BEDOK VIEW SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023

CANDIDATE NAME

REGISTER NUMBER

CLASS

ADDITIONAL MATHEMATICS
Secondary 4 Express / 5 Normal Academic

4049/01
29 August 2023
2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Setter(s): Mrs Kuek

This document consists of **22** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta ABC = \frac{1}{2} ab \sin C$$

1 **(a)** Express $-\frac{1}{2}x^2 - 4x + 3$ in the form $a(x+b)^2 + c$ where a, b and c are constants. [2]

(b) The line $y = p$ intersects $y = -\frac{1}{2}x^2 - 4x + 3$ at 2 real and distinct points. Find the range of values of p. [1]

(c) Student B stated that the curve $y = -\frac{1}{2}x^2 - 4x + 3$ lies below the x - axis. Do you agree or disagree with student B? Explain your answer. [1]

2 Express $\dfrac{8x^4+5x^3-43x^2-36x+27}{2x^4-9x^2}$ in partial fractions. [6]

3 **(a)** **(i)** Find the period of $f(x) = 3\tan\frac{x}{2}$. [1]

(ii) Find the amplitude and period of $g(x) = 1 - \sin x$. [2]

(b) Sketch, on the same axes, the graphs $f(x)$ and $g(x)$ for $0 \le x \le 2\pi$. [4]

(c) Hence, state the range of values of x, between 0 and π, for which $f(x)$ and $g(x)$ are both increasing as x increases. [1]

(d) Find the equation of a straight line to draw in order to find the solution of the equation $3\pi \tan\frac{x}{2} + x = 2\pi$. [1]

4 **(a)** **(i)** Explain why the powers of $\left(ax-\frac{3}{2x}\right)^{11}$ is always odd, where a is a constant. [3]

(ii) Given that the coefficient of $\frac{1}{x}$ in the expansion of $\left(ax-\frac{3}{2x}\right)^{11}$ is 168399, find the value of a. [4]

(b) **(i)** Find, in ascending powers of x, the first four terms in the expansion of $\left(1+2x^2\right)^n$, leaving your answer in terms of n. [2]

(ii) If the ratio of the coefficient of x^2 to the coefficient of x^6 in the expansion of $\left(1+2x^2\right)^n$ is $1:28$, find the value of n. [3]

5 A particle P which moves in a straight line, starts from a fixed point O so that its velocity, v ms^{-1}, is given by $v = e^{2t} - 4e^{-2t}$, where t is the time in seconds after leaving O.

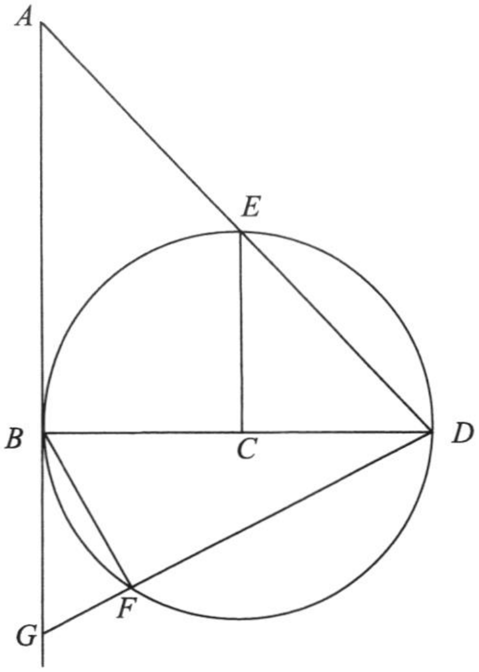
(a) Find the expression for the displacement in terms of t. [3]

(b) Find the acceleration of P when it attains its greatest distance. [4]

Continue working for part (b)

(c) Find the total distance travelled in the first second. [2]

6 In the diagram below, the line ABG is a tangent to the circle with BD as a diameter and C as the centre. $AE = ED$ and triangle BDF is inscribed in the circle where DFG is a straight line.



(a) Prove that $AB = 2EC$. [1]

(b) Prove that $FB \times BD = FD \times GB$. [3]

7 The straight line $3y-x=6$ intersects the curve $2y=2(2x-1)^2+n$ at the points $A(m,2.5)$ and B.

(a) Find the value of *m* and of *n*. [4]

(b) Find the coordinates of point *B*. [3]

8 The set of values of x which satisfy the inequality $x^2+3x+q\leq 2x+7$ is $p\leq x\leq 1$, where p and q are constants.

(a) Show that $q=5$. [2]

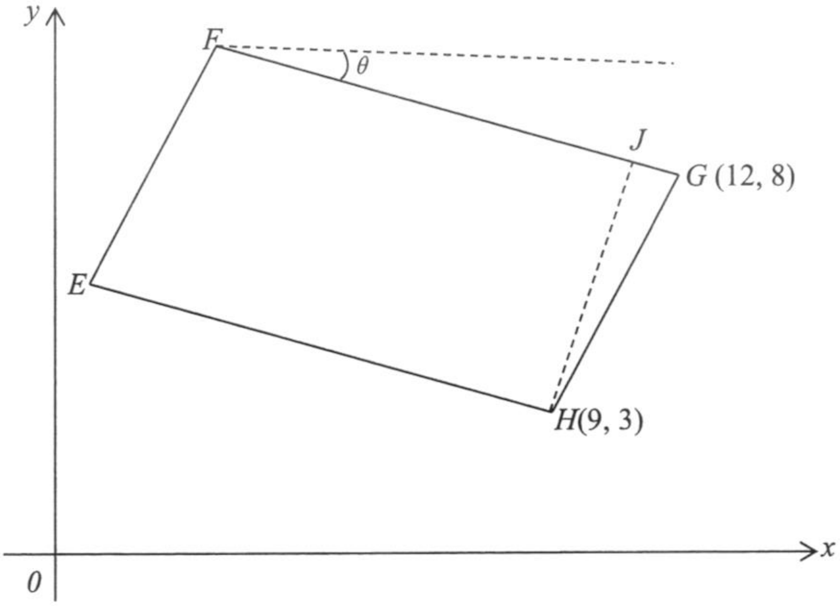
(b) Hence, find the value of p. [2]

9 A curve is such that its gradient function is given as $\frac{\cos 2x+1}{k}$, where k is a non-zero constant. The curve passes through point $W\left(\frac{\pi}{6},\frac{\sqrt{3}}{4}\right)$. The tangent to the curve at the point W is parallel to the line $4y-3x=6$.

(a) Show that the value of k is 2. [3]

(b) Hence find the equation of the curve, leaving your answer in its exact form. [4]

10 The diagram below shows a parallelogram *EFGH* in which the coordinates of the points *G* and *H* are (12, 8) and (9, 3) respectively. The line *FG* makes an angle θ with the horizontal and $\tan\theta = \frac{1}{2}$. The point *J* lies on *FG* such that *HJ* is the shortest distance from *H* to *FG*.



(a) Show that the equation of the line *FG* is $2y = 28 - x$. [3]

(b) Find the equation of the line *HJ*. [3]

(c) Find the coordinates of point *J*. [2]

(d) Given that $FJ = 19JG$, find the coordinates of point F. [2]

(e) Given that there is a point M on the line FG such that the area of triangle GHM is 19.5 units^2, find the coordinates of point M. [4]

11 **(a)** Find the range of values of m where the curve $y = mx^2 + 7$ lies entirely above the straight line $y = \sqrt{5}x + 7 - m$ where m is a real number, leaving your answer in its exact form. [4]

(b) State the values of m where the straight line $y = \sqrt{5}x + 7 - m$ is a tangent to the curve $y = mx^2 + 7$. [1]

12 The table below shows the 5 recordings of an experiment where the variables of x and y are related by the equation $e^y = ax^b$ where a and b are constants.

Recordings	1st	2nd	3rd	4th	5th
x	1	2	3	5	7
y	1.10	2.28	2.50	3.89	4.41

(a) Explain how a straight line graph may be drawn to represent the given equation. [2]

(b) Draw the straight line on the graph paper provided for the given data. [2]

(c) Due to some error, one of the y readings has been recorded wrongly. From the straight line graph, estimate

(i) the correct reading of y, [1]

(ii) the value of a and of b, [3]

(iii) the value of y when $x = 4$. [1]

END OF PAPER

BEDOK VIEW SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023

CANDIDATE NAME			
REGISTER NUMBER		CLASS	

ADDITIONAL MATHEMATICS
Secondary 4 Express / 5 Normal Academic
Paper 2

4049/02
12 September 2023
2 hours 15 minutes

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Setter: Ms S. Ang

Parent's / Guardian's Signature: ....

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1 By using a suitable substitution, solve the equation $3^{\frac{x}{2}} = 2 + 24(3)^{-\frac{x}{2}}$. [4]

2 **(a)** The function of a curve is given by $y = (ax + 1)\, e^{3x-1}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3e^{3x-1}(6x + 7)$.

Find the value of a. [4]

(b) Using the value of a found in part **(a)**, find the gradient of the curve at the x-axis. [2]

3 The term containing the highest power of x in the polynomial $\mathrm{P}(x)$ is $3x^4$. Two of the roots of $\mathrm{P}(x) = 0$ are k and -1 and $x^2 - 2x + 6$ is a quadratic factor of $\mathrm{P}(x)$. When divided by x, $\mathrm{P}(x)$ has a remainder of -36.

(a) Show that the value of k is 2. [3]

(b) Find the number of real roots of $\mathrm{P}(x) = 0$, justifying your answer. [3]

3 **(c)** Hence, solve the equation $\mathrm{P}\left(e^{y}-1\right)=0$. [2]

4 **(a)** Prove that $\sin 3\theta + \sin\theta = 4\sin\theta\cos^2\theta$. [4]

4 **(b)** Hence, solve $\sin 3\theta + \sin\theta - 2\cos\theta = 0$ for $0 \leq \theta \leq 2\pi$.

Leave your answers in terms of π. [5]

5 The value, V dollars, of a vehicle depreciates over time. It is given that $V = 84000e^{kt}$,where t is the time in years since it was bought, and k is a constant.

(a) Find the initial value of the vehicle. [1]

(b) Calculate the value of k if, after 5 years, the value of the vehicle has been halved. [3]

5 **(c)** Hence, calculate the value of t when the value of the vehicle is one-fifth its original value. [3]

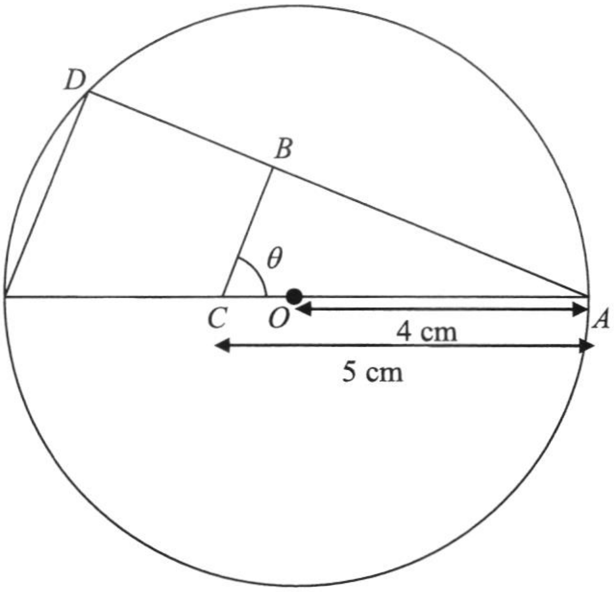
(d) Sketch the graph of $V = 84000\mathrm{e}^{kt}$. [2]

6 **(a)** Using the double angle formula, show that

$\cos^2 22.5° = \frac{2+\sqrt{2}}{4}$ and $\sin^2 22.5° = \frac{2-\sqrt{2}}{4}$. [5]

6 **(b)** Hence, find the exact value of $\cos^4 22.5° - \sin^4 22.5°$. [3]

7 The diagram shows two triangles, triangle ABC and triangle ADE which are inside a circle with centre O and the radius is 4 cm. Triangle ADE is similar to triangle ABC, $AC = 5$ cm and angle $ACB = \theta°$.



(a) Explain why angle $ABC = 90°$. [1]

(b) Show that $CB + ED + DB = 13\cos\theta + 3\sin\theta$. [3]

7 **(c)** Express $13\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$. [2]

(d) Find the value of θ for which $CB + ED + DB = 10$ cm. [3]

8 A circle C_1 with centre E passes through points $A(-1, 7)$ and $B(0, 8)$.

(a) Explain why the perpendicular bisector of the line AB will pass through E. [1]

(b) Given that E lies on the line $y = 2x - 2$, show that E is (3, 4). [4]

8 **(c)** Hence find the equation of the circle. [2]

(d) A second circle, C_2, has equation $(x-7)^2+(y-4)^2=4$.

Explain why the point (5, 6) lies only in one of the circles, C_1 or C_2. [3]

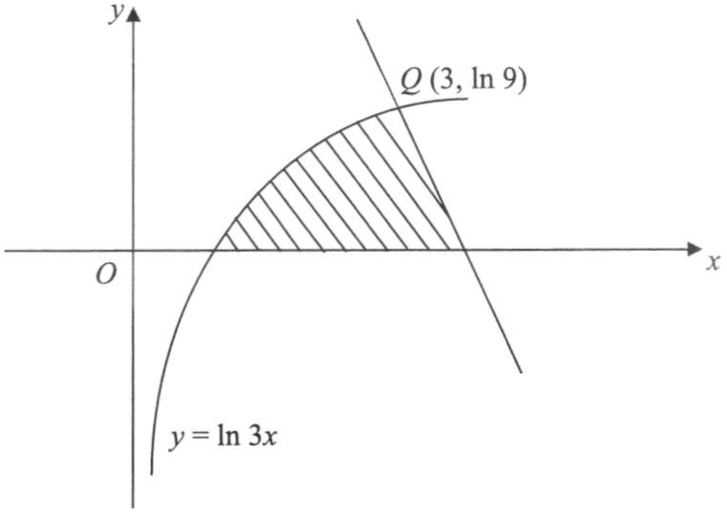
9 The equation of a curve is $y = x\ln x - x$.

(a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x - x) = \ln x$. [2]

(b) Use your result from part **(a)**, show that

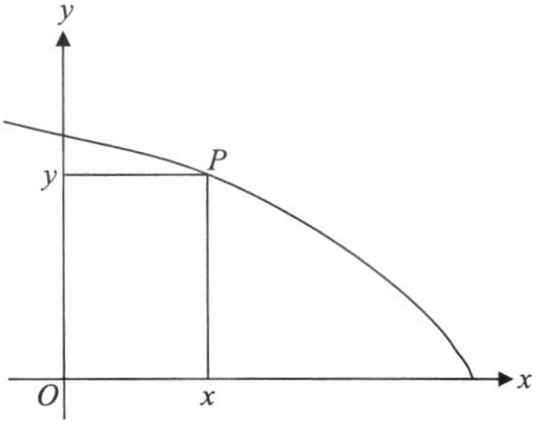
$\int_a^b \ln 3x \ \mathrm{d}x = (\ln 3 - 1)(b - a) + b\ln b - a\ln a$. [5]

9 **(c)** The diagram shows part of the curve $y = \ln 3x$ and the normal to the curve at point Q. Find the normal to the curve at the point $Q(3, \ln 9)$. [3]



(d) Hence or otherwise, find the area of the shaded region. [4]

10 The diagram shows part of the graph of $y = \sqrt{6-x}$. O is the origin and $P(x, y)$ is a point on the curve such that O and P are opposite vertices of the rectangle.



(a) Write down an expression for the area of the rectangle, A units^2, in terms of x. [2]

(b) Show that $\frac{dA}{dx} = \frac{12-3x}{2\sqrt{6-x}}$. [3]

10 **(c)** Hence, find the maximum area of the rectangle for $0 < x < 6$. [4]

10 **(d)** A particle moves along the curve $y = \sqrt{6 - x}$. At the point *P*, the *x*-coordinate of the particle is increasing at a rate of 0.08 units/sec and the *y*-coordinate is decreasing at 0.09 units/sec. Find the coordinates of point *P*. [4]

END OF PAPER

Bedok View Secondary School

Mathematics Department

Marking Scheme

Year	2023	**Level & Stream**	S4E & 5N
Type of Exam	Prelim	**Subject**	Add Math P1

No.	Workings		Remarks
1	**(a)**	$-\frac{1}{2}x^2 - 4x + 3$ $= -\frac{1}{2}(x^2 + 8x - 6)$ $= -\frac{1}{2}\left[(x+4)^2 - 6 - 16\right]$ **[M1] completing the square** $= -\frac{1}{2}\left[(x+4)^2 - 22\right]$ $= -\frac{1}{2}(x+4)^2 + 11$ **[A1]**	
	(b)	Max $y = 11$ $p < 11$ **[B1]**	
	(c)	Disagree with student B because the y-intercept is (0, 3) which lies above the x-axis/ the max pt is $(-4, 11)$/ Max $y = 11$ **[B1]**	

[Total: 4m]

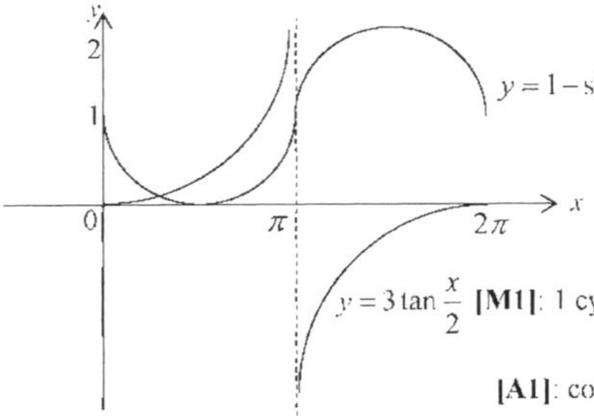
Bedok View Secondary School

Mathematics Department

Marking Scheme

No.	Workings	Remarks
2	$\qquad\qquad 4$ $2x^4+0x^3-9x^2\,\overline{)\,8x^4+5x^3-43x^2-36x+27}$ $\qquad -(8x^4+0x^3-36x^2)$ **[M1]**: long division $\qquad\qquad 5x^3-7x^2-36x+27$ Let $\frac{5x^3-7x^2-36x+27}{x^2(2x^2-9)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{2x^2-9}$ **[M1] FT**: formula $5x^3-7x^2-36x+27=Ax(2x^2-9)+B(2x^2-9)+(Cx+D)x^2$ When $x=0$, $27=-9B$ $B=-3$ Compare x term, $36=-9A$ $A=4$ Compare x^3 term, $5=2A+C$ $5=2(4)+C$ $C=-3$ Compare x^2 term, $-7=2B+D$ $-7=2(-3)+D$ $D=-1$ **[M2] FT**: either sub in values or compare term $\frac{5x^3-7x^2-36x+27}{x^2(2x^2-9)}=\frac{4}{x}-\frac{3}{x^2}-\frac{-3x-1}{2x^2-9}$ $4-\frac{5x^3-7x^2-36x+27}{x^2(2x^2-9)}=4-\frac{4}{x}-\frac{3}{x^2}-\frac{3x+1}{2x^2-9}$ **[A2]**	Award M1 if students found 2 unknowns by either sub in values or compare term Award M2 if students found 4 unknowns by correct method Any 2 fractions/ terms correct award A1 Minus P if didn't simplify

[Total: 6m]

No.	Workings			Remarks
3	(a)	(i)	$f(x)\quad 3\tan\frac{x}{2}$ Period $\frac{\pi}{\frac{1}{2}}$ $= 2\pi$ **[B1]**	
		(ii)	$g(x) = 1\quad \sin x$ Amplitude $\quad 1$ **[B1]** Period $\quad 2\pi$ **[B1]**	
	(b)	 $y = 1 - \sin x$ **[M1]**: 1 cycle of sine & correct shape **[A1]**: correct turning points $y = 3\tan\frac{x}{2}$ **[M1]**: 1 cycle of tangent & correct shape **[A1]**: correct x and y-intercepts		
	(c)	$\frac{\pi}{2} < x < \pi$ **[B1]**		
	(d)	$3\pi\tan\frac{x}{2} \cdot x = 2\pi$ $3\pi\tan\frac{x}{2} = 2\pi - x$ $3\tan\frac{x}{2} = 2\quad \frac{x}{\pi}$ $y = 2\quad \frac{x}{\pi}$ **[B1]**		

[Total: 9m]

Bedok View Secondary School

Mathematics Department — Marking Scheme

No.	Workings			Remarks
4	**(a)**	(i)	General term of $\left(ax-\frac{3}{2x}\right)^{11}$ $\binom{11}{r}(ax)^{11-r}\left(-\frac{3}{2x}\right)^{r}$ **[M1]**: general term $\binom{11}{r}a^{11-r}\left(-\frac{3}{2}\right)^{r}x^{11-2r}$ **[M1]**: simplify terms $2r$ is an even number $11-2r$ is always odd because odd number 11 minus an even number $2r$ will always give an odd number. **[A1]**	
		(ii)	$x^{11-2r}=x^{-1}$ **[M1] FT**: compare $\frac{1}{x}$ $11-2r=-1$ $r=6$ **[M1]**: to find r $\binom{11}{6}a^{11-6}\left(-\frac{3}{2}\right)^{6}=168399$ **[M1]FT**: sub in r $a^5=32$ $a=2$ **[A1]**	
	(b)	(i)	$\left(1+2x^2\right)^n$ $=\binom{n}{0}1^n\left(2x^2\right)^0+\binom{n}{1}1^{n-1}\left(2x^2\right)^1+\binom{n}{2}1^{n-2}\left(2x^2\right)^2+\binom{n}{3}1^{n-3}\left(2x^2\right)^3+\ldots$ **[M1]: binomial thm** $=1+(n)\left(2x^2\right)+\frac{n(n-1)}{1\times 2}\left(4x^4\right)+\frac{n(n-1)(n-2)}{1\times 2\times 3}\left(8x^6\right)+\ldots$ $=1+2nx^2+2n(n-1)x^4+\frac{4n(n-1)(n-2)}{3}x^6+\ldots$ **[A1]**	Minus P if didn't write "…"

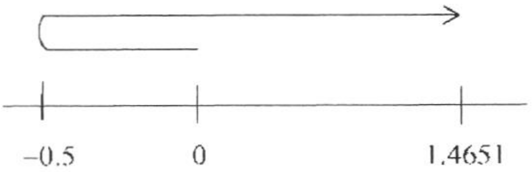
		(ii)	$28\times\binom{n}{1}(2)=\binom{n}{3}(8)$ **[M1] FT**: equate $56\times n=8\times\frac{n(n-1)(n-2)}{3\times2\times1}$ $42=(n-1)(n-2)$ $42=n^2-3n+2$ $0=n^2-3n-40$ **[M1]**: simplification $0=(n-8)(n+5)$ $n=8$ or -5(rej) **[A1]**	

[Total: 12m]

Bedok View Secondary School

Mathematics Department Marking Scheme

No.	Workings		Remarks
5	**(a)**	$v = e^{2t} - 4e^{-2t}$ $s = \int\left(e^{2t} - 4e^{-2t}\right)dt$ $= \frac{1}{2}e^{2t} + 2e^{-2t} + c$ **[M1]**: integrate exponential function When $t = 0$, $s = 0$ $0 = \frac{1}{2}e^{2(0)} + 2e^{-2(0)} + c$ **[M1] FT**: Sub $t = 0$, $s = 0$ $c = -\frac{5}{2}$ $s = \frac{1}{2}e^{2t} + 2e^{-2t} - \frac{5}{2}$ **[A1]**	
	(b)	Greatest distance implies $v = 0$, $e^{2t} - 4e^{-2t} = 0$ **[M1]**: greatest distance implies $v = 0$ $e^{2t} = \frac{4}{e^{2t}}$ $e^{4t} = 4$ $e^{2t} = 2$ or $e^{2t} = -2$ $2t = \ln 2$ no solution $t = \frac{1}{2}\ln 2$ **[M1]**: solve for t $a = \frac{d}{dt}\left(e^{2t} - 4e^{-2t}\right)$ $= 2e^{2t} + 8e^{-2t}$ **[M1]**: differentiate exponential functions $2e^{2\left(\frac{1}{2}\ln 2\right)} + 8e^{-2\left(\frac{1}{2}\ln 2\right)}$ $= 8$ m/s^2 **[A1]**	

	(c)	Total distance $\int_0^{\frac{1}{2}\ln 2} v\mathrm{d}t + \int_{\frac{1}{2}\ln 2}^{1} v\mathrm{d}t$ $= \left.\frac{1}{2}e^{2t} + 2e^{-2t} - \frac{5}{2}\right\|_0^{\frac{1}{2}\ln 2} + \left\|\frac{1}{2}e^{2t} + 2e^{-2t} - \frac{5}{2}\right\|_{\frac{1}{2}\ln 2}^{1}$ $2-\frac{1}{2}+3.965-2$ **[M1]**: find distance travelled $=\frac{1}{2}-1.9651$ 2.47 m (3 s.f.) **[A1]** Alternative method When $t=\frac{1}{2}\ln 2$, $s=-0.5$ When $t-1$, $s-1.4651$  Total distance $0.5+0.5+1.4651$ **[M1]**: find distance travelled $=2.4651$ $=2.47$ m (3 s.f.) **[A1]**	

[Total: 9m]

Bedok View Secondary School

Mathematics Department

Marking Scheme

No.	Workings		Remarks
6	(a)	Since $AE = ED$ and $BC = CD$, by mid-point theorem, $EC = \frac{1}{2}AB$. Therefore $AB = 2EC$ (proven) **[B1]**	
	(b)	$\angle GBF = \angle BDF$ ($\angle$s in alt. seg.) (A) $\angle DFB = \angle BFG = 90°$ (rt. $\angle$ in semicircle) (A) } **[M1]** for both properties ΔDFB is similar to ΔBFG (AA-similarity) **[M1]** for AA-similarity $\frac{FD}{FB} = \frac{BD}{GB}$ $FB \times BD = FD \times GB$ (Proven) } **[A1]**	

[Total: 4m]

No.	Workings		Remarks
7	**(a)**	Sub $y = 2.5,\ 3(2.5)\quad m = 6$ **[M1]**: sub in values $m = 3(2.5)\quad 6$ $m - 1.5$ **[A1]** Sub $(1.5, 2.5)$, $2(2.5) - 2\left(2(1.5) - 1\right)^2 - n$ **[M1] FT**: sub in values $n - -3$ **[A1]**	
	(b)	From $3y\quad x = 6$, $x = 3y\quad 6$ -----------(1) $2y - 2\left(2x - 1\right)^2 - 3$ -----------(2) Sub (1) into (2), $2y - 2\left(2(3y - 6) - 1\right)^2 - 3$ **[M1] FT**: solve simultaneous eqn $2y - 2\left(36y^2 - 156y + 169\right) - 3$ $2y - 72y^2 - 312y + 338 - 3$ $0 - 72y^2 - 314y + 335$ **[M1]**: simplification $y - \frac{-(-314) \pm \sqrt{(-314)^2 - 4(72)(335)}}{2(72)}$ $y = 1.8611$ or 2.5 $x -\quad 0.41667$ $B = (-0.417, 1.86)$ (3 s.f.) **[A1]**	

[Total: 7m]

No.	Workings		Remarks
8	**(a)**	$x^2+3x+q<2x+7$ $x^2-x+q-7<0$ Sub $x-1$, $(1)^2+1+q-7-0$ **[M1]**: sub in value $q=5$ (shown) **[A1]**	
	(b)	$x^2-3x-5\le 2x+7$ $x^2 \cdot x \quad 2\le 0$ $(x-2)(x-1)\le 0$ **[M1]**: factorisation $2\le x\le 1$ $p--2$ **[A1]**	

[Total: 4m]

No.	Workings		Remarks
9	**(a)**	$4y \quad 3x = 6$ $y - \frac{3}{4}x - \frac{3}{2}$ Gradient of tangent $\frac{3}{4}$ **[M1]**: find gradient When $x = \frac{\pi}{6}$, $\frac{\cos 2\left(\frac{\pi}{6}\right) + 1}{k} = \frac{3}{4}$ **[M1]**: sub in value $k = \frac{\frac{1}{2} + 1}{\frac{3}{4}}$ $k - 2$ (Shown) **[A1]**	

	(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\cos 2x+1}{2}$ $= \frac{1}{2}\cos 2x + \frac{1}{2}$ $y = \int\left(\frac{1}{2}\cos 2x + \frac{1}{2}\right)\mathrm{d}x$ $y = \frac{1}{4}\sin 2x + \frac{1}{2}x + c$ **[M1]**: integration Sub $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{4}\right)$, $\frac{\sqrt{3}}{4} = \frac{1}{4}\sin 2\left(\frac{\pi}{6}\right) - \frac{1}{2}\left(\frac{\pi}{6}\right) - c$ **[M1] FT**: sub in values $\frac{\sqrt{3}}{4} = \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{12} + c$ **[M1] FT**: special angles $\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{8} - \frac{\pi}{12} = c$ $c = \frac{\sqrt{3}}{8} \quad \frac{\pi}{12}$ $y = \frac{1}{4}\sin 2x + \frac{1}{2}x + \frac{\sqrt{3}}{8} \quad \frac{\pi}{12}$ **[A1]**	

[Total: 7m]

No.	Workings		Remarks
10	(a)	Since $\tan\theta = \frac{1}{2}$, gradient of $FG \quad \frac{1}{2}$ **[M1]**: find gradient Eqn of FG: $y = \quad \frac{1}{2}x \quad c$ Sub (12, 8), $8 - -\frac{1}{2}(12)+c$ **[M1]**: sub in values $c - 14$ $y = \quad \frac{1}{2}x + 14$ $2y = 28 \quad x$ (shown) **[A1]**	Alternative Method $y - 8 = \quad \frac{1}{2}(x \quad 12)$ **[M1]** $y \quad 8 = \quad \frac{1}{2}x \quad 6$ $y = -\frac{1}{2}x - 14$ $2y - -x + 28$(shown) **[A1]**
	(b)	Gradient of $HJ \quad -1 \div \left(-\frac{1}{2}\right)$ $= 2$ **[M1]**: $m_1 \times m_2 = -1$ Gradient of HJ: $y \quad 2x + c$ Sub (9, 3), $3 = 2(9)+c$ **[M1]**: sub in values $c = \quad 15$ Eqn of HJ: $y - 2x - 15$ **[A1]**	Alternative Method $y \quad 3 = 2(x \quad 9)$ **[M1]** $y \quad 3 = 2x \quad 18$ $y - 2x - 15$ **[A1]**
	(c)	Eqn of FG: $2y = 28 \quad x$ ---------------(1) Eqn of HJ: $y = 2x \quad 15$ ---------------(2) Sub (2) into (1), $2(2x-15) - 28 - x$ **[M1] FT**: solve simultaneous $4x \quad 30 - 28 \quad x$ $5x - 58$ $x = 11\frac{3}{5}$, $y = 8\frac{1}{5}$ $J \quad \left(11\frac{3}{5}, 8\frac{1}{5}\right)$ **[A1]**	

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	(d)	x-coordinates of $F = 12 \quad 20\left(12 \quad 11\frac{3}{5}\right)$ 4 **[M1]FT**: use similar Δ y-coordinates of $F = 8 \cdot 20\left(8\frac{1}{5} \quad 8\right)$ 12 $F \quad (4, 12)$ **[A1]**	
	(e)	Let M be $(28 \quad 2y, y)$ since M is on FG $\frac{1}{2}\begin{vmatrix}9 & 12 & 29 \cdot 2y & 9\\ 3 & 8 & y & 3\end{vmatrix}=19.5$ **[M1]:** shoelace mtd $(72 \cdot 12y \cdot 3(28 - 2y)) \quad (36 + 8(28-2y) + 9y)=39$ **[M1]** $(156 + 6y)-(260 \quad 7y)=39$ $13y \quad 104=39$ $13y=143$ $y=11$ **[M1]:** solve for unknown $x=28 \quad 2(11)$ $x-6$ $M-(6,11)$ **[A1]**	

[Total: 14m]

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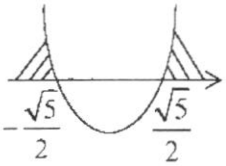
No.	Workings		Remarks
11	**(a)**	$mx^2+7>\sqrt{5}x+7-m$ **[M1]**: write inequality $mx^2-\sqrt{5}x+m>0$ $b^2 \quad 4ac<0$ $\left(\sqrt{5}\right)^2 \quad 4(m)(m)<0$ **[M1] FT**: discriminant <0 $5-4m^2<0$ $4m^2-5>0$  $\left(2m-\sqrt{5}\right)\left(2m+\sqrt{5}\right)>0$ **[M1]**: factorisation $m<-\frac{\sqrt{5}}{2}/-1.12$ (rej) or $m>\frac{\sqrt{5}}{2}/1.12$ **[A1]**	Accept decimal Do not award A1 if didn't reject
	(b)	$m=\pm\frac{\sqrt{5}}{2}/\pm1.12$ **[B1]**	

[Total: 5m]

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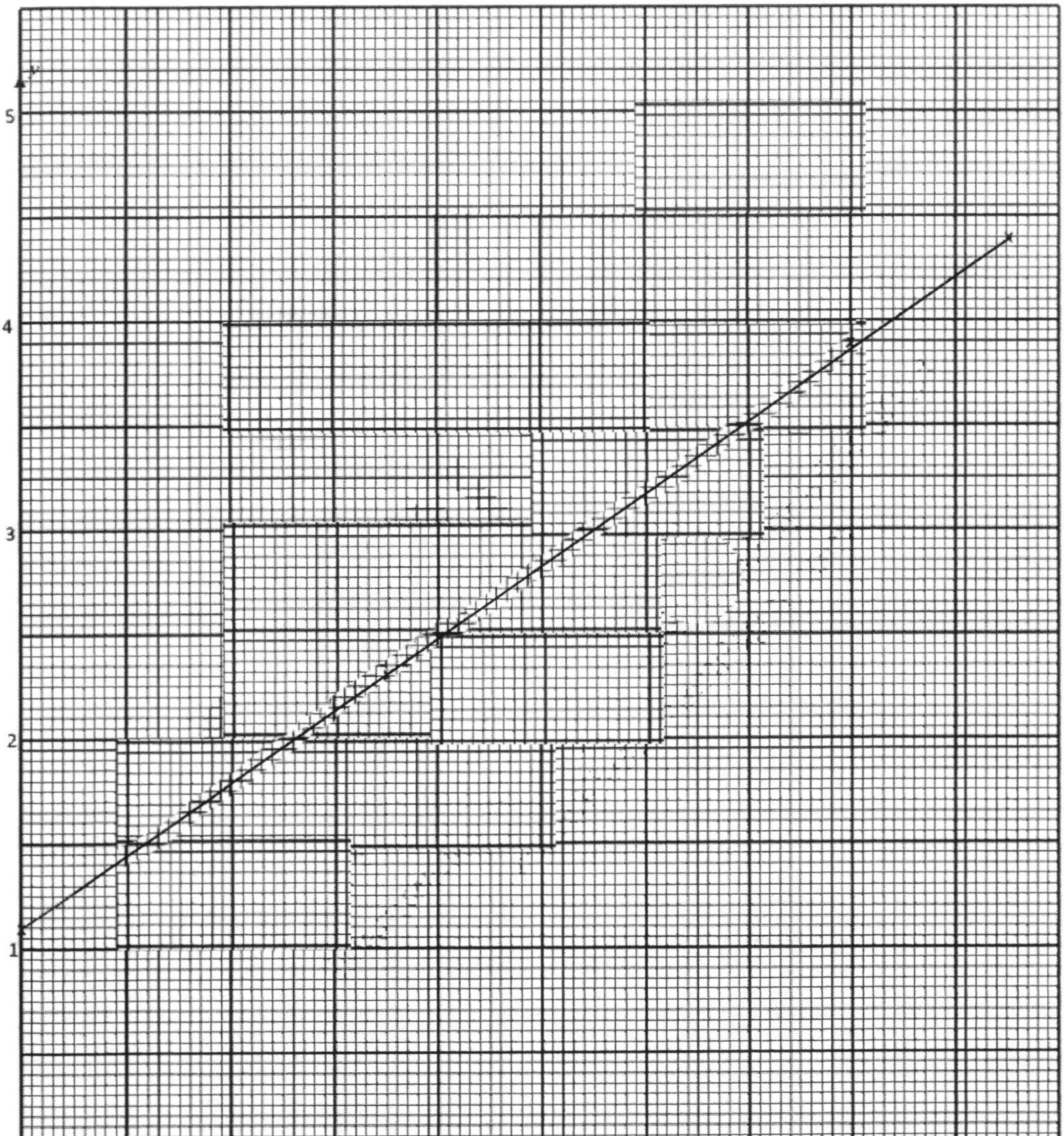
<table>
<tr><td>No.</td><td colspan="3">Workings</td><td>Remarks</td></tr>
<tr><td>12</td><td>(a)</td><td colspan="2">$e^y = ax^b$
$\ln e^y = \ln ax^b$ **[M1]**: apply ln to both sides
$y = \ln a + b\ln x$
$y = b\ln x + \ln a$
Draw a graph of y against $\ln x$ **[A1]**</td><td></td></tr>
<tr><td></td><td>(b)</td><td colspan="2"><table><tr><td>$\ln x$</td><td>0</td><td>0.693</td><td>1.10</td><td>1.61</td><td>1.95</td></tr><tr><td>y</td><td>1.10</td><td>2.28</td><td>2.50</td><td>3.89</td><td>4.41</td></tr></table>Correct plots **[B1]**
Best fit line **[B1] FT**</td><td></td></tr>
<tr><td></td><td>(c)</td><td>(i)</td><td>Correct $y = 3.025 \pm 0.1$ **[B1] FT**</td><td></td></tr>
<tr><td></td><td></td><td>(ii)</td><td>$\ln a = 1.10$
$a = e^{1.10}$
$a = 3.00$ (3 s.f) **[B1]**
$b = \frac{4.2 - 1.45}{1.78 - 0.2}$ **[M1] FT**: find gradient
$= 1.74$ (3 s.f.) (± 0.5) **[A1]**</td><td></td></tr>
<tr><td></td><td></td><td>(iii)</td><td>Read off the graph at $X = \ln 4$
1.39
$y = 3.5$ (± 0.1) **[B1]**</td><td></td></tr>
</table>

[Total: 9m]


y
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
ln x

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Year	2023	**Level & Stream**	4E5N
Type of Exam	Prelim P2	**Subject**	Add Math

No.	Working	Remarks
1	$3^{\frac{x}{2}} \quad 2+24(3)^{\frac{x}{2}}$ $3^{\frac{x}{2}} \quad 2+24\left(\frac{1}{3^{\frac{x}{2}}}\right)$ Let $3^{\frac{x}{2}} - y$ $y = 2 + \frac{24}{y}$ **[M1]** $y^2 = 2y + 24$ $y^2 \quad 2y \quad 24 = 0$ $y - 6$ or -4 **[M1]** $3^{\frac{x}{2}} = 6$ or $\quad 4$ $\frac{x}{2}\lg 3 = \lg 6$ or $\lg(\quad 4)$ (NA) **[M1]** $x = \frac{2\lg 6}{\lg 3}$ $x = 3.26$ (3 sf) **[A1]**	Sub method Solve for y lg both sides to solve and reject -ve values Accept either exact or 3 sf ans

[Total : 4m]

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No.	Working		Remarks
2	(a)	$y=(ax+1)e^{3x-1}$ $\frac{dy}{dx}=ae^{3x-1}+(ax+1)3e^{3x-1}$ **[M1]** $=e^{3x-1}(a+3ax+3)$ $\frac{d^2y}{dx^2}=3e^{3x-1}(a+3ax+3)+e^{3x-1}(3a)$ **[M1 FT]** $=3e^{3x-1}(3ax+3+2a)$ **[M1 FT]** $\frac{d^2y}{dx^2}=3e^{3x-1}(6x+7)$ By comparing, $3a=6$ Hence $a=2$ **[A1]**	Differentiate using product rule Find 2nd derivative Express in the form to compare
	(b)	At x-axis, $y=0$ $\left.\begin{array}{l}0=(2x+1)e^{3x-1}\\x=-0.5\end{array}\right\}$ **[M1 FT]** Sub $x=-0.5$: $\frac{dy}{dx}=e^{3(-0.5)-1}[6(-0.5)+5]$ $=0.16416$ $=0.164$ (3 sf) **[A1]**	Find x.

[Total : 6m]

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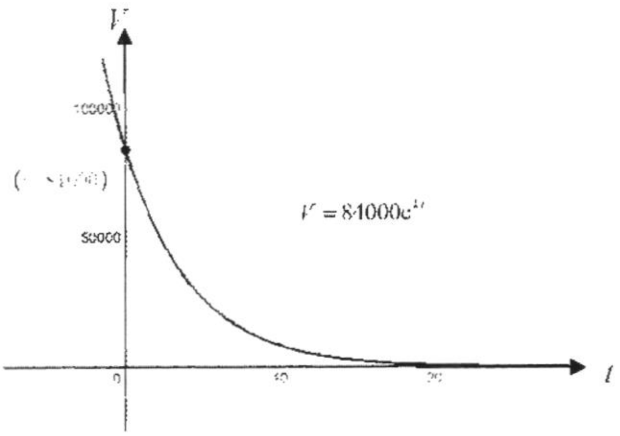
No.	Working		Remarks
3	**(a)**	$P(x)=3(x+1)(x-k)(x^2-2x+6)$ **[M1]** $P(0)=-36$ **[M1]** $3(1)(-k)(6)=-36$ $k=2$ (Shown) **[A1]**	
	(b)	When $P(x)=0$, $3(x+1)(x-2)(x^2-2x+6)=0$ $x=2$ or -1 **[M1 FT]** For $(x^2-2x+6)=0$, $b^2-4ac=(-2)^2-4(1)(6)$ $=-20$ <0 which means it has no real roots **[M1]** Hence when $P(x)=0$, it has 2 real roots. **[A1]**	Solve for x Use discriminant to prove that the quad equation has no roots. Accept other methods like quadratic formula
	(c)	$P\left(e^y-1\right)=0$ $e^y-1=2$ or -1 **[M1 FT]** $e^y=3$ or 0 (NA) $y=\ln 3$ **[A1]** $=1.10$ (3sf)	Sub $x=e^y-1$ Accept either exact value or 3 sf ans

[Total : 8m]

No.	Working		Remarks
4	**(a)**	$\text{LHS} = \sin 3\theta + \sin\theta$ $= \sin(2\theta + \theta) + \sin\theta$ **[M1]** $= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta + \sin\theta$ **[M1]** $= \cos\theta(2\sin\theta\cos\theta) + \sin\theta(2\cos^2\theta - 1) + \sin\theta$ **[M1]** $= 2\sin\theta\cos^2\theta + 2\cos^2\theta\sin\theta - \sin\theta + \sin\theta$ $= 4\sin\theta\cos^2\theta$ $= \text{RHS}$ (proven) **[A1]**	Change $3\theta = 2\theta + \theta$ Use of identity for sum of angles Use of identities for double angle
	(b)	$\sin 3\theta + \sin\theta - 2\cos\theta = 0$ $4\sin\theta\cos^2\theta - 2\cos\theta = 0$ **[M1]** $2\cos\theta(2\sin\theta\cos\theta - 1) = 0$ **[M1]** $2\cos\theta = 0$ or $2\sin\theta\cos\theta - 1 = 0$ **[M1 FT]** $2\cos\theta = 0$ or $\sin 2\theta = 1$ **[M1]** $\alpha = \frac{\pi}{2}$ or $\alpha = \frac{\pi}{2}$ $\theta = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$ or $2\theta = \frac{\pi}{2}, 2\pi + \frac{\pi}{2}$ $\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{\pi}{4}, \frac{5\pi}{4}$ **[A1]**	Use (a) Factorise common terms Solve equation Use identity For all answers

[Total : 9m]

No.	Working		Remarks
5	(a)	Initial value $= 84000e^0$ $= 84000$ **[B1]**	
	(b)	$84000e^{5kt} = 42000$	
		$e^{5k} = \frac{1}{2}$ **[M1]**	Simplify
		$5k = \ln\frac{1}{2}$ **[M1 FT]** $k = \frac{1}{5}\ln\frac{1}{2}$ 0.139 (3 sf) **[A1]**	Take ln on both sides
	(c)	$84000e^{kt} = 16800$ **[M1]** $e^{\frac{1}{5}\ln\frac{1}{2}t} = 0.2$	Equate to 1/5 of 84000
		$\frac{1}{5}\ln\frac{1}{2}t = \ln 0.2$ **[M1 FT]** $t = 11.610$ $t = 11.6$ (3 sf) **[A1]**	Take ln on both sides
	(d)		**B1** for shape **B1 FT** for correct y-intercept and asymptote

[Total : 9m]

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No.	Working		Remarks
6	(a)	By double angle formula,	
		$\cos 45^\circ = 2\cos^2 22.5^\circ - 1$ **[M1]**	Double angle formula
		$\cos^2 22.5^\circ = \frac{1+\cos 45^\circ}{2}$	
		$= \frac{1+\frac{1}{\sqrt{2}}}{2}$ **[M1]**	Correct ratio
		$= \frac{1}{2} + \frac{\sqrt{2}}{4}$	
		$= \frac{2+\sqrt{2}}{4}$ (Shown) **[A1]**	
		$\sin^2 22.5^\circ = 1 - \cos^2 22.5^\circ$	
		$= 1 - \left(\frac{1}{2} + \frac{1}{2\sqrt{2}}\right)$ **[M1]**	Sub using above
		$= \frac{1}{2} - \frac{\sqrt{2}}{4}$	
		$= \frac{2-\sqrt{2}}{4}$ (Shown) **[A1]**	
	(b)	$\cos^4 22.5^\circ - \sin^4 22.5^\circ$	Accept other methods
		$= \left(\cos^2 22.5^\circ\right)^2 - \left(\sin^2 22.5^\circ\right)^2$	
		$= (\cos^2 22.5^\circ + \sin^2 22.5^\circ)(\cos^2 22.5^\circ - \sin^2 22.5^\circ)$ **[M1]**	Factorise
		$= (1)\left[\left(\frac{1}{2} + \frac{\sqrt{2}}{4}\right) - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right)\right]$ **[M1] FT**	Sub using (a)
		$= \frac{\sqrt{2}}{2}$ **[A1]**	

[Total : 8m]

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No.	**Working**		**Remarks**
7	**(a)**	$\angle ADE = 90^\circ$ (right $\angle$ in a semicircle) $\angle ABC = \angle ADE$ $= 90^\circ$ (corr. $\angle$ of similar Δs) **[B1]**	
	(b)	$\cos\theta = \frac{CB}{5}$ $CB = 5\cos\theta$ $\cos\theta = \frac{DE}{8}$ $DE = 8\cos\theta$ $\sin\theta = \frac{AB}{5}$ $AB = 5\sin\theta$ $\sin\theta = \frac{AD}{8}$ $AD = 8\sin\theta$ **[M1]** $CB + DE + DB = 5\cos\theta + 8\cos\theta + 8\sin\theta - 5\sin\theta$ **[M1 FT]** $= 13\cos\theta + 3\sin\theta$ (shown) **[A1]**	Use TOA CAH SOH Add the sides together
	(c)	$R = \sqrt{13^2 + 3^2}$ $= \sqrt{178}$ $\alpha = \tan^{-1}\frac{3}{13}$ $= 12.995^\circ$ $= 13.0^\circ$ (1 dp) **[M1]** $13\cos\theta + 3\sin\theta = \sqrt{178}\cos(\theta - 13.0^\circ)$ **[A1]**	Find R and α.
	(d)	$\sqrt{178}\cos(\theta - 12.995^\circ) = 10$ **[M1 FT]** $\cos(\theta - 12.995^\circ) = 0.74953$ $\alpha = 41.450^\circ$ $\theta - 12.995^\circ = 41.450^\circ$ **[M1 FT]** $\theta = 54.4^\circ$ **[A1]**	Use R-formula to equate to 10 equating to 1st quad only

[Total : 9m]

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No.		Working	Remarks
8	(a)	**Perpendicular from centre of a circle to a chord bisects the chord** (symmetrical properties of circle) hence the perpendicular bisector of the **line AB which is the chord** will pass through E. **[B1]**	Accept other relevant explanations.
	(b)	$\text{Mid-point of } AB = \left(\frac{-1+0}{2}, \frac{7+8}{2}\right)$ $= \left(-\frac{1}{2}, \frac{15}{2}\right)$ $\text{Gradient of } AB = \frac{8-7}{0-(-1)}$ $= 1$ Gradient of $\perp$ bisector of $AB = -1$ **[M1]**	Find midpoint and gradient
		When $x = -\frac{1}{2}, y = \frac{15}{2}$, $\frac{15}{2} = -\left(-\frac{1}{2}\right) + c$ $c = 7$ Eqn of perpendicular bisector: $y = -x + 7$ **[M1]**	Find eqn of perpendicular bisector
		$y = -x + 7 \quad -(1)$ $y = 2x - 2 \quad -(2)$ Sub. (1) into (2), $-x + 7 = 2x - 2$ $3x = 9$ $x = 3$ Sub. $x = 3$ into (1), $y = -3 + 7$ $= 4$ **[M1 FT]**	Solve simultaneously to find E
		The coordinates of E is (3, 4). (Shown) **[A1]**	

	(c)	Radius = $\sqrt{(3-0)^2+(8-4)^2}$ **[M1]** $= 5$ units Equation of circle, $(x-3)^2+(y-4)^2=5^2$ $(x-3)^2+(y-4)^2=25$ or $x^2-6x+y^2-8y=0$ **[A1]**	Find radius
	(d)	Radius of $C_2 = 2$ units Centre of $C_2 = (7, 4)$ Distance between E and (5, 6) $=\sqrt{(5-3)^2+(6-4)^2}$ **[M1]** $=\sqrt{8}$ units < Radius of $C_1 = 5$ Distance between centre of C_2 and (5, 6) $=\sqrt{(7-5)^2+(4-6)^2}$ **[M1]** $=\sqrt{8}$ units > Radius of $C_2 = 2$ Therefore (5, 6) lies in C_1 only. **[A1]**	Find distance to prove Find distance to prove

[Total : 10m]

No.	Working		Remarks
9	**(a)**	$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x - 1$ **[M1]** $= \ln x$ (shown) **[A1]**	Techniques of differentiation
	(b)	$\frac{d}{dx}(x \ln x - x) = \ln x$ $\int_a^b \ln x \, dx = [x \ln x - x]_a^b$ **[M1]** $= b \ln b - b - a \ln a + a$ $\int_a^b \ln 3x \, dx = \int_a^b \ln 3 \, dx + \int_a^b \ln x \, dx$ **[M1]** $= [(\ln 3)x]_a^b + [x \ln x - x]_a^b$ **[M1]** $\left.\begin{aligned} &= (\ln 3)b - (\ln 3)a + b \ln b - b - a \ln a + a \\ &= (\ln 3)b - b - (\ln 3)a + a + b \ln b - a \ln a \\ &= (\ln 3 - 1)b - (\ln 3 - 1)a + b \ln b - a \ln a \end{aligned}\right\}$ **[M1]** $= (\ln 3 - 1)(b - a) + b \ln b - a \ln a$ (shown) **[A1]**	Use integration as reverse of differentiation Use of product law to split ln $3x$ Integrate ln 3 correctly Use of grouping method to factorise
	(c)	$\frac{dy}{dx} = \frac{3}{3x} = \frac{1}{x}$ **[M1]** Sub $x = 3$ $\frac{dy}{dx} = \frac{1}{3}$ m of normal $= -3$ **[M1 FT]** Eqn of normal: $y - \ln 9 = -3(x - 3)$ $y = -3x + 9 + \ln 9$ **[A1]**	Differentiation of ln Find normal gradient Accept $y = -3x + 11.2$ (3sf)

	(d)	At $y = 0$, $0 = -3x + 9 + \ln 9$ $x = 3 + \frac{\ln 9}{3}$ **[M1]**	Sub $y = 0$ to find x.
		Shaded Area $= \int_{\frac{1}{3}}^{3} \ln 3x \, dx + \text{Area of triangle}$ $= (\ln 3 - 1)\left(3 - \frac{1}{3}\right) + 3\ln 3 - \frac{1}{3}\ln\frac{1}{3} + \left[\frac{1}{2}(\ln 9)\left(3 + \frac{\ln 9}{3} - 3\right)\right]$ $= 0.262966 + 3.66204 + 0.80463$ $= 4.7296$ $= 4.73 \text{ units}^2$ (3 sf) **[A1]**	**[M1, M1 FT]** Use (b) and find area of triangle Accept other methods

[Total : 14m]

<table>
<tr><th>No.</th><th colspan="2">Working</th><th>Remarks</th></tr>
<tr><td>10</td><td>(ai)</td><td>$A = xy$ [M1]
$= x\sqrt{6-x}$ [A1]</td><td>Find area</td></tr>
<tr><td></td><td>(b)</td><td>$\frac{\mathrm{d}A}{\mathrm{d}x} = (6-x)^{\frac{1}{2}} + x\left[\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right]$ [M1]
$= (6-x)^{-\frac{1}{2}}\left(6-x-\frac{x}{2}\right)$ [M1]
$= \frac{12-2x-x}{2\sqrt{6-x}}$
$= \frac{12-3x}{2\sqrt{6-x}}$ (shown) [A1]</td><td>Differentiate using product rule
Factorise to simplify</td></tr>
<tr><td></td><td>(c)</td><td>At stat point, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$
$\frac{12-3x}{2\sqrt{6-x}} = 0$ [M1]
$x = 4$ [M1]
First derivative test,<table><tr><td>3.9</td><td>4</td><td>4.1</td></tr><tr><td>+</td><td>0</td><td>−</td></tr><tr><td>/</td><td>—</td><td>\</td></tr></table>[M1]
A is max at $x = 4$.
max $A = 4\sqrt{6-4}$
$= 4\sqrt{2}$ cm^2 [A1]</td><td>Equate to 0 at stat point.
Find x.
Prove that it is max. Accept 2nd derivative test
Accept 5.66 cm^2 (3 sf)</td></tr>
</table>

	(d)	$\frac{dy}{dx}=\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)$ **[M1]** $=-\frac{1}{2}(6-x)^{-\frac{1}{2}}$	Differentiate using chain rule
		$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-0.09=-\frac{1}{2}(6-x)^{-\frac{1}{2}}\times 0.08$ **[M1 FT]** $\frac{9}{8}=\frac{1}{2}(6-x)^{-\frac{1}{2}}$ $(6-x)^{\frac{1}{2}}=\frac{4}{9}$ $6-x=\frac{16}{81}$	Connected rate of change
		$x=\frac{470}{81}$ **[M1 FT]** $y=\sqrt{6-\frac{470}{81}}$ $=\frac{4}{9}$	Find x.
		coordinates of $P=\left(\frac{470}{81},\ \frac{4}{9}\right)$ **[A1]**	Accept (5.80, 0.444)

[Total : 13m]