Visit

FREETESTPAPER.com

for more papers

Website: freetestpaper.com
Facebook.com/freetestpaper
Twitter.com/freetestpaper

1 **(a)** Sketch the graph of $y = \ln x$. [1]

(b) Determine the equation of the straight line that would be drawn on the graph of $y = \ln x$ to obtain the graphical solution of the equation $x = e^{2-\frac{3}{2}x}$. [2]

(c) Find the number of solutions of the equation $x = e^{2-\frac{3}{2}x}$. [1]

2 **(a)** Without using a calculator, solve the equation $\sqrt{x+3} = 2 + \sqrt{5}$, leaving your answer in exact form. [2]

(b) A triangle ABC is such that its area is $\left(8 + 3\sqrt{2}\right)$ cm^2, the length of AB is $\left(4 + \sqrt{8}\right)$ cm and angle ABC is 90°.

Without using a calculator, find the length of BC, in cm, in the form $a + b\sqrt{2}$, where a and b are integers. [3]

3 **(a)** **(i)** State the values between which the principal value of $\cos^{-1} x$ must lie. [1]

(ii) Find the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ in radian in exact form. [1]

(b) Given that $\operatorname{cosec}\theta = -\frac{2}{\sqrt{3}}$ and that $\tan\theta$ and $\sin\theta$ have opposite signs, without evaluating θ, find the exact value of each of the following.

(i) $\cos(-\theta)$, [1]

(ii) $\tan 2\theta$. [2]

4 Express $\dfrac{x^2-3x+5}{(x+1)(2x^2-x)}$ in partial fractions. [5]

5 The table below shows experimental values of two variables x and y.

x	1	1.5	2	2.5	3
y	5.00	4.90	4.95	5.06	5.20

(a) Given that x and y are related by the equation $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$, where a and b are constants, draw the straight line graph of $y\sqrt{x}$ against x. [2]

(b) Use your graph to estimate the value of a and of b. [3]

(c) Explain how the graph could be used to find the value of y when x is 0.5. [2]

6 A curve is such that $\frac{d^2y}{dx^2} = 3x - 2$ and the equation of the tangent to the curve at the point (1, −5) is $y = -2 - 3x$. Find the equation of the curve. [6]

7 A stone is thrown upwards from a cliff, 40 m above the ground. It will reach a maximum vertical height and then fall back to the ground. The height of the stone from the ground at time t is h m, which is given by $h = -8t^2 + 64t + 40$, where t is the time in seconds.

(a) Find the time interval when the stone is below 160 m. [3]

(b) Express h in the form of $k(t-p)^2 + q$, where k, p and q are constants. [2]

(c) Hence, find the maximum height that the stone can reach and the time taken to reach this maximum height. [2]

8 Diagram I shows a spherical ornament of radius r cm.
Diagram II shows the cross-sectional view of an open cylindrical box containing 7 such identical spherical ornaments touching one another and the side of the box. The height of the box is h cm.
A paper label is wrapped round the box which exactly covers its vertical surface. The area of paper label is 152 cm^2.

Diagram I

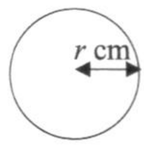
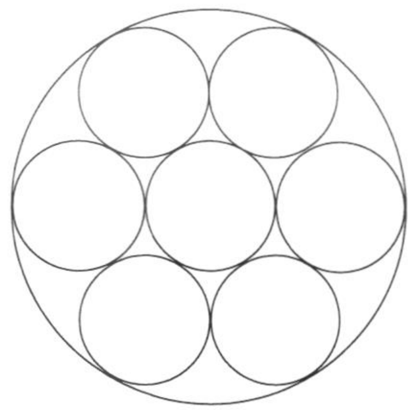


Diagram II



(a) Express h in terms of r. [1]

(b) Show that V cm^3, the volume of the space in the box which is not occupied by the ornaments is

$$V = 228r - \frac{28}{3}\pi r^3 .$$

[3]

(c) Given that r can vary, find the stationary value of V. [3]

9 **(a)** Prove the identity $\sin 2x - \tan x = \tan x \cos 2x$. [3]

(b) Solve the equation $3\cot^2\theta + 10 \operatorname{cosec}\theta = 5$ for $0 \leq \theta \leq 2\pi$. [5]

10 The expression $f(x) = 3x^3 + ax^2 + 2x + b$, where a and b are constants, is exactly divisible by $x - 2$ and leaves a remainder of -8 when divided by $x - 1$.

(a) Find the value of a and b. [4]

(b) Show that the equation $f(x) = 0$ has only one real root. [4]

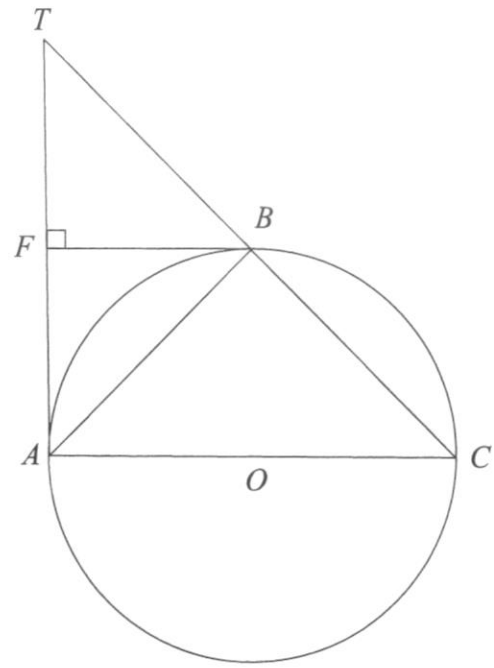
11 A circle passes through the points A, B and C are $(6, 0)$, $(2, 2)$ and $(-3, -3)$.

The equation of the perpendicular bisector of the line segment BC is $y = -x - 1$.

(a) Find the equation of the perpendicular bisector of the line segment AB. [3]

(b) Hence, find the equation of the circle that passes through the three points A, B and C. [4]

12 In the diagram, a circle with centre *O* passes through the vertices of triangle *ABC*.
AT is a tangent to the circle at *A* and *BF* is perpendicular to *AT*.
CB produced and *AF* produced meet at *T*.



Prove that

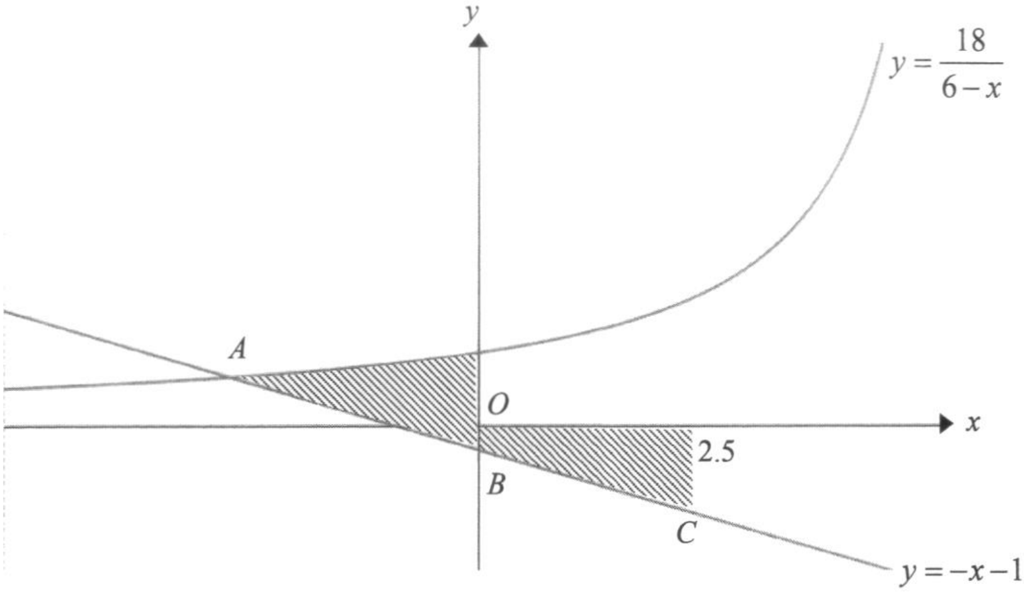
(a) triangle *ABC* is similar to triangle *BFA*, [2]

(b) $AB^2 = AC \times BF$, [1]

(c) $AB^2 + TB^2 = TC^2 - AC^2$. [2]

13 The diagram shows part of a curve $y = \dfrac{18}{6-x}$ and the line $y = -x - 1$.

A lies on the curve.
B and C lie on the line such that B is on the y-axis and C has a x-coordinate of 2.5.
Find the area of the shaded region. [7]



Continuation of working space for question **13**.

14 A particle, A, starts at a point 2 m away from O and travels in a straight line. Its velocity, v m/s, is given by $v = 4t - t^2$, where t is the time in seconds measured from the start of motion.

Calculate

(a) the time when the particle is instantaneously at rest, [2]

(b) total distance travelled by particle A in the first 5 seconds. [3]

Point X is 8 m from point O. Particle B travels in the same direction along the same straight line as Particle A. It travels with an acceleration of $a = -8t + 4$, where t is the time after passing point X. The velocity of Particle B when it passes through Point X is 7 m/s.

(c) Calculate the time when particle B collides with particle A. [4]

- End of paper –

1 It is given that $y = 7x^3e^{-2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Show that $e^{2x}\left(p\frac{dy}{dx} + \frac{d^2y}{dx^2} + 4y\right) = qx$

where p and q are constants to be determined. [5]

2 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(x\cos 4x)$. [2]

(b) Without using a calculator, find the value of each of the constant a and b for which

$$\int_0^{\frac{\pi}{12}} x\sin 4x \, \mathrm{d}x = a\pi + b$$ [4]

3 **(a)** The coefficient of x^{13} in the binomial expansion of $\left(ax^2 - \frac{1}{x}\right)^{11}$ is -42240.

Find the value of a. [3]

(b) In the expansion of $(3+7x)^n$, the coefficient of x^3 and x^4 are the in the ratio 12 : 35.

Find the value of n. [4]

4 A curve has equation $y = \frac{4x^3 - 1}{x^2}$, $x \neq 0$.

(a) Determine the range of values of x such that y is a decreasing function. [4]

(b) Determine the coordinates of the point where the tangent of the curve at $x = -1$ cuts the x-axis. [3]

5 **(a)** Solve the equation $\log_2(y-4)+\log_2 y=\log_2 2y-3$. [4]

(b) Solve the equation $\log_6 x=3-\log_x 36$. [4]

6 The equation of a curve is $y = kx^2 + 4x + 3k - 5$, where k is a constant.

The line $y = 2kx - 3$ is a tangent to the curve at the point H.

(a) Find the possible values of k. [4]

(b) Given that the gradient of the tangent is positive, find the coordinates of H. [2]

7 When a piece of steak is pan-fried, the internal temperature of the steak, T°C, can be modelled by the formula $T = 20 + me^{kt}$, where t is the time in minutes from the time the steak is placed in the pan to be cooked and k is a constant.

The steak must be in room temperature when it is placed in the pan.

The table below shows some information of how a piece of steak may be cooked to the preference of doneness*.

Doneness of Steak	**Colour & Firmness**	**Internal Temperature**	**Estimated Time to cook a piece of steak**
Medium-Rare	Red warm centre, springy firmness	52 °C	9 minutes
Medium	Hot pink centre, less springy than medium-rare	57 °C	
Medium-Well	Slight colour, cooked throughout, firm	63 °C	
Well-Done	No colour left, very firm	71 °C	14 minutes

***Doneness** is a gauge of how thoroughly cooked a piece of steak is based on its color, firmness, and internal temperature.

(a) Using the information in the table, find the value of k and of m. [5]

(b) Kenny prefers his steak to be done medium-well.
How long should he pan-fry his steak? [3]

(c) Explain why the model cannot be used for large values of t. [1]

8 **(a)** A particle moves along the curve $y = \ln\sqrt{\dfrac{5x-3}{x^2+1}}$ in such a way that the y-coordinate is decreasing at a constant rate of $\dfrac{1}{7}$ units per second. Find the rate at which the x-coordinate of the particle is increasing at the instant when $x = 2$. [4]

(b) A curve is defined by the equation $y = 10 - \left(\frac{x}{3} - 2\right)^5$.

(i) Find the coordinates of stationary point. [3]

(ii) By considering the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$, determine the nature of the stationary point. [2]

9 It is given that $y_1 = -3\cos 2x$ and $y_2 = \tan 2x$

(a) State the amplitude and period, in degrees, of y_1. [2]

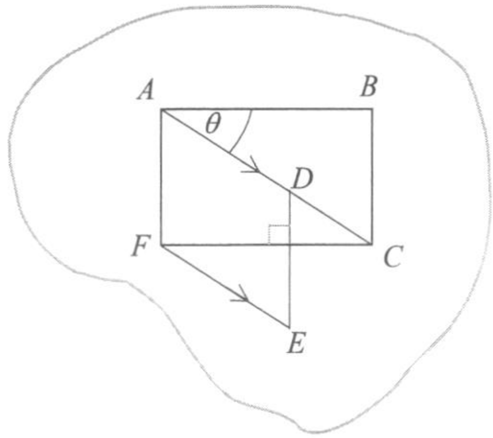
(b) Explain why the amplitude of y_2 does not exist. [1]

For the interval $0° \leq x \leq 180°$,

(c) solve the equation $y_1 = \dfrac{1}{y_2}$. [4]

(d) sketch on the same diagram, the graphs of y_1 and y_2. [4]

10



The diagram shows an enclosure in a miniature park. The lines represent the possible tracks of automated moving cars. *ABCF* is a rectangle and *AC* is parallel to *FE*. *D* is a point on *AC* and *DE* is perpendicular to *FC*. Angle *BAC* is θ radians and *AC* is 10 metres. A car moves along the track *ABCDEFA* to complete one cycle.

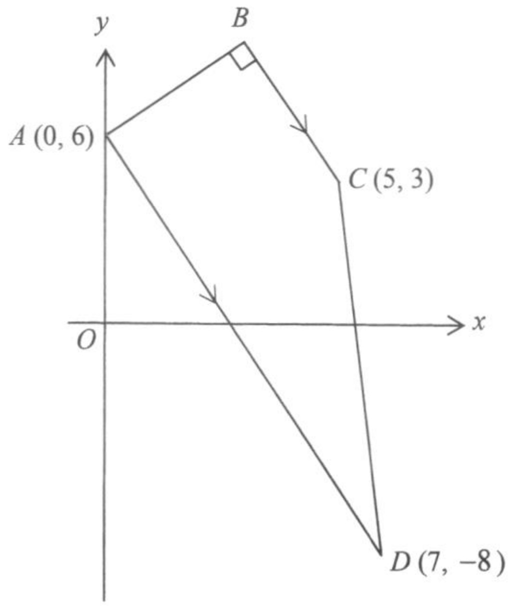
(a) Show that the distance, *S* m, travelled by the car per cycle is given by

$$S = 10\cos\theta + 30\sin\theta + 10.$$ [3]

(b) Express $10\cos\theta + 30\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute. [3]

(c) Find the value of θ if S is 36 m. [2]

11



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC and angle $ABC = 90°$. The coordinates of A, C and D are (0, 6), (5, 3) and (7, −8) respectively.

(a) Is $AB = BC$? Justify your answer with working. [7]

Continuation of working space for Question 11a.

(b) The point E lies on the line AD produced such that $\frac{\text{area of } \Delta ACD}{\text{area of } \Delta ACE} = \frac{4}{5}$.

Find the coordinates of point E. [2]

(c) A quadrilateral $OACF$ is formed such that the area is 50 square units. Given that DF is parallel to the x-axis, find the coordinates of F. [3]

End of paper

Answer Key
2023 4E5N Prelim Additional Math Paper 1

1a	graph	1b	$y-2-\frac{3}{2}x$
1c	1 solution		
2a	$x-6+4\sqrt{5}$	2b	$5\ \ \sqrt{2}$
3ai	$0\le\cos^{-1}x\le\pi$	3aii	$\frac{\pi}{3}$
3bi	-0.5	3bii	$-\sqrt{3}$
4	$\frac{3}{x+1}+\frac{5}{2x\ \ 1}-\frac{5}{x}$		
5a	graph	5b	$a=2, b=3$
5c	Draw a vertical line X = 0.5 From the graph, the corresponding Y 4 y 5.66		
6	$y=\frac{x^3}{2}\ \ x^2\ \ \frac{5}{2}x\ \ 2$		
7a	$0\le t<3$ $5<t\le 8.58$	7b	$-8(t-4)^2+168$
7c	Max height = 168 m Time taken - 4 sec		
8a	$h=\frac{76}{3\pi r}$	8b	show question
8c	245 cm^3		
9a	prove question	9b	3.39 or 6.03
10a	$a=-5, b=\ \ 8$	10b	show question
11a	$y-2x-7$	11b	$(x-2)^2+(y+3)^2-25$
12a	prove question	12b	prove question
12c	prove question		
13	11.4 units^2		
14a	$t=0$ or 4	14b	13 m
14c	$t=3$		

BP~712

2023 4E5N AM P2 Answer Key

1a) $e^{2x}(-14x^3+21x^2)$ b) $p=4 \quad q=42$

2a) $-4x\sin 4x+\cos 4x$ b) $a=-\frac{1}{96} \quad b=\frac{\sqrt{3}}{32}$ (do not accept 3 s.f.)

3a) $a = \pm 2$ b) $n = 8$

4a) $\sqrt[3]{\frac{1}{2}} < x < 0$ b) $\left(\frac{3}{2}, 0\right)$

5a) $\frac{17}{4}$ b) 6 or 36

6a) $k=-2$ or 1 b) $H(-1,-5)$

7a) $k=\frac{1}{5}\ln\left(\frac{51}{32}\right)$ o.e $m = 13.8$ b) 12.2 minutes

7c) Any logical reasons:

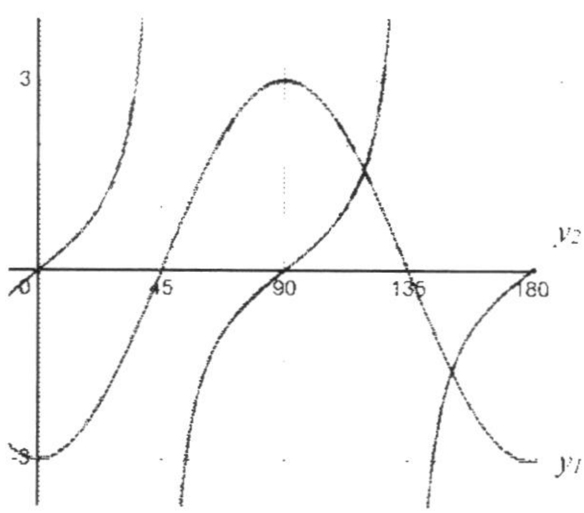
For large values of t, the temperature will rise to a very high value according to the model. The steak will be burnt and will not be edible/ the steak does not need to be cooked to such high temperature/ the internal temperature of the steak will reach a certain level and will not continue to rise anymore.

8a) $\frac{10}{3}$ units per second b) (6, 10) c) point of inflexion

9a) amp = 3, period = 180°

9b) $y_2 = \tan 2x$ do not have maximum or minimum values c) 99.7° , 170.3°

9d)



10a) To be shown   b) $10\sqrt{10}\cos(\theta - 1.25)$ C) .644

11a) No , $B\left(\frac{14}{5}, \frac{37}{5}\right)$, $AB = \sqrt{\frac{49}{5}}$ units is not to equal to $BC = \sqrt{\frac{121}{5}}$ units

11b) $E\left(8\frac{3}{4}, -11\frac{1}{2}\right)$ c) F (10 -8)