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TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2021
Secondary 4

CANDIDATE
NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

Thursday 19 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

1(i) Find the set of values of the constant k for which $y = kx - 2$ meets the curve $y^2 = 4x - x^2$.

[4]

(ii) Given that $2x^2 + bx + c$ is always positive, write down and simplify the relationship between b and c . Explain why c is never negative.

[3]

- 2 Given that θ is an acute angle and $\sin \theta = m$, express in terms of m ,
(i) $\cos \theta$,

[1]

- (ii) $\cot \theta$.

[1]

Another acute angle α is such that $\tan(\theta - \alpha) = -\frac{1}{2}$.

(iii) What can be deduced about the size of θ relative to α ? [1]

(iv) Given further that $\tan \alpha = \frac{3}{2}$, show $\tan \theta = \frac{4}{7}$.

Hence, find the value of m . [3]

- 3 Given that $y = A - B \cos 4x - \frac{1}{2} \sin 2x$ and $\frac{d^2y}{dx^2} + 4y = 3 \cos 4x + 1$, find the value of each of the following constants A and B . [4]

4 Given that $y = \frac{4}{5} \left(\frac{x}{12} - 1 \right)^6$ and that both x and y vary with time, find the value

of y when the rate of change of y is $12\frac{4}{5}$ times the rate of change of x . [5]

- 5 Given that $y = \frac{6x^2}{2-5x}$, find the range of values of x for which y is a decreasing function of x .

[4]

- 6 (i) A virus was spreading at a chicken farm such that 2% of chickens were infected in one day. The infected chickens were culled daily. If N is the total number of chickens before the start of the virus infection, and assuming that the virus continues to spread at the same rate, explain why the number of chickens expected to be alive after n days is given by $(0.98)^n N$. [2]

- (ii) It is known that the virus will infect x % of chicken in 7 days. Calculate to the nearest 2 significant figures, the value of x . [2]

- (iii) Given that the number of chickens expected to be alive after n days can be expressed as Ne^{kn} , find the value of the constant k . [2]

7 (i) Write down the general term in the binomial expansion of $\left(x + \frac{5}{x}\right)^n$. [1]

(ii) Given that there is a constant term in the expansion of $\left(x + \frac{5}{x}\right)^n$, show that n must be an even number. [2]

(iii) Find the term in x^6 in the expansion of $\left(x + \frac{5}{x}\right)^{10}$. [2]

(iv) Hence or otherwise, find the value of a , given that the coefficient of x^6 is 43875

in the expansion $(ax^2 - 1)\left(x + \frac{5}{x}\right)^{10}$.

[3]

- 8 A gardener plans to construct two flower beds, one a circle of radius r m, the other a square of side x m. To protect the young flowers, the two beds are to be surrounded by wire netting. The total length of wire netting to be used is 30 m.

(i) Express x in terms of r and π . [1]

(ii) The combined area of the two flower beds is A m².

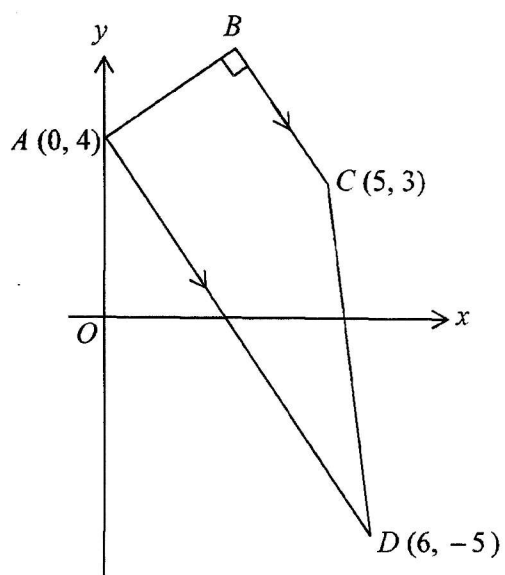
Show that $A = \left(\frac{1}{4}\pi^2 + \pi\right)r^2 - \frac{15}{2}\pi r + \frac{225}{4}$. [2]

(iii) Given that r can vary, find the value of r which gives a stationary value of A . [3]

(iv) Show that, when A is stationary, the side of the square is equal in length to the diameter of the circle. [2]

(v) Determine whether the stationary value of A is a maximum or minimum. [2]

9



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC and angle $ABC = 90^\circ$. The coordinates of A , C and D are $(0, 4)$, $(5, 3)$ and $(6, -5)$ respectively.

(i) Show that the coordinates of B are $(3, 6)$.

[4]

- (ii) The point E lies on the line DA produced such that $\frac{\text{area of } \triangle BED}{\text{area of } \triangle BAD} = \frac{4}{3}$.

Find the coordinates of point E .

[2]

- (iii) Given that $ECDF$ form a kite in which FC is perpendicular to ED , find the coordinates of F . Hence, find the area of $ECDF$.

[4]

10 A particle moves in a straight line, so that t seconds after passing through a fixed point O , its acceleration, $a \text{ ms}^{-2}$, is given by $a = -12e^{2t}$. The initial velocity is 9 ms^{-1} .

(i) Show that the particle is instantaneously at rest at $t = \frac{1}{2} \ln \frac{15}{6}$. [5]

(ii) Find an expression for the displacement of the particle from O in terms of t .

Hence, find the total distance travelled by the particle when it returns to O .

[4]

11 It is given that $f(x) = 2 - 3\sin 3x$ and $g(x) = 4\cos\left(\frac{x}{2}\right) - 1$.

(i) State the least and greatest values of $f(x)$. [2]

(ii) State the least and greatest values of $g(x)$. [2]

(iii) State the period of $f(x)$. [1]

(iv) State the period of $g(x)$. [1]

- (v) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0^\circ \leq x \leq 180^\circ$. [4]

- (vi) Find the number of solutions to the equation $2 \cos \frac{x}{2} + \frac{3}{2} \sin 3x = \frac{3}{2}$ for $0^\circ \leq x \leq 180^\circ$. [2]

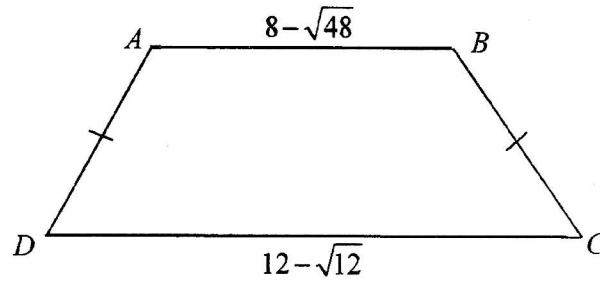
12 (a) Find the values of x and y which satisfy the equations

$$2^x \div 4^y = \frac{1}{8},$$

$$9^y (\sqrt{3})^{2x} = 3\sqrt{3}.$$

[5]

(b)



$ABCD$ is a trapezium with an area of $12 + 11\sqrt{3}$ cm², $AB = 8 - \sqrt{48}$ cm, $DC = 12 - \sqrt{12}$ cm and $AD = BC$. Without using a calculator, obtain an expression for the perpendicular distance between AB and DC in the form $a + b\sqrt{3}$, where a and b are integers.

[4]

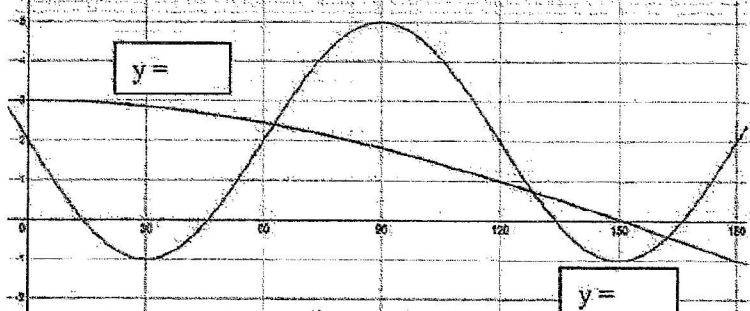
End of Paper



ANSWERS

Qns	Solutions
1(i)	$k \geq 0$
(ii)	$2x^2 + bx + c > 0$ $b^2 - 4(2)(c) < 0$ $b^2 < 8c$ $8c > b^2$ $c > \frac{b^2}{8}$ since $b^2 \geq 0$, $\frac{b^2}{8} \geq 0, c \geq 0$ Hence, c is never negative
2(i)	$\cos \theta = \sqrt{1 - m^2}$
(ii)	$\cot \theta = \frac{\sqrt{1 - m^2}}{m}$
(iii)	$\alpha > \theta$
(iv)	$m = \frac{4}{\sqrt{65}}$
3	$B = \frac{1}{4}, A = \frac{1}{4}$
4	$y = 51.2$
5	$x < 0$ or $x > \frac{4}{5}$
6(i)	Number of chicken alive after 1 day $= (1 - 0.02)N$ $= (0.98)N$ Number of chicken alive after 2 days $= (0.98)(0.98)N$ $= (0.98)^2 N$ Number of chicken alive after 3 days $= (0.98)(0.98)^2 N$ $= (0.98)^3 N$ Therefore the number of chickens expected to be alive after n days is $(0.98)^n N$.
(ii)	$x = 13$
(iii)	$k = -0.0202(3sf)$

Qns	Solutions
7(i)	$\binom{n}{r} 5^r (x)^{n-2r}$
(ii)	For constant term, x^0 $n - 2r = 0$ $n = 2r$ Hence, n must be an even number
(iii)	$1125x^6$
(iv)	$a = 3$
8(i)	$x = \frac{15}{2} - \frac{\pi r}{2}$
(ii)	$x = \frac{15}{2} - \frac{\pi r}{2}$ $A = \pi r^2 + x^2$ $= \pi r^2 + \left(\frac{15}{2} - \frac{\pi r}{2}\right)^2$ $= \pi r^2 + \left(\frac{225}{4} - \frac{15\pi r}{2} + \frac{\pi^2 r^2}{4}\right)$ $A = \left(\frac{1}{4}\pi^2 + \pi\right)r^2 - \frac{15}{2}\pi r + \frac{225}{4}$
(iii)	$r = 2.1004$
(iv)	Diameter, $2r = 2(2.1004) = 4.2007$ $x = \frac{15}{2} - \frac{\pi(2.1004)}{2} = 4.2007$ Hence, $x = \text{diameter}$
(v)	Area is a minimum
9(ii)	$E = (-2, 7)$
(iii)	Area $ECDF = 52$ units ²
10(ii)	4.74 m
11(i)	least value of $f(x) = -1$ greatest value of $f(x) = 5$
(ii)	least value of $g(x) = -5$ greatest value of $g(x) = 3$
(iii)	period of $f(x) = 120^\circ$
(iv)	period of $g(x) = 720^\circ$

Qns	Solutions
(v)	
(vi)	Number of intersections = 3
12a	$x = \frac{-3}{4}, y = 1\frac{1}{8}$
12b	$height = 3 + 2\sqrt{3}$



Additional Mathematics Paper 1

Qns	Solutions	Marks	Remarks
1(i)	$y^2 = 4x - x^2$ meets $y = kx - 2$ $(kx - 2)^2 = 4x - x^2$ $k^2x^2 - 4kx + 4 - 4x + x^2 = 0$ $(k^2 + 1)x^2 - (4k + 4)x + 4 = 0$ $D \geq 0$ $(4k + 4)^2 - 4(k^2 + 1)(4) \geq 0$ $16k^2 + 32k + 16 - 16k^2 - 16 \geq 0$ $32k \geq 0$ $k \geq 0$	 B1 B1 B1 B1	 Equate both eqns Discriminant ≥ 0 Correct substitution of their a, b and c
(ii)	$2x^2 + bx + c > 0$ $b^2 - 4(2)(c) < 0$ $b^2 < 8c$ $8c > b^2$ $c > \frac{b^2}{8}$ since $b^2 \geq 0$, $\frac{b^2}{8} \geq 0, c \geq 0$ Hence, c is never negative	 B1 B1 B1	 Attempt to use D relationship
2(i)	$\sin \theta = m$ $\cos \theta = \sqrt{1 - m^2}$	B1	
(ii)	$\tan \theta = \frac{m}{\sqrt{1 - m^2}}$ $\cot \theta = \frac{\sqrt{1 - m^2}}{m}$	B1	
(iii)	$\tan(\theta - \alpha) = -\frac{1}{2}$ $\alpha > \theta$	B1	
(iv)	$\frac{\tan \theta - \tan \alpha}{1 + \tan \theta \tan \alpha} = -\frac{1}{2}$ $\frac{\tan \theta - \frac{3}{2}}{1 + \frac{3}{2} \tan \theta} = -\frac{1}{2}$ $\tan \theta = \frac{4}{7}$	B1 AG	Substitute ratio using compound angle formula

Qns	Solutions	Marks	Remarks
	$\frac{4}{7} = \frac{m}{\sqrt{1-m^2}}$ $4\sqrt{1-m^2} = 7m$ $16(1-m^2) = 49m^2$ $m^2 = \frac{16}{65}$ $m = \frac{4}{\sqrt{65}} \quad (\text{reject } \frac{-4}{\sqrt{65}})$	M1 A1	Equate from reciprocal of their (ii)
3	$y = A - B \cos 4x - \frac{1}{2} \sin 2x$ $\frac{dy}{dx} = 4B \sin 4x - \cos 2x$ $\frac{d^2y}{dx^2} = 16B \cos 4x + 2 \sin 2x$ $\frac{d^2y}{dx^2} + 4y$ $= 16B \cos 4x + 2 \sin 2x + 4 \left[A - B \cos 4x - \frac{1}{2} \sin 2x \right]$ $= 12B \cos 4x + 4A$ $\therefore 12B \cos 4x + 4A = 3 \cos 4x + 1$ $B = \frac{1}{4}, A = \frac{1}{4}$	B1 B1 M1 A1	Either $4B \sin 4x$ or $-\cos 2x$ see Second derivative Their second derivative $+ 4y$ Both correct

Qns	Solutions	Marks	Remarks
4	$y = \frac{4}{5} \left(\frac{x}{12} - 1 \right)^6$ $\frac{dy}{dx} = \frac{24}{5} \left(\frac{x}{12} - 1 \right)^5 \left(\frac{1}{12} \right) = \frac{2}{5} \left(\frac{x}{12} - 1 \right)^5$ <p>given $\frac{dy}{dt} = \frac{64}{5} \times \frac{dx}{dt}$</p> $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $\frac{64}{5} \times \frac{dx}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $\therefore \frac{dy}{dx} = \frac{64}{5}$ $\frac{2}{5} \left(\frac{x}{12} - 1 \right)^5 = \frac{64}{5}$ $\left(\frac{x}{12} - 1 \right)^5 = 32$ $x = 36$ $y = 51.2$	 B1 B1 M1 A1 A1	 1 st derivative using chain rule Form connected rates of change Or $dy/dx = 64/5$ Their 1 st derivative to $64/5$
5	$y = \frac{6x^2}{2-5x}$ $\frac{dy}{dx} = \frac{12x(2-5x) - 6x^2(-5)}{(2-5x)^2}$ $= \frac{24x - 60x^2 + 30x^2}{(2-5x)^2}$ $= \frac{24x - 30x^2}{(2-5x)^2}$ <p>For decreasing function,</p> $\frac{dy}{dx} < 0$ $\frac{24x - 30x^2}{(2-5x)^2} < 0$ $(2-5x)^2 > 0$ $24x - 30x^2 < 0$ $6x(4-5x) < 0$ $x < 0 \text{ or } x > \frac{4}{5}$	 M1 B1 M1 A1	 Quotient rule with $(2-5x)() - 6x^2()$ seen Their $dy/dx < 0$ Their nume < 0 provided deno is squared.

Qns	Solutions	Marks	Remarks
	$(ax^2 - 1)\left(x + \frac{5}{x}\right)^{10}$ $= (ax^2 - 1)(\dots\dots\dots + 1125x^6 + 15000x^4 + \dots\dots\dots)$ $15000a - 1125 = 43875$ $a = 3$	M1 A1	compare terms to form x^6 term
8(i)	$2\pi r + 4x = 30$ $x = \frac{15}{2} - \frac{\pi r}{2}$	B1	
(ii)	$x = \frac{15}{2} - \frac{\pi r}{2}$ $A = \pi r^2 + x^2$ $= \pi r^2 + \left(\frac{15}{2} - \frac{\pi r}{2}\right)^2$ $= \pi r^2 + \left(\frac{225}{4} - \frac{15\pi r}{2} + \frac{\pi^2 r^2}{4}\right)$ $A = \left(\frac{1}{4}\pi^2 + \pi\right)r^2 - \frac{15}{2}\pi r + \frac{225}{4}$	B1 B1 AG	Substitute their (i) Expand quadratic
(iii)	$\frac{dA}{dr} = 2\left(\frac{1}{4}\pi^2 + \pi\right)r - \frac{15}{2}\pi$ $\frac{dA}{dr} = 0$ $2\left(\frac{1}{4}\pi^2 + \pi\right)r = \frac{15}{2}\pi$ $r = 2.1004$	B1 M1 A1	1 st derivative Their $dA/dr = 0$
(iv)	Diameter, $2r = 2(2.1004) = 4.2008$ $x = \frac{15}{2} - \frac{\pi(2.1004)}{2} = 4.2007$ Hence, $x = \text{diameter}$	M1 A1	Use their r to find either diameter or length of square side Correct values of x and r seen
(v)	$\frac{d^2 A}{dr^2} = 2\left(\frac{1}{4}\pi^2 + \pi\right)$ $\frac{d^2 A}{dr^2} > 0$ Area is a minimum	B1 ✓ B1	o.c. first derivative test with table conclusion

Qns	Solutions	Marks	Remarks
9(i)	$\text{gradient } AD = \frac{4 - (-5)}{0 - 6} = \frac{-3}{2}$ $\text{gradient } BC = \frac{-3}{2}$ $\text{Eqn } BC: y - 3 = \frac{-3}{2}(x - 5)$ $y = \frac{-3}{2}x + 10 \frac{1}{2} \text{ ----- (1)}$ $\text{Eqn } AB: y - 4 = \frac{2}{3}x$ $y = \frac{2}{3}x + 4 \text{ ----- (2)}$ $(1) - (2): 0 = -2\frac{1}{6} + 6\frac{1}{2}$ $x = 3$ $y = \frac{2}{3}(3) + 4$ $= 6$ $\therefore B = (3, 6)$	M1 M1 M1	Attempt to find eqn BC or AB Use reciprocal of gradient to find another eqn Solve their eqns Both x and y correct with working shown, leading to answer
(ii)	$\frac{ED}{AD} = \frac{4}{3}$ $E = (-2, 7)$	M1 A1	Any method including drawing on diagram with ration seen.
(iii)	By counting, $F = (-1, -1)$ Area ECDF $= 2 \times \frac{1}{2} \begin{vmatrix} -2 & -1 & 6 & -2 \\ 7 & -1 & -5 & 7 \end{vmatrix}$ $= [2 + 5 + 42] - [10 + 0 + 7]$ $= 49 + 3$ $= 52 \text{ units}^2$	B1 M1 M1 A1	Coor of F Determinant method Or Find ED Evaluate determinant or find area of triangles o.e.

Qns	Solutions	Marks	Remarks
10(i)	$a = -12e^{2t}$ $v = \int -12e^{2t} dt$ $= -6e^{2t} + c$ $t = 0, V = 9,$ $9 = -6e^{2(0)} + c$ $c = 15$ $\therefore v = -6e^{2t} + 15$ $v = 0,$ $-6e^{2t} + 15 = 0$ $e^{2t} = \frac{15}{6}$ $2t = \ln \frac{15}{6}$ $t = \frac{1}{2} \ln \frac{15}{6}$	B1 M1 A1 M1 A1	-6e ^{2t} seen Attempt to find c Their v = 0 Taking ln, leading to CAG
(ii)	$v = -6e^{2t} + 15$ $s = \int -6e^{2t} + 15 dt$ $= -3e^{2t} + 15t + c$ $t = 0, s = 0,$ $0 = -3e^{2(0)} + 15(0) + c$ $c = 3$ $\therefore s = -3e^{2t} + 15t + 3$ Distance when particle stops to change direction: Sub $t = \frac{1}{2} \ln \frac{15}{6}$, $s = 2.372\text{m}$ When particle returns to O, total distance travelled = $2(2.372) = 4.74\text{m}$	B1 M1 A1 B1	Integrate their v to find s Attempt to find c B1
11(i)	least value of $f(x) = -1$ greatest value of $f(x) = 5$	B1 B1	
(ii)	least value of $g(x) = -5$ greatest value of $g(x) = 3$	B1 B1	
(iii)	period of $f(x) = 120$	B1	Accept $\frac{2\pi}{3}$
(iv)	period of $g(x) = 720$	B1	Accept 4π

Qns	Solutions	Marks	Remarks
(v)		<p>G1</p> <p>G1</p> <p>G1</p> <p>G1</p>	<p>For cosine graph fully correct</p> <p>For cosine graph 1.5 cycles seen</p> <p>Starts and ends at 2</p> <p>Graph is fully correct</p> <p>-1 mark for not labelling</p>
(vi)	$2 \cos \frac{x}{2} + \frac{3}{2} \sin 3x = \frac{3}{2}$ $4 \cos \frac{x}{2} + 3 \sin 3x = 3$ $4 \cos \frac{x}{2} + 3 \sin 3x = 2 + 1$ $4 \cos \frac{x}{2} - 1 = 2 - 3 \sin 3x$ <p>Number of intersections = 3</p>	<p>M1</p> <p>A1 ✓</p>	<p>Manipulation seen</p> <p>Their number of intersections provided that manipulation is seen</p>
12a	$2^x + 4^y = \frac{1}{8}$ $2^x + 2^{2y} = 2^{-3}$ $x - 2y = -3 \quad \text{-----(1)}$ $9^y (\sqrt{3})^{2x} = 3\sqrt{3}$ $3^{2y} \times 3^{\frac{1}{2}(2x)} = 3^{\frac{3}{2}}$ $2y + x = \frac{3}{2} \quad \text{-----(2)}$ <p>(1) + (2): $2x = -1.5$</p> $x = \frac{-3}{4}$ $y = 1\frac{1}{8}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Change to same base for either eqn</p> <p>Index rule seen for either eqn</p> <p>Comparing index</p>

Qns	Solutions	Marks	Remarks
12b	$\frac{1}{2}(8 - \sqrt{48} + 12 - \sqrt{12})height = 12 + 11\sqrt{3}$	M1	Forming equation
	$\frac{1}{2}(8 - 2\sqrt{12} + 12 - \sqrt{12})height = 12 + 11\sqrt{3}$	M1	Attempt to reduce / simplify surds (take out largest perfect square o.e.)
	$(20 - 3\sqrt{12})height = 24 + 22\sqrt{3}$		
	$(20 - 6\sqrt{3})height = 24 + 22\sqrt{3}$		
	$height = \frac{12 + 11\sqrt{3}}{(10 - 3\sqrt{3})} \times \frac{(10 + 3\sqrt{3})}{(10 + 3\sqrt{3})}$ $= \frac{120 + 36\sqrt{3} + 110\sqrt{3} + 99}{100 - 27}$ $= \frac{219 + 146\sqrt{3}}{73}$ $= 3 + 2\sqrt{3}$	M1	Simplify with their conjugate seen
		A1	



TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2021
Secondary 4

CANDIDATE
NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/02

Paper 2

Tuesday 24 August 2021

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The total number of marks for this paper is 90.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

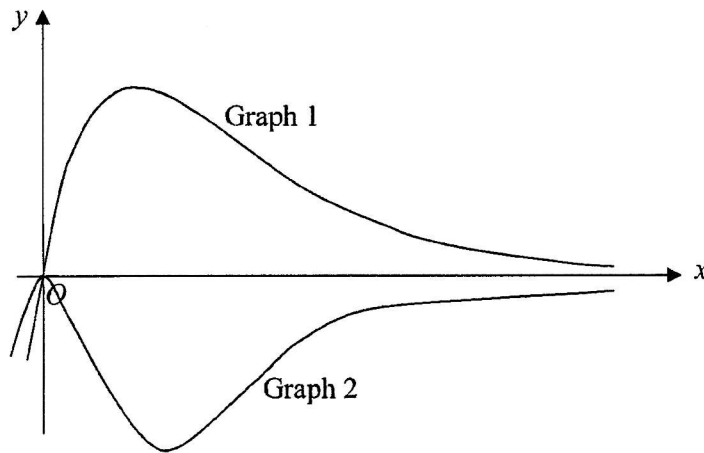
Answer **all** questions.

1 (i) Differentiate $2xe^{1-2x}$ with respect to x . [2]

(ii) Hence find the exact value of $\int_0^1 2xe^{1-2x} dx$. [4]

(iii) Determine which one of two graphs below is that of $y = 2xe^{1-2x}$. Explain your answer.

[2]



2 (i) Prove the identity $\frac{\tan^2 y - \sin^2 y}{\sec^2 y (\operatorname{cosec}^2 y - 1)} = \tan^2 y \sin^4 y$.

[4]

(ii) Hence solve the equation $\frac{\tan^2 y - \sin^2 y}{\sec^2 y (\operatorname{cosec}^2 y - 1)} = \frac{1}{16} \tan^2 y$ for $0^\circ \leq y \leq 180^\circ$. [4]

3 (i) Show that $x-2$ is a factor of $x^3-12x+16$. [1]

(ii) Express $\frac{2+x^2}{x^3-12x+16}$ as the sum of three partial fractions. [7]

- 4 (a) Given that $\log_a 8 + 3\log_a N = 3$ and $N > \frac{1}{4}$, find the range of values of a . [4]

- (b) Solve the equation $\log_3 x - \log_x 3x - 2 = 0$. [5]

- 5 It is given that $f'(x) = \sin 3x - \frac{1}{2x+1}$ and $f(0) = \frac{2}{3}$.

Find an expression for $6f(x) + f''(x)$.

[6]

6 (i) Show that $\frac{d}{d\theta}(\cos^4 \theta + \sin^4 \theta) = p \sin q\theta$, where p and q are integers. [4]

(ii) Hence find the exact value of θ , for which the graph $y = \cos^4 \theta + \sin^4 \theta$ has a horizontal tangent for $0 < \theta < \frac{\pi}{2}$. [2]

- 7 The equation of a circle, centre C , is $x^2 + y^2 + px + \left(\frac{p}{2} + 4\right)y + k = 0$, where p and k are constants. It is given that C lies on the line $3x - 2y - 8 = 0$.

(i) Show that $p = -4$. [2]

(ii) Find the coordinates of C . [1]

(iii) Find the value of k , given that $x = -8$ is a tangent to the circle. [3]

The coordinates of point A is $(14, -8)$.

(iv) Show that A lies outside of the circle.

[2]

(v) Point X lies on the circle such that it is furthest from A .

State the geometrical relationship between points A , C and X .

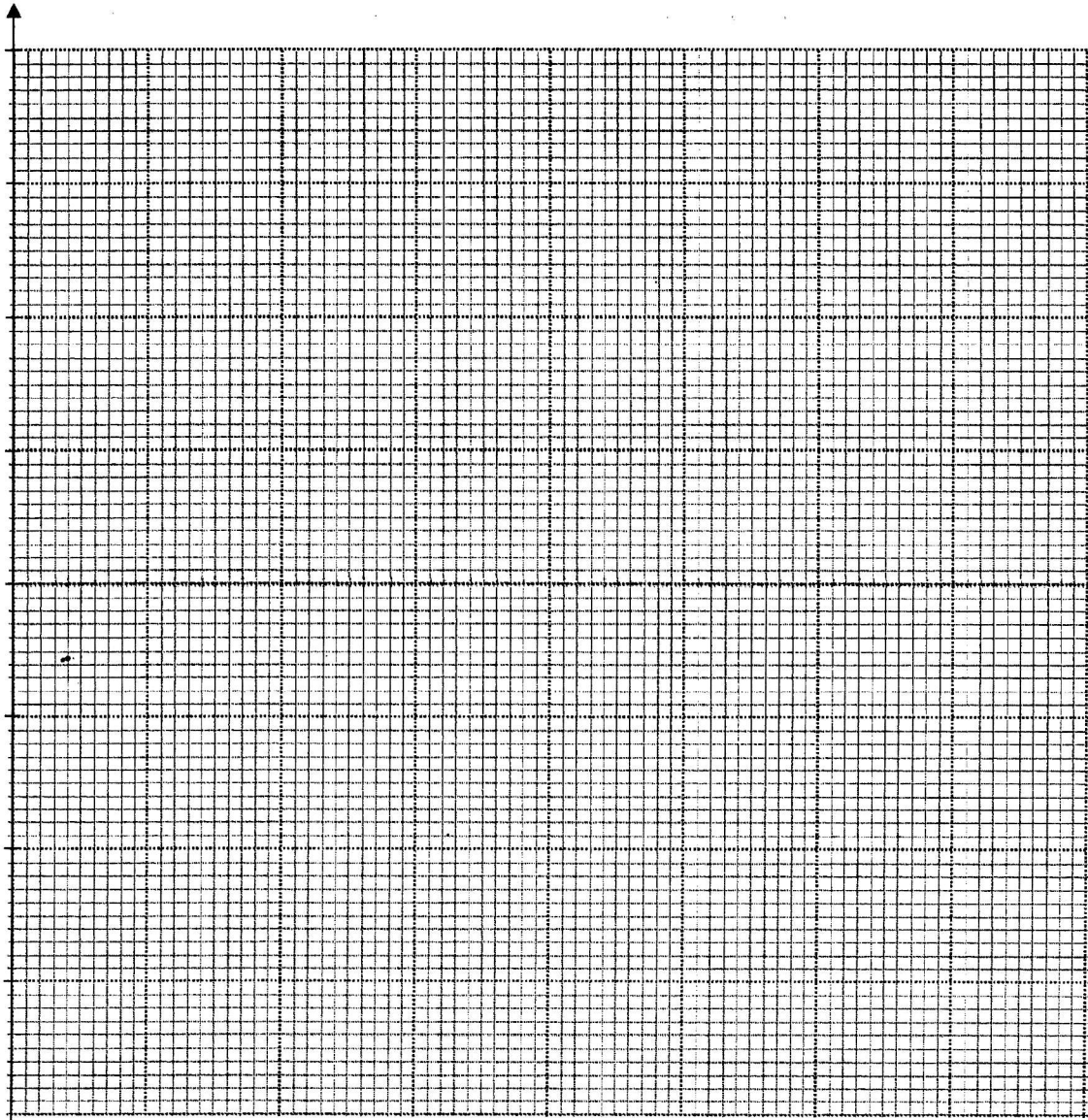
[1]

- 8 Measured values of x and y are given in the following table.

x	1.2	1.6	2.0	2.6	3.5	4.5
y	11.1	6.29	3.98	2.36	1.55	0.79

It is known that x and y are to obey the formula $x^a y = c$, where a and c are constants to be found.

- (i) Explain how a straight line graph may be drawn to represent the given formula. [2]



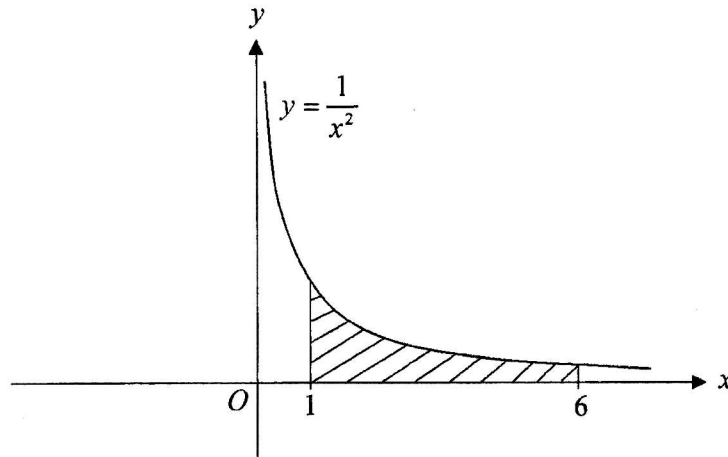
(ii) On the grid above, draw the straight line graph for the given data and show that 5 of the readings are a close fit to the formula.

[4]

(iii) Assuming that x is correct, state which value of y appears to be wrong and estimate what it should be to fit the formula.; [2]

(iv) Find the value of a and of c . [3]

- 9 The diagram shows part of the curve $y = \frac{1}{x^2}$.



- (i) Find the area bounded by the curve $y = \frac{1}{x^2}$, the lines $x=1$, $x=6$ and the x -axis. [3]

- (ii) Show that $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} < \frac{5}{6}$. [2]

10 (i) Show that $4 \sin x + 2 \cos\left(x + \frac{\pi}{6}\right) = a \cos x + b \sin x$, where a and b are exact values. [3]

(ii) Express $4 \sin x + 2 \cos\left(x + \frac{\pi}{6}\right)$ in the form $k \cos(x - \alpha)$ where $k > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Find the value of k and of α .

[3]

(iii) State the minimum value of $4 \sin x + 2 \cos\left(x + \frac{\pi}{6}\right)$ and the corresponding value of x . [2]

(iv) Solve the equation $4 \sin x + 2 \cos\left(x + \frac{\pi}{6}\right) = -2$ for $0 < x < 6$. [3]

11 (i) Find the gradient function of the curve $y = 2x^2 - \ln x + c$. [1]

(ii) The curve touches the positive x -axis at point T . Find the coordinates of T . [2]

(iii) Show that $c = -\ln 2\sqrt{e}$. [3]

(iv) Find the value of $\frac{d^2y}{dx^2}$ at T and hence determine the nature of point T . [3]

End of Paper

Answers

1(i) $2e^{1-2x} - 4xe^{1-2x}$

1(ii) $\frac{1}{2}\left(e - \frac{3}{e}\right)$

1(iii) $\frac{1}{2}\left(e - \frac{3}{e}\right) > 0$. Graph 1 as graph is above the x-axis.

2(ii) $y = 0^\circ, 30^\circ, 150^\circ, 180^\circ$

3(ii) $\frac{2+x^2}{x^3-12x+16} = \frac{1}{2(x-2)} + \frac{1}{(x-2)^2} + \frac{1}{2(x+4)}$

4(a) $a > \frac{1}{2}, a \neq 1$

4(b) $x = 37.7, 0.717$

5 $\cos 3x - 3 \ln(2x+1) + 6 + \frac{2}{(2x+1)^2}$

6(i) $\frac{d}{dx}(\cos^4 \theta + \sin^4 \theta) = -\sin 4\theta$

6(ii) $\theta = \frac{\pi}{4}$

7(ii) $C = (2, -1)$

7(iii) $k = -95$

7(v) Points A, C and X are collinear.

8(i) $\lg y = -a \lg x + \lg c$

By plotting $\lg y$ against $\lg x$, a straight line will be obtained with $-a$ as the gradient and $\lg c$ as the vertical intercept.

8(iii) $y = 1.30$

8(iv) $a = 2, c = 16.2$

9(i) $\frac{5}{6}$

10(i) $4 \sin x + 2 \cos\left(x + \frac{\pi}{3}\right) = \sqrt{3} \cos x + 3 \sin x$

10(ii) $k = 2\sqrt{3}, \alpha = \frac{\pi}{3}, 4 \sin x + 2 \cos\left(x + \frac{\pi}{6}\right) = 2\sqrt{3} \cos\left(x - \frac{\pi}{3}\right)$

10(iii) Minimum value $= -2\sqrt{3}, x = \frac{4\pi}{3}$

10(iv) $x = 3.23, 5.14$

11(i) $\frac{dy}{dx} = 4x - \frac{1}{x}$

11(ii) $T = \left(\frac{1}{2}, 0\right)$

11(iv) $\frac{d^2y}{dx^2} = 8$, minimum point

Qn	Working	
1(i)	$\frac{d}{dx} 2xe^{1-2x}$ $= 2e^{1-2x} + 2x(-2e^{1-2x})$ $= 2e^{1-2x} - 4xe^{1-2x}$ $= 2e^{1-2x}(1-2x)$	B1 for $2e^{1-2x}$ B1 for $2x(-2e^{1-2x})$ isw
1(ii)	$\int_0^1 2e^{1-2x} - 4xe^{1-2x} dx = [2xe^{1-2x}]_0^1$ $- 2 \int_0^1 2xe^{1-2x} dx = (2e^{-1} - 0) - \int_0^1 2e^{1-2x} dx$ $- 2 \int_0^1 2xe^{1-2x} dx = \frac{2}{e} - \left[\frac{2}{-2} e^{1-2x} \right]_0^1$ $\int_0^1 2xe^{1-2x} dx = -\frac{1}{e} + \frac{1}{2} [-e^{-1} + 1]$ $\int_0^1 2xe^{1-2x} dx = -\frac{1}{e} - \frac{1}{2e} + \frac{1}{2}$ $\int_0^1 2xe^{1-2x} dx = -\frac{3}{2e} + \frac{1}{2} = \frac{1}{2} \left(e - \frac{3}{e} \right)$	B1 Using diff for integral M1 Splitting the integrals, attempt to manipulate B1 Integrate $\int_0^1 2e^{1-2x} dx = -\frac{2}{2} e^{1-2x}$ A1 exact o.e.
1(iii)	<p>When $x > 0$,</p> <p>$2x > 0$ and $e^{1-2x} > 0$</p> <p>$\therefore 2xe^{1-2x} > 0$</p> <p>$y > 0$</p> <p>$\therefore$ Graph 1 is $y = 2xe^{1-2x}$</p> <p>Or</p> <p>Since area under the graph is positive because,</p> $\int_0^1 2xe^{1-2x} dx = -\frac{3}{2e} + \frac{1}{2} = \frac{1}{2} \left(e - \frac{3}{e} \right) > 0$ <p>thus, graph is above the x-axis.</p> <p>\therefore Graph 1 is $y = 2xe^{1-2x}$.</p>	B1 Any logical reasoning B1 Graph 1

<p>2(i)</p>	<p>Taking LHS,</p> $\frac{\tan^2 y - \sin^2 y}{\sec^2 y (\operatorname{cosec}^2 y - 1)}$ $= \frac{\tan^2 y - \sin^2 y}{\sec^2 y (\cot^2 y)}$ $= \frac{\tan^2 y - \sin^2 y}{\frac{1}{\cos^2 y} \left(\frac{\cos^2 y}{\sin^2 y} \right)}$ $= \sin^2 y (\tan^2 y - \sin^2 y)$ $= \sin^2 y \left(\frac{\sin^2 y}{\cos^2 y} \right) - \sin^4 y$ $= \sin^4 y \left(\frac{1}{\cos^2 y} - 1 \right)$ $= \sin^4 y (\sec^2 y - 1)$ $= \tan^2 y \sin^4 y \text{ (shown)}$	<p>B1 Reciprocal of sec/cosec</p> <p>B1 Use of any sq identities</p> <p>B1 $\tan y = \frac{\sin y}{\cos y}$</p> <p>B1 Manipulate leading to correct answer</p> <p>AG</p>
<p>2(ii)</p>	$\frac{\tan^2 y - \sin^2 y}{\sec^2 y (\operatorname{cosec}^2 y - 1)} = \frac{1}{16} \tan^2 y$ <p>From (i),</p> $\tan^2 y \sin^4 y = \frac{1}{16} \tan^2 y$ $16 \tan^2 y \sin^4 y - \tan^2 y = 0$ $\tan^2 y (16 \sin^4 y - 1) = 0$ $\tan^2 y = 0 \quad \text{or} \quad 16 \sin^4 y - 1 = 0$ $\tan y = 0 \quad \sin y = \pm \frac{1}{2}$ $y = 0^\circ, 180^\circ \quad y = 30^\circ, 150^\circ$ <p>$\therefore y = 0^\circ, 30^\circ, 150^\circ, 180^\circ$</p>	<p>B1 Using part (i)</p> <p>B1 Factorisation</p> <p>B2 2 sets of answers</p>

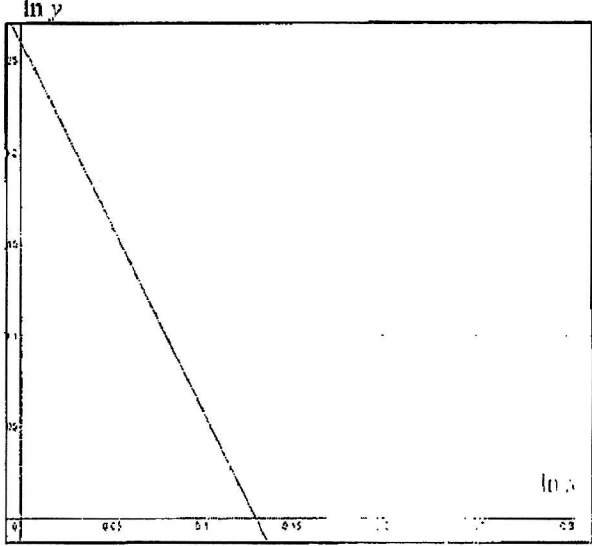
<p>3(i)</p>	<p>Let $f(x) = x^3 - 12x + 16$ Sub. $x = 2$, $f(2) = (2)^3 - 12(2) + 16$ $= 0$ By Factor Theorem since $f(2) = 0$. $(x - 2)$ is a factor. (shown)</p>	<p>B1 Substitution and Factor Theorem/ Remainder Theorem must be quoted</p>
<p>3(ii)</p>	<p>$x^3 - 12x + 16 = (x - 2)(x^2 + px - 8)$</p> <p>By comparing coefficient of x^2. $0 = p - 2$ $p = 2$</p> <p>$\therefore f(x) = (x - 2)(x^2 + 2x - 8)$ $= (x - 2)(x - 2)(x + 4)$ $= (x - 2)^2(x + 4)$</p> $\frac{2 + x^2}{x^3 - 12x + 16} = \frac{A}{x - 2} + \frac{B}{(x - 2)^2} + \frac{C}{x + 4}$ $2 + x^2 = A(x - 2)(x + 4) + B(x + 4) + C(x - 2)^2$ <p>Sub $x = 2$, $B = 1$ Sub $x = -4$, $C = \frac{1}{2}$ Sub $x = 0$, $A = \frac{1}{2}$</p> $\frac{2 + x^2}{x^3 - 12x + 16} = \frac{1}{2(x - 2)} + \frac{1}{(x - 2)^2} + \frac{1}{2(x + 4)}$	<p>M1 Inspection/Long Div</p> <p>B1 Quad factor soi</p> <p>B1 3 Factors soi <i>Lose 3m if use calculator</i></p> <p>B1 Correct cases \checkmark (must be cubic)</p> <p>M1 Subs/Comparing coef</p> <p>A2 A, B, C</p> <p>[- 1m of not stated]</p>

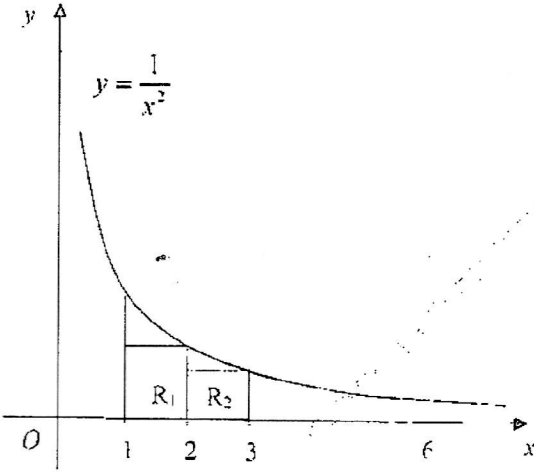
<p>4(a)</p>	$\log_a 8 + 3 \log_a N = 3$ $\log_a 8N^3 = 3$ $a^3 = 8N^3$ $a = 2N$ $N = \frac{a}{2}$ $\frac{a}{2} > \frac{1}{4} \quad \text{since } N > \frac{1}{4}$ $a > \frac{1}{2}, \quad a \neq 1$	<p>M1 $\log_a 8N^3$ (combine)</p> <p>M1 $a^3 = 8N^3$ (remove log)</p> <p>M1 (Their N) $\frac{a}{2} > \frac{1}{4}$</p> <p>A1 Answer</p>
<p>4(b)</p>	$\log_3 x - \log_x 3x - 2 = 0$ $\log_3 x - \frac{\log_3 3x}{\log_3 x} - 2 = 0$ $(\log_3 x)^2 - \log_3 3x - 2 \log_3 x = 0$ $(\log_3 x)^2 - (\log_3 3 + \log_3 x) - 2 \log_3 x = 0$ $(\log_3 x)^2 - 1 - 3 \log_3 x = 0$ <p>Let $u = \log_3 x$,</p> $u^2 - 3u - 1 = 0$ $u = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$ $u = 3.30278 \quad \text{or} \quad u = -0.30278$ $\log_3 x = 3.30278 \quad \text{or} \quad \log_3 x = -0.30278$ $x = 3^{3.30278} \quad \quad \quad x = 3^{-0.30278}$ $x = 37.7 \text{ (3 sf)} \quad \quad \quad x = 0.717 \text{ (3 sf)}$	<p>B1 Change base soi</p> <p>B1 Addition log law soi</p> <p>M1 Forming equation (Subst)</p> <p>M1 Solving Quad</p> <p>A1 both answers</p>

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<p>7(i)</p>	$x^2 + y^2 + px + \left(\frac{p}{2} + 4\right)y + k = 0$ <p>Centre, $C = \left(-\frac{p}{2}, -\frac{p}{4} - 2\right)$</p> <p>Sub into eqn of line;</p> $3\left(-\frac{p}{2}\right) - 2\left(-\frac{p}{4} - 2\right) - 8 = 0$ $3p - p - 8 + 16 = 0$ $p = -4 \text{ (shown)}$	<p>B1 Centre of circle</p> <p>B1 Substitution ✓ (wrt centre)</p> <p>AG</p>
<p>7(ii)</p>	$C = (2, -1)$	<p>B1 ✓</p>
<p>7(iii)</p>	<p>Radius = 10</p> $10 = \sqrt{(2)^2 + (-1)^2} - k$ $100 = 4 + 1 - k$ $k = -95$	<p>B1 ✓</p> <p>M1 Subs of formula (ignore r)</p> <p>A1</p>
<p>7(iv)</p>	$\text{Length } CA = \sqrt{(2-14)^2 + (-1-(-8))^2}$ $= 13.89 > 10$ <p>∴ A lies outside of the circle. (shown)</p>	<p>M1 Finding length of point from centre of circle</p> <p>A1 cao</p>
<p>7(v)</p>	<p>ACX will be a straight line.</p> <p>OR</p> <p>Points A, C and X are collinear.</p>	<p>B1 Statement</p>

8(i)	$x^a y = c$ $\lg x^a + \lg y = \lg c$ $\lg y = -a \lg x + \lg c$ By plotting $\lg y$ against $\lg x$, a straight line will be obtained with $-a$ as the gradient and $\lg c$ as the vertical intercept.	B1 $Y = mX + c$ B1 Statement																												
8(ii)	<table border="1" data-bbox="405 568 1378 636"> <tr> <td>$\lg x$</td> <td>0.08</td> <td>0.20</td> <td>0.30</td> <td>0.41</td> <td>0.54</td> <td>0.65</td> </tr> <tr> <td>$\lg y$</td> <td>1.05</td> <td>0.80</td> <td>0.60</td> <td>0.37</td> <td>0.19</td> <td>-0.10</td> </tr> </table> <div data-bbox="405 658 1018 1227"> </div> <p data-bbox="405 1256 437 1285">Or</p> $x^a y = c$ $\ln x^a + \ln y = \ln c$ $\ln y = -a \ln x + \ln c$ <p data-bbox="405 1451 935 1554">By plotting $\ln y$ against $\ln x$, a straight line will be obtained with $-a$ as the gradient and $\ln c$ as the vertical intercept.</p> <table border="1" data-bbox="405 1585 1378 1653"> <tr> <td>$\ln x$</td> <td>0.18</td> <td>0.47</td> <td>0.69</td> <td>0.96</td> <td>1.25</td> <td>1.50</td> </tr> <tr> <td>$\ln y$</td> <td>2.41</td> <td>1.84</td> <td>1.38</td> <td>0.86</td> <td>0.44</td> <td>-0.24</td> </tr> </table>	$\lg x$	0.08	0.20	0.30	0.41	0.54	0.65	$\lg y$	1.05	0.80	0.60	0.37	0.19	-0.10	$\ln x$	0.18	0.47	0.69	0.96	1.25	1.50	$\ln y$	2.41	1.84	1.38	0.86	0.44	-0.24	<p data-bbox="1034 725 1315 815">T1 New table of values Accept only 2, 3, 4 dp accuracy</p> <p data-bbox="1034 824 1315 853">P1 Pts correctly plotted</p> <p data-bbox="1034 862 1251 891">L1 Line of best fit</p> <p data-bbox="1034 900 1331 967">L1 Vertical intercept seen with one outlier point</p>
$\lg x$	0.08	0.20	0.30	0.41	0.54	0.65																								
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$\ln y$	2.41	1.84	1.38	0.86	0.44	-0.24																								

	 <p>As shown from the plotted graph, one point is an outlier point from the straight line $Y = mX + C$ which is at $\lg x = 0.54 / \ln x = 1.25$.</p>	<p>Not required. Teacher to reiterate when papers are returned - explain qns</p>
<p>8(iii)</p>	<p>$y = 1.55$ For graph of $\lg y$ against $\lg x$, $\lg y = 0.11$ $y = 10^{0.11}$ $y = 1.29$ Or For graph of $\ln y$ against $\ln x$, $\ln y = 0.26$ $y = e^{0.26}$ $y = 1.30$</p>	<p>B1 Incorrect value of y</p> <p>B1 [1.26 to 1.32]</p>
<p>8(iv)</p>	<p>For graph of $\lg y$ against $\lg x$, $\lg c = 1.71$ $c = 10^{1.71}$ $c = 16.2$ (3sf)</p> <p>$-a = \frac{0.9 - 0.2}{0.15 - 0.5}$ $a = 2$ Or For graph of $\ln y$ against $\ln x$, $\lg c = 2.76$ $c = e^{2.76}$ $c = 15.8$ (3sf)</p> <p>$-a = \frac{2.36 - 0.4}{0.2 - 1.18}$ $a = 2$</p>	<p>M1 Vertical intercept used [1.20 to 1.22]</p> <p>A1 [15.5 to 16.5]</p> <p>A1 [-1.94 to -2.06]</p> <p>[2.74 to 2.78]</p>

<p>9(i)</p>	$\text{Area} = \int_1^6 \frac{1}{x^2} dx$ $= \left[-\frac{1}{x} \right]_1^6$ $= -\frac{1}{6} + \frac{1}{1}$ $= \frac{5}{6}$	<p>BI $\int_1^6 \frac{1}{x^2} dx$ seen</p> <p>BI $-\frac{1}{x}$ soi</p> <p>BI</p>
<p>9(ii)</p>	 <p>Area of $R_1 = \text{Length} \times \text{Breadth}$</p> $= 1 \times \frac{1}{2^2}$ $= \frac{1}{2^2}$ <p>Area of $R_2 = \text{Length} \times \text{Breadth}$</p> $= 1 \times \frac{1}{3^2}$ $= \frac{1}{3^2}$ <p>Area of $(R_1 + R_2 + R_3 + R_4 + R_5) < \text{Shaded area}$</p> $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} < \frac{5}{6} \quad (\text{shown})$	<p>BI Identify the rectangles</p> <p>BI Conclusion</p> <p>AG</p>

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<p>10(i)</p>	$4 \sin x + 2 \cos \left(x + \frac{\pi}{6} \right)$ $= 4 \sin x + 2 \left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6} \right)$ $= 4 \sin x + 2 \left(\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x \right)$ $= 4 \sin x + \sqrt{3} \cos x - \sin x$ $= \sqrt{3} \cos x + 3 \sin x$	<p>M1 soi</p> $\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}$ <p>B1 $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ soi</p> <p>A1</p>
<p>10(ii)</p>	<p>Let $\sqrt{3} \cos x + 3 \sin x = k \cos(x - \alpha)$</p> $k = \sqrt{(\sqrt{3})^2 + 3^2}$ $k = \sqrt{12}$ $k = 2\sqrt{3}$ $\alpha = \tan^{-1} \frac{3}{\sqrt{3}}$ $\alpha = \frac{\pi}{3}$ $4 \sin x + 2 \cos \left(x + \frac{\pi}{6} \right) = 2\sqrt{3} \cos \left(x - \frac{\pi}{3} \right)$	<p>B1 Find k o.e.</p> <p>B1 Finding α o.e.</p> <p>B1 Answer \surd o.e.</p>
<p>10(iii)</p>	<p>Minimum value $= -\sqrt{12} = -2\sqrt{3}$</p> <p>Occurs when $x = \frac{4\pi}{3}$</p>	<p>B1 \surd o.e.</p> <p>B1</p>
<p>10(iv)</p>	$\sqrt{12} \cos \left(x - \frac{\pi}{3} \right) = -2$ $\cos \left(x - \frac{\pi}{3} \right) = \frac{-2}{\sqrt{12}} \quad (2^{\text{nd}} / 3^{\text{rd}} \text{ quads})$ <p>Basic angle of $x - \frac{\pi}{3} = 0.95532$</p> $x - \frac{\pi}{3} = 2.1863, 4.0969$ $x = 3.23, 5.14 \quad (3 \text{sf})$ <p>Domain: $0 < x < 6$</p> $-1.0472 < x - \frac{\pi}{3} < 4.9528$	<p>B1 Statement \surd o.e.</p> <p>B2 Answers Accept 5.15</p>

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11(i)	$y = 2x^2 - \ln x + c$ $\frac{dy}{dx} = 4x - \frac{1}{x}$	B1 $4x - \frac{1}{x}$ soi
11(ii)	$4x - \frac{1}{x} = 0$ $4x^2 - 1 = 0$ $x = \frac{1}{2} \quad (\because x > 0)$ $T = \left(\frac{1}{2}, 0\right)$	B1 = 0 ✓ B1 Coordinates
11(iii)	$\text{Sub } \left(\frac{1}{2}, 0\right),$ $0 = 2\left(\frac{1}{2}\right)^2 - \ln \frac{1}{2} + c$ $c = \ln \frac{1}{2} - 2\left(\frac{1}{4}\right)$ $c = \ln 2^{-1} - \frac{1}{2}$ $c = \ln 2^{-1} - \frac{1}{2} \ln e$ $c = \ln 2^{-1} - \ln e^{\frac{1}{2}}$ $c = \ln \frac{2^{-1}}{\sqrt{e}}$ $c = \ln \frac{1}{2\sqrt{e}}$ $c = \ln (2\sqrt{e})^{-1}$ $c = -\ln 2\sqrt{e}$	M1 Substitution M1 Combine 2 logs A1 $\ln x^a = y \ln x$ o.e. AG
11(iv)	$\frac{d^2y}{dx^2} = 4 + \frac{1}{x^2}$ $\text{Sub } x = \frac{1}{2},$ $\frac{d^2y}{dx^2} = 4 + \frac{1}{\left(\frac{1}{2}\right)^2}$ $\frac{d^2y}{dx^2} = 8 > 0$ <p>Since $\frac{d^2y}{dx^2} > 0$, therefore point T is a minimum point.</p>	M1 Finding $\frac{d^2y}{dx^2}$ ✓ M1 Value of $\frac{d^2y}{dx^2}$ ✓ A1 Minimum point seen