ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2020

SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4047/01

PAPER 1

15 September 2020

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

For Examiner's Use
80

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Find the values of *x* and *y* which satisfy the equations.

$$3^x \times \sqrt{3^y} = 1$$

$$4^{x-4} \div 32^y = 16^{\frac{1}{x}}$$

[5]

(b) Without using a calculator, find the values of a and b such that $7-3\sqrt{3}=\left(a+b\sqrt{3}\right)\left(2+\sqrt{3}\right)$, where a and b are integers. [4]

2 Find the set of values of the constant k for which the curve $y = (k+2)x^2 - 10x + 2k + 1$ lies completely below the line $y = 2x + 3$. [4]

3 Given that $\sin A = \frac{2}{3}$ and $\tan B = -\frac{1}{\sqrt{3}}$ where angles A and B lie in the same quadrant, leaving your answers in exact form, calculate the value of

(a) $\cos 2B$ [2]

(b) $\sec(A - B)$ [3]

4 In view of a contagious virus, the government of a particular country has imposed a 'Stay-Home-Notice' on her people to reduce the number of human-to-human transmission cases. It is estimated that the percentage of the population, P, complying to the Stay-Home-Notice is given by the equation $P = 100\left(1 - e^{-0.15t}\right)$, where t is the number of days after the imposition.

(i) Find the percentage of population complying to the 'Stay-Home-Notice' after 5 days of the imposition. [1]

(ii) Find the number of complete days after the imposition that it will take for at least 90% of the population to comply. [2]

(iii) Is it possible for the percentage of this country's population complying to the 'Stay-Home-Notice' to reach 100%? Explain your answer. [1]

5 **(i)** In the expansion of $\left(2-\frac{x}{3}\right)^n$, show that the ratio of coefficient of the 2^{nd} term to that of the 4^{th} term can be simplified to the expression $\frac{216}{(n-1)(n-2)}$. [4]

(ii) Find the value of n if the ratio in **(i)** is $108:55$. [2]

(iii) Hence, find the term in x^5. [2]

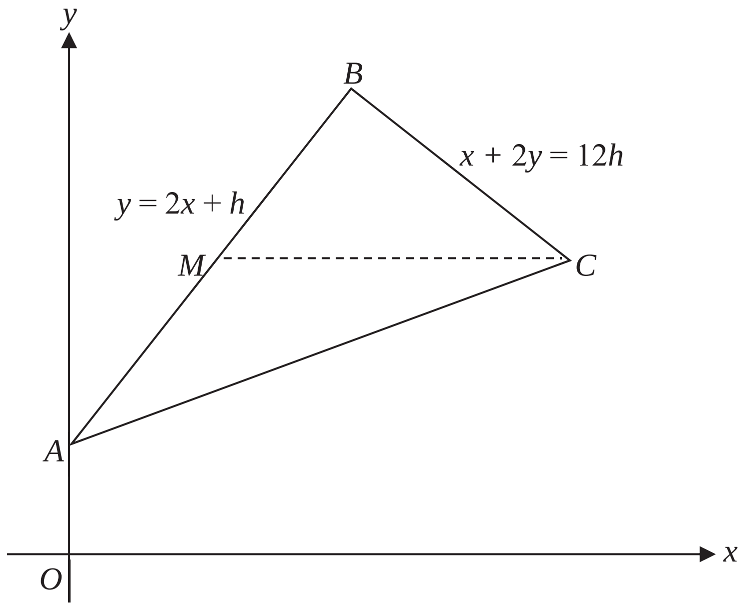
6 The equation of a curve is $y = \dfrac{2x-9}{\sqrt{x^2+1}}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{9x+2}{\sqrt{\left(x^2+1\right)^3}}$. [3]

(ii) Find the coordinates of the stationary point of the curve, leaving your answer in exact form. [3]

(iii) Find the nature of this stationary point. [2]

7



The diagram shows a triangle *ABC* in which *A* lies on the *y*-axis.
The equation of *AB* and *BC* are $y = 2x + h$ and $x + 2y = 12h$.
M is midpoint of *AB* and *MC* is parallel to the *x*-axis.

(i) Explain why *AB* is perpendicular to *BC*. [2]

(ii) Show that coordinates of *B* is $(2h, 5h)$. [2]

(iii) Express the coordinates of M and of C in terms of h. [3]

(iv) Find the value of h if the area of triangle AMC is 125 square units. [2]

8 The equation of a curve is $y = \dfrac{x-6}{x+4} + 4$, where $x \neq -4$.

Find the

(i) gradient of the tangent at the point where the curve passes through the x-axis, [4]

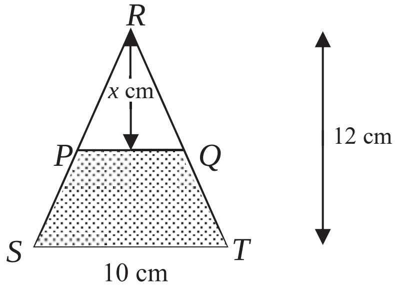
(ii) equation of another tangent to the curve that is parallel to the tangent in **(i)**. [4]

9 The equation of a curve is $y = \cos^3 x + \sin 3x$.

(i) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Given that $\dfrac{d^2y}{dx^2} + 9y = A\cos x$, find the value of A. [2]

10 The diagram shows an isosceles triangle RST with height 12 cm and $ST = 10$ cm. PQ moves towards ST at a steady rate of 0.5 cm/s, keeping parallel to ST.



If PQ is x cm from R,

(i) Show that $PQ = \dfrac{5x}{6}$ cm. [1]

(ii) Find the area, A cm^2, of the shaded region in terms of x. [2]

(iii) Find the rate of change of the shaded area when PQ is halfway towards ST from R. [3]

11 It is given that $f(x) = 3\cos 2x - 1$ and $g(x) = \dfrac{2x}{\pi} - 2$.

(i) State the least and greatest values of $f(x)$. [2]

(ii) State the period of $f(x)$. [1]

(iii) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $-\pi \le x \le \pi$. [4]

(iv) State, with detailed workings, the number of solutions of the equation $3\pi\cos 2x = 2x - \pi$ for $-\pi \le x \le \pi$. [2]

12 The function *f* is defined by $y = (x+1)^3 e^{2x-3}$, for $x > -\frac{5}{2}$, and $x \ne -1$. Explain, with working, whether *f* is an increasing or decreasing function. [4]

End of paper

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4047/02

PAPER 2

17 September 2020
2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use
100

This question paper consists of **19** printed pages (including this cover page)

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer <u>all</u> questions

1 **(i)** Differentiate $\ln(\cos x)$ with respect to x. [1]

(ii) Hence find $\int \tan x \, dx$. [2]

(iii) Differentiate $x \tan x$ with respect to x. [2]

(iv) Using the results from (ii) and (iii), hence find $\int x \sec^2 x \, dx$. [3]

2 **(i)** Prove that $\dfrac{(\sin A+\cos A)(1-\sin A\cos A)}{\sin^3 A}=1+\cot^3 A$ [4]

(ii) Hence solve the equation $(\sin A+\cos A)(1-\sin A\cos A)=2\sin^3 A$ for $0° \le A \le 180°$. [3]

3 **(i)** The expression $3x^3 + hx^2 + kx - 4$, where h and k are constants, has a factor of $3x - 1$ and leaves a remainder of -4 when divided by $x + 1$.
By forming 2 equations, of h and of k, show that the value of $h = 11$ and $k = 8$. [4]

(ii) Hence, express $\dfrac{-x^2+6x+9}{3x^3+hx^2+kx-4}$ as the sum of three partial fractions [7]

4 A particle, moving in a straight line, passes through a fixed point *O* with a speed of 4 m/s. The acceleration, a m/s^2, of the particle, *t* s after passing through *O*, is given by $a = -\frac{4}{15}e^{-\frac{t}{60}}$.

(i) Find the exact value of *t* when the particle is at instantaneous rest. [6]

(ii) Find an expression in term of *t*, for the displacement, from *O*, of the particle *t* seconds after passing *O*. [3]

4 **(iii)** Hence find the distance of the particle from *O* when it is at instantaneous rest. [1]

(iv) Show that the particle is again at *O* at some instant during the thirty-seventh second after first passing through *O*. [2]

5 **(i)** The equation $\log_3 x - \frac{1}{2}\log_{27} x = \log_2 8$ has the solution $x = 3^k$.

Find the value of k. [4]

(ii) Solve the equation $2\log_4(x-2) - \log_4(x+10) = \frac{1}{2}$. [4]

6 The roots of the quadratic equation $2x^2 = x + 3$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [2]

(ii) Find the quadratic equation whose roots are $\dfrac{\beta}{\alpha^2}$ and $\dfrac{\alpha}{\beta^2}$. [5]

7 The points $A(2, 1)$ and $B(11, -2)$ lie on a circle.

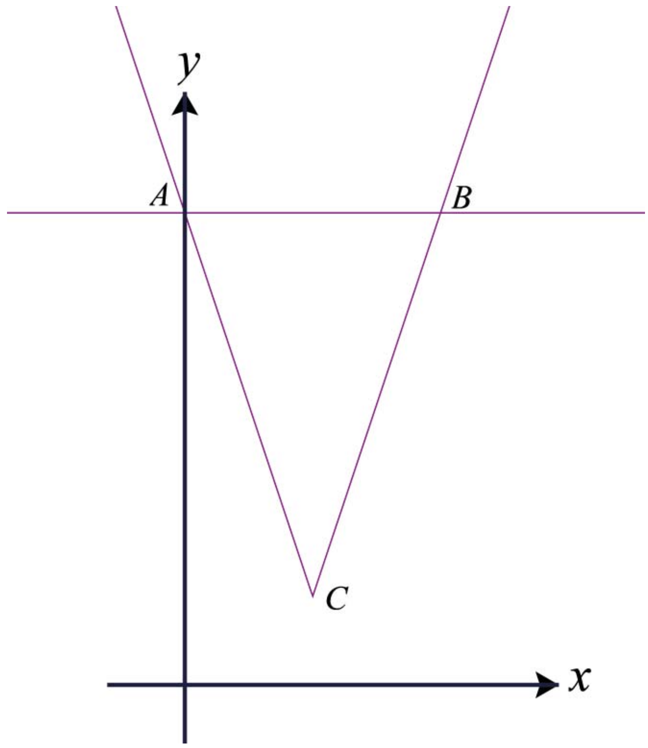
(i) Find the equation of the perpendicular bisector of the chord AB. [4]

The line with equation $y = \frac{4}{3}x - 10$ is a normal to the circle.

(ii) Hence, find the equation of the circle. [5]

(iii) Find the coordinates of the point on the circle which is at the greatest distance from the x-axis. [1]

8



The diagram shows part of the graph of $y = |3x - 7| + 2$. A horizontal line is drawn from *A* to intersect the graph of $y = |3x - 7| + 2$ at *B*.

(i) Find the coordinates of *A*, *B* and *C*. [3]

8 **(ii)** Find the set of values of m for which the graph of $y = |3x - 7| + 2$ and the line $y = mx$ intersects at 2 points. [2]

(iii) Solve the equation $|3x - 7| + 2 = 4x - 1$ [3]

9 The resistance to motion, R newtons, of a plank towed through water and its speed, V m/s is given by $R = AV^n$, where A and n are constants.
The table shows the corresponding values of V and R.

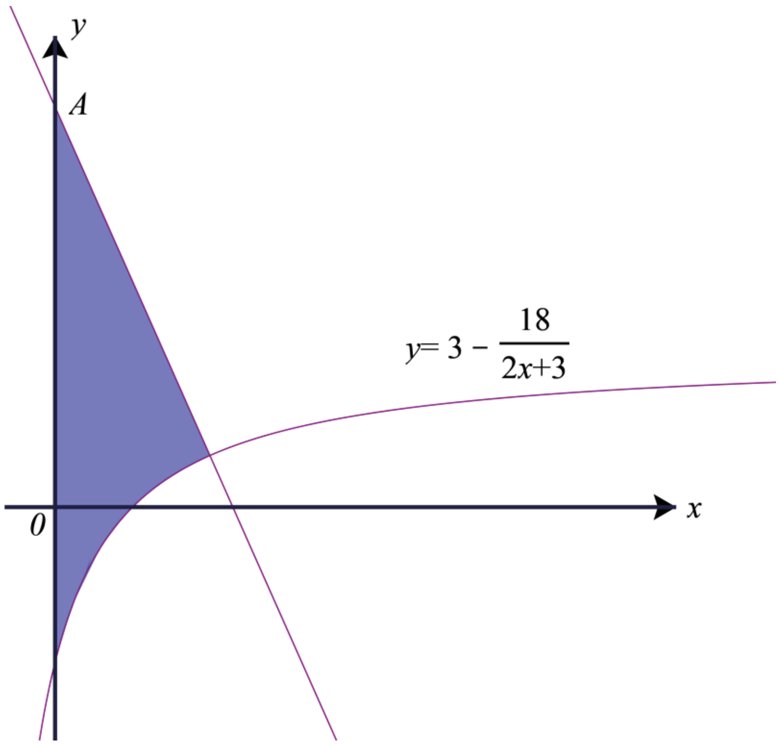
V	1.65	2.46	3.68	6.05
R	1.87	3.70	7.33	17.1

(i) On the grid on page 15, draw the graph of ln R against ln V, using a scale of 4 cm for 0.5 unit on the ln V-axis and a scale of 5 cm to 1 unit on the ln R-axis. [3]

(ii) Use the graph to estimate the value of each of the constants A and n, giving your answers correct to 1 decimal place. [4]

(iii) By drawing a suitable line on your graph, solve the equation $AV^n = V^{1.2}$, giving your answer correct to 1 decimal place. [3]

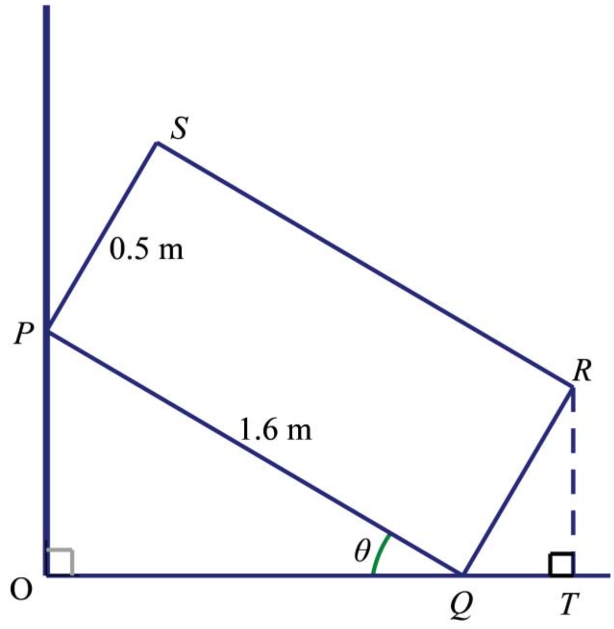
10



The diagram show part of the curve $y = 3 - \dfrac{18}{2x+3}$. The normal to the curve at $x = 3$ intersects the y-axis at A. Find the exact area of the shaded region. [11]

Continuation of Working Space for Question 10.

11



A rectangular table *PQRS* is positioned at a corner of a room. Given that $PQ = 1.6\text{m}$ $PS = 0.5$ m, angle $OQP = \theta$ and angle QOP = angle $RTQ = 90°$, where θ varies.

(i) Show that $OT = 1.6 \cos\theta + 0.5 \sin\theta$. [3]

11 **(ii)** Find the value of θ for which $OT = 1.2$ m [5]

End of Paper