpha}+2\right)=5+2(16)=37$ $x^2-(20)x+37=0$ $x^2-20x+37=0$	M1 for sum of roots M1 for product of roots A1

2	The mass, *m* grams, of a radioactive substance, present at time *t* days after first being observed, is given by the formula $m = 30\,e^{-0.025t}$. **(i)** Find the mass remaining after 30 days. [2] **(ii)** Find the number of days required for the mass to drop to half of its value at $t = 0$. Give your answer correct to the nearest integer. [2] **(iii)** State the value *m* approaches when *t* becomes large. [1]	
2(i)	$m = 30\,e^{-0.025(30)}$ $= 14.171$ $= 14.2$ The remaining mass after 30 days is 14.2g.	M1 A1 – 0 mark for omission of unit in answer
(ii)	$15 = 30\,e^{-0.025t}$ $e^{-0.025t} = \frac{1}{2}$ $-0.025t = \ln\frac{1}{2}$ $t = 27.726$ $t = 28$ The number of days required is 28 days.	B1 A1
(iii)	As $t \to \infty$, $30\,e^{-0.025t} \to \infty$, the value of *m* approaches 0.	A1

3	**(a)** Find, in ascending powers of x, the first three terms in the expansion of $(2-x)^7$. [2] Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3] **(b)** In the expansion of $\left(x^2-\frac{1}{2x^4}\right)^n$ in descending powers of x, the sixth term is independent of x. Find the value of n and the term independent of x. [4]	

3(a)(i)	$(2-x)^7 = 2^7 - \binom{7}{1}(2^6)x + \binom{7}{2}(2^5)x^2 + \ldots$	M1 A1
	$= 128 - 448x + 672x^2 + \ldots$	
	$(a-x)(2-x)^7$	M1
	$= (a-x)(128 - 448x + 672x^2 + \ldots)$	
(ii)	$= \ldots + 672ax^2 + 448x^2 + \ldots$	M1
	coefficient of x^2: $672a + 448 = 616$	A1
	$a = \frac{1}{4}$	
(b)	$\left(x^2-\frac{1}{2x^4}\right)^n$	
	$T_6 = \binom{n}{5}\left(x^2\right)^{n-5}\left(-\frac{1}{2x^4}\right)^5$	M1
	$= \binom{n}{5}\left(x^{2n-10}\right)\left(-\frac{1}{2}\right)^5 x^{-20}$	
	$= \binom{n}{5}\left(x^{2n-30}\right)\left(-\frac{1}{2}\right)^5$	A1
	As it is independent of x $2n - 30 = 0$ $n = 15$	M1
	Value of the term $= \binom{15}{5}\left(-\frac{1}{2}\right)^5$	
	$= -\frac{3003}{32}$ or $-93\frac{27}{32}$	A1

<table>
<tr><td colspan="2">4</td><td colspan="2">Solutions to this question by accurate drawing will not be accepted.


The diagram shows a parallelogram ABCD in which A (– 2, 6), B (6, 8) and C (4, – 1) are the coordinates of its vertices. Find the

(i) equation of AD, [2]
(ii) coordinates of D, [2]
(iii) equation of the perpendicular bisector of the line AD, [2]
(iv) area of the parallelogram ABCD, [2]
(v) acute angle the line AB makes with the y-axis. [2]</td></tr>
<tr><td>4(i)</td><td colspan="2">Gradient of AD = Gradient of BC
$= \frac{8+1}{6-4}$
$= \frac{9}{2}$

Equation of AD :
$\frac{y-6}{x+2} = \frac{9}{2}$
$y = \frac{9}{2}x + 15$</td><td>M1

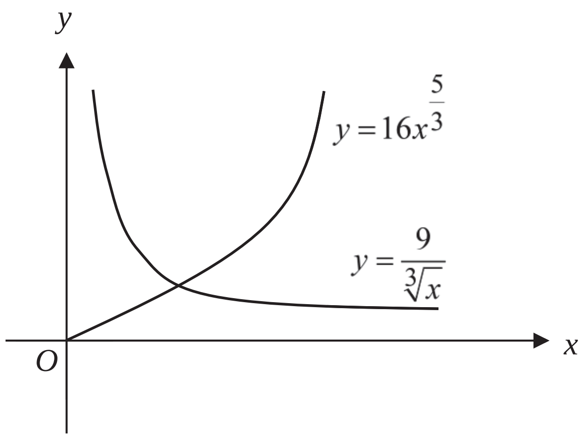
A1</td></tr>
<tr><td>(ii)</td><td colspan="2">Coordinates of $D = (-2-2, 6-9)$
$= (-4, -3)$</td><td>M1
A1</td></tr>
</table>

(iii)	Gradient of Perpendicular Bisector $= -\frac{2}{9}$ Midpoint of $AD = \left(\frac{-2+(-4)}{2}, \frac{-3+6}{2}\right)$ $= \left(-3, \frac{3}{2}\right)$ Equation of Perpendicular Bisector $y - \frac{3}{2} = -\frac{2}{9}(x+3)$ $y = -\frac{2}{9}x + \frac{5}{6}$	M1 A1
(iv)	Area of $ABCD$ $= \frac{1}{2}\begin{vmatrix} -2 & -4 & 4 & 6 & -2 \\ 6 & -3 & -1 & 8 & 6 \end{vmatrix}$ $= \frac{1}{2}\left[(6+4+32+36)-(-24-12-6-16)\right]$ $= 68 \text{ units}^2$	M1 clockwise and -68 (max 1 mark) A1
(v)	Angle made by line AB with y-axis $= \tan^{-1}\left(\frac{6+2}{8-6}\right)$ $= 76.0° \text{ (1 d.p.)}$	M1 A1 accept both degrees and radians

5	**(i)** Show that $\frac{d}{dx}\left(\tan^3 5x\right) = 15\sec^4 5x - 15\sec^2 5x$. [3] **(ii)** Use your answers to part **(i)**, find $\int \sec^4 5x \, dx$. [4]	
(i)	$\frac{d}{dx}\left(\tan^3 5x\right) = 3\left(\tan^2 5x\right)\left(\sec^2 5x\right)(5)$ $= 15\left(\tan^2 5x\right)\left(\sec^2 5x\right)$ $= 15\left(\sec^2 5x - 1\right)\left(\sec^2 5x\right)$ $= 15\sec^4 5x - 15\sec^2 5x$	M1 – power M1 – chain rule differentiate tan $5x$ M1

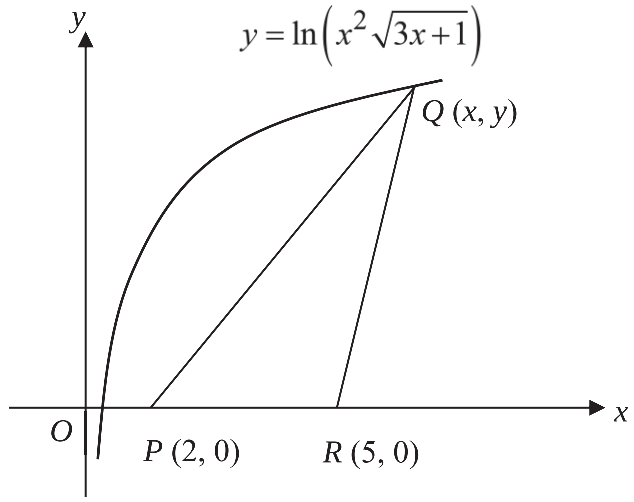
(ii)	$\frac{d}{dx}\left(\tan^3 5x\right) = 15\sec^4 5x - 15\sec^2 5x$ Integrate w.r.t. x $\tan^3 5x + C_1 = \int\left(15\sec^4 5x - 15\sec^2 5x\right) dx$ $\tan^3 5x + C_1 = 15\int \sec^4 5x\, dx - 15\int \sec^2 5x\, dx$ $\tan^3 5x + C_1 = 15\int \sec^4 5x\, dx - 15\left(\frac{\tan 5x}{5}\right)$ $\tan^3 5x + 3\tan 5x + C_1 = 15\int \sec^4 5x\, dx$ $\int \sec^4 5x\, dx = \frac{1}{15}\tan^3 5x + \frac{1}{5}\tan 5x + C_2$	M1 M1 M1 A1 If everything is C, minus 1 mark for overall presentation

6	**(i)** Given that $y=\dfrac{3x}{\sqrt{5-4x}}$, express $\dfrac{dy}{dx}$ in the form $\dfrac{ax+b}{\sqrt{(5-4x)^n}}$ where a, b and n are real constants. [4] **(ii)** Hence find the equation of the normal to the curve $y=\dfrac{3x}{\sqrt{5-4x}}$ at the point on the curve where $x=1$. [2]	
6(i)	$y=\dfrac{3x}{\sqrt{5-4x}}$ $\dfrac{dy}{dx}=\dfrac{\sqrt{5-4x}\dfrac{d}{dx}(3x)-(3x)\dfrac{d}{dx}\sqrt{5-4x}}{\left(\sqrt{5-4x}\right)^2}$ $=\dfrac{\sqrt{5-4x}(3)-(3x)\frac{1}{2}(5-4x)^{-\frac{1}{2}}(-4)}{5-4x}$ $=\dfrac{\sqrt{5-4x}(3)+\dfrac{(3x)2}{\sqrt{5-4x}}}{5-4x}$ $=\dfrac{\sqrt{5-4x}(3)+\dfrac{6x}{\sqrt{5-4x}}}{5-4x}\times\dfrac{\sqrt{5-4x}}{\sqrt{5-4x}}$ $=\dfrac{(5-4x)(3)+6x}{(5-4x)\sqrt{5-4x}}$ $=\dfrac{15-6x}{\sqrt{(5-4x)^3}}$	M1 – Quotient Rule M1 – Chain Rule M1 A1
(ii)	$x=1,\ y=\dfrac{3(1)}{\sqrt{5-4(1)}}=3$ Gradient of the tangent $=\dfrac{15-6(1)}{\sqrt{(5-4(1))^3}}$ $=9$ Gradient of the normal $=-\dfrac{1}{9}$ Equation of normal $y-3=-\dfrac{1}{9}(x-1)$ $y=-\dfrac{1}{9}x+\dfrac{28}{9}$ $9y+x=28$	M1 – find gradient of tangent and normal A1

7	(i) On the same diagram, sketch the graphs of $y=16x^{\frac{5}{3}}$ and $y=\frac{9}{\sqrt[3]{x}}$ for $x>0$. [3] (ii) Find the $x-$coordinate of the intersection point of the two graphs. . [2]	
		B1 – For shape of $y=\frac{9}{\sqrt[3]{x}}$ B1 – For asymptotes B1 – For shape of $y=16x^{\frac{5}{3}}$ and pass through origin
(ii)	$\frac{9}{\sqrt[3]{x}}=16x^{\frac{5}{3}}$ $\frac{9}{16}=x^{\frac{5}{3}}\times\sqrt[3]{x}$ $x^{\frac{5}{3}}\times x^{\frac{1}{3}}=\frac{9}{16}$ $x^2=\frac{9}{16}$ $x=\frac{3}{4}\quad(x>0)$	M1 A1

8	Given that $y = \text{cosec } x \tan x$, **(i)** show that $\frac{dy}{dx} = \sin x \sec^2 x$. [2] **(ii)** determine where y is decreasing for $0 \le x \le 2\pi$. [2]	
8(i)	$y = \text{cosec } x \tan x$ $= \frac{1}{\sin x} \times \frac{\sin x}{\cos x}$ $= \frac{1}{\cos x}$ $= (\cos x)^{-1}$ $\frac{dy}{dx} = (-1)(\cos x)^{-2}(-\sin x)$ $= \frac{1}{\cos^2 x}(\sin x)$ $= \sec^2 x \sin x$ Or $y = \text{cosec } x \tan x$ $= \frac{1}{\sin x} \times \tan x$ $= (\sin x)^{-1} \tan x$ $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \sec^2 x + \tan x\left(-\sin^{-2} x \cos x\right)$ $= \frac{1}{\sin x} \cdot \sec^2 x - \frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin^2 x}\right)$ $= \frac{1}{\sin x} \cdot \sec^2 x - \frac{1}{\sin x}$ $\frac{dy}{dx} = \frac{\sec^2 x - 1}{\sin x}$ $= \frac{\tan^2 x}{\sin x}$ $= \frac{1}{\sin x}\left(\frac{\sin^2 x}{\cos^2 x}\right)$ $= \frac{1}{\cos^2 x}(\sin x)$ $= \sin x \sec^2 x$ Or	M1 M1 – chain rule and differentiation of trigonometric functions M1 M1

	$y=\operatorname{cosec} x \tan x$ $=\dfrac{\tan x}{\sin x}$ $\dfrac{dy}{dx}=\dfrac{\sin x\sec^2 x-\tan x\cos x}{\sin^2 x}$ $=\dfrac{\sin x\sec^2 x-\sin x}{\sin^2 x}$ $=\dfrac{\sin x\left(\sec^2 x-1\right)}{\sin^2 x}$ $=\dfrac{\sin x\left(\tan^2 x\right)}{\sin^2 x}$ $=\dfrac{1}{\sin x}\left(\dfrac{\sin^2 x}{\cos^2 x}\right)$ $=\dfrac{1}{\cos^2 x}\left(\sin x\right)$ $=\sin x\sec^2 x$	M1 M1
(ii)	For a decreasing function, $\dfrac{dy}{dx}<0$ For $0\le x\le 2\pi$, $\sec^2 x>0,\ x\ne\dfrac{\pi}{2},\dfrac{3\pi}{2}$, $\sin x<0$ for $\pi<x<2\pi$ Hence $\dfrac{dy}{dx}<0$ for $\pi<x<2\pi$, $x\ne\dfrac{3\pi}{2}$	B1 B1 - if didn't write $x\ne\dfrac{3\pi}{2}$, its acceptable.

<table>
<tr><td>9</td><td colspan="2">The diagram shows the curve $y=\ln\left(x^2\sqrt{3x+1}\right)$ and three points $P(2,\ 0)$, $Q(x,\ y)$ and $R(5,\ 0)$. The point $Q(x,\ y)$ lies on the curve.

(i) Show that the area, A units², of the triangle PQR is given by
$A=3\ln x+\frac{3}{4}\ln(3x+1)$ [2]
(ii) Given that x is increasing at a rate of 0.2 units/s, find the rate at which the area, A, is changing at the instant when $x=15$ units. [3]</td></tr>
<tr><td>9(i)</td><td>$A=\frac{1}{2}\times PR\times y=\frac{1}{2}\times 3\times\ln\left(x^2\sqrt{3x+1}\right)$
$=\frac{3}{2}\ln\left(x^2\sqrt{3x+1}\right)$
$A=\frac{3}{2}\left(\ln x^2+\ln\sqrt{3x+1}\right)$
$=\frac{3}{2}\left(2\ln x+\ln(3x+1)^{\frac{1}{2}}\right)$
$=\frac{3}{2}\left(2\ln x+\frac{1}{2}\ln(3x+1)\right)$
$=3\ln x+\frac{3}{4}\ln(3x+1)$</td><td>M1

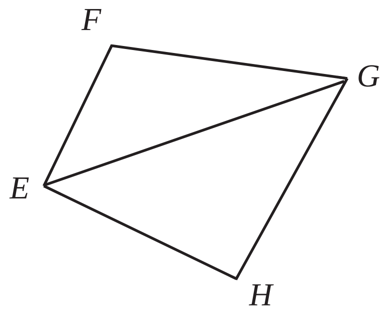
M1 – apply the laws for logarithms</td></tr>
<tr><td>(ii)</td><td>$\frac{dx}{dt}=0.2$ m/s and $x=15$

$A=3\ln x+\frac{3}{4}\ln(3x+1)$

$\frac{dA}{dx}=3\left(\frac{1}{x}\right)+\frac{3}{4}\left(\frac{1}{3x+1}\right)(3)$
$=\frac{3}{x}+\frac{9}{4(3x+1)}$</td><td>M1 – differentiation of logarithms</td></tr>
</table>

	$\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ $=\left(\frac{3}{(15)}+\frac{9}{4\left(3(15)+1\right)}\right)\times(0.2)$ $=0.0498 \text{ units}^2/\text{s}$ The area is increasing at 0.0498 units^2/s.	M1 A1

10



EFGH is a plot of land that comprises two smaller plots, triangle *EFG* and triangle *EGH*. *EF* and *EH* are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

(i) Show that the area, A m^2, of *EFGH* can be expressed as

$$A = 1320\sin\theta + 1155\cos\theta. \qquad [2]$$

(ii) Express A in the form in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of θ if the area is 1231 m^2. [2]

10(i)	Area of triangle $EFG = \frac{1}{2} \times EF \times EG \sin\angle FEG$ $= \frac{1}{2} \times 48 \times 55 \sin\theta$ $= 1320\sin\theta$ Area of triangle $EGH = \frac{1}{2} \times EH \times EG \sin\angle FEG$ $= \frac{1}{2} \times 42 \times 55 \sin(90° - \theta)$ $= 1155(\sin 90° \cos\theta - \cos 90° \sin\theta)$ $= 1155\cos\theta$ $A =$ Area of triangle $EFG +$ Area of triangle EGH $= 1320\sin\theta + 1155\cos\theta$	M1 – area of triangle M1 – use of trigonometry
(ii)	$1320\sin\theta + 1155\cos\theta = R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $R = \sqrt{1320^2 + 1155^2} = \sqrt{3076425}$ $\tan\alpha = \frac{1155}{1320}, \quad \alpha = \tan^{-1}\left(\frac{1155}{1320}\right) = 41.1859°$ $A = \sqrt{3076425}\sin(\theta + 41.1859°)$ Or $A \approx 165\sqrt{113}\sin(\theta + 41.2°)$	M1 – finding R and α A1 A1

(iii)	$\sqrt{3076425}\sin(\theta+41.1859°)=1231$ $\sin(\theta+41.1859°)=\frac{1231}{\sqrt{3076425}}$ $\text{Basic angle}=\sin^{-1}\left(\frac{1231}{\sqrt{3076425}}\right)$ $=44.57449°$ $\theta+41.1859°=44.57449°,\ 180-44.57449°$ $\theta=3.3885°,\ \ 94.2397°(\text{NA})$ $\theta=3.4°$	M1 A1

11	**(a)** The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through (2, 9) and (5, 3). Find an expression for y in terms of x. [4]	
11(i)	Let $Y=\frac{x}{y}, X=\frac{1}{x}$ Gradient = $\frac{9-3}{2-5}=-2$ $\therefore Y-3=-2(X-5)$ $Y=-2X+13$ $\frac{x}{y}=-\frac{2}{x}+13$ $\frac{x}{y}=\frac{13x-2}{x}$ $y=\frac{x^2}{13x-2}$	M1 M1 M1 A1

(b) The table shows experimental values of two variables, x and y.

x	2	4	6	8
y	8.48	5.99	4.90	4.24

It is known that x and y are related by the equation $x^n y = k$, where n and k are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of n and k. [6]

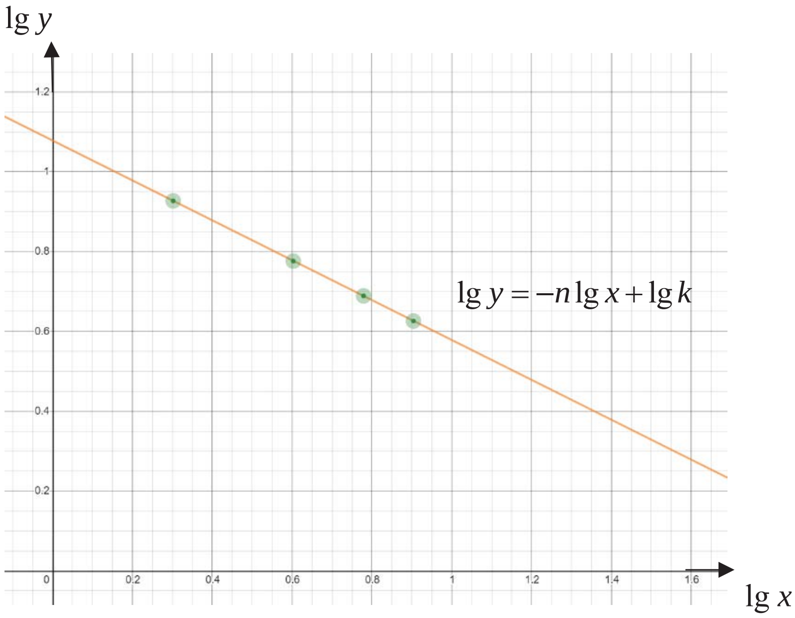
(ii)

$x^n y = k$

$\lg y = -n \lg x + \lg k$ — M1

x	2	4	6	8
y	8.48	5.99	4.90	4.24
$\lg x$	0.301	0.602	0.778	0.903
$\lg y$	0.928	0.777	0.690	0.627

B1



G1 – correct points

G1 – y-intercept

Deduct 1 mark for labels

$\lg k = 1.08$ $(1.06 - 1.10)$

$k = 12.0$ $(11.5 - 12.6)$ — A1

$$-n = \frac{1.00 - 0.60}{0.15 - 0.95}$$

$= -0.500$

$\therefore n = 0.500$ $(0.45 - 0.55)$ — A1

12	Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x, in radians. [5]	
	$3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$ $3\operatorname{cosec}^2 x \sin x = 5\cos x + 5\sin x$ $3\operatorname{cosec}^2 x = \frac{5\cos x + 5\sin x}{\sin x}$ $= \frac{5\cos x}{\sin x} + \frac{5\sin x}{\sin x}$ $= 5\cot x + 5$ $3\operatorname{cosec}^2 x = 5\cot x + 5$ $3(\cot^2 x + 1) - 5\cot x - 5 = 0$ $3\cot^2 x + 3 - 5\cot x - 5 = 0$ $3\cot^2 x - 5\cot x - 2 = 0$ $(3\cot x + 1)(\cot x - 2) = 0$ $\cot x = -\frac{1}{3}$ $\cot x = 2$ $\tan x = -3$ or $\tan x = \frac{1}{2}$ $x = -1.25$ $x = 0.464$	M1 – use of $\cot x = \frac{\cos x}{\sin x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$, $\tan x = \frac{\sin x}{\cos x}$ etc M1 – use identity to change into an equation with single trigonometric term M1 – reach quadratic equation and factorize, or use formula A2 – deduct one mark if more than the principal values are given.

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020

S4

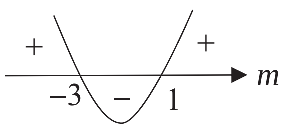
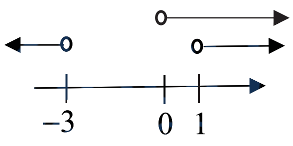
ADDITIONAL MATHEMATICS **4047/02**

Paper 2

28 August 2020 Friday

2 hours 30 minutes

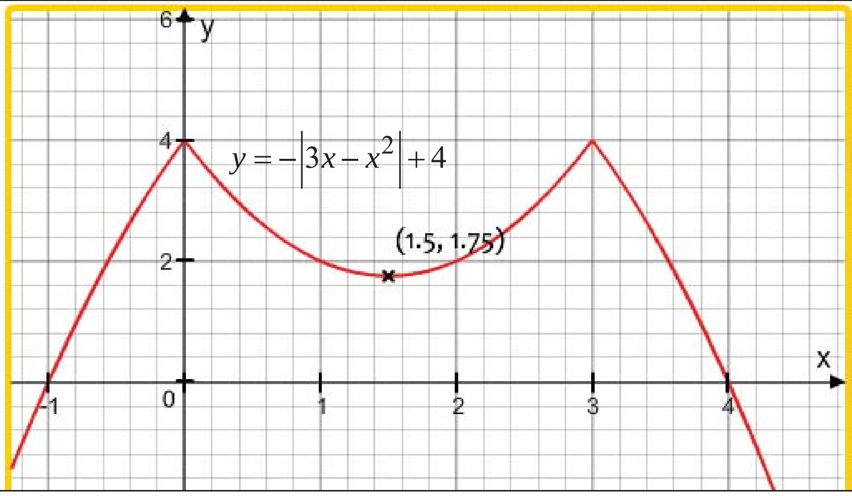
Marking Scheme

1a	The equation of a curve is $y = mx^2 + (m-3)x + m$, where m is a constant. Find the range of values of m for which the curve lies completely above the x-axis.	[5]
	Since the curve is completely above the x-axis, the curve must be a quadratic curve with minimum point , $a > 0$ $\therefore m > 0$ And there will be no point of intersection with the x-axis, $b^2 - 4ac < 0$ $(m-3)^2 - 4(m)(m) < 0$ $m^2 - 6m + 9 - 4m^2 < 0$ $-3m^2 - 6m + 9 < 0$ $m^2 + 2m - 3 > 0$ $(m+3)(m-1) > 0$ $m < -3$ or $m > 1$   Hence, $m > 1$	B1 M1 M1 A1 A1
1b	Given that $y = ax^2 - 4x + c$ is always negative, give an example of values of a and c which satisfy the condition.	[2]
	$b^2 - 4ac < 0$ $(-4)^2 - 4ac < 0$ $ac > 4,\quad a < 0$ $a = -1$ and $c = -5$ or $a = -2$ and $c = -2$ or $a = -5$ and $c = -1$ or any pairs of values that fulfill the above criteria	M1 A1

2a	Given that $2x^4+3x^3+ax^2-9x+9=\left(x^2-1\right)(x-2)\mathrm{Q}(x)-3x^2+bx+c$ is an identity, state, with reason, the degree of $\mathrm{Q}(x)$.	[1]
	Since degree of dividend = degree of quotient + degree of divisor, degree of $\mathrm{Q}(x)=1$ ALTERNATIVE: Must multiply with x3 to give degree 4 in the polynomial on the left.	B1 for reason and answer
2b	Find the value of a, of b and of c.	[5]
	$2x^4+3x^3+ax^2-9x+9=\left(x^2-1\right)(x-2)\mathrm{Q}(x)-3x^2+bx+c$ When $x = 1$, $2+3+a-9+9=-3+b+c$ $a=-8+b+c \quad --------(1)$ When $x = -1$, $2-3+a+9+9=-3-b+c$ $a=-20-b+c \quad ---------(2)$ (1) = (2): $-20-b+c=-8+b+c$ $-12=2b$ $b=-6$ When $x = 2$, $32+24+4a-18+9=-12-12+c$ $c=47+4a+24$ $c=4a+71 \quad ---------(3)$ Sub (3) and b = –6 into (2) $a=-20+6+4a+71$ $a=4a+57$ $a=-19$ Sub a = –19 into (2) $c=4(-19)+5$ $c=-5$ $a=-19, \quad b=-6, \quad c=-5$	M1 M1 M1 M1 for solving A1 for 3 answers
2c	Hence, find the remainder when $2x^4+3x^3+ax^2-9x+9$ is divided by $(3x-1)$.	[1]
	Let $\mathrm{f}(x)=2x^4+3x^3-19x^2-9x+9$	

	$f\left(\frac{1}{3}\right)=2\left(\frac{1}{3}\right)^4+3\left(\frac{1}{3}\right)^3-19\left(\frac{1}{3}\right)^2-9\left(\frac{1}{3}\right)+9$ $f\left(\frac{1}{3}\right)=\frac{326}{81}$ Therefore the reminder is $\frac{326}{81}$.	B1
3a	Given that $p=3^x$ and $q=3^y$, express $\log_3 \frac{p^7 q}{243}$ in terms of *x* and *y*.	[4]
	$p=3^x \Rightarrow \quad x=\log_3 p \qquad q=3^y \Rightarrow \quad y=\log_3 q$ $\log_3 \frac{p^7 q}{243}=7\log_3 p+\log_3 q-\log_3 3^5$ $\log_3 \frac{p^7 q}{243}=7x+y-5$	[M1 for change to log] M2 for using rules A1
3b	Given that $\log_2 x-\log_x x^2=\frac{1}{3}-\log_8 2x$, find the value of *x* by leaving your answer in index form.	[4]
	$\log_2 x-\log_x x^2=\frac{1}{3}-\log_8 2x$ $\log_2 x-2\log_x x=\frac{1}{3}-(\log_8 2+\log_8 x)$ $\log_2 x-2(1)=\frac{1}{3}-\frac{1}{3}\log_8 8-\frac{\log_2 x}{\log_2 8}$ $\log_2 x-2=\frac{1}{3}-\frac{1}{3}-\frac{\log_2 x}{3}$ $3\log_2 x-6=-\log_2 x$ $4\log_2 x=6$ $\log_2 x=\frac{3}{2}$ $x=2^{\frac{3}{2}}$ or $x=8^{\frac{1}{2}}$ or $x=64^{\frac{1}{4}}$	M1 for power rule and product rule M1 for change of base rule M1 A1
4a	Without using a calculator, express $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}$ in the form of $a\sqrt{10}+b\sqrt{3}$.	[4]
		M1 for rationalizati on

	$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}=\frac{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{15}-\sqrt{2}\right)}{15-2}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}=\frac{\sqrt{90}-\sqrt{12}-\sqrt{75}+\sqrt{10}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}=\frac{3\sqrt{10}-2\sqrt{3}-5\sqrt{3}+\sqrt{10}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}=\frac{4\sqrt{10}-7\sqrt{3}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}=\frac{4}{13}\sqrt{10}-\frac{7}{13}\sqrt{3}$	M1 for expansion M1 for simplifying A1
4b	Without the use of a calculator, solve the equation $\sqrt[3]{27^x}-81^{x+1}=0$.	[3]
	$\sqrt[3]{27^x}-81^{x+1}=0$ $27^{\frac{x}{3}}-81^{x+1}=0$ $\left(3^3\right)^{\frac{x}{3}}-\left(3^4\right)^{x+1}=0$ $3^x-3^{4x+4}=0$ $3^x=3^{4x+4}$ By comparing powers, $x=4x+4$ $x=-\frac{4}{3}$	M1 for converting all terms to base 3 M1 for using if $a^m=a^n$ then $m=n$ [A1]
5ai	Given the curve $y=-\left\|3x-x^2\right\|+4$, find the *x*-coordinates of the points where the curve meets the *x*-axis.	[2]
	When $y=0$, $0=-\left\|3x-x^2\right\|+4$ $3x-x^2=4$ or $3x-x^2=-4$ $3x-x^2-4=0$ or $3x-x^2+4=0$ $x^2-3x+4=0$ or $x^2-3x-4=0$ $b^2-4ac=(-3)^2-4(1)(4)$ or $(x-4)(x+1)=0$ $b^2-4ac=-7$ or $x=4$ or $x=-1$	M1 for considering positive and negative A1 for answers
5aii	Sketch the curve $y=-\left\|3x-x^2\right\|+4$, giving the coordinates of the maximum point and of the points where the curve meets the axes..	[3]

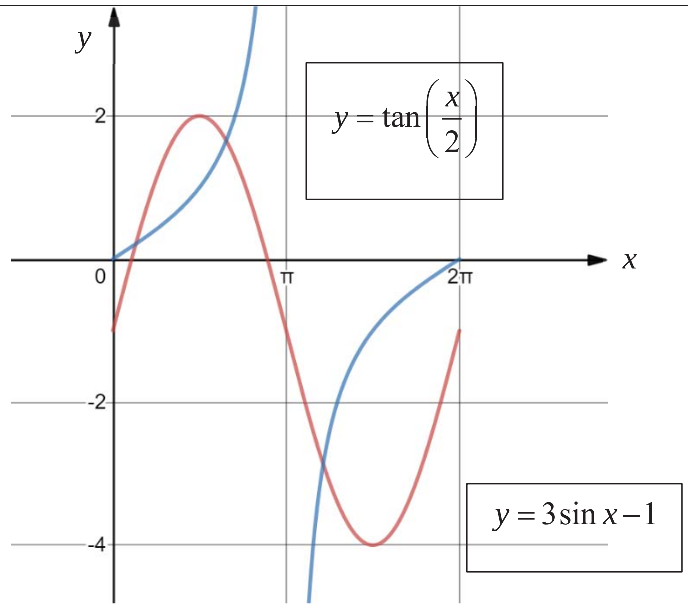
		B1 for shape B1 for coordinates of max pt and *x*-intercepts. B1 for labelling.
5b	Explain why there are only two solutions to the equation $-\left\|3x-x^2\right\| = k-4$ for k < 1.75.	[2]
	$-\left\|3x-x^2\right\| = k-4$ $-\left\|3x-x^2\right\| + 4 = k$ $y = k$ $y = k$ is a **horizontal line** and when k is lesser than 2.25, it will be **below the turning point** and so it will only **intersect the curve at the two outer arms thereby giving two solutions only.**	B1 B1
5c	Determine the maximum value of m for which the line $y = mx + 1$ intersects the graph of $y = -\left\|3x - x^2\right\| + 4$ in three points.	[1]
	$m = \dfrac{4-1}{3-0}$ $m = 1$	B1
6(i)	Express $\dfrac{7x+11}{(x-1)(x+2)^2}$ in partial fractions.	[4]
	Let $\dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2} = \dfrac{7x+11}{(x-1)(x+2)^2}$ $A(x+2)^2 + B(x-1)(x+2) + C(x-1) = 7x+11$ when $x = 1$, $9A = 7+11$ $A = 2$ when $x = -2$ $-3C = -14+11$ $C = 1$ when $x = 0$ $4A - 2B - C = 11$ $4(2) - 2B - 1 = 11$	[M1 combining fractions together and equating the numerator] [M1 for substitution or any other method]

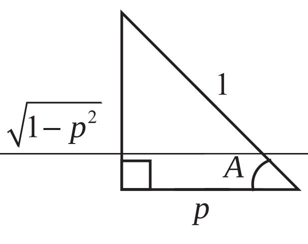
	$-2B=4$ $B=-2$ $\therefore \frac{7x+11}{(x-1)(x+2)^2}=\frac{2}{x-1}-\frac{2}{x+2}+\frac{1}{(x+2)^2}$	[A2 for fully correct partial fractions. A1 only if at least two terms are correct]
6(ii)	Hence, find $\int \frac{7x+11}{2(x-1)(x+2)^2}\,dx$.	[3]
	$\int \frac{7x+11}{2(x-1)(x+2)^2}\,dx=\frac{1}{2}\int \frac{2}{x-1}-\frac{2}{x+2}+\frac{1}{(x+2)^2}\,dx$ $=\ln(x-1)-\ln(x+2)-\frac{1}{2(x+2)}+c$	M1 for taking out coefficient A2 for all correct, A1 if at least two terms are correct
7	A piece of wire which has a fixed length of *k* cm long is bent to form a rectangle. Show that the area of the rectangle is a maximum when it is a square.	[5]
	Let the length of the rectangle be *x*. Breadth of rectangle = $\frac{k-2x}{2}$ Let area of the rectangle be *A*. $A=x\left(\frac{k-2x}{2}\right)$ $A=\frac{xk-2x^2}{2}$ $A=\frac{1}{2}xk-x^2$ $\frac{dA}{dx}=\frac{1}{2}k-2x$ At the stationary value, $\frac{dA}{dx}=0$ $\frac{1}{2}k-2x=0$ $2x=\frac{k}{2}$	[M1] [M1] [M1]

	$x=\frac{k}{4}$ $\frac{d^2A}{dx^2}=-2<0$ Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square.	[M1] [A1]
8	Given a circle with the equation $(2x+5)(x+2)+(2y+1)(y-5)=0$,	
8(i)	Express the equation of the circle in standard form.	[5]
	$(2x+5)(x+2)+(2y+1)(y-5)=0$ $2x^2+9x+10+2y^2-9y-5=0$ $2x^2+2y^2+9x-9y+5=0$ $x^2+y^2+\frac{9}{2}x-\frac{9}{2}y+\frac{5}{2}=0$ $x^2+\frac{9}{2}x+\left(\frac{9}{4}\right)^2+y^2-\frac{9}{2}x+\left(\frac{9}{4}\right)^2=-\frac{5}{2}+\left(\frac{9}{4}\right)^2+\left(\frac{9}{4}\right)^2$ $\left(x+\frac{9}{4}\right)^2+\left(y-\frac{9}{4}\right)^2=\frac{61}{8}$ Coordinates of centre = $\left(-\frac{9}{4},\frac{9}{4}\right)$ Radius of circle = $\sqrt{\frac{61}{8}}$ units Equation of circle, $\left(x+\frac{9}{4}\right)^2+\left(y-\frac{9}{4}\right)^2=\frac{61}{8}$	M1 for expansion and simplification M1 – for getting the centre and radius [A1] [A1] [A1]
8(ii)	Find the length of the chord when the line $y=-2x$ cuts the circle.	[5]
	$y=-2x$ -(1) $\left(x+\frac{9}{4}\right)^2+\left(y-\frac{9}{4}\right)^2=\frac{61}{8}$ -(2) Sub (1) into (2) $\left(x+\frac{9}{4}\right)^2+\left(-2x-\frac{9}{4}\right)^2=\frac{61}{8}$	[M1 for substitution]

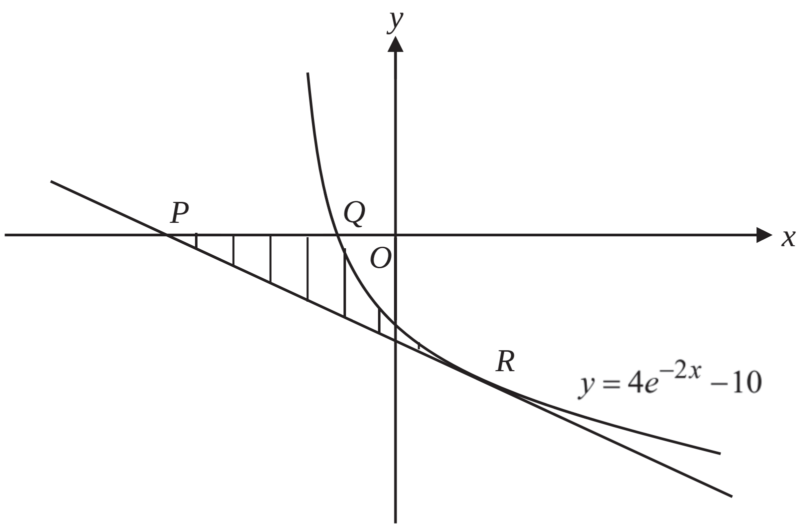
	$5x^2+\frac{27}{2}x+\frac{5}{2}=0$ $10x^2+27x+5=0$ $(5x+1)(2x+5)=0$ $x=-\frac{1}{5}$ *or* $x=-\frac{5}{2}$ when $x=-\frac{1}{5}$, $y=\frac{2}{5}$ when $x=-\frac{5}{2}$, $y=5$ Thus the coordinates of the end points of the chord are $\left(-\frac{1}{5},\frac{2}{5}\right)$ and $\left(-\frac{5}{2},5\right)$ Length of chord = $\sqrt{\left(2\frac{3}{10}\right)^2+\left(-4\frac{3}{5}\right)^2}$ = 5.14 units (to 3 s.f.)	[M1 for solving] [A1 for correct coordinates] [M1] [A1]
9ai	Prove the identity $\sin x\cos x+\cot x\cos^2 x=\cot x$.	[4]
	LHS = $\sin x\cos x+\cot x\cos^2 x$ $=\cos x(\sin x+\cot x\cos x)$ $=\cos x\left(\sin x+\frac{\cos x}{\sin x}\cos x\right)$ $=\cos x\left(\sin x+\frac{\cos^2 x}{\sin x}\right)$ $=\cos x\left(\frac{\sin^2 x+\cos^2 x}{\sin x}\right)$ $=\cos x\left(\frac{1}{\sin x}\right)$ $=\cot x=$ RHS Or LHS = $\sin x\cos x+\cot x\cos^2 x$ $=\sin x\cos x+\frac{\cos x}{\sin x}\cos^2 x$ $=\frac{\sin^2 x\cos x}{\sin x}+\frac{\cos^3 x}{\sin x}$	[M1] [M1] [M1] [A1] [M1]

	$= \dfrac{\sin^2 x \cos x + \cos^3 x}{\sin x}$ $= \dfrac{\cos x\left(\sin^2 x + \cos^2 x\right)}{\sin x}$ $= \dfrac{\cos x(1)}{\sin x}$ $= \cot x = \text{RH}$	[M1] [M1] [A1]
9aii	Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \le x \le \pi$.	[3]
	Since $\sin x \cos x + \cot x \cos^2 x = \cot x$ Therefore, $\sin 3x \cos 3x + \cot 3x \cos^2 3x = \cot 3x$ and $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1 \quad \Rightarrow \quad \cot 3x = 1$ $0 \le x \le \pi$ $0 \le 3x \le 3\pi$ Let the basic angle be α $\tan \alpha = 1$ $\alpha = \dfrac{\pi}{4}$ $3x = \dfrac{\pi}{4}, \pi + \dfrac{\pi}{4}, 2\pi + \dfrac{\pi}{4}$ $3x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}$ $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{3\pi}{4}$ rad	[M1] [M1] [A1]
9bi	On the same axes, sketch the graphs of $y = 3\sin x - 1$ and $y = \tan\dfrac{x}{2}$ for $0 \le x \le 2\pi$.	[5]
		For tan curve, [B1] for shape, [B1] for asymptote For Sine curve, [B1] for shape [B1 for correct max and

		minimum values [B1] for correct period for both graphs [subtract 1mark for wrong or no labels for axes or functions]
9bii	Hence, state the number of solutions of $3\sin x - 1 = \tan\frac{x}{2}$ for $0 \le x \le 2\pi$.	[1]
	From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \le x \le 2\pi$	[B1]
10a	Two particles, *A* and *B*, leaves a point *O* at the same time and travel in the same direction along the same straight line. Particle *A* starts with a velocity of 9 m/s and moves with a constant acceleration of 2 m/s^2. Particle *B* starts from rest and moves with an acceleration of *a* m/s^2, where $a = 1 + \frac{t}{3}$ and *t* seconds is the time since leaving *O*. Find an expression for the velocity of each particle in terms of *t*,	[3]
	For Particle *A*, $v_A = \int 2\, dt$ $v_A = 2t + c$ When $t = 0$, $v_A = 9$ $c = 9$ $v_A = 2t + 9$ For Particle *B*, $v_B = \int \left(1 + \frac{t}{3}\right) dt$ $v_B = t + \frac{1}{6}t^2 + c$ When $t = 0$, $v_B = 0$, $c = 0$ $v_B = t + \frac{1}{6}t^2$	B1 M1 A1
10b	an expression for the displacement of each particle in terms of *t*,	[3]

	For Particle A, $s_A = \int (2t+9)\, dt$ $s_A = t^2 + 9t + c$ When $t = 0$, $s_A = 0$ $c = 0$ $s_A = t^2 + 9t$ For Particle B, $s_B = \int \left(t + \frac{1}{6}t^2 \right) dt$ $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3 + c$ When $t = 0$, $s_B = 0$, $c = 0$ $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$	B1 M1 A1
10c	the distance from O at which particle B collides with A,	[3]
	When particle B collides with particle A, $s_A = s_B$ $t^2 + 9t = \frac{1}{2}t^2 + \frac{1}{18}t^3$ $18t^2 + 162t = 9t^2 + t^3$ $t^3 - 9t^2 - 162t = 0$ $t\left(t^2 - 9t - 162\right) = 0$ $t(t-18)(t+9) = 0$ $t = 0(\text{N.A.})$ or $t = 18$ or $t = -9(\text{N.A.})$ Distance from O = $(18)^2 + 9(18) = 486$ m	M1 M1 A1
10d	the speed of each particle at the point of collision.	[2]
	Speed of particle A = $2(18) + 9 = 45$ m/s Speed of particle B = $(18) + \frac{1}{6}(18)^2 = 72$ m/s	B1 B1
11	Given that $\cos A = p$ and that A is acute, express the following in terms of p.	
i	$\sin 2A$	[3]
		M1 for getting the length of opposite side

	$\sin 2A = 2\sin A\cos A$ $= 2p\sqrt{1-p^2}$ Or $\cos^2 A + \sin^2 A = 1$ $\sin^2 A = 1 - \cos^2 A$ $\sin^2 A = 1 - p^2$ $\sin A = \sqrt{1-p^2}$ (reject negative as A is acute) $\sin 2A = 2\sin A\cos A$ $= 2p\sqrt{1-p^2}$	M1 A1 M1 M1 A1
ii	$\cos\frac{A}{2}$	[3]
	$\cos A = 2\cos^2\frac{A}{2} - 1$ $\cos\frac{A}{2} = \pm\sqrt{\frac{\cos A + 1}{2}}$ $\cos\frac{A}{2} = \pm\sqrt{\frac{p+1}{2}}$ $\cos\frac{A}{2} = -\sqrt{\frac{p+1}{2}}$ (rejected) or $\cos\frac{A}{2} = \sqrt{\frac{p+1}{2}}$	M1 M1 for rejection A1
12	The diagram shows the curve, $y = 4e^{-2x} - 10$. The curve crosses the $x-$axis at Q. The line PR is a tangent to the curve at R and intersects the $x-$axis at P. The $x-$coordinate of R is $\ln 2$. Find the area of the shaded region, PQR , which is the region enclosed by curve, the $x-$axis and the line $x-$axis and the line PR correct to 3 significant figures.	[11]

		
	At R, $x=\ln 2$ $y=4e^{-2\ln 2}-10=-9$ $R(\ln 2,-9)$	B1
	$y=4e^{-2x}-10$ $\frac{dy}{dx}=4(-2)e^{-2x}=-8e^{-2x}$ Gradient of line $PR=-8e^{-2\ln 2}=-2$ Equation of line PR $y-(-9)=-2(x-\ln 2)$ $y=-2x+2\ln 2-9$ $y=-2x-7.6137$	M1 A1
	At P, $y=0$ $0=-2x+2\ln 2-9$ $x=\ln 2-\frac{9}{2}$ $=-3.8069$ $P\left(\ln 2-\frac{9}{2},0\right)$	B1
	At Q, $y=0$ $0=4e^{-2x}-10$ $4e^{-2x}=10$ $e^{-2x}=\frac{5}{2}$ $-2x=\ln\frac{5}{2}$ $x=-\frac{1}{2}\ln\frac{5}{2}=-0.45815$ $Q\left(-\frac{1}{2}\ln\frac{5}{2},0\right)$	M1 – change index form to log form A1
	Area of the shaded region, PQR = Area of the triangle – Area between the curve and the x – axis	M1

	$\text{Area} = \frac{1}{2}\times\left((\ln 2)-\left(\ln 2-\frac{9}{2}\right)\right)\times(9)-\left|\int_{-\frac{1}{2}\ln\frac{5}{2}}^{\ln 2}\left(4e^{-2x}-10\right)dx\right|$	M1
	$=\frac{81}{4}-\left|\left[-2e^{-2x}-10x\right]_{-\frac{1}{2}\ln\frac{5}{2}}^{\ln 2}\right|$	M1
	$=\frac{81}{4}-\left|\left(\left[-2e^{-2\ln 2}-10\ln 2\right]-\left[-2e^{-2\left(-\frac{1}{2}\ln\frac{5}{2}\right)}-10\left(-\frac{1}{2}\ln\frac{5}{2}\right)\right]\right)\right|$	M1
	$=\frac{81}{4}-\left|-7.01295\right|$	
	$=13.23705=13.2\ \text{units}^2$	A1