CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	

CLASS	4		INDEX NUMBER		

CENTRE/ INDEX NO					/				

ADDITIONAL MATHEMATICS

4049/01

Paper 1

22 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Two cylinders are such that the ratio of their heights is $\sqrt{7} : 1$.

The height of the smaller cylinder is $\dfrac{2\sqrt{7}-1}{\left(2-\sqrt{7}\right)^2}$ cm.

Without using a calculator, find the height of the larger cylinder, expressing your answer in the form $\left(a+b\sqrt{7}\right)$ cm, where a and b are integers. [4]

2 The profit, $\$y$ of a company can be modelled by the equation $y = a(x-h)^2 + k$, where x is the number of goods sold and a, h and k are constants.

The company obtained the maximum profit of $24 500 when 800 goods were sold.

The company incurred a loss of $7 500 when no goods were sold.

(a) State the value of h and of k. [2]

(b) Using the values of h and k found in part **(a)**, find the value of a. [2]

(c) Find the range of the number of goods the company needed to sell to earn a profit. [2]

3 Solve the equation $2\log_3 p + \dfrac{1}{\log_{27} 3} - 1 = \log_3(8p+1)$. [5]

4 Solve the equation $5^{x+1} = 5^{2x-1} - 30$. [5]

5 Given that $y = x^3 - px^2 - 45x + 10$ is decreasing for $-3 < x < q$, find the value of the constants p and q. [5]

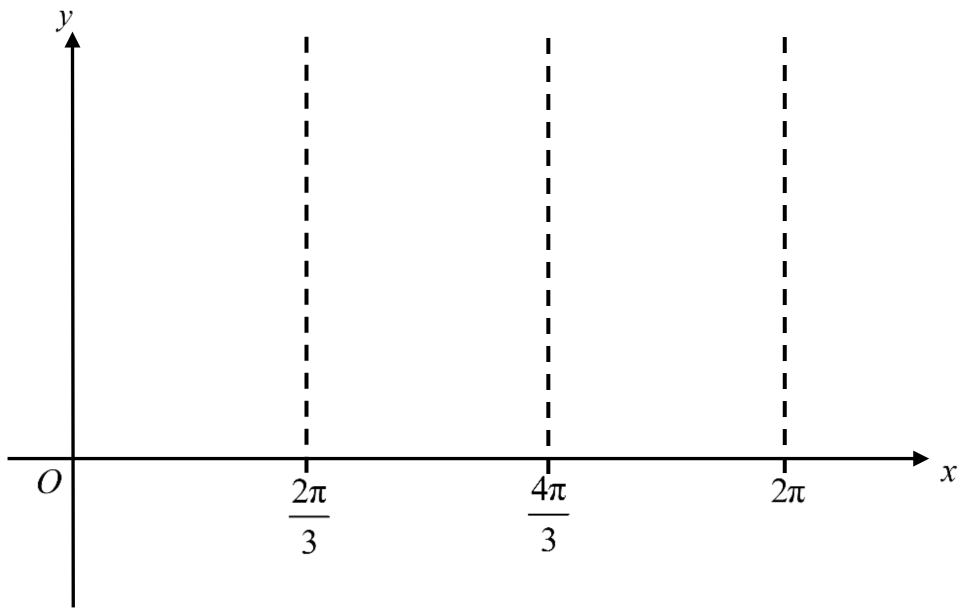
6 The graph $y = a + b\cos(cx)$ is defined for $0 \le x \le 2\pi$, where a, b and c are constants.

The graph has a period of $\dfrac{4\pi}{3}$ and a minimum value of 1 when $x = \dfrac{4\pi}{3}$.

The graph also passes through the point $\left(\dfrac{\pi}{3},\ 3\right)$.

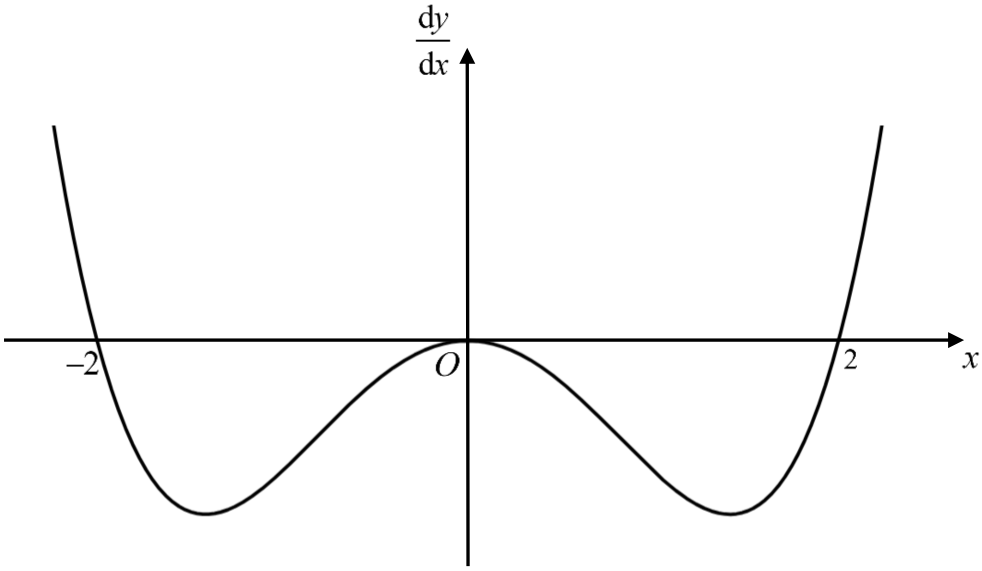
(a) Show that $a = 3$, $b = -2$ and $c = \dfrac{3}{2}$. [3]

(b) Sketch the graph of $y = a + b\cos(cx)$ for $0 \le x \le 2\pi$. [2]



(c) Given that there are exactly 3 solutions for the equation $b\cos(cx) = k - a$, for $0 \le x \le 2\pi$, state the range of values of the constant k. [1]

7



The graph of $\frac{dy}{dx}$ of a function $y = f(x)$ is shown in the diagram, passing through the x-axis at $(-2, 0)$, $(0, 0)$ and $(2, 0)$.

(a) State the number of stationary points of the graph of $y = f(x)$. [1]

(b) State the x-coordinate of the **minimum** point of $y = f(x)$ and explain why it is the minimum point.

[2]

8 The curve $y = ax + \frac{b}{(2x-1)^3}$, where a and b are constants, has a stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$.

(a) Find the value of a and of b. [4]

(b) Find the x-coordinate of the other stationary point. [3]

(c) Find $\frac{d^2y}{dx^2}$. [1]

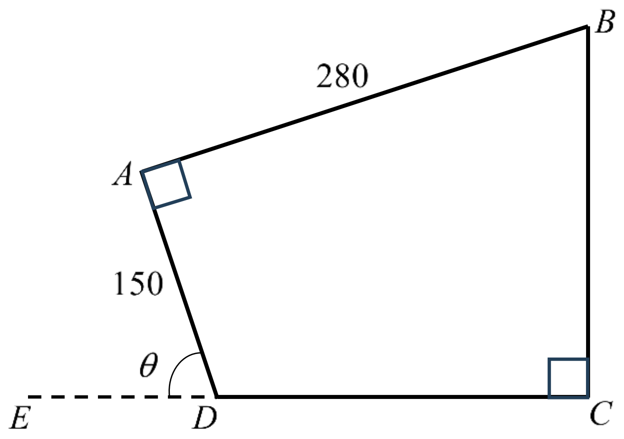
(d) Hence, determine the nature of the stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$. [2]

9 It is given that $y = \ln\left(\dfrac{2x}{5-x}\right)$, where $0 < x < 5$.

(a) If y is decreasing at a rate of 10 units/s , find the rate of change of x when $y = \ln 8$. [5]

(b) The equation of the normal to the curve at point A is $8y = 8\ln 2 + 25 - 10x$.
Find the x-coordinate of point A. [4]

10



The diagram shows a running track *ABCD*. *D* is a point on the straight line *EC*. It is given that $AB = 280$ m, $AD = 150$ m, angle BCD = angle $DAB = 90°$ and angle $ADE = \theta$, where $0° < \theta < 90°$.

(a) Show that *L* m, the length of the running track, is given by $L = p + 130\cos\theta + 430\sin\theta$, where *p* is a constant. [3]

(b) Express *L* in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(c) Xin Ting ran five complete laps around the track.
She claims that she ran a total of 5 km. Explain why she might be wrong. [2]

(d) Given that the running track is 700 m long, find the value of θ. [3]

11 After n years, the population, P, **in thousands**, of a certain species of foxes can be modelled by an equation $P = Ae^{kn}$, where A and k are constants.

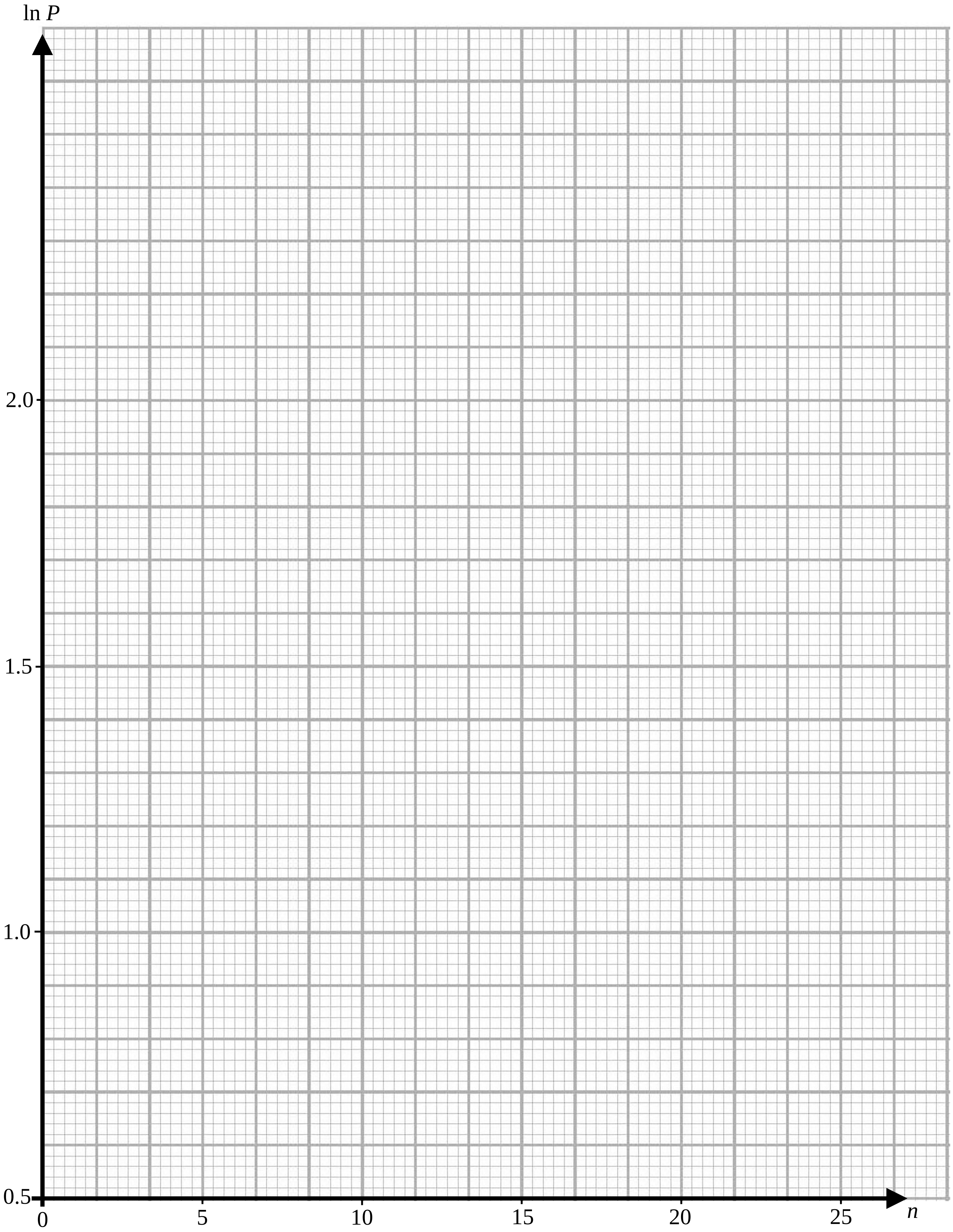
The table below shows the values of n and P.

n	5	10	15	20	25
P	6.23	4.85	3.78	2.94	2.29

(a) On the grid on the next page, plot $\ln P$ against n and draw a straight line graph. [2]

(b) Use your graph to estimate the initial population of the foxes and the value of k. [4]

(c) If this model for the population remains valid, after how many whole years would the population of the foxes first decreased to 1000? [3]

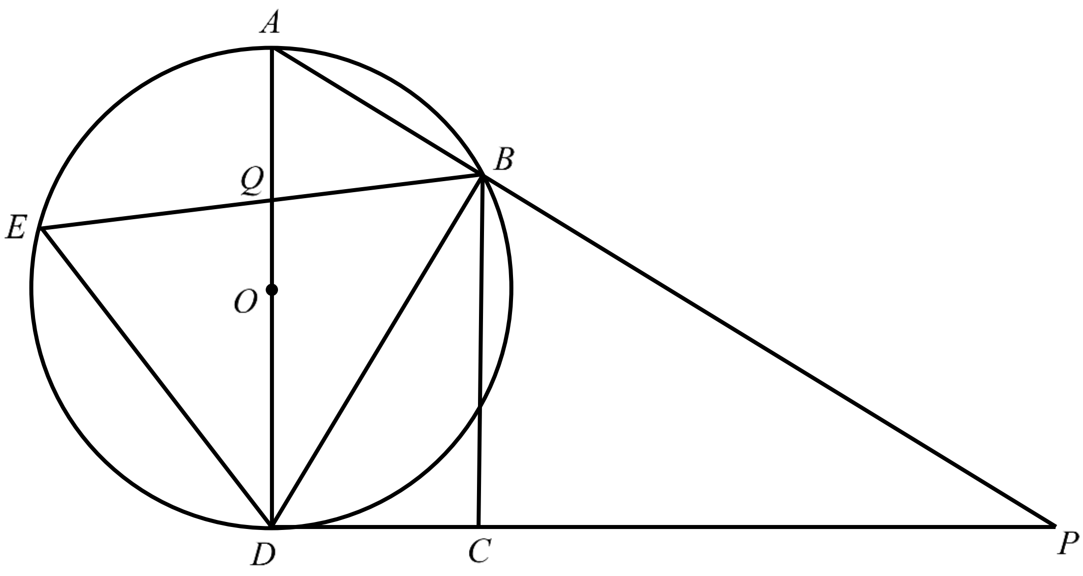

ln P
2.0
1.5
1.0
0.5
0
5
10
15
20
25
n

12 **(a)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\ln 2x}{3x^2}\right)=\dfrac{1}{3x^3}-\dfrac{2\ln 2x}{3x^3}$. [3]

(b) Integrate $\frac{\ln 2x}{x^3}$ with respect to *x*. [3]

(c) Given that the curve $y = \text{f}(x)$ passes through the point $\left(\frac{1}{2}, 1\right)$ and is such that $\text{f}'(x) = \frac{\ln 2x}{x^3}$, find $\text{f}(x)$. [3]

13



The diagram shows a circle, centre *O*. Points *A*, *B*, *D* and *E* lie on the circle.
The line *AD* is a diameter of the circle and *DCP* is the tangent of the circle at *D*.
The lines *EB* and *AD* intersect at point *Q*.
The chord *AB* is produced to meet the tangent *DCP* at *P*.
It is given that angle *CBP* = angle *DEB*.

(a) Show that angle *BCD* = $90°$. [4]

(b) Show that $DP^2 = BP^2 + DA \times BC$. [4]

End of Paper

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ADDITIONAL MATHEMATICS — 4049/01

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

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Without using a calculator, find the height of the larger cylinder, expressing your answer in the form $\left(a+b\sqrt{7}\right)$ cm, where a and b are integers. [4]

Height of larger cylinder = $\dfrac{2\sqrt{7}-1}{\left(2-\sqrt{7}\right)^2}\times\sqrt{7}$

$$= \frac{14-\sqrt{7}}{\left(2-\sqrt{7}\right)^2}$$

$$= \frac{14-\sqrt{7}}{4-4\sqrt{7}+7}$$

$$= \frac{14-\sqrt{7}}{11-4\sqrt{7}}\times\frac{11+4\sqrt{7}}{11+4\sqrt{7}}$$

$$= \frac{154+56\sqrt{7}-11\sqrt{7}-28}{9}$$

$$= \frac{126+45\sqrt{7}}{9}$$

$$= 14+5\sqrt{7} \text{ cm}$$

2 The profit, $$y$ of a company can be modelled by the equation $y = a(x-h)^2 + k$,
where x is the number of goods sold and a, h and k are constants.
The company obtained the maximum profit of $24 500 when 800 goods were sold.
The company incurred a loss of $7 500 when no goods were sold.

(a) State the value of h and of k. [2]

Since maximum profit of $24500 occurs when 800 goods were sold,

$$y = a(x-800)^2 + 24500$$

Therefore, $h = 800$

$k = 24500$

(b) Using the values of h and k found in part **(a)**, find the value of a. [2]

$$y = a(x-800)^2 + 24500$$

When $x = 0$, $y = -7500$

$$-7500 = a(-800)^2 + 24500$$

$$640\,000a = -32\,000$$

$$a = -\frac{1}{20} \quad \text{or} \quad -0.05$$

(c) Find the range of the number of goods the company needed to sell to earn a profit. [2]

$$y = -\frac{1}{20}(x-800)^2 + 24500$$

For the company to be profitable, $y > 0$

$$-\frac{1}{20}(x-800)^2 + 24500 > 0$$

$$(x-800)^2 - 490\,000 < 0$$

$$(x-100)(x-1500) < 0$$

$$100 < x < 1500$$

3 Solve the equation $2\log_3 p + \dfrac{1}{\log_{27} 3} - 1 = \log_3(8p+1)$. [5]

$$2\log_3 p + \frac{1}{\log_{27} 3} - 1 = \log_3(8p+1)$$

$$\log_3 p^2 + \log_3 27 - \log_3 3 = \log_3(8p+1)$$

$$\log_3 \frac{27p^2}{3} = \log_3(8p+1)$$

$$\log_3 9p^2 = \log_3(8p+1)$$

$$9p^2 = 8p+1$$

$$9p^2 - 8p - 1 = 0$$

$$(p-1)(9p+1) = 0$$

$p = 1$ or $p = -\dfrac{1}{9}$ (reject)

4 Solve the equation $5^{x+1} = 5^{2x-1} - 30$. [5]

$5^{x+1} = 5^{2x-1} - 30$

$5\left(5^{x}\right) = \frac{1}{5}\left(5^{2x}\right) - 30$

$\frac{1}{5}\left(5^{2x}\right) - 5\left(5^{x}\right) - 30 = 0$

$5^{2x} - 25\left(5^{x}\right) - 150 = 0$

Let 5^{x} be y

$y^{2} - 25y - 150 = 0$

$\left(y - 30\right)\left(y + 5\right) = 0$

$y = 30$ or $y = -5$

$5^{x} = 30$ or $5^{x} = -5$ (reject)

$x\lg 5 = \lg 30$

$x = \frac{\lg 30}{\lg 5} = 2.11$ (3 s.f)

5 Given that $y = x^{3} - px^{2} - 45x + 10$ is decreasing for $-3 < x < q$, find the value of the constants p and q. [5]

$$\frac{dy}{dx} = 3x^2 - 2px - 45$$

Since y is decreasing, $3x^2 - 2px - 45 < 0$

For y is decreasing for $-3 < x < q$,

$3(x+3)(x-q) < 0$

$3x^2 - 3qx + 9x - 9q < 0$

$3x^2 + (9-3q)x - 9q < 0$

Comparing coefficients,

$-9q = -45$

$q = 5$

$-2p = 9 - 3q$

$-2p = 9 - 15$

$p = 3$

6 The graph $y = a + b\cos(cx)$ is defined for $0 \le x \le 2\pi$, where a, b and c are constants.

The graph has a period of $\frac{4\pi}{3}$ and a minimum value of 1 when $x = \frac{4\pi}{3}$.

The graph also passes through the point $\left(\frac{\pi}{3},\ 3\right)$.

(a) Show that $a = 3$, $b = -2$ and $c = \frac{3}{2}$. [3]

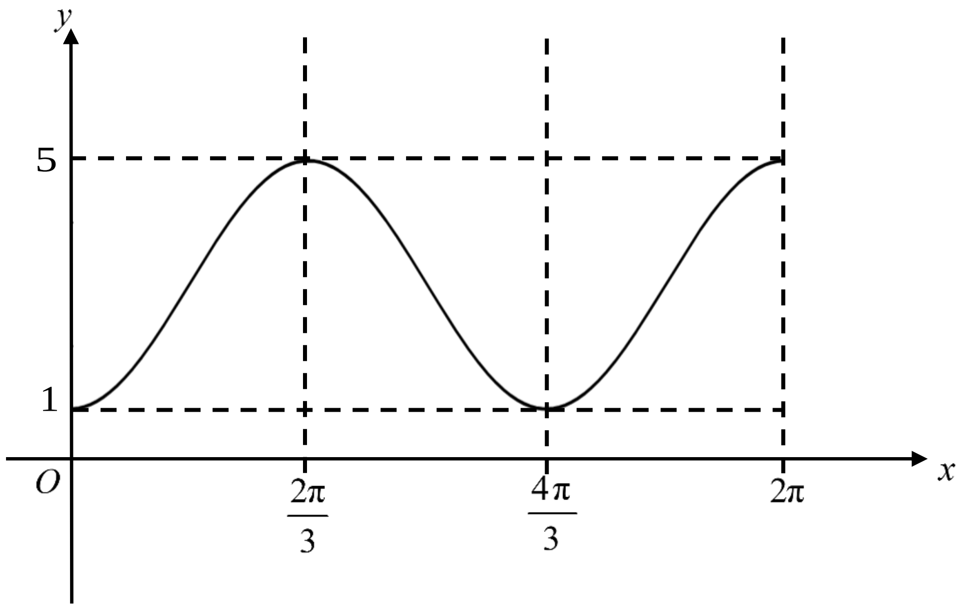
Period = $\frac{2\pi}{c} = \frac{4\pi}{3}$ ➔ $c = \frac{3}{2}$

When $x = \frac{\pi}{3}$, $y = 3$ ➔ $a = 3$

When $x = \frac{4\pi}{3}$, $y = 1$,

$1 = 3 + b$ ➔ $b = -2$

(b) Sketch the graph of $y = a + b\cos(cx)$ for $0 \le x \le 2\pi$. [2]

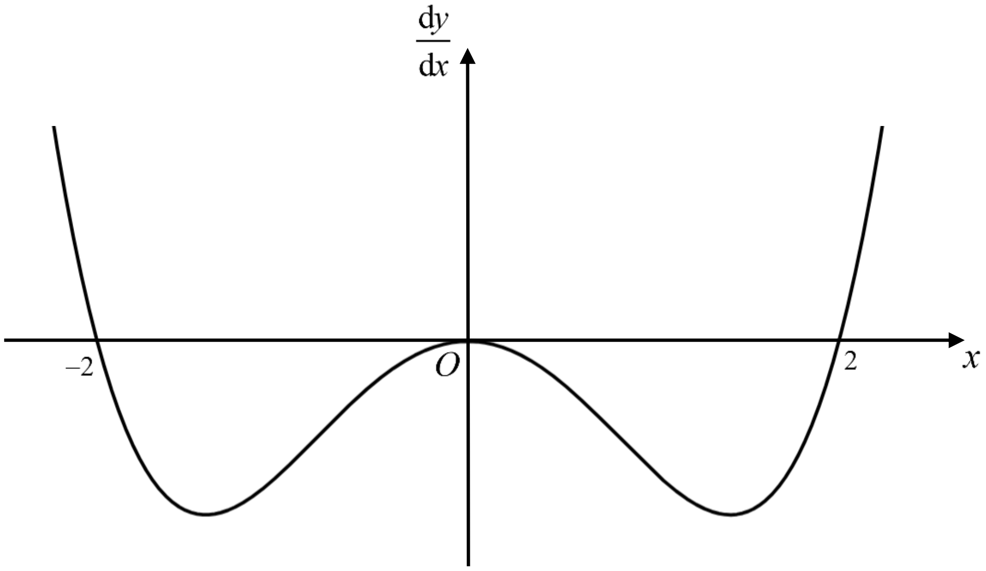


(c) Given that there are exactly 3 solutions for the equation $b\cos(cx) = k - a$, for $0 \le x \le 2\pi$, state the range of values of the constant *k*. [1]

$-2\cos\left(\frac{3}{2}x\right) + 3 = k$

For 3 points of intersections, $1 < k < 5$

7



The graph of $\frac{\mathrm{d}y}{\mathrm{d}x}$ of a function $y = \mathrm{f}(x)$ is shown in the diagram, passing through the x-axis at $(-2, 0)$, $(0, 0)$ and $(2, 0)$.

(a) State the number of stationary points of the graph of $y = \mathrm{f}(x)$. [1]

3

(b) State the x-coordinate of the **minimum** point of $y = \mathrm{f}(x)$ and explain why it is the minimum point.

[2]

$x = 2$

The gradient is <u>negative before</u> the stationary point and is <u>positive after</u> the stationary point. Hence, the point is a minimum point.

8 The curve $y = ax + \frac{b}{(2x-1)^3}$, where a and b are constants, has a stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$.

(a) Find the value of a and of b. [4]

When $x = \frac{3}{2}, y = \frac{11}{2}$

$\frac{11}{2} = \frac{3a}{2} + \frac{b}{8}$

$44 = 12a + b$

$b = 44 - 12b$ ------ (1)

$\frac{dy}{dx} = a - 3b(2x-1)^{-4}(2)$

$= a - \frac{6b}{(2x-1)^4}$

When $x = \frac{3}{2}, \frac{dy}{dx} = 0$

$a - \frac{6b}{16} = 0$

$a = \frac{3b}{8}$ ------(2)

Solving (1) and (2),

$a = 3, b = 8$

(b) Find the *x*-coordinate of the other stationary point. [3]

$$\frac{dy}{dx} = 3 - \frac{48}{(2x-1)^4}$$

To find stationary points, $\frac{dy}{dx} = 0$

$$3 - \frac{48}{(2x-1)^4} = 0$$

$$(2x-1)^4 = 16$$

$$2x - 1 = 2 \quad \text{or} \quad -2$$

$$x = \frac{3}{2} \quad \text{or} \quad -\frac{1}{2}$$

The *x*-coordinate of the other stationary point is $-\frac{1}{2}$.

(c) Find $\frac{d^2y}{dx^2}$. [1]

$$\frac{d^2y}{dx^2} = 192(2x-1)^{-5}(2)$$

$$= \frac{384}{(2x-1)^5}$$

(d) Hence, determine the nature of the stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$. [2]

When $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = \frac{384}{2^5}$

$= 12 \ (> 0)$

Therefore, the stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$ is a minimum point.

9 It is given that $y = \ln\left(\frac{2x}{5-x}\right)$, where $0 < x < 5$.

(a) If y is decreasing at a rate of $10\,\text{units/s}$, find the rate of change of x when $y = \ln 8$. [5]

$$\ln 8 = \ln\left(\frac{2x}{5-x}\right)$$

$$8 = \frac{2x}{5-x}$$

$$x = 4$$

$$y = \ln\left(\frac{2x}{5-x}\right)$$

$$= \ln 2x - \ln(5-x)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{5-x}$$

When $x = 4$, $\frac{dy}{dx} = \frac{5}{4}$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$-10 = \frac{5}{4} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -8$$

Rate of change of x is $-8\,\text{units/s}$

(b) The equation of the normal to the curve at point A is $8y = 8\ln 2 + 25 - 10x$.
Find the x-coordinate of point A. [4]

$$8y = 8\ln 2 + 25 - 10x$$

$$y = -\frac{5}{4}x + \frac{25}{8} + \ln 2$$

Gradient of normal at point A = $-\frac{5}{4}$

Gradient of tangent at point A = $\frac{4}{5}$

$$\frac{1}{x} + \frac{1}{5-x} = \frac{4}{5}$$

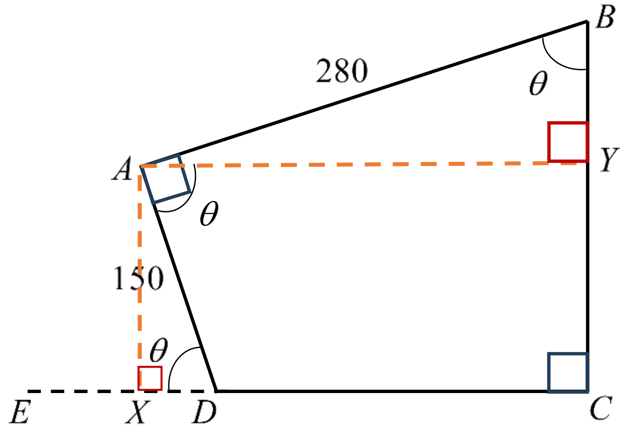
$$\frac{5-x+x}{x(5-x)} = \frac{4}{5}$$

$$4x^2 - 20x + 25 = 0$$

$$(2x-5)^2 = 0$$

$$x = \frac{5}{2}$$

10



The diagram shows a running track $ABCD$. D is a point on the straight line EC. It is given that $AB = 280$ m, $AD = 150$ m, angle BCD = angle $DAB = 90°$ and angle $ADE = \theta$, where $0° < \theta < 90°$.

(a) Show that L m, the length of the running track, is given by $L = p + 130\cos\theta + 430\sin\theta$, where p is a constant. [3]

$\cos\theta = \frac{BY}{280}$ $\qquad$ $\cos\theta = \frac{XD}{150}$

$BY = 280\cos\theta$ $\qquad$ $XD = 150\cos\theta$

$\sin\theta = \frac{AY}{280}$ $\qquad$ $\sin\theta = \frac{AX}{150}$

$AY = 280\sin\theta$ $\qquad$ $AX = 150\sin\theta$

Therefore, $BC = BY + AX$
$= 280\cos\theta + 150\sin\theta$

$CD = AY - XD$
$= 280\sin\theta - 150\cos\theta$

$L = AB + BC + CD + DA$

$= 280 + 280\cos\theta + 150\sin\theta + 280\sin\theta - 150\cos\theta + 150$

$= 430 + 130\cos\theta + 430\sin\theta$

(b) Express L in the form $p+R\cos(\theta-\alpha)$, where $R>0$ and α is an acute angle. [3]

$$430+130\cos\theta+430\sin\theta=430+R\cos(\theta-\alpha)$$

$$R=\sqrt{130^2+430^2}=10\sqrt{2018} \text{ or } 449.22$$

$$\tan\ \alpha=\frac{430}{130}$$

$$\alpha=73.179^\circ$$

$$430+130\cos\theta+430\sin\theta=430+449\cos(\theta-73.2^\circ)$$

(c) Xin Ting ran five complete laps around the track.
She claims that she ran a total of 5 km. Explain why she might be wrong. [2]

Maximum distance of one lap = 430 + 449 = 879 m

Maximum total distance ran = $5\times879=4395\,\text{m}$ (<5000)

Hence, it is not possible to run 5 km.

(d) Given that the running track is 700 m long, find the value of θ. [3]

$$430+449.22\cos(\theta-73.179^\circ)=700$$

$$\cos(\theta-73.179^\circ)=\frac{270}{449.22}$$

$$\theta-73.179^\circ=53.055^\circ \quad \text{or} \quad 306.945^\circ$$

$$\theta=126.234^\circ \quad \text{or} \quad \theta=380.124^\circ$$

Since θ is an acute angle, $\theta=380.124^\circ-360^\circ=20.1^\circ$

11 After n years, the population, P, **in thousands**, of a certain species of foxes can be modelled by an equation $P = Ae^{kn}$, where A and k are constants.

The table below shows the values of n and P.

n	5	10	15	20	25
P	6.23	4.85	3.78	2.94	2.29

(a) On the grid on the next page, plot $\ln P$ against n and draw a straight line graph. [2]

n	5	10	15	20	25
$\ln P$	1.83	1.58	1.33	1.08	0.83

(b) Use your graph to estimate the initial population of the foxes and the value of k. [4]

$$\ln P = \ln\left(Ae^{kn}\right)$$
$$\ln P = \ln A + \ln e^{kn}$$

$$\ln P = \ln A + kn$$

From the graph, vertical intercept = 2.08

$$\ln A = 2.08$$

$A = 8.00$ (3 sf) ➔ Initial population is 8000

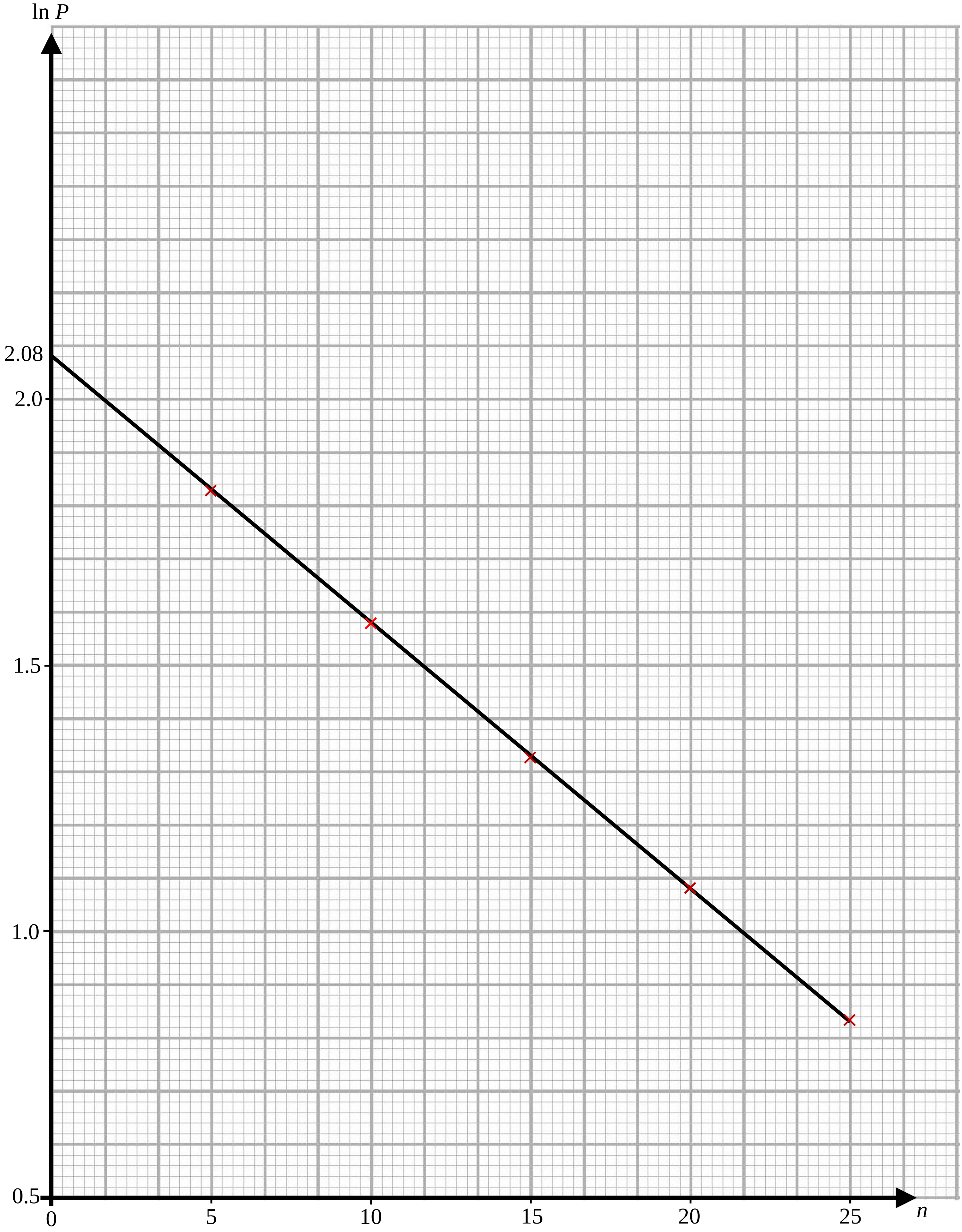
$$\text{Gradient} = \frac{2.08 - 1.08}{0 - 20}$$
$$= -0.05 \quad ➔ \quad k = -0.05$$

(c) If this model for the population remains valid, after how many whole years would the population of the foxes first decreased to 1000? [3]

$$P = 8e^{-0.05n}$$

$$8e^{-0.05n} = 1$$
$$e^{-0.05n} = \frac{1}{8}$$
$$-0.05n = \ln\left(\frac{1}{8}\right)$$
$$n = 41.6$$

After <u>**42**</u> years, the population would have first decreased to 1000.


ln P
2.08
2.0
1.5
1.0
0.5
0
5
10
15
20
25
n

12 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{3x^2}\right)=\frac{1}{3x^3}-\frac{2\ln 2x}{3x^3}$. [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{3x^2}\right)$$

$$=\frac{\left(3x^2\right)\left(\frac{2}{2x}\right)-(6x)(\ln 2x)}{9x^4}$$

$$=\frac{3x-(6x)(\ln 2x)}{9x^4}$$

$$=\frac{3x(1-2\ln 2x)}{9x^4}$$

$$=\frac{1-2\ln 2x}{3x^3}$$

$$=\frac{1}{3x^3}-\frac{2\ln 2x}{3x^3}$$

(b) Integrate $\dfrac{\ln 2x}{x^3}$ with respect to x. [3]

$$\int \frac{1}{3x^3} - \frac{2\ln 2x}{3x^3}\ \mathrm{d}x = \frac{\ln 2x}{3x^2} + c,$$ where c is an arbitrary constant.

$$\frac{2}{3}\int \frac{\ln 2x}{x^3}\ \mathrm{d}x = \int \frac{1}{3x^3}\ \mathrm{d}x - \frac{\ln 2x}{3x^2} + c_1$$

$$\frac{2}{3}\int \frac{\ln 2x}{x^3}\ \mathrm{d}x = -\frac{1}{6x^2} - \frac{\ln 2x}{3x^2} + c_1$$

$$\int \frac{\ln 2x}{x^3}\ \mathrm{d}x = -\frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + c_2$$

(c) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{1}{2}, 1\right)$ and is such that $\mathrm{f}'(x) = \dfrac{\ln 2x}{x^3}$, find $\mathrm{f}(x)$. [3]

$$\mathrm{f}(x) = \int \frac{\ln 2x}{x^3}\ \mathrm{d}x$$

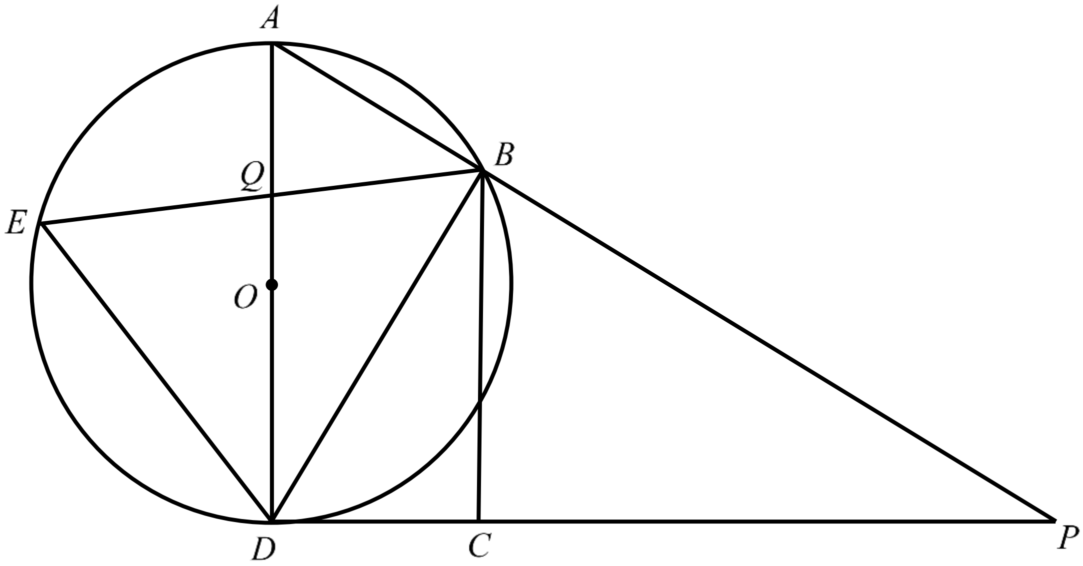
$$= -\frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + c$$

$$1 = -\frac{1}{4\left(\frac{1}{2}\right)^2} - 0 + c$$

$$c = 2$$

Equation of curve: $y = 2 - \dfrac{1}{4x^2} - \dfrac{\ln 2x}{2x^2}$

13



The diagram shows a circle, centre *O*. Points *A*, *B*, *D* and *E* lie on the circle.
The line *AD* is a diameter of the circle and *DCP* is the tangent of the circle at *D*.
The lines *EB* and *AD* intersect at point *Q*.
The chord *AB* is produced to meet the tangent *DCP* at *P*.
It is given that angle *CBP* = angle *DEB*.

(a) Show that angle *BCD* = $90°$. [4]

$\angle ABD = 90°$ (right $\angle$ in semicircle)

Let $\angle CDB = x$

$\angle DEB = \angle CDB = x$ (tangent chord theorem)

Since $\angle CBP = \angle DEB$

$\angle CBP = x$

$\angle DBC = 180° - 90° - x$ (adj $\angle$s on str line)
$= 90° - x$

$\angle BCD = 180° - x - (90° - x)$ ($\angle$ sum of Δ)
$= 90°$

(b) Show that $DP^2 = BP^2 + DA \times BC$. [4]

$\angle ABD = \angle DCB = 90°$

$\angle CDB = \angle DAB$ (tangent chord theorem)

Therefore, Triangle ABD is similar to triangle DCB
(all corresponding angles are equal)

$$\frac{BD}{CB} = \frac{AD}{BD}$$

$BD^2 = DA \times BC$

Since $BD^2 = DP^2 - BP^2$ (Pythagoras' Theorem),

$DP^2 - BP^2 = DA \times BC$

$DP^2 = BP^2 + DA \times BC$

End of Paper

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CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	

CLASS	4		INDEX NUMBER	

CENTRE/ INDEX NO					/				

ADDITIONAL MATHEMATICS

4049/02

Paper 2

26 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(a)** The curve $y = mx^2 - 12x + 2(x^2 + m - 1)$ lies entirely below the x-axis for all real values of x. Find the largest integer value of m. [5]

(b) Show that the roots of the equation $2x^2 - 3(1 - x) = -p$ are real if $p \leq 4\frac{1}{8}$. [4]

2 Let $\mathrm{f}(x) = \dfrac{3-3x^2}{(2x+1)(x+2)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence find the value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. [5]

3 **(a)** **(i)** Prove $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$. [3]

(ii) Hence, solve $\operatorname{cosec} 4\theta - \cot 4\theta = -\sqrt{3}$ for $0 < \theta < \pi$. [2]

(b) The angles A and B are such that

$$\sin(A + 45°) = \left(2\sqrt{2}\right)\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [5]

4 The fifth term in the expansion of $\left(px - \frac{q}{x}\right)^n$, where p and q are positive numbers, is independent of x.

(a) Show that $n = 8$. [2]

(b) Hence, explain why the fifth term is a positive constant. [1]

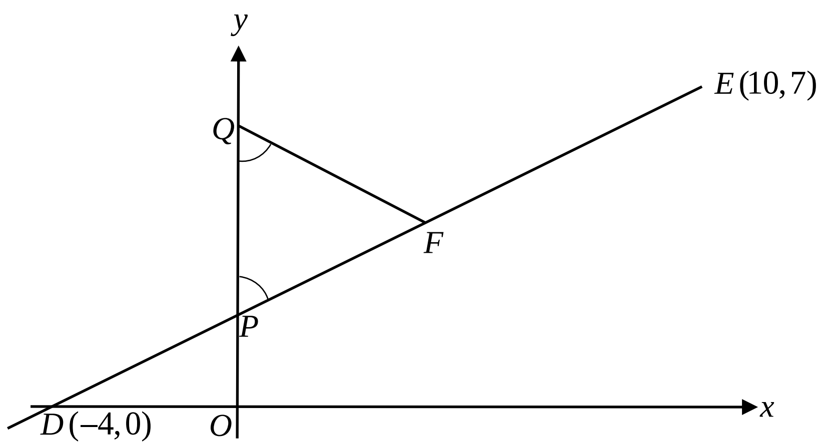
It is given that $p = 3$ and $q = 1$.

(c) Find the term independent of x in $\left(2 + \frac{1}{x^2}\right)\left(px - \frac{q}{x}\right)^n$ for $n = 8$. [4]

5 The diagram shows a line DE which cuts the y-axis at P and a line through F meets the y-axis at Q such that angle FPQ = angle FQP.

The coordinates of D and E are $(-4, 0)$ and $(10, 7)$ respectively.

F is a point on the line DE such that $DF : FE = 4 : 3$.

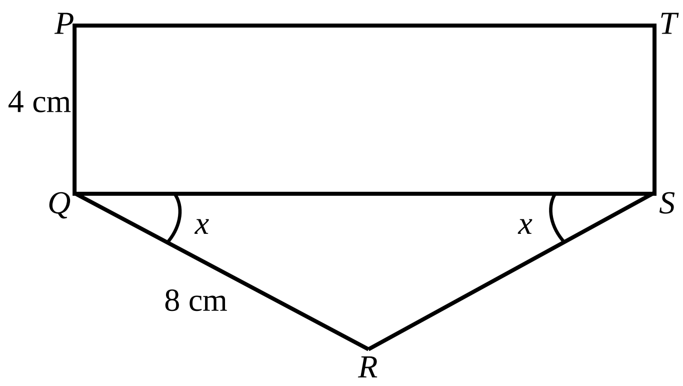


(a) Find the coordinates of F. [3]

(b) Find the equation of the straight line *FQ*. [2]

(c) Find the area of the triangle *QFE*. [3]

6



The diagram shows a figure *PQRST* which consists of a rectangle *PQST* and an isosceles triangle *QRS*.
It is given that $PQ = 4$ cm and $QR = 8$ cm.

(a) Give angle $SQR =$ angle $QSR = x$ radians and the area of *PQRST* is given by A cm^2, show that $A = 64\cos x + 32\sin 2x$. [4]

(b) Find the value of x for which A has a stationary value. [3]

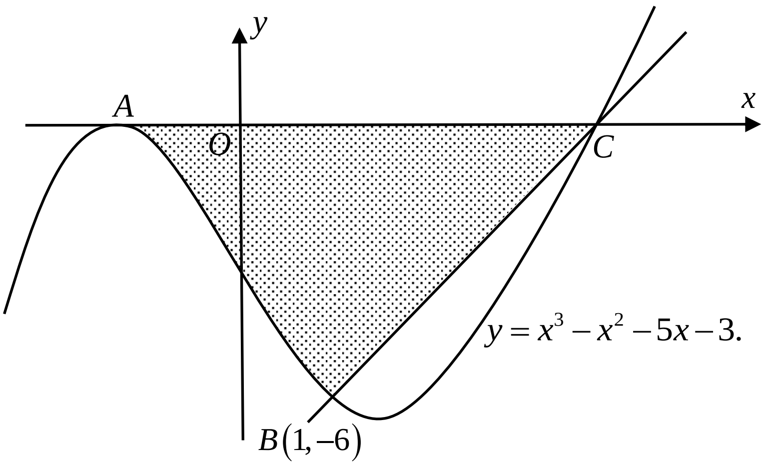
(c) Hence find the exact stationary value of A and determine whether it is a maximum or a minimum. [2]

7 It is given that $f(x)=x^3+ax^2-5x+b$, where a and b are constants, has a factor of $x+1$ and leaves a remainder of -24 when divided by $(x+3)$.

(a) Show that $a=-1$ and $b=-3$. [4]

(b) Hence factorise f (x) completely and write down the solutions for $\mathrm{f}(x)=0$. [3]

From part **(a)**, it is known that $f(x) = x^3 - x^2 - 5x - 3$.
The diagram below shows part of the curve $y = f(x)$ and a line that cuts the curve at *B* and *C*. *B* has coordinates $(1, -6)$.
A and *C* are points on the curve and the *x*-axis.



(c) Find the area of the shaded region. [5]

(d) Show that the equation of the tangent at the minimum point of the curve is $y = -\frac{256}{27}$. [3]

8 A circle has a diameter *AB*. The point *A* has coordinates (1, − 6) and the equation of the tangent to the circle at *B* is $3x + 4y = k$.

(a) Show that the equation of the normal to the circle at the point *A* is $4x - 3y = 22$. [3]

It is also given also that the line $x = -1$ touches the circle at the point (–1, –2).

(b) Find the coordinates of the centre and the radius of the circle. [4]

Continuation of working space for question **8 (b).**

(c) Find the value of *k*. [3]

9 Mr Chan was driving a car along a straight road. He was 35 m away from the stop line when he applied his brakes near a traffic light. His acceleration, a m/s^2, after applying the brakes was given by $a = -7.5e^{-\frac{t}{2}}$, where t is the time in seconds after he applied the brakes.

(a) Explain mathematically why $a < 0$ for all $t \geq 0$ and the significance of $a < 0$. [2]

(b) Mr Chan's car was travelling at 14 m/s just before he applied his brakes. Express the velocity of his car, v m/s, in terms of t. [3]

(c) Hence find the time taken for his car to come to a complete stop. [2]

(d) Obtain an expression, in terms of *t*, for the displacement of Mr Chan's car from the point he applied the brakes [3]

(e) Determine if Mr Chan's car was able to come to a complete stop before reaching the stop line. Explain your answer. [2]

End of Paper

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CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	**WORKED SOLUTIONS**
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ADDITIONAL MATHEMATICS **4049/02**

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26 August 2024

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$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(a)** The curve $y = mx^2 - 12x + 2\ (x^2 + m - 1)$ lies entirely below the x-axis for all real values of x. Find the largest integer value of m. [5]

$y = mx^2 - 12x + 2\ (x^2 + m - 1)$
$= mx^2 - 12x + 2x^2 + 2m - 2$
$= (m+2)x^2 - 12x + 2m - 2$
Since the curve lies entirely below the x-axis,
① Discriminant, $b^2 - 4ac < 0$

$(-12)^2 - 4(m+2)(2m-2) < 0$
$144 - 4(2m^2 + 2m - 4) < 0$
$-8m^2 - 8m + 160 < 0$
$m^2 + m - 20 > 0$
$(m+5)(m-4) > 0$
$m < -5$ or $m > 4$

② Coefficient of x^2, $m + 2 < 0 \Rightarrow m < -2$
Combining the inequalities, $m < -5$
Largest integer value of $m = -6$

(b) Show that the roots of the equation $2x^2 - 3(1-x) = -p$ are real if $p \le 4\frac{1}{8}$. [4]

$2x^2 - 3(1-x) = -p \Rightarrow 2x^2 + 3x + p - 3 = 0$
$b^2 - 4ac = 9 - 8(p-3) = 33 - 8p$

Since $p \le 4\frac{1}{8}$, $p \le \frac{33}{8}$, therefore $33 - 8p \ge 0$
$b^2 - 4ac = 9 - 8(p-3) = 33 - 8p \ge 0$
Hence the roots are real.

2 Let $\mathrm{f}(x)=\dfrac{3-3x^2}{(2x+1)(x+2)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

$$\frac{3-3x^2}{(2x+1)(x+2)^2}=\frac{A}{2x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$$

$3-3x^2=A(x+2)^2+B(2x+1)(x+2)+C(2x+1)$

Let $x=-2$, $-9=-3C\Rightarrow C=3$

Let $x=-\dfrac{1}{2}$, $\dfrac{9}{4}=\dfrac{9}{4}A\Rightarrow A=1$

Comparing coefficients of x^2, $-3=A+2B\Rightarrow B=-2$

$$\frac{3-3x^2}{(2x+1)(x+2)^2}=\frac{1}{2x+1}-\frac{2}{x+2}+\frac{3}{(x+2)^2}$$

(b) Hence find the value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $a+b\ln c$, where a, b and c are integers. [5]

$$\int_0^4 \mathrm{f}(x)\,\mathrm{d}x = \int_0^4 \left(\frac{1}{2x+1} - \frac{2}{x+2} + \frac{3}{(x+2)^2}\right)\mathrm{d}x$$

$$= \left[\frac{\ln(2x+1)}{2} - 2\ln(x+2) - \frac{3}{(x+2)}\right]_0^4$$

$$= \left[\frac{\ln 9}{2} - 2\ln(6) - \frac{3}{(6)}\right] - \left[\frac{\ln 1}{2} - 2\ln 2 - \frac{3}{2}\right]$$

$$= \frac{2\ln 3}{2} - 2\ln 6 + 2\ln 2 + 1$$

$$= 1 + \ln\frac{3(4)}{36}$$

$$= 1 + \ln\frac{1}{3}$$

$$= 1 + \ln 1 - \ln 3 = 1 - \ln 3$$

3 **(a)** **(i)** Prove $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$. [3]

$$LHS = \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$$
$$= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

(ii) Hence, solve $\operatorname{cosec} 4\theta - \cot 4\theta = -\sqrt{3}$ for $0 < \theta < \pi$. [2]

Hence $\tan 2\theta = -\sqrt{3}$ [Making use of the identity]

Basic angle = $\frac{\pi}{3} = 1.0472$

2θ lies in the 2nd or 4th quadrant and $0 < 2\theta < 2\pi$.

$2\theta = \pi - \frac{\pi}{3}$ or $2\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3} = 1.05$ or $\theta = \frac{5\pi}{6} = 2.62$

(b) The angles A and B are such that

$$\sin(A+45°)=\left(2\sqrt{2}\right)\cos A \quad \text{and} \quad 4\sec^2 B+5=12\tan B.$$

Without using a calculator, find the exact value of $\tan(A-B)$. [5]

$\sin A\cos 45°+\cos A\sin 45°=(2\sqrt{2})\cos A$

$\frac{\sqrt{2}}{2}\sin A+\frac{\sqrt{2}}{2}\cos A=(2\sqrt{2})\cos A$ [Use of addition formula]

$\frac{\sqrt{2}}{2}\sin A+=\frac{3\sqrt{2}}{2}\cos A$

$\tan A=\frac{3\sqrt{2}}{2}\times\frac{2}{\sqrt{2}}=3$

$4(1+\tan^2 B)+5=12\tan B$

$4\tan^2 B-12\tan B+9=0$ [Use of $\sec^2 B=1+\tan^2 B$ and form a QE.]

$\left(2\tan B-3\right)^2=0$

$\tan B=\frac{3}{2}$

$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$=\frac{3-\frac{3}{2}}{1+3\left(\frac{3}{2}\right)}=\frac{3}{11}$

4 The fifth term in the expansion of $\left(px - \frac{q}{x}\right)^n$, where p and q are positive numbers, is independent of x.

(a) Show that $n = 8$. [2]

Since it is fifth term, $T_5 = T_{4+1} = \binom{n}{4}(px)^{n-4}\left(-\frac{q}{x}\right)^4$ [for $r = 4$]

and it is independent, powers of $x = n - 4 - 4 = 0$
Therefore $n = 8$.

(b) Hence, explain why the fifth term is a positive constant. [1]

Since p and q are positive, $(p)^4 > 0$ and $(-q)^4 > 0$, the fifth term is a positive constant.

It is given that $p = 3$ and $q = 1$.

(c) Find the term independent of x in $\left(2+\frac{1}{x^2}\right)\left(px-\frac{q}{x}\right)^n$ for n = 8. [4]

$$\left(2+\frac{1}{x^2}\right)\left(3x-\frac{1}{x}\right)^8 = \left(2+\frac{1}{x^2}\right)\left(\ldots + \text{Independent term + Term in } x^2 + \ldots\right)$$

General term of $\left(3x-\frac{1}{x}\right)^8 = \binom{8}{r}(3x)^{8-r}\left(-\frac{1}{x}\right)^r$

For independent term, $8-2r=0$

$r=4$

Independent term = $\binom{8}{4}(3)^4(-1)^4 = 5670$

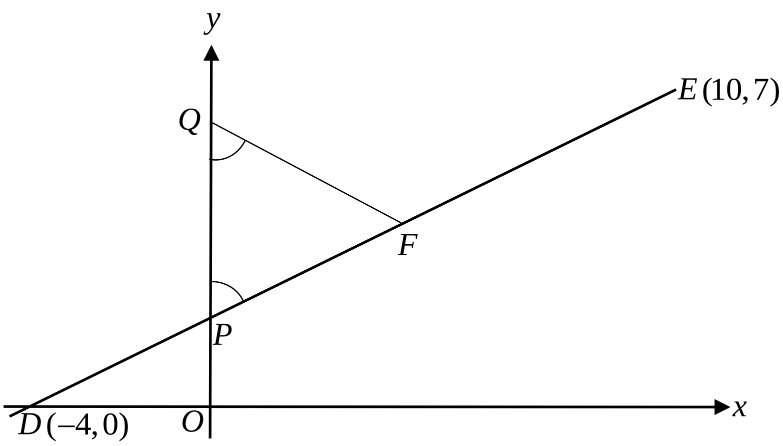
For term in x^2, $8-2r=2$

$r=3$

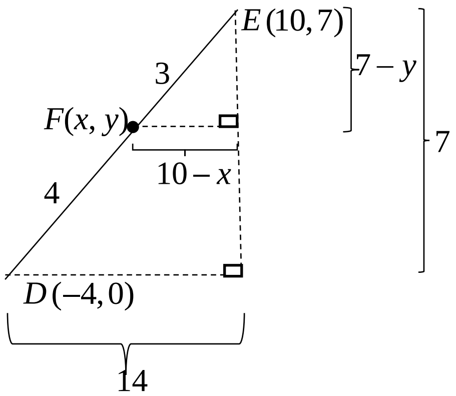
Term in $x^2 = \binom{8}{3}(3)^5(-1)^3 = -13608x^2$

Independent term in $\left(2+\frac{1}{x^2}\right)\left(3x-\frac{1}{x}\right)^8 = 2(5670)-13608 = -2268$

5 The diagram shows a line DE which cuts the y-axis at P and a line through F meets the y-axis at Q such that angle FPQ = angle FQP.
The coordinates of D and E are $(-4, 0)$ and $(10, 7)$ respectively.
F is a point on the line DE such that $DF : FE = 4 : 3$.



(a) Find the coordinates of F. [3]



$$\frac{7-y}{7} = \frac{3}{7} \Rightarrow y = 4$$ M1

$$\frac{10-x}{14} = \frac{3}{7} \Rightarrow x = 4$$ M1

Coordinates of F = (4, 4)

(b) Find the equation of the straight line *FQ*. [2]

Gradient of *FQ* = $-\frac{1}{2}$

Equation of *FQ* : $y-4=-\frac{1}{2}(x-4)$

$$y=-\frac{1}{2}x+6$$

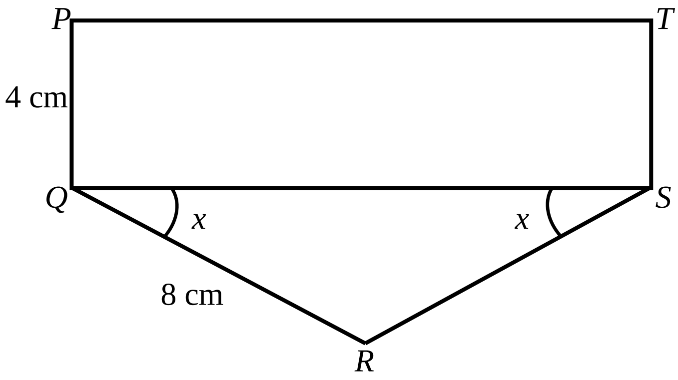
(c) Find the area of the triangle *QFE*. [3]

When $x = 0$, $y = 6$, Coordinates of *Q* = $(0,6)$

Area of triangle *QFE* = $\frac{1}{2}\begin{vmatrix} 0 & 4 & 10 & 0 \\ 6 & 4 & 7 & 6 \end{vmatrix}$

= 12 sq units

6



The diagram shows a figure *PQRST* which consists of a rectangle *PQST* and an isosceles triangle *QRS*.
It is given that *PQ* = 4 cm and *QR* = 8 cm.

(a) Given angle $SQR =$ angle $QSR = x$ radians and the area of *PQRST* is given by A cm^2, show that $A = 64\cos x + 32\sin 2x$. [4]

$$\cos x = \frac{\frac{1}{2}QS}{8}$$

$$\Rightarrow QS = 16\cos x$$

$$\text{Area of } \Delta QRS = \frac{1}{2}\times 8\times(16\cos x)\sin x$$

$$= 64\sin x\cos x \text{ cm}^2$$

$$\text{Area of } \square PQST = 4\times 16\cos x$$

$$= 64\cos x \text{ cm}^2$$

$$\therefore A = \text{area of } \Delta QRS + \text{area of } \square PQST$$

$$= 64\cos x + 64\sin x\cos x$$

$$= 64\cos x + 32\sin 2x \quad \text{(shown)}$$

(b) Find the value of x for which A has a stationary value. [3]

$A = 64\cos x + 32\sin 2x$

$\frac{dA}{dx} = -64\sin x + 64\cos 2x$

For stationary value, $\frac{dA}{dx} = 0$

$-64\sin x + 64\cos 2x = 0$

$\cos 2x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$ (rejected)

$x = \frac{\pi}{6} = 0.524$

(c) Hence find the exact stationary value of A and determine whether it is a maximum or a minimum. [2]

$A = 64\cos\frac{\pi}{6} + 32\sin 2\left(\frac{\pi}{6}\right)$

$= 64\left(\frac{\sqrt{3}}{2}\right) + 32\left(\frac{\sqrt{3}}{2}\right)$

$= 32\sqrt{3} + 16\sqrt{3}$

$= 48\sqrt{3}$ cm^2

$\frac{d^2A}{dx^2} = -64\cos x - 128\sin 2x$

when $x = \frac{\pi}{6}$, $\frac{d^2A}{dx^2} < 0 \Rightarrow A$ has a maximum value.

7 It is given that $f(x) = x^3 + ax^2 - 5x + b$, where a and b are constants, has a factor of $x+1$ and leaves a remainder of -24 when divided by $(x+3)$.

(a) Show that $a = -1$ and $b = -3$. [4]

Since x + 1 is a factor, $f(-1) = 0$

$$f(-1) = (-1)^3 + a(-1)^2 - 5(-1) + b = 0$$
$$a + b = -4$$

Since R = -24 when divided by $(x+3)$, $f(-3) = -24$

$$f(-3) = (-3)^3 + a(-3)^2 - 5(-3) + b = -24$$
$$9a + b = -12$$
$$a = -1 \text{ and } b = -3.$$

(b) Hence factorise f (x) completely and write down the solutions for $f(x)=0$. [3]

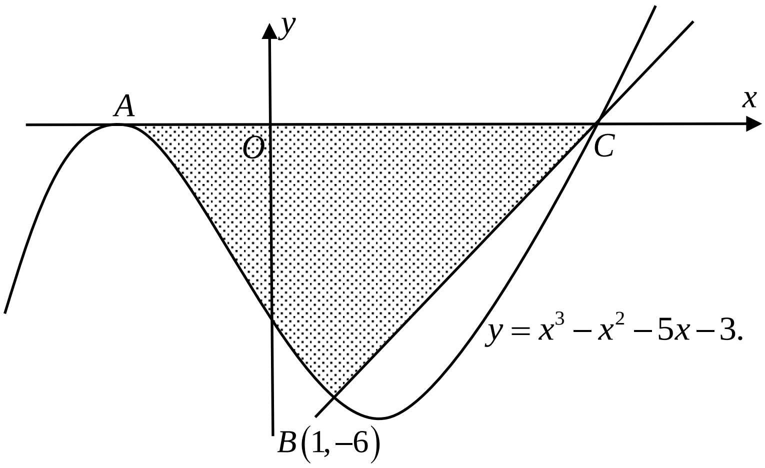
$$x+1 \,\overline{\big)\, x^3-x^2-5x-3.}\quad \text{quotient: } x^2-2x-3$$

$f(x)=(x+1)^2(x-3)$

$f(x)=(x+1)^2(x-3)=0$

$x=-1$ or $x=3$

From part **(a)**, it is known that $\mathrm{f}(x) = x^3 - x^2 - 5x - 3$.
The diagram below shows part of the curve $y = \mathrm{f}(x)$ and a line that cuts the curve at B and C. B has coordinates $(1, -6)$.
A and C are points on the curve and the x-axis.



(c) Find the area of the shaded region. [5]

$A = (-1, 0)$
$C = (3, 0)$
} Either one

Area of shaded region $= \left|\int_{-1}^{1} \left(x^3 - x^2 - 5x - 3\right) \mathrm{d}x\right| + \frac{1}{2} \times (3-1) \times 6$

[Area = Integration under curve + Area of triangle]

$$= \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} - 3x\right]_{-1}^{1}\right| + 6$$

$$= \left|\frac{(1)^4}{4} - \frac{(1)^3}{3} - \frac{5(1)^2}{2} - 3(1) - \left(\frac{(-1)^4}{4} - \frac{(-1)^3}{3} - \frac{5(-1)^2}{2} - 3(-1)\right)\right| + 6$$

$= \left|-\frac{2}{3} - 6\right| + 6 = 12\frac{2}{3}$ sq units [Final Area]

(d) Show that the equation of the tangent at the minimum point of the curve is $y = -\frac{256}{27}$. [3]

$f'(x) = 3x^2 - 2x - 5$

For minimum point, $3x^2 - 2x - 5 = 0$

$(3x-5)(x+1) = 0$

$x = \frac{5}{3}$ or $x = -1$

Equation of the tangent: $y = \left(\frac{5}{3}\right)^3 - \left(\frac{5}{3}\right)^2 - 5\left(\frac{5}{3}\right) - 3 = -\frac{256}{27}$

8 A circle has a diameter *AB*. The point *A* has coordinates (1, − 6) and the equation of the tangent to the circle at *B* is $3x+4y=k$.

(a) Show that the equation of the normal to the circle at the point *A* is $4x-3y=22$. [3]

Since the normal at *A* will pass through the centre of the circle and ultimately *B*, it will be perpendicular to the tangent at *B*.

$3x+4y=k \Rightarrow y=-\frac{3x}{4}+\frac{k}{4}$

Gradient of normal at *A* = $\frac{-1}{-3/4}=\frac{4}{3}$ [Attempt to find gradient of normal]

Equation of normal at *A*: $y-(-6)=\frac{4}{3}(x-1) \Rightarrow y+6=\frac{4}{3}x-\frac{4}{3}$

$4x-3y=22$

It is also given that the line $x=-1$ touches the circle at the point (–1, –2).

(b) Find the coordinates of the centre and the radius of the circle. [4]

Since the line *x* = –1 touches the circle at the point (–1, –2), so the equation of the normal at (–1, –2) is $y=-2$ and this passes through the centre of the circle. Solving the equations $4x-3y=22$ and *y* = –2 provides the *x* coordinate of centre of circle and the *y* coordinate of centre of circle is -2.

$4x-3(-2)=22 \Rightarrow 4x=16 \Rightarrow x=4$

Coordinates of the centre = $(4,-2)$

Radius of circle = $\sqrt{(4-1)^2+(-6-(-2))^2}$

= 5 units

Or Radius of circle (from (–1, –2) to $(4,-2)$ = 1+ 4= 5 units

Continuation of working space for question **8 (b).**

(c) Find the value of *k*. [3]

Let the coordinates of *B* be (a, b).

$$\left(\frac{a+1}{2}, \frac{b-6}{2}\right) = (4, -2)$$

$a = 2(4) - 1 = 7$ and $b = 2(-2) + 6 = 2$

Coordinates of $B = (7, 2)$,

Sub. (7, 2) into $3x + 4y = k$,

$k = 3(7) + 4(2) = 29$

9 Mr Chan was driving a car along a straight road. He was 35 m away from the stop line when he applied his brakes near a traffic light. His acceleration, a m/s^2, after applying the brakes was given by $a = -7.5e^{-\frac{t}{2}}$ where t is the time in seconds after he applied the brakes.

(a) Explain mathematically why $a < 0$ for all $t \geq 0$ and the significance of $a < 0$. [2]

As $e^{-\frac{t}{2}} > 0$ for all $t \geq 0$, then $a = -7.5e^{-\frac{t}{2}} < 0$. [Both concepts]
This means that the car is decelerating.

(b) Mr Chan's car was travelling at 14 m/s just before he applied his brakes. Express the velocity of his car, v m/s, in terms of t. [3]

$$v = \frac{-7.5e^{-\frac{t}{2}}}{-\frac{1}{2}} + c$$

$$v = 15e^{-\frac{t}{2}} + c$$

When $t = 0$, $v = 14$

$14 = 15(1) + c \Rightarrow c = -1$

$$v = 15e^{-\frac{t}{2}} - 1$$

(c) Hence find the time taken for his car to come to a complete stop. [2]

When it comes to a complete stop, $v = 0$.

$$15e^{-\frac{t}{2}} - 1 = 0$$
$$t = -2\ln\frac{1}{15}$$
$$t = -2\ln\frac{1}{15} = 5.4216 = 5.42$$

Time taken = 5.42 s

(d) Obtain an expression, in terms of t, for the displacement of Mr Chan's car from the point he applied the brakes. [3]

$$s = \int v \, dt = -30e^{-\frac{t}{2}} - t + c$$

When $t = 0$, $s = 0$, $0 = -30(1) - 0 + c \Rightarrow c = 30$

$$s = -30e^{-\frac{t}{2}} - t + 30$$

(e) Determine if Mr Chan's car was able to come to a complete stop before reaching the stop line. Explain your answer. [2]

When $t = 5.4161$, $s = -30e^{-\frac{5.4161}{2}} - 5.4161 + 30 = 22.6 < 35$

Yes, his car came to a complete stop before reaching the stop line.

End of Paper

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