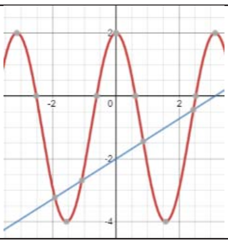
Answer Key

1a	$x = -\frac{1}{3}, y = \frac{2}{3}$ or $x = 1, y = -2$	7i	Since the product of 2 gradients is – 1, *AB* is perpendicular to *BC*.
b	$a = 23, \qquad b = -13$	ii	Proof
2	$k < -5$	iii	$M = (h, 3h) \quad C(6h, 3h)$
3a	$\frac{1}{2}$	iv	$h = 5$
b	$\frac{6\sqrt{15} - 12}{11}$	8i	$\left.\frac{dy}{dx}\right\|_{x=-2} = \frac{5}{2}$
4i	52.8%	ii	$2y = 5x + 50$
ii	16	9i	$\frac{dy}{dx} = -3cos^2xsinx + 3cos3x$ $\frac{d^2y}{dx^2} = -3cos^3x + 6sin^2xcosx - 9sin3x$
iii	As t gets very large, $e^{-0.15t}$ approaches zero but will never reach zero. Therefore, the percentage will not reach 100%.	ii	A = 6
5i	Proof	10i	Proof
ii	$n = 12$	ii	$60 - \frac{5}{12}x^2$
iii	$-\frac{11264}{27}x^5$	iii	2.5cm^2/s
6i	Proof	11i	Least value = −4 Greatest value = 2
ii	$\left(-\frac{2}{9}, -\sqrt{85}\right)$	ii	π
iii	minimum point	iii	
		iv	4 solutions
		12	$\frac{dy}{dx} = e^{2x-3}(x+1)^2[5+2x]$ $x > -\frac{5}{2}, 2x + 5 > 0, (x+1)^2$ *and* $e^{2x-3} > 0$
			$\frac{dy}{dx} > 0,$ *the curve is increasing*

Answer Key

1(i)	$-\tan x$	8(i)	$A=(0,9)$, $B=\left(\frac{7}{3},\ 2\right)$, $C=\left(\frac{14}{3},\ 9\right)$
(ii)	$-\ln(\cos x)+c$	(ii)	$\frac{6}{7}<m<3$
(iii)	$\tan x+x\sec^2 x$	(iii)	$x=-4$(rejected) or $x=\frac{10}{7}$
(iv)	$x\tan x+\ln(\cos x)+\mathrm{c}$	9(ii)	$n=\text{gradient}=1.7$, $A=\mathrm{e}^{-0.24}=0.8$
2(ii)	45°	(iii)	1.6
3(i)	$h+3k=35$, $h-k=3$	10	$\frac{33}{8}+9\ln 3$
(ii)	$\frac{2}{3x-1}-\frac{1}{x+2}+\frac{1}{(x+2)^2}$	11(ii)	61.6°
4(i)	$-60\ln\frac{3}{4}$		
(ii)	$s=-960\mathrm{e}^{-\frac{t}{60}}-12t+960$		
(iii)	32.9 m		
5(i)	$\frac{18}{5}$		
(ii)	$x=8$ or $x=-2$(rejected)		
6(i)	$\frac{13}{4}$		
(ii)	$x^2-\frac{19}{18}x-\frac{2}{3}=0$		
7(i)	$y=3x-20$		
(ii)	$(x-6)^2+(y+2)^2=25$		
(iii)	$(6,\ -7)$		

ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2020
SECONDARY 4E/5N

Candidate's Name	Class	Register Number
MARK SCHEME		

ADDITIONAL MATHEMATICS 4047/01

PAPER 1

15 September 2020
2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

For Examiner's Use
80

This question paper consists of **21** printed pages (including this cover page)

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1	**(a)**	Find the values of *x* and *y* which satisfy the equations.	
		$3^x \times \sqrt{3^y} = 1$ $4^{x-4} \div 32^y = 16^{\frac{1}{x}}$	[5]
		$3^x \times \sqrt{3^y} = 1$ $\qquad$ $4^{x-4} \div 32^y = 16^{\frac{1}{x}}$	
		$3^x \times 3^{\frac{1}{2}y} = 3^0$ $\qquad$ $2^{2x-8} \div 2^{5y} = 2^{\frac{4}{x}}$	[M1] applying laws of indices correctly
		$3^{x+\frac{1}{2}y} = 3^0$ $\qquad$ $2^{2x-8-5y} = 2^{\frac{4}{x}}$	
		$x + \frac{1}{2}y = 0$ $\qquad$ $2x - 8 - 5y = \frac{4}{x}$	
		$x = -\frac{1}{2}y$ --- (1) $\qquad$ $2x^2 - 5xy - 8x - 4 = 0$ --- (2)	[A1] for either (1) or (2) correctly
		Sub (1) into (2):	
		$2\left(-\frac{1}{2}y\right)^2 - 5\left(-\frac{1}{2}y\right)y - 8\left(-\frac{1}{2}y\right) - 4 = 0$	[M1] substitution
		$2\left(\frac{1}{4}y^2\right) + \frac{5}{2}y^2 + 4y - 4 = 0$	
		$3y^2 + 4y - 4 = 0$	
		$(3y-2)(y+2) = 0$	
		$y = \frac{2}{3}$ $\quad$ or $\quad$ $y = -2$	
		$y = \frac{2}{3}$ $\qquad$ $x = -\frac{1}{2}\left(\frac{2}{3}\right) = -\frac{1}{3}$	[A1] both *x* and *y* correct
		When	
		$y = -2$ $\qquad$ $x = -\frac{1}{2}(-2) = 1$	[A1] both *x* and *y* correct

	(b)	Without using a calculator, find the values of *a* and *b* such that $7-3\sqrt{3}=\left(a+b\sqrt{3}\right)\left(2+\sqrt{3}\right)$, where *a* and *b* are integers.	[4]
		$7-3\sqrt{3}=\left(a+b\sqrt{3}\right)\left(2+\sqrt{3}\right)$	
		$a+b\sqrt{3}=\dfrac{7-3\sqrt{3}}{2+\sqrt{3}}$	
		$=\dfrac{7-3\sqrt{3}}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$	[M1] rationalisation
		$=\dfrac{7\left(2-\sqrt{3}\right)-3\sqrt{3}\left(2-\sqrt{3}\right)}{4-3}$	
		$=14-7\sqrt{3}-6\sqrt{3}+9$	[M1] expansion/simplification
		$=23-13\sqrt{3}$	
		$a=23, \qquad b=-13$	[A1] [A1]
		Alternative solution	
		$a\left(2+\sqrt{3}\right)+b\sqrt{3}\left(2+\sqrt{3}\right)=7-3\sqrt{3}$	
		$2a+a\sqrt{3}+2b\sqrt{3}+3b=7-3\sqrt{3}$	
		$2a+3b+a\sqrt{3}+2b\sqrt{3}=7-3\sqrt{3}$	
		$2a+3b=7$ --- (1)	[M1] grouping
		$a+2b=-3$ --- (2)	
		(2) into (1):	
		$2(-3-2b)+3b=7$	[M1] simultaneous equation
		$-6-4b+3b=7$	
		$b=-13$	[A1]
		$a=23$	[A1]

2	Find the set of values of the constant k for which the curve $y=(k+2)x^2-10x+2k+1$ lies completely below the line $y=2x+3$.	[4]
	$(k+2)x^2-10x+2k+1<2x+3$	
	$(k+2)x^2-10x+2k+1-2x-3<0$	
	$(k+2)x^2-12x+2k-2<0$	[M1] eliminate y
	$b^2-4ac<0$ and $k+2<0$	[B1] use of correct discriminant
	$(-12)^2-4(k+2)(2k-2)<0$ and $k<-2$	
	$144-(8k^2+8k-16)<0$	
	$-8k^2-8k+160<0$	
	$k^2+k-20>0$	
	$(k-4)(k+5)>0$	
	$k<-5$ or $k>4$	[A1]
	$\therefore k<-5$	[A1]

3	Given that $\sin A = \frac{2}{3}$ and $\tan B = -\frac{1}{\sqrt{3}}$ where angles A and B lie in the same quadrant, leaving your answers in exact form, calculate the value of		
	(a)	$\cos 2B$ $\cos 2B = 2\cos^2 B - 1$ $= 2\left(-\frac{\sqrt{3}}{2}\right)^2 - 1$ $= 2\left(\frac{3}{4}\right) - 1$ $= \frac{1}{2}$	[M1] Applying double angle [A1]
	(b)	$\sec(A-B)$ $\sec(A-B) = \frac{1}{\cos(A-B)}$ $= \frac{1}{\cos A \cos B + \sin A \sin B}$ $= 1 \div \left(-\frac{\sqrt{5}}{3} \times -\frac{\sqrt{3}}{2} + \frac{2}{3} \times \frac{1}{2}\right)$ $= 1 \div \left(\frac{\sqrt{15}}{6} + \frac{1}{3}\right)$ $= 1 \div \left(\frac{\sqrt{15}+2}{6}\right)$ $= \frac{6}{\sqrt{15}+2}$ $= \frac{6\sqrt{15}-12}{11}$	[B1] change from sec to cos [M1] addition formula [A1]

4	In view of a contagious virus, the government of a particular country has imposed a 'Stay-Home-Notice' on her people to reduce the number of human-to-human transmission cases. It is estimated that the percentage of the population, P, complying to the Stay-Home-Notice is given by the equation $P=100\left(1-e^{-0.15t}\right)$, where t is the number of days after the imposition.		
	(i)	Find the percentage of population complying to the 'Stay-Home-Notice' after 5 days of the imposition.	[1]
		$P=100\left(1-e^{-0.15t}\right)$ $=100\left(1-e^{-0.15\times 5}\right)$ $=52.763$ $=52.8\%$	[B1]
	(ii)	Find the number of days after the imposition that it will take for at least 90% of the population to comply.	[2]
		$90=100\left(1-e^{-0.15t}\right)$ $0.9=1-e^{-0.15t}$ $0.1=e^{-0.15t}$ $\ln e^{-0.15t}=\ln 0.1$ $-0.15t=\ln 0.1$ $t=15.35$ ≈ 15.4 No. of complete days = 16	[M1] take ln [A1]
	(iii)	Is it possible for the percentage of this country's population complying to the 'Stay-Home-Notice' to reach 100%? Explain your answer.	[1]
		As t gets very large, $e^{-0.15t}$ approaches zero but will never reach zero. Therefore, the percentage will not reach 100%.	[B1]

5	**(i)**	In the expansion of $\left(2-\frac{x}{3}\right)^n$, show that the ratio of coefficient of the 2nd term to that of the 4th term can be simplified to the expression $\frac{216}{(n-1)(n-2)}$.	[4]
		$\left(2-\frac{x}{3}\right)^n$ $=2^n-\binom{n}{1}2^{n-1}\left(\frac{x}{3}\right)+\binom{n}{2}2^{n-2}\left(\frac{x}{3}\right)^2-\binom{n}{3}2^{n-3}\left(\frac{x}{3}\right)^3+\ldots$ $=2^n-\frac{2^{n-1}nx}{3}+\frac{2^{n-2}n(n-1)x^2}{2\times 9}-\frac{2^{n-3}n(n-1)(n-2)x^3}{6\times 27}+\ldots$ Cofficient of $\frac{T_2}{T_4}=\frac{-\frac{2^{n-1}n}{3}}{-\frac{2^{n-3}n(n-1)(n-2)}{162}}$ $=\frac{54\times 2^{n-1-n+3}}{(n-1)(n-2)}$ $=\frac{54\times 2^2}{(n-1)(n-2)}$ $=\frac{216}{(n-1)(n-2)}$	[M1] binomial theorem or general term used [B1] [B1] [M1] simplification leading to correct answer A.G.

	(ii)	Find the value of n if the ratio in **(i)** is $108:55$.	[2]
		$\frac{216}{(n-1)(n-2)}=\frac{108}{55}$ $11880=108\left(n^2-3n+2\right)$ $110=n^2-3n+2$ $n^2-3n-108=0$ $(n-12)(n+9)=0$ $n=12$ or $n=-9$ (rejected)	[M1] [A1]
	(iii)	Hence, find the term in x^5.	[2]
		$T_6=\binom{12}{5}2^{12-5}\left(-\frac{x}{3}\right)^5$ $=-\frac{11264}{27}x^5$	[M1] [A1]

6	The equation of a curve is $y = \frac{2x-9}{\sqrt{x^2+1}}$.		
	(i)	Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9x+2}{\sqrt{\left(x^2+1\right)^3}}$.	[3]
		$y = \frac{2x-9}{\sqrt{x^2+1}}$	
		$\frac{dy}{dx} = \frac{\left(x^2+1\right)^{\frac{1}{2}}(2)-(2x-9)\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)}{x^2+1}$	[M1] quotient rule [B1] correct expression
		$= \frac{2\left(x^2+1\right)^{\frac{1}{2}}-x(2x-9)\left(x^2+1\right)^{-\frac{1}{2}}}{x^2+1}$	
		$= \frac{\left(x^2+1\right)^{-\frac{1}{2}}\left[2\left(x^2+1\right)-x(2x-9)\right]}{x^2+1}$	[A1] factorisation leading to correct answer
		$= \frac{\left(x^2+1\right)^{-\frac{1}{2}}\left[2x^2+2-2x^2+9x\right]}{x^2+1}$	
		$= \frac{2+9x}{\left(x^2+1\right)^{\frac{3}{2}}}$	[A.G]

(ii) Find the coordinates of the stationary point of the curve, leaving your answer in exact form. [3]

For stationery points,

$$\frac{dy}{dx}=0$$ [M1]

$$\frac{2+9x}{\left(x^2+1\right)^{\frac{3}{2}}}=0$$

$$2+9x=0$$

$$x=-\frac{2}{9}$$ [A1]

$$\therefore y=\frac{2\left(-\frac{2}{9}\right)-9}{\sqrt{\left(-\frac{2}{9}\right)^2+1}}$$

$$=\frac{-\frac{4}{9}-9}{\sqrt{\frac{85}{81}}}$$

$$=\frac{-\frac{85}{9}}{\frac{\sqrt{85}}{9}}$$

$$=-\frac{85}{\sqrt{85}}=-\sqrt{85}$$

Coordinates = $\left(-\frac{2}{9},\ -\sqrt{85}\right)$ [A1]

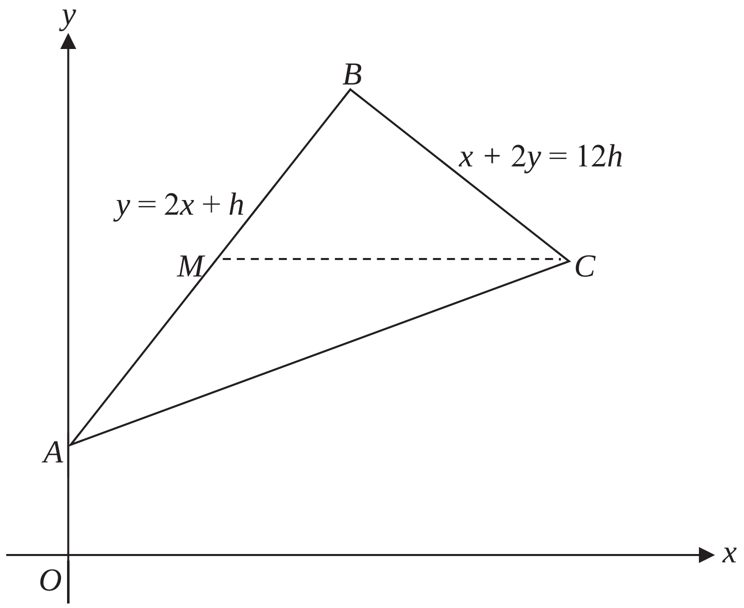
(iii) Find the nature of this stationary point. [2]

x	$\left(-\frac{2}{9}\right)^-$	$-\frac{2}{9}$	$\left(-\frac{2}{9}\right)^+$
Sign	-	0	+
slope	╲	—	╱

[M1]

$\left(-\frac{2}{9},\ -\sqrt{85}\right)$ is a minimum point. [A1]

7



The diagram shows a triangle *ABC* in which *A* lies on the *y*-axis.
The equation of *AB* and *BC* are $y = 2x + h$ and $x + 2y = 12h$.
M is midpoint of *AB* and *MC* is parallel to *x*-axis.

(i) Explain why *AB* is perpendicular to *BC*. [2]

Line *AB*: Gradient = 2
Line *BC*: $x + 2y = 12h$

$$2y = -x + 12h$$

$$y = -\frac{1}{2}x + 6h$$

Gradient = -0.5

$$m_{AB} \times m_{BC} = 2 \times \left(-\frac{1}{2}\right)$$ [M1]

$$= -1$$

Since the product of 2 gradients is – 1,
AB is perpendicular to *BC*. [A1] with conclusion

(ii) Show that coordinates of *B* is ($2h, 5h$). [2]

Solve:

$$2x + h = -\frac{1}{2}x + 6h$$ [M1]

$$\frac{5}{2}x = 5h$$

$$x = 5h \times \frac{2}{5}$$

$$= 2h$$ [A1] either one found correctly

$$y = 2(2h) + h$$

$$= 5h$$

$B(2h, 5h)$ A.G.

	(iii)	Express the coordinates of *M* and of *C* in terms of *h*.	[3]
		$A(0,h)$	
		$M=\left(\frac{2h+0}{2},\frac{5h+h}{2}\right)$	
		$=(h,3h)$	[B1]
		When $y=3h$,	
		$x+2(3h)=12h$	[M1]
		$x+6h=12h$	
		$x=6h$	
		$C(6h,3h)$	[A1]
	(iv)	Find the value of *h* if the area of triangle *AMC* is 125 square units.	[2]
		Area	
		$=\frac{1}{2}\begin{vmatrix}0 & 6h & h & 0\\ h & 3h & 3h & h\end{vmatrix}$	[M1]
		$=\frac{1}{2}(18h^2+h^2-6h^2-3h^2)$	
		$=5h^2$	
		$\therefore 5h^2=125$	
		$h^2=25$	
		$h=\sqrt{25}$	
		$=5$	[A1]

8	The equation of a curve is $y = \frac{x-6}{x+4} + 4$, where $x \neq -4$. Find the		
	(i)	gradient of the tangent at the point where the curve passes through the x-axis,	[4]

When $y = 0$ [M1]

$$\frac{x-6}{x+4} + 4 = 0$$

$$x - 6 = -4x - 16$$

$$5x = -10$$

$$x = -2$$

$$\frac{dy}{dx} = \frac{x+4-(x-6)}{(x+4)^2}$$ [M1]

$$= \frac{10}{(x+4)^2}$$ [A1]

$$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{10}{(-2+4)^2}$$

$$= \frac{5}{2}$$ [A1]

	(ii)	equation of another tangent to the curve that is parallel to the tangent in **(i)**.	[4]

$$\frac{dy}{dx} = \frac{5}{2}$$ [M1]

$$\frac{10}{(x+4)^2} = \frac{5}{2}$$

$$(x+4)^2 = 4$$

$$x+4 = \pm 2$$

$$x = -2 \quad \text{or} \quad x = -6$$ [A1]

$$x = -6, \qquad y = \frac{-6-6}{-6+4} + 4 = 10$$

Equation:

$$\frac{y-10}{x+6} = \frac{5}{2}$$ [M1]

$$2y - 20 = 5x + 30$$

$$2y = 5x + 50$$ [A1]

9	The equation of a curve is $y = \cos^3 x + \sin 3x$.		
	(i)	Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.	[4]

$$y = cos^3x + sin3x$$

$$\frac{dy}{dx} = 3cos^2x(-sinx) + 3cos3x \qquad [M1]\ \ chain\ rule\ seen$$

$$\frac{dy}{dx} = -3cos^2xsinx + 3cos3x \qquad [A1]$$

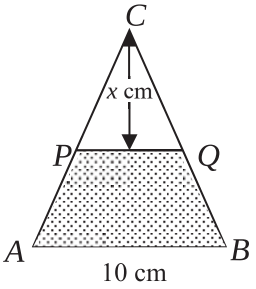
$$\frac{d^2y}{dx^2} = -3[cos^2xcosx + sinx(2cosx)(-sinx)] - 9sin3x\ \ [M1]\ \ product\ rule\ seen$$

$$= -3cos^3x + 6sin^2xcosx - 9sin3x\ \ [A1]$$

	(ii)	Given that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}+9y=A\cos x$, find the value of *A*.	[2]

$-3cos^3x + 6sin^2xcosx - 9sin3x + \ 9cos^3x + 9sin3x$
$= 6cos^3x + 6sin^2xcosx$
$= 6cosx(cos^2x + sin^2x)$ $[M1]$ *identity seen*
$= 6cosx(1)$
$= 6cosx$
$A = 6$ [A1]

10 The diagram shows an isosceles triangle ABC with height 12 cm and $AB = 10$ cm. PQ moves towards AB at a steady rate of 0.5 cm/s, keeping parallel to AB.



If PQ is x cm from C,

(i) Show that $PQ = \frac{5x}{6}$ cm. [1]

By similar triangles

$$\frac{x}{12} = \frac{PQ}{10} \qquad [B1]$$
$$10x = 12PQ$$
$$PQ = \frac{5}{6}x \qquad (A.G)$$

(ii) Find the area, A cm^2, of the shaded region in terms of x. [2]

Area of triangle $CPQ = \frac{1}{2}x\left(\frac{5}{6}x\right) \qquad [M1]$

$= \frac{5}{12}x^2$

Area of shaded region $= \frac{1}{2}(10)(12) - \frac{5}{12}x^2$

$= 60 - \frac{5}{12}x^2 \qquad [A1]$

(iii) Find the rate of change of the shaded area when *PQ* is halfway towards *AB* from *C*. [3]

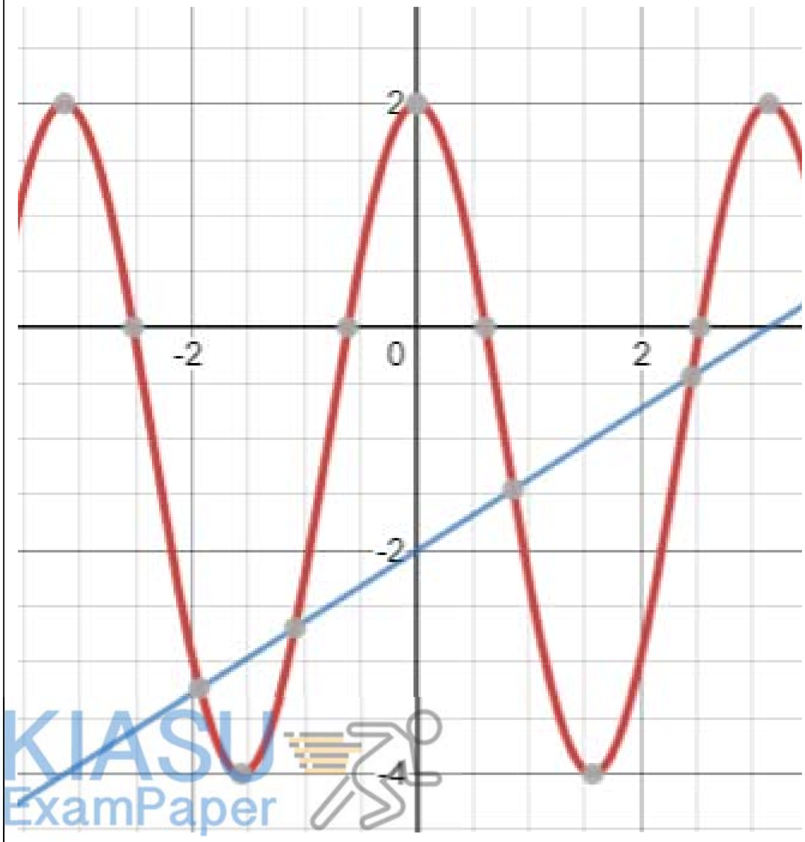
$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt} \qquad [B1] \quad \textit{use of chain rule correctly}$$

$\frac{dA}{dx} = -\frac{10}{12}x$ [B1]

$$\frac{dA}{dt} = -\frac{10}{12}(6) \times (0.5)$$

$= -2.5 \qquad [B1]$

Area is decreasing at a rate of 2.5cm^2/s

11	It is given that $f(x)=3\cos 2x-1$ and $g(x)=\frac{2x}{\pi}-2$.		
	(i)	State the least and greatest values of $f(x)$.	[2]
		Least value = -4 [B1] Greatest value = 2 [B1]	
	(ii)	State the period of $f(x)$.	[1]
		Period = π [B1]	
	(iii)	Sketch, on the same axes, the graphs of $y=f(x)$ and $y=g(x)$ for $-\pi \le x \le \pi$.	[4]
		 [B1] shape of curve [B1] Max and Min points seen [B1] correct domain and period seen [B1] correct line, passing through $y = 2$	

	(iv)	State, with detailed workings, the number of solutions of the equation $3\pi\cos 2x = 2x - \pi$ for $-\pi \le x \le \pi$.	[2]

$3\pi cos2x = 2x - \pi$

$3cos2x = \frac{2x}{\pi} - 1$ [B1] for manipulation

$3cos2x - 1 = \frac{2x}{\pi} - 2$

Since there are 4 intersection points, there are 4 solutions [B1] dep

12 The function *f* is defined by $y = (x+1)^3 e^{2x-3}$, for $x > -\frac{5}{2}$, and $x \ne -1$. Explain, with working, whether *f* is an increasing or decreasing function. [4]

$\frac{dy}{dx} = 3(x+1)^2 e^{2x-3} + 2(x+1)^3 e^{2x-3}$ $[M1]$ $product\ rule$ $[A1]$

$\frac{dy}{dx} = e^{2x-3}(x+1)^2[3 + 2(x+1)\ \ ]$

$\frac{dy}{dx} = e^{2x-3}(x+1)^2[5 + 2x]$

Given that

$x > -\frac{5}{2},\ \ 2x + 5 > 0, (x+1)^2\ \ and\ \ e^{2x-3} > 0,\ \ [M1]$

$\therefore \frac{dy}{dx} > 0,\ the\ curve\ is\ increasing$ $[A1]$ dep.

End of paper

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1	(i) Differentiate $\ln(\cos x)$ with respect to *x*.	[1]

Answer	Marks	Guidance
$\frac{d}{dx}\ln(\cos x) = \frac{1}{\cos x} \times (-\sin x)$		
$= \quad = -\frac{\sin x}{\cos x}$ $= -\tan x$	B1	

1	(ii) Hence find $\int \tan x \ dx$	[2]

Answer	Marks	Guidance
$\int \tan x \ dx$		
$= -\int -\tan x \ dx$	M1	Reverse differentiation
$= -\ln(\cos x) + c$	A1	

1	(iii) Differentiate $x\tan x$ with respect to *x*.	[2]

Answer	Marks	Guidance
$\frac{d}{dx}(x\tan x) = \tan x + x\sec^2 x$	M1 A1	Product rule

1	(iv) Using the results from (ii) and (iii), hence find $\int x\sec^2 x \ dx$	[3]

Answer	Marks	Guidance
$x\tan x \ + c_1 = \int \tan x + x\sec^2 x \ dx$	M1	Reverse differentiation
$x\tan x + c_1 - \int \tan x \ dx = \int x\sec^2 x \ dx$	M1	Split to 2 integrals
$\int x\sec^2 x \ dx = x\tan x - (-\ln(\cos x)) + c$		
$= x\tan x + \ln(\cos x) + c$	A1	

2	(i) Prove that $\frac{(\sin A+\cos A)(1-\sin A\cos A)}{\sin^3 A}=1+\cot^3 A$	[4]

Answer	Marks	Guidance
LHS= $\frac{\sin A+\cos A-\sin^2 A\cos A-\sin A\cos^2 A}{\sin^3 A}$	M1	Correct expansion
$= \frac{\sin A+\cos A-(1-\cos^2 A)\cos A-\sin A(1-\sin^2 A)}{\sin^3 A}$	B1	Use of $\cos^2 A+\sin^2 A=1$
$= \frac{\sin A+\cos A-\cos A+\cos^3 A-\sin A+\sin^3 A}{\sin^3 A}$	B1	
$= \frac{\cos^3 A+\sin^3 A}{\sin^3 A}$		
$= \frac{\cos^3 A}{\sin^3 A}+\frac{\sin^3 A}{\sin^3 A}$	B1	
$= 1+\cot^3 A$		

Alternative solution

Answer	Marks	Guidance
LHS= $\frac{(\sin A+\cos A)}{\sin A}\times\frac{(1-\sin A\cos A)}{\sin^2 A}$	B1	
$= (1+\cot A)\times\left(\frac{1}{\sin^2 A}-\frac{\sin A\cos A}{\sin^2 A}\right)$	B1	Use of $\frac{\cos A}{\sin A}$
$= (1+\cot A)\times\left(\cos ec^2 A-\cot A\right)$		
$= (1+\cot A)\times\left(1+\cot^2 A-\cot A\right)$	B1	Use of $1+\cot^2 A=\cos ec^2 A$
$= 1+\cot A-\cot^2 A+\cot^3 A-\cot A-\cot^2 A$	B1	Correct expansion
$= 1+\cot^3 A$		

2	(ii) Hence solve the equation $(\sin A+\cos A)(1-\sin A\cos A)=2\sin^3 A$ for $0° \leq A \leq 180°$.	[3]

Answer	Marks	Guidance
$\dfrac{(\sin A+\cos A)(1-\sin A\cos A)}{\sin^3 A}=2$		
$1+\cot^3 A=2$	B1	
$\cot^3 A=1$		
$\cot A=1$	M1	Taking cube root without negative sign
$\tan A=1$		
$A=45°$	A1	

3	**(i)** The expression $3x^3+hx^2+kx-4$, where h and k are constants, has a	
	factor of $3x-1$ and leaves a remainder of -4 when divided by $x+1$.	
	By forming 2 equations of h and of k, show that the value of h = 11 and k = 8.	[4]

Answer	Marks	Guidance
Let $\mathrm{f}(x)=3x^3+hx^2+kx-4$		
$(3x-1)$ is a factor of $\mathrm{f}(x)$		
$\mathrm{f}\left(\dfrac{1}{3}\right)=0$	M1	Realise $\mathrm{f}\left(\dfrac{1}{3}\right)=0$
$3\times\left(\dfrac{1}{3}\right)^3+h\left(\dfrac{1}{3}\right)^2+k\left(\dfrac{1}{3}\right)-4=0$		
$\dfrac{1}{9}+\dfrac{h}{9}+\dfrac{k}{3}-4=0$		
$h+3k=35$ ------------(1)		
Divisor $=x+1$, reminder $=-4$		
$\mathrm{f}(-1)=-4$		
$-3+h-k-4=-4$		
$h-k=3$ -----------(2)	M1	Realise $\mathrm{f}(-1)=-4$
$(1)-(2)$, $4k=32$		
$k=8$		DM on correct equation (1) and (2)
$h=3+k$	DM1 B1	
$=3+8=11$		

3	**(ii)** Hence, express $\dfrac{-x^2+6x+9}{3x^3+hx^2+kx-4}$ as the sum of three partial fractions	[7]

Answer	Marks	Guidance
Let $3x^3+11x^2+8x-4=(3x-1)\left(x^2+bx+4\right)$		
comparing the coefficient of x^2,		
$11=-1+3b$		
$b=4$		
$3x^3+11x^2+8x-4=(3x-1)\left(x^2+4x+4\right)$	M1	Long division or inspection
$=(3x-1)(x+2)^2$	A1	
$\dfrac{-x^2+6x+9}{(3x-1)(x+2)^2}=\dfrac{A}{3x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$	M1	
$-x^2+6x+9=A(x+2)^2+B(3x-1)(x+2)+C(3x-1)$	M1	
Let $x=-2$		
$-(-2)^2+6(-2)+9=C(-7)$		
$-4-12+9=-7C$		
$C=1$		
Let $x=\dfrac{1}{3}$		
$-\dfrac{1}{9}+\dfrac{6}{3}+9=A\left(\dfrac{7}{3}\right)^2$		
$\dfrac{98}{9}=\dfrac{49A}{9}$		
$A=2$		
comparing coefficients of x^2,		A1 for each correct *A*, *B*, *C*
$-1=A+3B$		
$-1-2=3B$		
$B=-1$		
$\dfrac{-x^2+6x+9}{(3x-1)(x+2)^2}=\dfrac{2}{3x-1}-\dfrac{1}{x+2}+\dfrac{1}{(x+2)^2}$		

4	A particle, moving in a straight line, passes through a fixed point O with a speed of 4 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O, is given by $a = -\frac{4}{15}\mathrm{e}^{-\frac{t}{60}}$.	
	(i) Find the exact value of *t* when the particle is at instantaneous rest.	[6]

Answer	Marks	Guidance
$v = \int -\frac{4}{15}\mathrm{e}^{-\frac{t}{60}}\,\mathrm{d}x$	M1	Integrating *a*
$= -\frac{4}{15} \times \frac{\mathrm{e}^{-\frac{t}{60}}}{-\frac{1}{60}} + c$	A1	Correct integration (accept without + +c)
$v = 16\mathrm{e}^{-\frac{t}{60}} + \mathrm{c}$ when $t = 0,\ v = 4$ $4 = 16\mathrm{e}^{0} + \mathrm{c}$ $\mathrm{c} = -12$		
$v = 16\mathrm{e}^{-\frac{t}{60}} - 12$ At instantaneous rest, $v = 0$	A1	
$16\mathrm{e}^{-\frac{t}{60}} - 12 = 0$ $\mathrm{e}^{-\frac{t}{60}} = \frac{12}{16}$	M1	Equating *v* to zero
$-\frac{t}{60} = \ln\frac{3}{4}$	M1	Taking logarithm
$t = -60\ln\frac{3}{4}$	A1	

4	(ii) Find an expression in term of *t*, for the displacement, from O, of the	
	particle *t* seconds after passing O.	[3]

Answer	Marks	Guidance
$s = \int 16e^{-\frac{t}{60}} - 12 \, dt$	M1	Integrating to get *s*
$= \dfrac{16e^{-\frac{t}{60}}}{-\frac{1}{60}} - 12t + c$		
$= -960e^{-\frac{t}{60}} - 12t + c$	A1	
when $t = 0, s = 0$ $0 = -960e^{0} - 0 + c$ $960 = c$		
$s = -960e^{-\frac{t}{60}} - 12t + 960$	A1	

	(ii) Hence find the distance of the particle from O when it is at instantaneous rest.	[1]

Answer	Marks	Guidance
when $v = 0, t = -60\ln\frac{3}{4}$		
$s = -960 \times \frac{3}{4} + 720\ln\frac{3}{4} + 960$ $= 32.9$ m	A1	

	(iv) Show that the particle is again at *O* at some instant during the	[2]
	thirty-sixth second after first passing through *O*.	

Answer	Marks	Guidance
when $t = 36, \ s = -960e^{-\frac{36}{60}} - 12(36) + 960$ $= 1.14$ m		
when $t = 37, \ s = -960e^{-\frac{37}{60}} - 12(37) + 960$ $= -2.15$ m	B1	
When t=36, particle is on the right side of O with displacement of 1.14 and when t =37, particle is on the left side of O with s=−2.15. Hence at some instant during the 37th second the particle is again at O.	B1	

5	**(i)** The equation $\log_3 x - \frac{1}{2}\log_{27} x = \log_2 8$ has the solution $x = 3^k$.	
	Find the value of *k*.	[4]

Answer	Marks	Guidance
$\log_3 x - \frac{1}{2} \times \frac{\log_3 x}{\log_3 27} = \log_2 2^3$	M1	Change of base law
$\log_3 x - \frac{1}{6}\log_3 x = 3$		
$\frac{5}{6}\log_3 x = 3$	M1	Simplify to single log
$\log_3 x = \frac{18}{5}$		
$x = 3^{\frac{18}{5}}$	A1	
By comparing with 3^k		
$k = \frac{18}{5}$ or $k = 3.6$	A1	

Alternatively

Answer	Marks	Guidance
since $x = 3^k$ is a solution,		
$\log_3 3^k - \frac{1}{2} \times \log_{27} 3^k = \log_2 2^3$	M1	Substitution
$k\log_3 3 - \frac{1}{2} \times \frac{\log_3 3^k}{\log_3 27} = 3$	M1	Applying power law
$k - \frac{1}{2} \times \frac{k\log_3 3}{\log_3 3^3} = 3$		
$k - \frac{1}{2} \times \frac{k\log_3 3}{3\log_3 3} = 3$		
$k - \frac{1}{2} \times \frac{k}{3} = 3$		
$\frac{5k}{6} = 3$	B1	Reduce to a linear equation
$k = \frac{18}{5} = 3.6$	A1	

5	**(ii)** Solve the equation $2\log_4(x-2)-\log_4(x+10)=\frac{1}{2}$.	[4]

Answer	Marks	Guidance
$2\log_4(x-2)-\log_4(x+10)=\frac{1}{2}$		
$\log_4\frac{(x-2)^2}{(x+10)}=\frac{1}{2}$	M1	Apply power or subtraction law
$\frac{(x-2)^2}{(x+10)}=4^{\frac{1}{2}}$	M1	convert to index form or equivalent.
$(x-2)^2=2(x+10)$ $x^2-4x+4-2x-20=0$ $x^2-6x-16=0$ $(x-8)(x+2)=0$		
$x=8$ or $x=-2$(rejected)	A1 A1	$x=-2$, must be rejected

6	The roots of the quadratic equation $2x^2 = x+3$ are α and β.	
	(i) Find the value of $\alpha^2+\beta^2$.	[4]

Answer	Marks	Guidance
$2x^2 = x+3$		
$\alpha+\beta = -\left(\frac{-1}{2}\right) = \frac{1}{2}$		
$\alpha\beta = -\frac{3}{2}$	M1	
$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$		
$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$		
$= \left(\frac{1}{2}\right)^2 - 2\times\left(-\frac{3}{2}\right)$		
$= \frac{13}{4}$	A1	

6	**(ii)** Find the quadratic equations whose roots are $\frac{\beta}{\alpha^2}$ and $\frac{\alpha}{\beta^2}$.	[3]

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Answer	Marks	Guidance
sum of roots $=\dfrac{\beta}{\alpha^2}+\dfrac{\alpha}{\beta^2}$		
$=\dfrac{\beta^3+\alpha^3}{\alpha^2\beta^2}$	M1	
$=\dfrac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha^2\beta^2}$	B1	or equivalent
$=\dfrac{\left(\frac{1}{2}\right)\left(\frac{13}{4}-\left(-\frac{3}{2}\right)\right)}{\frac{9}{4}}$		
$=\dfrac{1}{2}\times\dfrac{19}{4}\times\dfrac{4}{9}$		
$=\dfrac{19}{18}$	A1	
product of roots $=\dfrac{\beta}{\alpha^2}\times\dfrac{\alpha}{\beta^2}=\dfrac{1}{\alpha\beta}=-\dfrac{2}{3}$	A1	
Quadratic equation is $x^2-\dfrac{19}{18}x-\dfrac{2}{3}=0$	A1	

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7	The points A(2 , 1) and B(11 , −2) lie on a circle.	
	(i) Find the equation of the perpendicular bisector of the chord AB.	[4]

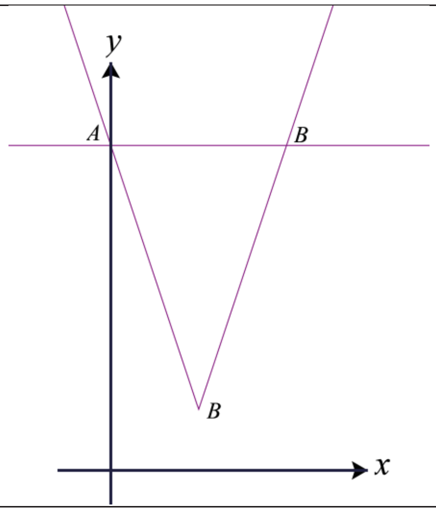
Answer	Marks	Guidance
gradient of $AB = \frac{3}{-9} = -\frac{1}{3}$		
gradient of the perpendicular bisector = $-\frac{1}{-\frac{1}{3}}$ $= 3$	M1A1	
midpoint of $AB = \left(\frac{13}{2}, -\frac{1}{2}\right)$	B1	
Equation of perpendicular bisector is $y + \frac{1}{2} = 3\left(x - \frac{13}{2}\right)$ $y = 3x - 20$	A1	

7	The line with equation $y = \frac{4}{3}x - 10$ is a normal to the circle.	
	(ii) Hence, find the equation of the circle.	[5]

Answer	Marks	Guidance
For centre of circle, $\frac{4}{3}x - 10 = 3x - 20$ $\frac{5}{3}x = 10$ $x = 6$ $y = 18 - 20 = -2$	M1A1	Equating eqn of normal to eqn of perpendicular bisector A1 for the value of *x*.
centre $= (6, -2)$	B1	
radius, $r = \sqrt{(6-2)^2 + (-2-1)^2}$ or $r = 11 - 6$ $= 5$ $= 5$	A1	
Equation of circle is $(x-6)^2 + (y+2)^2 = 25$	A1	

7	(iii) Find the coordinates of the point on the circle which is at the greatest distance from the *x*-axis.	[1]

Answer	Marks	Guidance
point $= (6,\ -7)$	A1	

8		
	The diagram shows part of the graph of $y = \lvert 3x-7 \rvert + 2$. A horizontal line is drawn	
	from *A* to intersect the graph of $y = \lvert 3x-7 \rvert + 2$ at *C*.	
	(i) Find the coordinates of *A*, *B* and *C*.	[3]

Answer	Marks	Guidance
At *A*, $y = 0,\ x = 9$ $A = (0,9)$	A1	
At *C*, $y = 9$, $9 = \lvert 3x-7 \rvert + 2$ $\lvert 3x-7 \rvert = 7$ $3x - 7 = 7$ $x = \dfrac{14}{3}$ $C = \left(\dfrac{14}{3},\ 9\right)$	A1	
$B = \left(\dfrac{7}{3},\ 2\right)$	A1	

8	**(ii)** Find the set of values of *m* for which the graph of $y=\lvert 3x-7\rvert+2$ and the line	[2]
	$y=mx$ intersects at 2 points.	

Answer	Marks	Guidance
gradient $OB=\frac{2}{7/3}=\frac{6}{7}$	M1	
$\frac{6}{7}<m<3$	A1	

8	(iii) Solve the equation $\lvert 3x-7\rvert+2=4x-1$	[3]

Answer	Marks	Guidance
$\lvert 3x-7\rvert+2=4x-1$		
$\lvert 3x-7\rvert=4x-3$		
$3x-7=\pm(4x-3)$	M1	
$3x-7=4x-3$ or $3x-7=-(4x-3)$		
$x=-4$(rejected) or	A1	Must be rejected
$7x=10$		
$x=\frac{10}{7}$	A1	

9 The resistance to motion, R newtons, of a plank towed through water and its speed, V m/s is given by $R = AV^n$, where A and n are constants.

The table shows the corresponding values of V and R.

V	1.65	2.46	3.68	6.05
R	1.87	3.70	7.33	17.1

(i) On the grid on page 15, draw the graph of ln R against ln V, using a scale of 4 cm for 0.5 unit on the ln V-axis and a scale of 5 cm to 1 unit on the ln R-axis. [3]

Answer	Marks	Guidance
$\ln R = \ln A + n \ln V$		

ln V	0.501	0.900	1.30	1.80
ln R	0.626	1.31	1.99	2.84

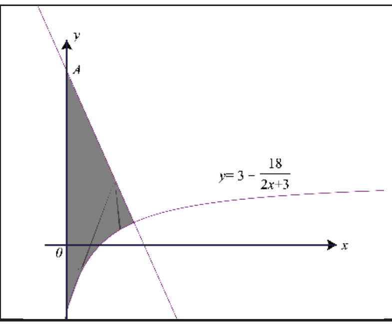
Axes and scale A1 All points plotted correctly join with a straight line	A2	Deduct 1 mark for any point plotted wrongly

(ii) Use the graph to estimate the value of each of the constants A and n, giving your answers to 1 decimal place. [4]

Answer	Marks	Guidance
From the graph, $\ln r$-intercept $= -0.24$		
$\ln A = \ln r$-intercept $= -0.24$	M1	
$A = \mathrm{e}^{-0.24} = 0.8$(to dec pl)	A1	
gradient $= \dfrac{2.84-(-0.24)}{1.80-0} = 1.7$	M1	Must indicate the 2 points used to find gradient
$n =$ gradient $= 1.7$ (to 1 dec pl)	A1	

(iii) By drawing a suitable line on your graph, solve the equation $AV^n = V^{1.2}$, giving your answer correct to 1 decimal place. [4]

Answer	Marks	Guidance
$\ln A + n\ln V = 1.2\ln V$		
Draw $\ln R = 1.2\ln V$	B1	Correct line drawn
From the graph, $\ln V = 0.475$ $V = \mathrm{e}^{0.475}$	M1	Reading off the lnV coordinate from the point of intersection
$= 1.6$ (to 1 dec pl)	A1	

10		The diagram show part of the curve $y=3-\frac{18}{2x+3}$. The normal to the curve at $x=3$ intersects the y-axis at A. Find the exact area of the shaded region. [11]	

Answer	Marks	Guidance
$y=3-\frac{18}{2x+3}$		
$\frac{dy}{dx}=-18(-1)\left(2x+3\right)^{-2}\times 2$	M1	Find $\frac{dy}{dx}$
$=\frac{36}{\left(2x+3\right)^2}$	M1 B1	Chain Rule
when x = 3. gradient of tangent = $\frac{36}{9^2}=\frac{4}{9}$		
when x = 3, $y=3-\frac{18}{9}=1$		
gradient of normal = $-\frac{9}{4}$	M1,B1	
Equation of normal is $y-1=-\frac{9}{4}(x-3)$		
$y=-\frac{9}{4}x+\frac{31}{4}$	B1	
Area of the shaded region = $\int_0^3\left(-\frac{9}{4}x+\frac{31}{4}\right)-\left(3-\frac{18}{2x+3}\right)\mathrm{d}x$	M1 B1	Integrate for area Correct Integral
$=\int_0^3-\frac{9}{4}x+\frac{19}{4}+\frac{18}{2x+3}\ \mathrm{d}x$		
$=\left[-\frac{9}{4}\times\frac{x^2}{2}+\frac{19}{4}x+18\times\frac{\ln\left(2x+3\right)}{2}\right]_0^3$	B1	$\int\frac{1}{2x+3}$ correctly
$=\left[-\frac{9}{8}x^2+\frac{19}{4}x+9\ln\left(2x+3\right)\right]_0^3$		
$=-\frac{9}{8}\times 3^2+\frac{19}{4}\times 3+9\ln 9-9\ln 3$	M1	
$=\frac{33}{8}+9\ln 3$	A1	Accept $\frac{33}{8}+9\ln 9-9\ln 3$

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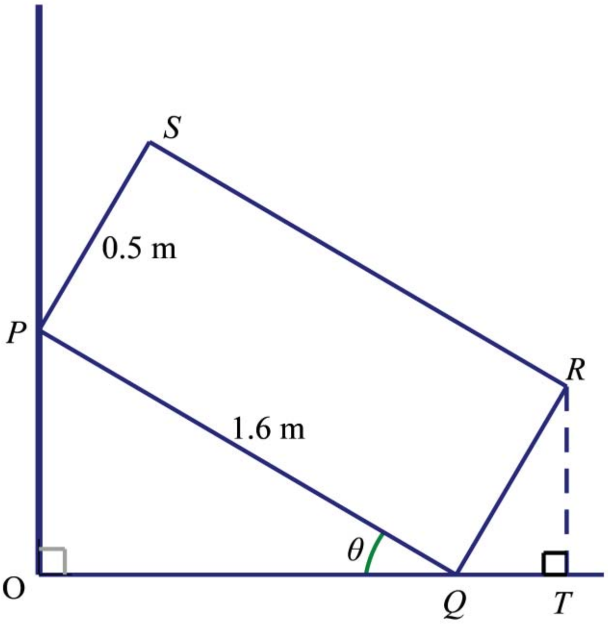
11	A rectangular table *PQRS* is positioned at a corner of a room. Given that $PQ = 1.6$m	
	$PS = 0.5$ m, angle $OQP = \theta$ and angle QOP = angle $RTQ = 90°$, where θ varies.	
	(i) Show that $OT = 1.6 \cos \theta + 0.5 \sin \theta$.	[3]
		

Answer	Marks	Guidance
$\angle RQT = 180° - 90° - \theta = 90° - \theta$ $\angle QRT = 180° - (90° + 90° - \theta) = \theta$	M1	$\angle QRT = \theta$ can be in the diagram
$\cos\theta = \dfrac{OQ}{1.6}$ $OQ = 1.6\cos\theta$	B1	
$\sin\theta = \dfrac{QT}{0.5}$ $QT = 0.5\sin\theta$	B1	
$OT = OQ + QT$ $= 1.6\cos\theta + 0.5\sin\theta$		Award 3 marks only if it is complete.

11	(ii) Find the value of θ for which $OT = 1.2$ m	[5]

Answer	Marks	Guidance
Let $1.6\cos\theta + 0.5\sin\theta = R\cos(\theta - \alpha)$	M1	
$R = \sqrt{1.6^2 + 0.5^2} = \sqrt{2.81}$	B1	
$\alpha = \tan^{-1}\dfrac{0.5}{1.6} = 17.354°$	B1	
$\sqrt{2.81}\cos(\theta - 17.354°) = 1.2$		
$\cos(\theta - 17.354°) = \dfrac{1.2}{\sqrt{2.81}}$	M1	
$\theta - 17.354° = 44.286°$		
$\theta = 61.6°$(to 1 dec pl)	A1	