Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination **4E5N**

ADDITIONAL MATHEMATICS **4049/1**

Paper 1

23 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.
Write your answers in the spaces provided on the question paper.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use
90

This document consists of **21** printed pages including the cover page.

[Turn Over

Setter: Ms Elene Phang

Mathematical Formula

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The height of an object, y metres, above the ground, x seconds after it had been thrown can be modelled by the equation $y=-2x^2+12x+k$, where k is a constant. The object reaches a maximum height of 20 metres.

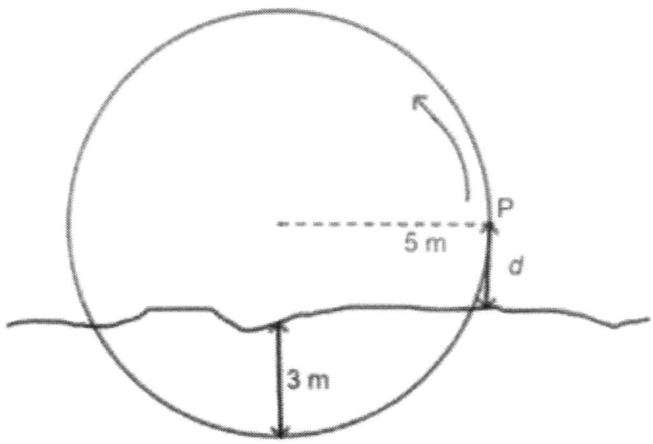
(a) Find the value of k. [3]

(b) Without calculating, explain if the object is moving upward or downward at the 5^{th} second.
State the other timing when the object is at the same height as it is at the 5^{th} second. [2]

2 **(a)** Explain why there is no even powers of x in the expansion of $\left(\frac{x^3}{2}+4x\right)^7$. [2]

(b) Given that $a>0$ and $(1-3x)^2(a+bx)^6=64+576x+cx^2$, find the values of a, b and c. [5]

3 A water wheel of radius 5 metres rests on a lake of height 3 metres. The water wheel makes 2 revolutions per minute. d is the distance, in metres, of the point P above the water level such that $d = a\sin(bt) + c$, where t is the time in seconds.



(a) Show that $b = \dfrac{\pi}{15}$. [1]

(b) State the value of a and of c. [2]

(c) Sketch $d = a\sin(bt) + c$ for $0 \le t \le 60$. [2]

(d) Find the time interval(s) when P is submerged in the water where $0 \le t \le 60$. [3]

4 Find the equation of the curve that passes through the origin and
$\frac{dy}{dx} = 3\tan^2(3x) + 2.$ [4]

5 A sample is taken out from the freezer in the science laboratory and the temperature $T^{\circ}C$ is given by $T = 15 - 28e^{-kt}$ where t is the number of hours after it is retrieved from the freezer.

(a) State the initial value of T. [1]

(b) After 3 hours, the temperature of the sample is $12.5^{\circ}C$, find the value of k. [2]

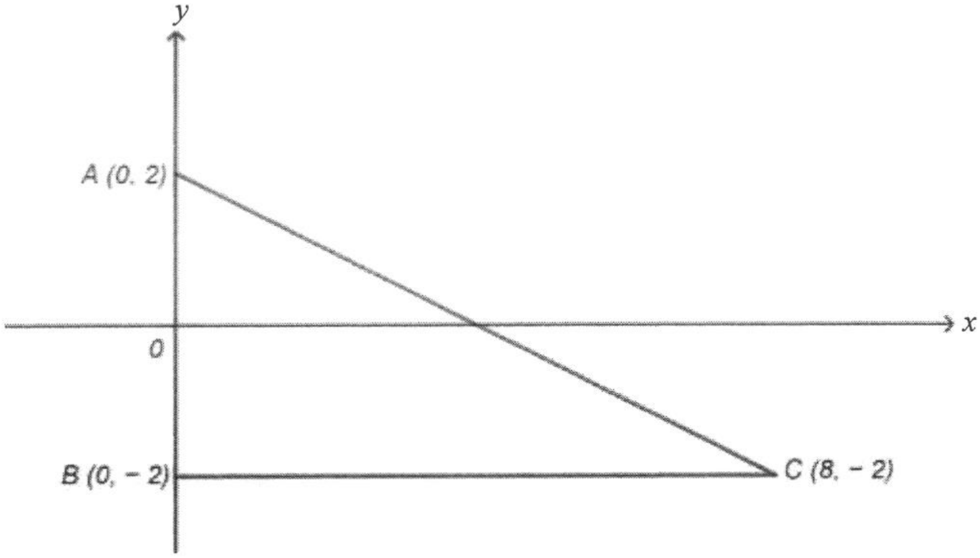
(c) Explain why the temperature is always below $15^{\circ}C$. [2]

(d) Sketch the curve for $T = 15 - 28e^{-kt}$. [2]

6 (a) Given $f(x)=\dfrac{7x-3}{x+1}, x\neq -1$, explain with workings whether $f(x)$ is an increasing or decreasing function. [3]

(b) A point P moves along the curve $y=f(x)$ such that the y-coordinate is decreasing at a constant rate of 0.4 units per second, find the rate of change of x when $x=2$. [3]

7 The diagram shows $A(0, 2)$, $B(0, -2)$ and $C(8, -2)$ and the line AC cuts the x-axis at $x = 4$.



(a) Line CD is perpendicular to AC and meets the line $-2x + 7y = 42$ at D. Find the coordinates of D. [4]

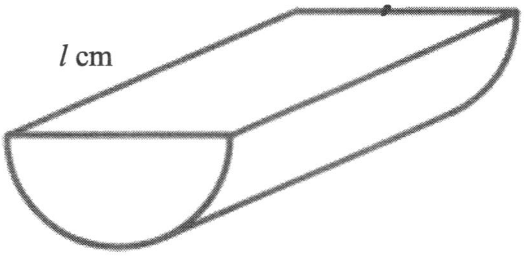
(b) Find the area of quadrilateral $ABCD$. [2]

8 **(a)** Given $\dfrac{\sin(A+B)}{\sin(A-B)}=\dfrac{5}{3}$, evaluate $\dfrac{\tan A}{\tan B}$. [2]

(b) **(i)** Express $7\cos^2 x-3\sin^2 x$ in the form $a+b\cos 2x$, where a and b are constants. [2]

(ii) Using the values of a and b, find $\int 7\cos^2 x - 3\sin^2 x \, dx$. Hence, evaluate $\int_{-\frac{\pi}{12}}^{\frac{\pi}{4}} 7\cos^2 x - 3\sin^2 x \, dx$, leaving your answers in terms of π. [4]

9 Mr Tan wants to make a trough that is in the shape of a half cylinder, with radius r cm, length l cm and has a volume of $45\,000\pi$ cm^3.



(a) Show that the exterior surface area of the trough, A, is $A = \dfrac{90\,000\pi}{r} + \pi r^2$. [3]

(b) Find the value of r for which A has a stationary value, leaving your answer in 2 decimal places. [3]

(c) Mr Tan would need to consider the material cost when making this trough. Advise him, based on calculations, if this trough is worth making. [2]

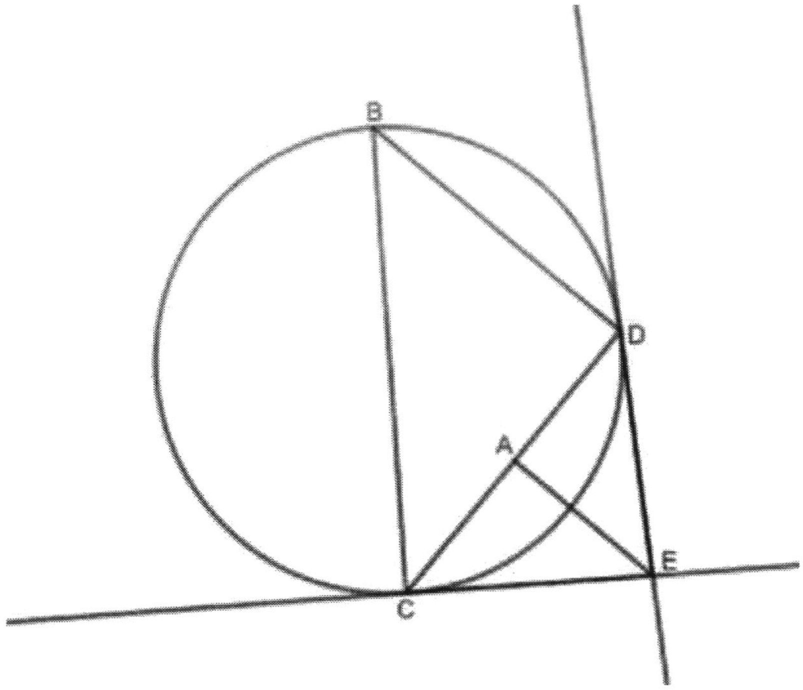
10 (a) The expression $3x^3+ax^2+bx+5$ has a factor of $(x+1)$ and leaves a remainder of -45 when divided by $(x-2)$.

(i) Find the value of a and of b. [4]

(ii) Using these values of a and b, solve the equation $3x^3+ax^2+bx+5=0$. [3]

(b) Justify why $3x^2 - 4y^3$ is a factor of $54x^6 - 128y^9$. [2]

11 In the diagram below, B, C and D lie on the circle and BC is the diameter. CE is a tangent at C and DE is a tangent at D. AE bisects angle CED.



(a) Show that triangle CED is an isosceles triangle. [2]

(b) Show that triangle ACE is similar to triangle DBC. [3]

(c) Show that $2AC^2 = AE \times BD$. [3]

12 **(a)** Show that $(\sec x - \tan x)(\cos ec x + 1) = \cot x$. [3]

(b) Hence solve $(\sec x - \tan x)(\cos ec x + 1) = 3\tan x$ for $0 < x < 2\pi$. [3]

13 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, v m/s is given by $v = 3t^2 - 23t + 30$.

(a) Find the values of t when the particle is instantaneous at rest. [2]

(b) Find the distance travelled in the first 7 seconds. [4]

(c) Find the minimum velocity. [2]

End of Paper

Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2023 4E5N

ADDITIONAL MATHEMATICS 4049/02

Paper 2

28 August 2023

2 hours 15 minutes

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Setter: Mdm Suzanne Lye

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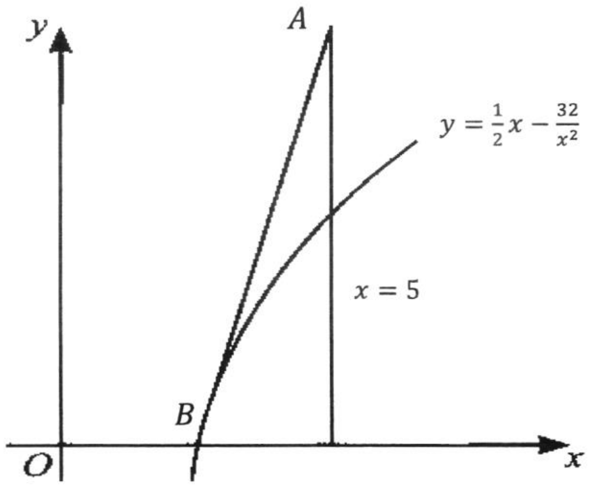
1 Express $\dfrac{2x^3-6}{x^3+3x}$ in partial fractions. [6]

2



The diagram shows part of the curve $y = \dfrac{1}{2}x - \dfrac{32}{x^2}$, which intersects the x-axis at B.
The tangent to the curve at B meets the line $x = 5$ at A.

(a) Find the equation of the line AB. [5]

(b) Find the area of the shaded region bounded by the tangent AB and the line $x = 5$. [5]

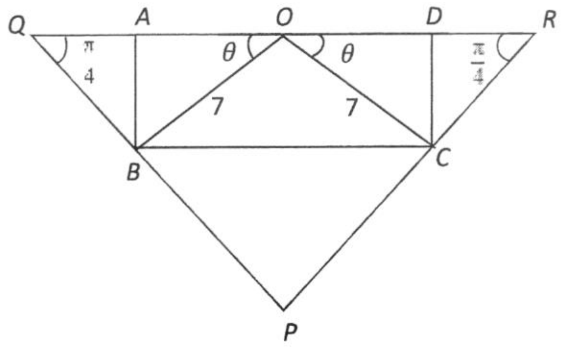
3 **(a)** Given that $y = x\ln(3x+1)$, find an expression for $\frac{\mathrm{dy}}{\mathrm{dx}}$. [2]

(b) Express $\frac{3x}{3x+1}$ in the form $a+\frac{b}{3x+1}$, where a and b are constants, and hence find $\int \frac{3x}{3x+1}\,\mathrm{dx}$. [3]

(c) Using your answers in **(a)** and **(b)**, find $\int \ln(3x+1)\,\mathrm{dx}$. [3]

4 The figure shows a rectangle $ABCD$ inscribed inside an isosceles triangle PQR.

It is given that $OC = OB = 7\text{cm}$, $\angle AOB = \angle DOC = \theta$ and $\angle AQB = \angle CRD = \frac{\pi}{4}$.



(a) Show that the length $QR = 14(\cos\theta + \sin\theta)$. [3]

(b) Express QR in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. [2]

(c) Find the maximum value of QR and the corresponding value of θ. [2]

(d) Find the value of θ when $QR = 15$. [2]

5 The equation of a curve is $y = e^{x^3 - 3x} + 3$ for $-1 \le x \le 1$.

(a) Find expressions for $\frac{\mathrm{dy}}{\mathrm{dx}}$ and $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}$. [4]

(b) Find the exact value of the coordinates of the stationary points. [5]

(c) Find the nature of the stationary points. [2]

6 The points P and Q both lie on a circle and have coordinates $(-2,8)$ and $(7,2)$ respectively. The centre of the circle lies on the line $y = x - 5$.

(a) Find the equation of the perpendicular bisector of PQ. [4]

(b) Find the equation of the circle. [5]

R is a point on the circle such that PR is the diameter of the circle.

(c) Find the coordinates of R. [2]

7 **(a)** Using suitable substitution, solve the equation $2^x - \left(\sqrt{2}\right)^{x+2} = 15$. [5]

(b) Solve $\lg(x-7) = \frac{1}{2}\lg 36 - \lg(x-6)$. [3]

(c) The equation $\log_5 x + 2\log_{25} x^2 = 4\log_3 9$ has the solution $x = 5^y$. Find the value of y. [3]

8 **(a)** The line $y = mx + c$ is drawn on the same axes as $y = 5x - x^2$. Given that the line has a negative y-intercept. Determine with workings, whether the line and the curve have 0, 1 or 2 points of intersections. [3]

(b) The curve $y = ax^2 + 6x + b$ lies completely above the x-axis.

(i) What conditions must apply to a and b? [2]

(ii) Give an example of values for a and b which satisfy the conditions in **(b)(i)**. [2]

9 **(a)** Given that $\frac{1}{5-6\sqrt{2}}=a+b\sqrt{2}$, where a and b are rational numbers, find the value of a and of b. [3]

(b) Represent the solution set $2(1+x)-8\geq x(1-2x)$ on a number line. [3]

10 Measured values of x and y are given in the following table.

x	1.6	1.8	2.2	2.8	3.7	4.7
y	8.32	6.29	3.98	2.36	1.55	0.79

It is known that x and y are to obey the formula $x^h y = k$, where h and k are constants.

(a) Explain how a straight line graph may be drawn to represent the given formula. [3]

(b) On the grid, draw the straight line graph for the given data and **show** that five of the readings are a close fit to the formula. [4]

(c) From the graph plotted in **(b)**, estimate a value of y to replace the incorrect reading of y. [2]

(d) Use your graph to estimate the value of h. [2]

- End of Paper -

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HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination **4E5N**

ADDITIONAL MATHEMATICS **4049/1**

Paper 1

23 August 2023

2 hours 15 minutes

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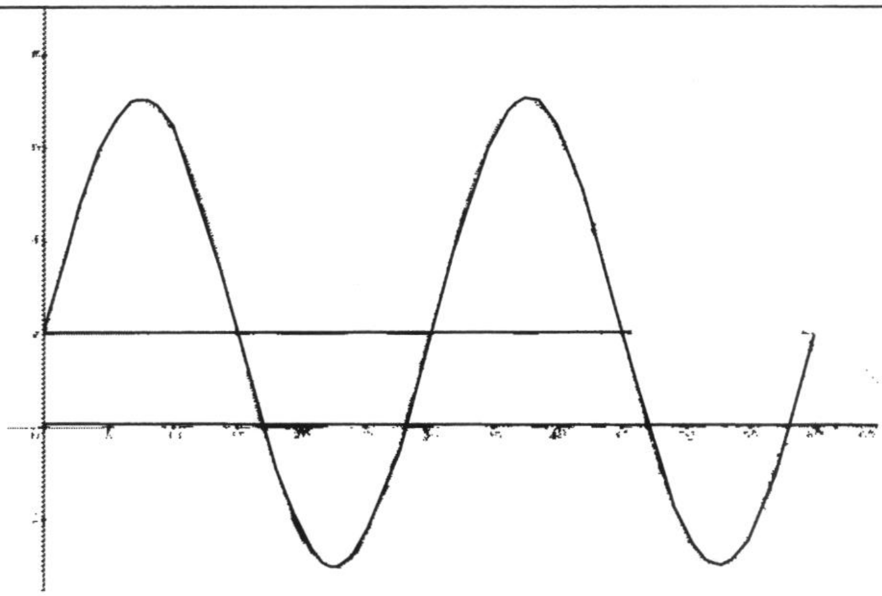
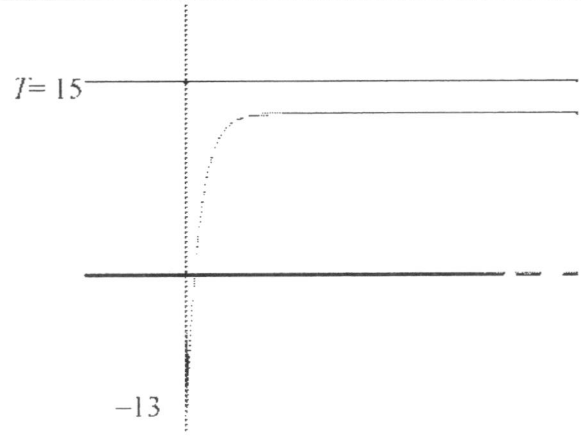
MARK SCHEME

1 (a)	$y = -2\left[x^2 - 6x\right] + k$ M1 factorise -2 correctly $= -2\left[x^2 - 6x + (3)^2 - (3)^2\right] + k$ $= -2\left[(x-3)^2 - 9\right] + k$ M1 for $(x-3)^2$ $= -2(x-3)^2 + 18 + k$ $k = 2$ A1 Alternatively: $\frac{dy}{dx} = -4x + 12$ M1 $\frac{dy}{dx} = 0,$ $x = 3$ M1 $20 = -2(3)^2 + 12(3) + k$ $k = 2$ A1
(b)	The maximum height was reached at the 3rd second, 5th second is **after the 3rd** second, therefore object is moving downward. A1 ECF At the **1st second**, the object is at the same height as it is at the 5th second. A1 ECF (symmetry property of quadratic curve)
2 (a)	$\binom{7}{r}\left(\frac{1}{2}x^3\right)^{7-r}(4x)^r$ $x^{21-3r} \times x^r = x^{21-2r}$ M1 For all integer values of r, $0 \le r < 7$, $2r$ is always even, $21 - 2r$ is always odd, therefore there is no even powers of x in the expansion. A1
(b)	$(1-3x)^2 = 1 - 6x + 9x^2$ M1 $(a+bx)^6 = a^6 + 6a^5bx + 15a^4b^2x^2 + \dots$ $1 \times a^6 = 64$ $a = 2$ $(a > 0)$ A1 $-6(64) + 6(32)(b) = 576$ M1 $b = 5$ A1 $15(2)^4(5)^2x^2 - 6x(960x) + 9x^2(64) = 816x^2$ $c = 816$ A1

3 (a)	$b=\frac{2\pi}{30}$ M1 $=\frac{\pi}{15}$
(b)	$a=5$ B1 $c=2$ B1
(c)	 M1 for 2 complete cycles of sine graph for $0 \le t \le 60$ M1 for correct maximum and minimum d values.
(d)	$5\sin\frac{\pi}{15}t+2=0$ $\sin\frac{\pi}{15}t=-\frac{2}{5}$ basic angle $=0.041151$ M1 $\frac{\pi}{15}t=3.5531,\quad 5.871668,\quad 9.83629,\quad 12.15485$ $t=16.9648,\quad 28.035,\quad 46.9648,\quad 58.03515$ $17.0 \le t \le 28.0$ A1 $47.0 \le t \le 58.0$ A1

4	$\frac{dy}{dx}=3\tan^2(3x)+2$ $=3\left[\sec^2(3x)-1\right]+2$ M1 $=3\sec^2(3x)-1$ $y=\int 3\sec^2(3x)-1\,dx$ $=\frac{3\tan(3x)}{3}-x+c$ M1 for integrating trigo correctly $=\tan(3x)-x+c$ M1 for + c $x=0,\ y=0,\ \therefore c=0$ $y=\tan(3x)-x$ A1
5 (a)	$T=-13$ B1
(b)	$12.5=15-28e^{-3k}$ $\frac{2.5}{28}=e^{-3k}$ M1 $-3k=\ln\left(\frac{2.5}{28}\right)$ $k=0.805$ A1
(c)	As ***t* becomes very large**, $28e^{-0.805t}$ becomes **very small**. [M1] When $28e^{-0.805t}$ is deducted from 15, the temperature T, where $T=15-28e^{-0.805t}$ becomes smaller than 15. [A1] **Or** $\frac{28}{e^{0.805t}}>0$ $-\frac{28}{e^{0.805t}}<0$ M1 $-\frac{28}{e^{0.805t}}+15<15$ A1

(d)	 M1: Shape and T-intercept M1: Asymptote or indicative value of $T = 15$
6 (a)	$f'(x) = \frac{7(x+1)-(7x-3)1}{(x+1)^2}$ M1 $= \frac{10}{(x+1)^2}$ M1 $10 > 0$ $(x+1)^2 > 0$ $f'(x) > 0 \therefore f'(x)$ is an increasing function. A1
(b)	$x = 2, \frac{dy}{dx} = \frac{10}{9}$ M1 $0.4 = \frac{10}{9} \times \frac{dx}{dt}$ M1 $\frac{dx}{dt} = \frac{9}{25}$ units/sec A1
7 (a)	gradient of $AC = \frac{1}{2}$ gradient of $CD = 2$ M1 $y = 2x + C$ $2 = 2(8) + C$ $C = -18$ $CD: y = 2x - 18$ M1 $2x + 7y = 42$ solve simultaneous equations M1 $x = 14$ $y = 10$ $D = (14, 10)$ A1

(b)	$\frac{1}{2}\begin{vmatrix} 0 & 0 & 8 & 14 & 0 \\ 2 & 2 & 2 & 10 & 2 \end{vmatrix}$ M1 $=\frac{1}{2}\left[(108)\ \ (\ 44)\right]$ -76 units^2 A1
8 (a)	$\frac{\sin A\cos B+\sin B\cos A}{\sin A\cos B\ \ \sin B\cos A}=\frac{5}{3}$ $3\sin A\cos B+3\sin B\cos A=5\sin A\cos B\ \ 5\sin B\cos A$ M1 $-2\sin A\cos B=-8\sin B\cos A$ $\tan A\cot B=4$ $\frac{\tan A}{\tan B}=4$ A1
(b) (i)	$7\cos^2 x\ \ 3(1\ \ \cos^2 x)$ M1 use of identity $=10\cos^2 x\ \ 3\ \ 2+2$ $=10\cos^2 x\ \ 5+2$ $=5(2\cos^2 x-1)+2$ $=5\cos 2x+2$ A1
(ii)	$\int 5\cos 2x+2\ \ dx$ $\frac{5\sin 2x}{2}+2x+C$ M1 for integrating trigo, M1 for 2 and + C (ECF) $\left[\frac{5\sin 2x}{2}+2x\right]_{-\frac{\pi}{12}}^{\frac{\pi}{4}}$ $\left[\frac{5\sin\frac{\pi}{2}}{2}+\frac{\pi}{2}\right]-\left[\frac{5\sin\left(-\frac{\pi}{6}\right)}{2}-\frac{\pi}{6}\right]$ $\frac{15}{4}+\frac{2\pi}{3}$ A1 for each correct term

9 (a)	$\frac{\pi r^2}{2} l = 45000\pi$ M1 $l = \frac{90000}{r^2}$ M1 $A = \pi r^2 + \frac{2\pi r l}{2}$ $= \pi r^2 + \pi r\left(\frac{90000}{r^2}\right)$ M1 $= \pi r^2 + \frac{90000\pi}{r}$
(b)	$\frac{dA}{dr} = \frac{-90000\pi}{r^2} + 2\pi r$ M1 $\frac{-90000\pi}{r^2} + 2\pi r = 0$ M1 $r^3 = 45000$ $r = 35.57$ A1
(c)	$\frac{d^2A}{dr^2} = \frac{18000\pi}{r^3} + 2\pi$ M1 $\frac{d^2A}{dr^2} > 0$, **Area is minimum**, material cost will be minimum, therefore it is worth making. A1
10 (a) (i)	Let $f(x) = 3x^3 + ax^2 + bx + 5$ $f(2) = -45$ $4a + 2b = -74$ M1 $f(-1) = 0$ $a = b - 2$ M1 Solving simultaneous equations: $b = -11$ A1 $a = -13$ A1
(ii)	$3x^3 - 13x^2 - 11x + 5 = (x+1)(3x^2 - 16x + 5)$ M1 $= (x+1)(3x-1)(x-5)$ M1 $f(x) = 0$ $x = -1, \frac{1}{3}, 5$ A1
(b)	$2(27x^6 - 64y^9)$ M1 $= 2\left[(3x^2)^3 - (4y^3)^3\right]$ $= 2(3x^2 - 4y^3)(9x^4 + 12x^2y^3 + 16y^6)$ A1

11 (a)	$CE = DE$ (tangent from ext. point) M1 $\therefore \Delta CED$ is an isosceles triangle A1
(b)	AE is angle bisector of isosceles triangle CED, $EAC = 90^\circ$ $CDB = 90^\circ = EAC$ (rt ∡ in semi-circle) M1 $ECA = CBD$ (alternate segmt theorem) M1 By AA property, ΔACE is similar to ΔDBC. A1
(c)	$\frac{CD}{BD} - \frac{AE}{AC}$ M1 $CD \times AC - AE \times BD$ $2AC \times AC = AE \times BD$ M1 $2AC^2 - AE \times BD$ A1
12 (a)	$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + 1\right)$ M1 $\left(\frac{1 - \sin x}{\cos x}\right)\left(\frac{1 + \sin x}{\sin x}\right)$ M1 combine fractions $\left(\frac{1 - \sin^2 x}{\cos x \sin x}\right)$ $= \left(\frac{\cos^2 x}{\cos x \sin x}\right)$ M1 $= \cot x$
(b)	$\cot x = 3\tan x$ $\frac{1}{\tan x} = 3\tan x$ M1 $\tan^2 x \quad \frac{1}{3}$ $\tan x = +\frac{1}{\sqrt{3}}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ A1 A1
13 (a)	$3t^2 - 23t + 30 \quad 0$ M1 $(3t \quad 5)(t \quad 6) = 0$ $t = \frac{5}{3}, \quad t = 6$ A1

(b)	$s=\int 3t^2-23t+30\,dt$ $=\frac{3t^3}{3}-\frac{23t^2}{2}+30t+C$ M1 $=t^3-\frac{23}{2}t^2+30t+C$ $t=0, s=0, \therefore C=0$ $t=\frac{5}{3}, s=\frac{1225}{54}$ M1 $t=6, s=-18$ M1 $t=7, s=\frac{-21}{2}$ distance $=\frac{1225}{54}\times 2+18+7.5$ $=70\frac{47}{54}$ m or 70.9 m A1	
(c)	$\frac{dv}{dt}=0$ $6t-23=0$ M1 $t=\frac{23}{6}$ minimum $v=-\frac{169}{12}$ m/s A1 OR $v=3\left(t-\frac{23}{6}\right)^2-\frac{169}{12}$ M1 for $\left(t-\frac{23}{6}\right)$ minimum $v=-\frac{169}{12}$ m/s A1	

Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2023 **4E5N**

Additional MATHEMATICS **4049/02**

28 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper
No additional materials are required.

Marking Scheme

This document consists of **17** printed pages including the cover page.

[Turn Over

Setter: Mdm Suzanne Lye

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + hx * c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 +$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r*1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 . \quad \cos \cdot \quad = 1$$

$$\sec 2 \equiv 1 + \tan^2 \#$$

$$\text{coseC } A = 1 + \cot^* A$$

$$\text{su}_\chi A \pm B) \quad \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos 4 \text{cosfi} \quad \sin A \sin b$$

$$\tan(\Lambda \pm B) \quad \frac{4 \_ \tan B}{\mp \tan A \tan B}$$

$$\text{sm}2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - yiP \; A = 2\cos^{\cdot} A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A \quad \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 Express $\frac{2x^3-6}{x^3+3x}$ in partial fractions. [6]

$= 2 + \frac{-6x-6}{x^3+3x}$ ---------------------------------------M1

$\frac{-6x-6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$ --------------------------------M1

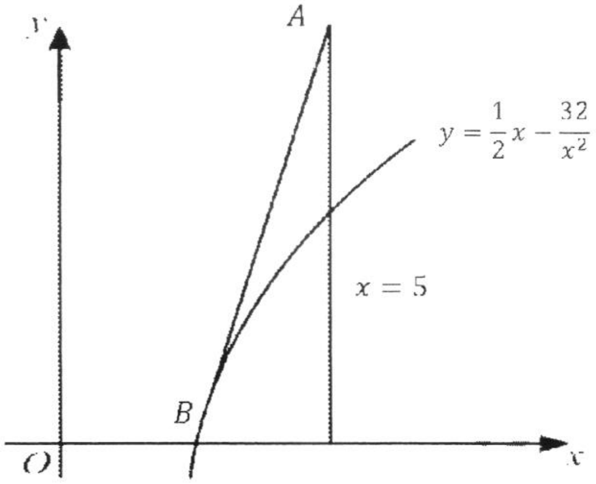
$-6x - 6 = A(x^2 + 3) + (Bx + C)x$ ---------M1

Comparing coefficients or substituting values of x,

$A = -2, B = 2, C = -6$ -------------------------A1A1A1

$$2 - \frac{2}{x} + \frac{2x-6}{x^2+3}$$

2



The diagram shows part of the curve $y = \frac{1}{2}x - \frac{32}{x^2}$, which intersects the x-axis at B.
The tangent to the curve at B meets the line $x = 5$ at A.

(a) Find the equation of the line AB. [5]

At B, $y = 0$,
$0 = 0.5x - \frac{32}{x^2}$ -------------------------------------M1
$x = 4$ ---A1
$\frac{dy}{dx} = 0.5 + \frac{64}{x^3}$ -------------------------------------M1
Gradient of $AB = 0.5 + \frac{64}{4^3} = \frac{3}{2}$-------M1
$y = \frac{3}{2}x + c$
Sub (4, 0) into the eqn.

$c = -6$
$y = \frac{3}{2}x - 6$ ----------------------------------A1

(b) Find the area of the shaded region bounded by the tangent AB and the line $x = 5$. [5]

Integrating curve $= \left|\frac{x^2}{4} + \frac{32}{x}\right|$ ----------------------------------M1

Limits 4 to 5 -> Area under curve $\frac{13}{20}$ unit^2 -----------------M1

Area under $AB = \frac{1}{2} \times 1 \times \frac{3}{2} = \frac{3}{4}$ ---------------------------M1

Shaded area $\frac{3}{4} - \frac{13}{20} = \frac{1}{10}$ or 0.10 units^2 --------------M1A1

3 (a) Given that $y = x\ln(3x+1)$, find an expression for $\frac{dy}{dx}$. [2]

$\frac{dy}{dx} = \frac{3x}{3x+1} + \ln(3x+1)$ ------------------------M1 (Use product rule) A1

(b) Express $\frac{3x}{3x+1}$ in the form $a + \frac{b}{3x+1}$ where a and b are constants, and hence find $\int \frac{3x}{3x+1} dx$. [3]

$\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$ -----------------------------------M1

$\int 1 - \frac{1}{3x+1} dx = x - \frac{\ln(3x+1)}{3} + c$ ----------------A1A1

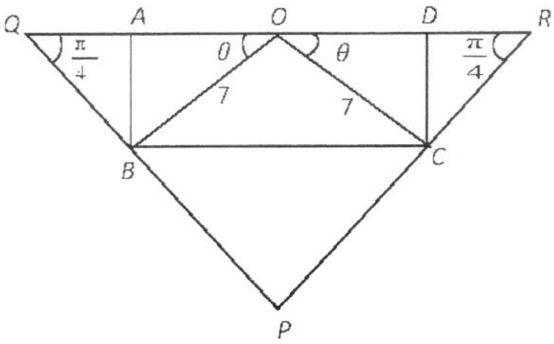
(c) Using your answers in **(a)** and **(b)**, find $\int \ln(3x+1)dx$ [3]

$$\frac{dy}{dx} = \frac{3x}{3x+1} + \ln(3x+1)$$

$y = \int \frac{3x}{3x+1} dx + \int \ln(3x+1)dx$ -----------------------------------M1

$\int \ln(3x+1)\, dx = x\ln(3x+1) - x + \frac{\ln(3x+1)}{3} + c$ --------------------A1A1

4 The figure shows a rectangle $ABCD$ inscribed inside an isosceles triangle PQR. It is given that $OC = OB = 7$ cm, $\angle AOB = \angle DOC = \theta$ and $\angle AQB = \angle CRD = \frac{\pi}{4}$.



(a) Show that the length $QR = 14(\cos\theta + \sin\theta)$. . [3]

$AO = OD = 7\cos\theta$ -------------------------M1
$CD = DR = AQ = 7\sin\theta$ -----------------M1
$QR = 2(7\cos\theta) + 2(\sin\theta) = 14(\cos\theta + \sin\theta)$ -----------------A1

(b) Express QR in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. [2]

$R = \sqrt{14^2 + 14^2} = \sqrt{392}$ ------------------------A1

$\alpha = tan^{-1}\left(\frac{14}{14}\right) = \frac{\pi}{4}$-----------------A1

$\sqrt{392}\sin\left(\theta + \frac{\pi}{4}\right)$

(c) Find the maximum value of QR and the corresponding value of θ. [2]

Max $QR = \sqrt{392}$ when $\sin\left(\theta + \frac{\pi}{4}\right) = 1$--------------------A1
$\theta = \frac{\pi}{4}$ ---A1

(minus overall 1 mark if answers in degree)

(d) Find the value of θ when $QR = 15$. [2]

$\sqrt{392}\sin\left(\theta + \frac{\pi}{4}\right) = 15$

$\sin\left(\theta + \frac{\pi}{4}\right) = \frac{15}{\sqrt{392}}$ ------------------------------------M1

$\theta + \frac{\pi}{4} = 0.8596504$

$\theta = 0.07$ rad --A1

5 The equation of a curve is $y = e^{x^3-3x} + 3$ for $-1 \leq x \leq 1$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

$\frac{dy}{dx} = (3x^2 - 3)e^{x^3-3x}$ -------------------------A1

$\frac{d^2y}{dx^2} = (6x).e^{x^3-3x} + (3x^2 - 3)^2 e^{x^3-3x}$--------------M1M1

$= e^{x^3-3x}(9x^4 - 18x^2 + 6x + 9)$--------------------------------A1

Or $e^{x^3-3x}(6x + (3x^2 - 3)^2)$

(b) Find the exact value of the coordinates of the stationary points. [5]

For stationary value, $\frac{dy}{dx} = (3x^2 - 3)e^{x^3-3x} = 0$

$(3x^2 - 3)e^{x^3-3x} = 0$ ----------------M1

Since $e^{x^3-3x} > 0, 3x^2 - 3 = 0$------M1

$x^2 = 1$

$x = \pm 1$---------------------------------M1

Coordinates of points are $(-1, 3 + e^2)$ and $(1, 3 + e^{-2})$ ---------------A1A1 (only A1 if never leave in exact)

(c) Find the nature of the stationary points. [2]

When $x = -1, \frac{d^2y}{dx^2} = -44.3 < 0, (-1, 3 + e^2)$ is a maximum point. -----------------A1

When $x = 1, \frac{d^2y}{dx^2} = 0.812 > 6 > 0, (1, 3 + e^{-2})$ is a minimum point.----------------A1

6 The points P and Q both lie on a circle and have coordinates $(-2, 8)$ and $(7, 2)$ respectively.
The centre of the circle lies on the line $y = x - 5$.

(a) Find the equation of the perpendicular bisector of PQ. [4]

Midpoint of PQ $(2.5, 5)$ --------------------------------M1

$m_{PQ} = -\frac{2}{3}$ --M1

$m_{\perp\ bisector} = \frac{3}{2}$--M1

$y - 5 = \frac{3}{2}(x - 2.5)$

$y = \frac{3}{2}x + \frac{5}{4}$ ---A1

(b) Find the equation of the circle. [5]

Solve simultaneous eqn -------------------------------M1

$y = \frac{3}{2}x + \frac{5}{4}$ and $y = x - 5$

$x - 5 = \frac{3}{2}x + \frac{5}{4}$

$x = -12.5$ ---A1

$(-12.5, -17.5)$ = centre of circle--------------------M1

Radius = $\sqrt{(-2 + 12.5)^2 + (8 + 17.5)^2}$

$= \sqrt{\frac{1521}{2}}$

$= \frac{39}{\sqrt{2}}$ --M1

$(x + 12.5)^2 + (y + 17.5)^2 = 760.5$ ----------------A1

R is a point on the circle such that PR is the diameter of the circle.

(c) Find the coordinates of R. [2]

$R = (a, b)$

$\left(\frac{-2+a}{2}, \frac{8+b}{2}\right) = (-12.5, -17.5)$ ----------------------M1

$R = (a, b) = (-23, -43)$---------------------------A1

7 **(a)** Using suitable substitution, solve the equation $2^x - (\sqrt{2})^{x+2} = 15$. [5]

$2^x - (2^{0.5})^{x+2} = 15$
$2^x - 2^{0.5x}.2^1 = 15$
$2^x - 2.2^{0.5x} - 15 \; 0$ ----------------------------M1
Let $y = 2^{0.5x}$
$y^2 - 2y - 15 = 0$ -------------------------------M1
$(y-5)(y+3) = 0$
$2^{0.5x} = 5$ or $2^{0.5x} = -3$ (reject) -----------------------M1 M1
$x = 2log \log_2 5 = 4.64$ -----------------------A1

(b) Solve $\lg(x-7) = \frac{1}{2}\lg 36 - \lg(x-6)$. [3]

$\lg(x-7) = \lg 6 - \lg(x-6)$
$\lg(x-7) = \lg\frac{6}{x-6}$ ---------------------------M1
$x - 7 = \frac{6}{x-6}$
$x^2 - 13x + 36 = 0$--------------------------M1
$x = 4\,(rej), x = 9$ --------------------------A1

(c) The equation $\log_5 x + 2\log_{25} x^2 = 4\log_3 9$ has the solution $x = 5^y$. [3]
Find the value of y.

$log_5 5^y + 2log_{25}(5^y)^2 = 8$ --------------------M1
$y + 2y = 8$ --M1
$3y - 8 = 0$
$y = \frac{8}{3}$ --------------------------------A1

8 **(a)** The line $y = mx + c$ is drawn on the same axes as $y = 5x - x^2$. Given that the line has a negative y-intercept. Determine with workings, whether the line and the curve have 0, 1 or 2 points of intersections. [3]

Equate $mx + c = 5x - x^2$ --------------------------M1
$x^2 - 5x + mx + c = 0$
Discriminant
$(m - 5)^2 - 4(1)(c)$
$= (m - 5)^2 - 4c > 0$ (since 4C >0) ---------------M1

Hence curve has 2 points of intersection -----------A1

(b) The curve $y = ax^2 + 6x + b$ lies completely above the x-axis.
(i) What conditions must apply to a and b? [2]
$b^2 - 4ac < 0$
$36 - 4(a)(b) < 0$--------------------M1
$ab > 9$ and $a > 0$ --------------------A1 (both correct)

(ii) Give an example of values for a and b which satisfy the conditions in **(b)(i)**. [2]

Any possible values of a and b that satisfies $ab > 9$ -----------------A1A1

9 **(a)** Given that $\frac{1}{5-6\sqrt{2}} = a + b\sqrt{2}$, where a and b are rational numbers, find the value of a and of b. [3]

$\frac{1}{5-6\sqrt{2}} \times \frac{5+6\sqrt{2}}{5+6\sqrt{2}}$ ---------------------------M1

$= \frac{5+6\sqrt{2}}{25-72}$

$-\frac{5}{47} - \frac{6}{47}\sqrt{2}$

$a = -\frac{5}{47}$, $b = -\frac{6}{47}$ ----------------A1 A1

(b) Represent the solution set $2(1+x) - 8 \geq x(1-2x)$ on a number line. [3]

$2(1-x) - 8 > x(1-2x)$

$2 + 2x - 8 > x - 2x^2$

$2x^2 - x - 6 > 0$ ------------------------------------ M1

$(2x-3)(x+2) \geq 0$ ----------------------------------M1

A1

10 Measured values of x and y are given in the following table.

x	1.6	1.8	2.2	2.8	3.7	4.7
y	8.32	6.29	3.98	2.36	1.55	0.79

It is known that x and y are to obey the formula $x^h y = k$, where h and k are constants.

(a) Explain how a straight line graph may be drawn to represent the given formula. [3]

$lgx^h + lgy = lgk$
$lgy = -hlgx + lgk$ ------------------------------------M1

Gradient $-h$ --A1
Vertical Intercept = lgk -----------------------------A1

(b) On the grid, draw the straight line graph for the given data and **show** that five of the readings are a close fit to the formula. [4]

$lg\,x$	0.204	0.255	0.342	0.447	0.568	0.672
lgy	0.920	0.799	0.600	0.373	0.190	-0102

Correct calculation of values for lgx and lgy (to 2 or 3 dp)--------------------M1

Plot points and line correctly on graph --------------------------------M2

Best fit Line passes through 5 of the points or as close as possible, leaving the 5[th] point out. -------------------------------------A1

(c) From the graph plotted in **(b)**, estimate a value of y to replace the incorrect reading of y. [2]

At $lgy = 0.11$ ---------------M1
$y = 1.29$ ----------------------A1

(d) Use your graph to estimate the value of h. [2]

$h = -\frac{0.92-0.2}{0.2-0.525}$------------------------M1
$= 2.22$ --------------------------------A1 (accept answer based on graph)

Q10(b)