NAME: ______________________ (   ) CLASS: ______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS

4049/01

Paper 1

Date: 21 August 2024

Duration: 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

Table of Penalties		Question Number		
Presentation	☐ 1 ☐ 2			
Rounding off	☐ 1		Parent's/Guardian's Signature	/ 90

This question paper consists of 22 printed pages

NAME: ____________________________ (   )   CLASS: ______

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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Answer **all** the questions.

1 **(i)** Express $4x^2 + 8x - 5$ in the form $p(x+q)^2 + r$, where p, q and r are constants to be found. [3]

(ii) Hence, state the coordinates of the turning point of the curve $y = 4x^2 + 8x - 5$. [1]

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2 Without using a calculator, find the values of a and b for which the solution of the equation $x\sqrt{24} = x\sqrt{3} + \sqrt{6}$ is $\dfrac{a+\sqrt{b}}{7}$. [5]

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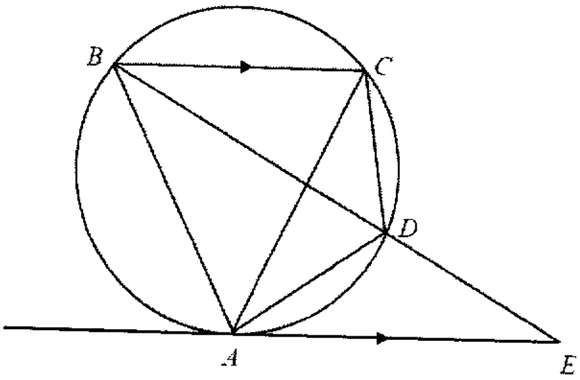
3 Points A, B, C and D lie on a circle. The tangent to the circle at A meets BD produced at E. AE is parallel to BC.



Prove that

(i) $AB = AC$, [3]

(ii) $\angle CDE = 2\angle ABC$. [3]

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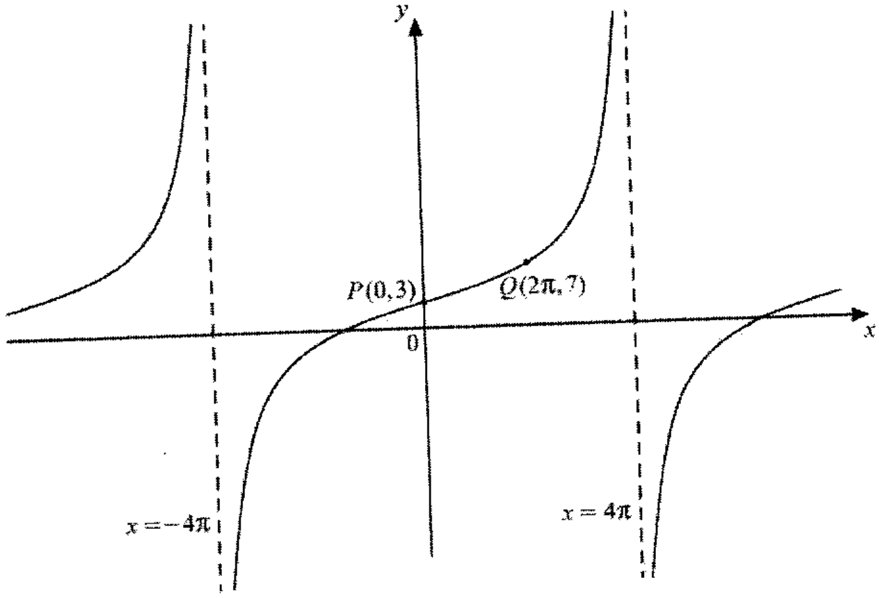
4 (a) The diagram shows part of the graph of $y = a\tan bx + c$. The graph has vertical asymptotes at $x = -4\pi$ and $x = 4\pi$ and passes through the points P and Q.



(i) Explain why $b = \dfrac{1}{8}$. [2]

(ii) Hence find the equation of the curve. [2]

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(b) The function $f(x)$ is defined by $f(x) = 4 + 3\sin 2x$ for $0° \le x \le 360°$.
Sketch the graph of $y = f(x)$ on the axes below. [2]

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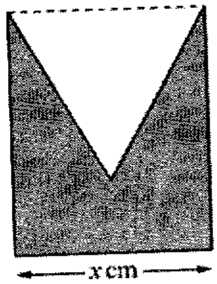
5 The diagram shows a shape made by cutting an equilateral triangle out of a rectangle of width x cm.



The perimeter of the shape is 20 cm.

(i) Show that the area, A cm^2, of the shape is given by $A = 10x - \left(\frac{6+\sqrt{3}}{4}\right)x^2$. [3]

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(ii) Given that x can vary, find the value of x which produces the maximum area and calculate this maximum area. Give your answers to 2 significant figures. [4]

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6 **(a)** Express $\frac{8x+13}{(1+2x)(2+x)^2}$ in partial fractions. [5]

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(b) Hence, evaluate $\int_1^2 \frac{8x+13}{(1+2x)(2+x)^2}\,\mathrm{d}x$. [3]

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7 The polynomial $\mathrm{f}(x)$ is such that $\mathrm{f}(x)=6x^3+ax^2-50x+b$, where a and b are integers. It is given that $\mathrm{f}(x)$ is divisible by $2x-3$ and that $\mathrm{f}'(1)=6$.

(a) Find the values of a and b. [5]

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(b) Using your values of a and b, solve the equation $f(x) = 0$. [3]

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8 **(a)** Show that the solution of the equation $2^{3x+4} \times 5^{2x-1} = 16^x \times 5^{3x}$ is $\lg\frac{16}{5}$. [3]

(b) Express $2\log_2 x - \log_2(x-4) = 3$ as a quadratic equation in x and explain why there are no real solutions. [5]

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9 (a) The variables x and y increase in such a way that when $x = 3$, the rate of increase of y with respect to time is three times the rate of increase of x with respect to time. Given that $y = k\sqrt{3x+7}$, where k is a constant, find the value of k. [4]

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(b) The mass, m grams, of a radioactive sample, present at time t days after being observed, is given by $m = 24e^{-0.02t}$.

Find

(i) the initial mass of the radioactive sample, [1]

(ii) the time taken for the sample to decrease to half its initial mass, [2]

(iii) the rate at which the mass is decreasing after 12 hours. [2]

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10 **(i)** Show that $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=\frac{2}{\sin x}$. [5]

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(ii) Hence solve the equation $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=1+3\sin x$ for $0^\circ \le x \le 360^\circ$. [4]

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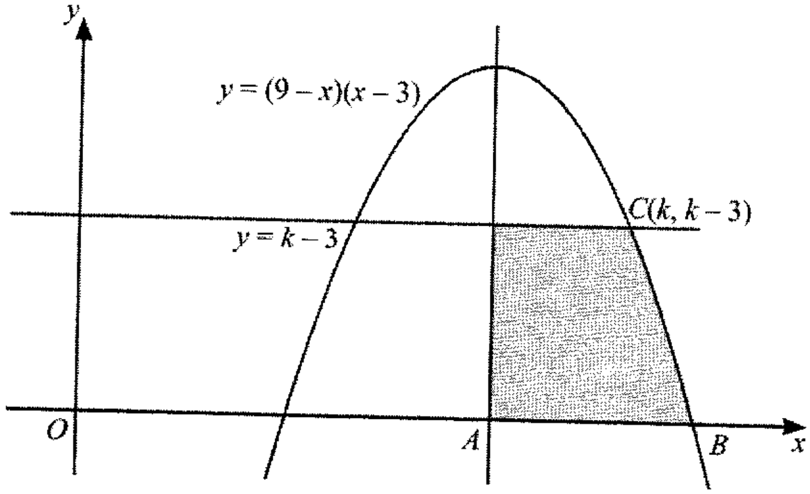
11 The diagram shows part of the curve $y=(9-x)(x-3)$ and the line $y=k-3$, where $k>3$. The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A. The curve meets the x-axis at B and the line $y=k-3$ meets the curve at the point $C(k,k-3)$.



(i) Show that the value of k is 8. [4]

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(ii) Find the area of the shaded region. [5]

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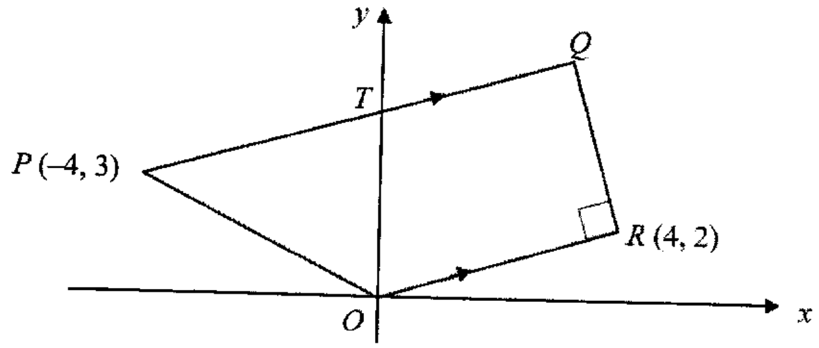
12 **Solutions to this question by accurate drawing will not be accepted.**

The diagram (not drawn to scale) shows a trapezium *OPQR* in which *PQ* is parallel to *OR* and $\angle ORQ = 90°$. The coordinates of *P* and *R* are (–4, 3) and (4, 2) respectively and *O* is the origin.



(i) Find the coordinates of *Q*. [5]

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(ii) PQ meets the y-axis at T. Show that triangle ORT is isosceles. [3]

The point S is such that $ORPS$ forms a parallelogram.

(iii) Find the coordinates of S. [1]

(iv) Find the area of the trapezium $OPQR$. [2]

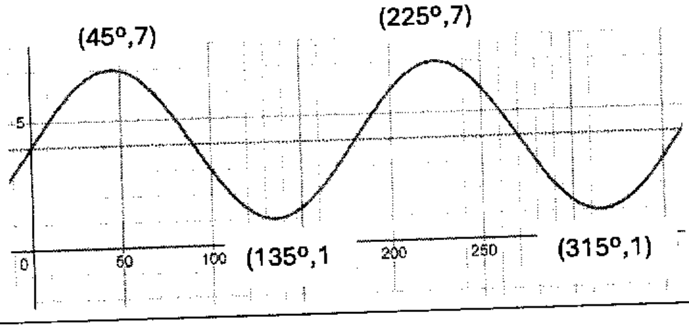
~ End of Paper ~

FMS(S) SEC 4Exp Additional Mathematics Paper 1 Preliminary Exam 2024 Marking Scheme

1i	$4x^2+8x-5=4(x^2+2x)-5$ $=4\left[(x+1)^2-1\right]-5$ $=4(x+1)^2-9$	M1 M1 complete square A1	AO1
ii	Turning point = (–1, –9)	B1	AO1

2	$x\sqrt{24}=x\sqrt{3}+\sqrt{6}$ $x\left(\sqrt{24}-\sqrt{3}\right)=\sqrt{6}$ $x=\dfrac{\sqrt{6}}{\left(\sqrt{24}-\sqrt{3}\right)}$ $x=\dfrac{\sqrt{6}}{\left(2\sqrt{6}-\sqrt{3}\right)}\times\dfrac{2\sqrt{6}+\sqrt{3}}{2\sqrt{6}+\sqrt{3}}$ $=\dfrac{2(6)+\sqrt{18}}{4(6)-3}$ $=\dfrac{12+3\sqrt{2}}{21}$ $=\dfrac{4+\sqrt{2}}{7}$ $a=4$ and $b=2$	M1 factorise x M1 Multiply by conjugate surd M1 simplification A1, A1	AO1

3i	$\angle CAE=\angle ABC$ (alt segment theorem/tangent chord thm) $\angle ACB=\angle CAE$ (alternate $\angle$s, $BC//AE$) $\angle ABC=\angle ACB$ (base $\angle$s of isos Δ) $\therefore AB=AC$ (*shown*)	B1 B1 AG1	AO3
3ii	$\angle BAC=180^\circ-2\angle ABC$ ($\angle$ sum of Δ) $\angle BDC=\angle BAC$ ($\angle$s in same segment) $\angle CDE=180^\circ-\angle BDC$ (adj $\angle$s on a straight line) $\angle CDE=180^\circ-(180^\circ-2\angle ABC)$ (adj $\angle$s on a straight line) $\angle CDE=2\angle ABC$ (shown)	M1 B1 AG1	AO3

4a i	Since the period = 8π $8\pi = \frac{\pi}{b}$ $b = \frac{1}{8}$	B1 AG1	AO3
ii	$c = 3$ $7 = a\tan\left(\frac{\pi}{4}\right) + 3$ $a = 4$ $y = 4\tan\left(\frac{x}{8}\right) + 3$	M1 A1	AO1
4b		B1 correct sinusoidal shape with correct turning points B1 two cycles	AO1
5i	Length of rectangle = $\frac{20-3x}{2}$ Area $= \left(\frac{20-3x}{2}\right)x - \frac{1}{2}x^2\sin 60°$ $= 10x - \frac{3}{2}x^2 - \frac{1}{2}x^2\frac{\sqrt{3}}{2}$ $= 10x - \left(\frac{6+\sqrt{3}}{4}\right)x^2$	B1 M1 AG1	AO3
5ii	$\frac{dA}{dx} = 10 - 2\left(\frac{6+\sqrt{3}}{4}\right)x$ For stationary point, $10 - 2\left(\frac{6+\sqrt{3}}{4}\right)x = 0$ $\left(6+\sqrt{3}\right)x = 20$ $x = \frac{20}{\left(6+\sqrt{3}\right)} = 2.5866 = 2.6$ (2 s.f.) $A = 10(2.5866) - \left(\frac{6+\sqrt{3}}{4}\right)(2.5866)^2 = 12.933$	B1 - diff M1 – equate to 0 A1 A1	AO1

	$A=13$		
6a	Let $\frac{8x+13}{(1+2x)(2+x)^2}=\frac{A}{1+2x}+\frac{B}{2+x}+\frac{C}{(2+x)^2}$ $8x+13=A(2+x)^2+B(1+2x)(2+x)+C(1+2x)$ $x=-2 \quad \Rightarrow \quad -3=-3C$ $C=1$ $x=-\frac{1}{2} \quad \Rightarrow \quad 9=\frac{9}{4}A$ $A=4$ $x=0 \quad \Rightarrow \quad 13=4A+2B+C$ $13=4(4)+2B+1$ $B=-2$ $\therefore \quad \frac{8x+13}{(1+2x)(2+x)^2}=\frac{4}{(1+2x)}-\frac{2}{(2+x)}+\frac{1}{(2+x)^2}$	B1 M1 – sub or compare coefficient A1 (anyone of A, B or C correct) A1 (all 3 values A, B & C) A1	AO1
6b	$\int_1^2 \frac{8x+13}{(1+2x)(2+x)^2}\,dx$ $=\int_1^2 \frac{4}{(1+2x)}-\frac{2}{(2+x)}+\frac{1}{(2+x)^2}\,dx$ $=\left[\frac{4}{2}\ln(1+2x)-2\ln(2+x)-\frac{1}{2+x}\right]_1^2$ $=\left[2\ln 5-2\ln 4-\frac{1}{4}\right]-\left[2\ln 3-2\ln 3-\frac{1}{3}\right]$ $=2\ln\frac{5}{4}+\frac{1}{12}$ or $\ln\frac{25}{16}+\frac{1}{12}$ or 0.530 (to 3 sig fig)	B2 [B1 for 2 correct ln term; B1 for the 3rd term] B1	AO2

7a	$f'(x)=18x^2+2ax-50$ Since $f'(1)=6$ $18+2a-50=6$ $a=19$ Given $f\left(\frac{3}{2}\right)=0$ $6\left(\frac{3}{2}\right)^3+19\left(\frac{3}{2}\right)^2-50\left(\frac{3}{2}\right)+b=0$ $20.25+42.75-75+b=0$	B1 M1 forming equation A1 M1 forming equation A1	AO2

	$b=12$		
7b	$f(x)=(2x-3)(3x^2+14x-4)$ When $f(x)=0$ $(2x-3)(3x^2+14x-4)=0$ $(2x-3)=0$ $x=\frac{3}{2}$ $(3x^2+14x-4)=0$ $x=\frac{-14\pm\sqrt{(14)^2-4(3)(-4)}}{2(3)}$ $x=0.270$ or $x=-4.94$ (3 sig fig)	B1 found using any valid method B1 B1	AO1

8a	$2^{3x+4}\times5^{2x-1}=16^x\times5^{3x}$ $2^{3x}\times2^4\times5^{2x}\times5^{-1}=2^{4x}\times5^{3x}$ $\frac{16}{5}=2^{4x}\div2^{3x}\times5^{3x}\div5^{2x}$ $\frac{16}{5}=2^x\times5^x$ $\frac{16}{5}=10^x$ $\lg\frac{16}{5}=\lg10^x$ $x=\lg\frac{16}{5}$ (shown)	M1: simplify base 2 and base 5 terms correctly M1: introduce log on both sides AG1	AO3
8b	$2\log_2 x-\log_2(x-4)=3$ $\log_2 x^2-\log_2(x-4)=3$ $\log_2\frac{x^2}{x-4}=3$ $\frac{x^2}{x-4}=2^3$ $x^2=8(x-4)$ $x^2-8x+32=0$ $x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(32)}}{2(1)}$	M1: power/quotient law M1: change to index form M1: form quadratic eqn M1 solve for x OR show $b^2-4ac=-64$	AO3

	$x=\frac{8\pm\sqrt{-64}}{2}$ $\therefore$ No real solution (shown)	AG1: explain $\sqrt{-64}$ does not exist or $b^2-4ac<0$ $\therefore$ no real solution	

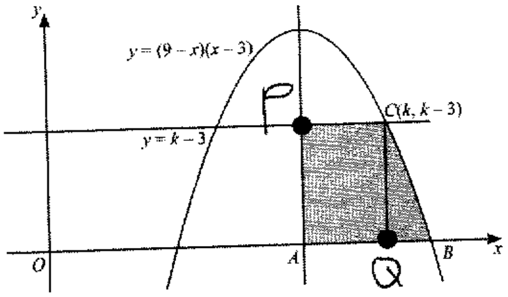
9a	$y=k\sqrt{3x+7}$, $\frac{dy}{dx}=\frac{3k}{2}(3x+7)^{-\frac{1}{2}}$ Given $\frac{dy}{dt}=3\frac{dx}{dt}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\frac{3dx}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $3=\frac{3k}{2}(3x+7)^{-\frac{1}{2}}$ Sub $x=3$ $1=\frac{k}{2}(3\times3+7)^{-\frac{1}{2}}$ $1=\frac{k}{2}\times\frac{1}{4}$ $k=8$	B1: correct differentiation M1: write the correct ratio M1: form chain rule correctly and sub $x=3$ A1	AO2
9b i	Sub $t=0$, $m=24$ g	B1	AO1
ii	$24e^{-0.02t}=12$ $\ln e^{-0.02t}=\ln 0.5$ $-0.02t=\ln 0.5$ $t=\frac{\ln 0.5}{-0.02}=34.7$ days	M1 A1	AO1
iii	$\frac{dm}{dt}=-0.48e^{-0.02t}$ Sub $t=0.5$, $\frac{dm}{dt}=-0.48e^{-0.02(0.5)}=-0.475$ (3 s.f.) Mass is decreasing at 0.475 g/day	B1 B1 (must write statement)	AO1

10 i	LHS $=\left[\frac{sinx}{\cos x}\div\left(1+\frac{1}{\cos x}\right)\right]+\left[\left(1+\frac{1}{\cos x}\right)\div\frac{\sin x}{\cos x}\right]$ $=\left[\frac{\sin x}{\cos x}\times\left(\frac{\cos x}{\cos x+1}\right)\right]+\left[\left(\frac{\cos x+1}{\cos x}\right)\times\frac{\cos x}{\sin x}\right]$ $=\frac{\sin x}{\cos x+1}+\frac{1+\cos x}{\sin x}$ $=\frac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$ $=\frac{\sin^2 x+1+2\cos x+\cos^2 x}{\sin x(1+\cos x)}$ $=\frac{2+2\cos x}{\sin x(1+\cos x)}$ $=\frac{2(1+\cos x)}{\sin x(1+\cos x)}$ $=\frac{2}{\sin x}$ (shown) OR LHS $=\frac{\tan^2 x+(1+\sec x)^2}{\tan x(1+\sec x)}$ $=\frac{\tan^2 x+1+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$ $=\frac{2\sec^2 x+2\sec x}{\tan x(1+\sec x)}$ $=\frac{2\sec x(\sec x+1)}{\tan x(1+\sec x)}$ $=\frac{2}{\cos x}\div\frac{\sin x}{\cos x}$ $=\frac{2}{\sin x}$ (shown)	M1: $\tan x=\frac{\sin x}{\cos x}$ & $\sec x=\frac{1}{\cos x}$ M1: simplify both [ ] correctly M1: add the fractions and expand correctly M1: factorise numerator AG1 **OR** M1: add fractions correctly M1: expand and add correctly M1: factorise numerator M1: $\sec x=\frac{1}{\cos x}$ & $\tan x=\frac{\sin x}{\cos x}$ AG1	AO2

10 ii	$\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=1+3\sin x$ $\frac{2}{\sin x}=1+3\sin x$ $3\sin^2 x+\sin x-2=0$ $(3\sin x-2)(\sin x+1)=0$	M1: form quadratic eqn M1: factorisation or general formula	AO1

	$\sin x = \frac{2}{3}$ or $\sin x = -1$ $x = 41.8^\circ, 138.2^\circ$ or $x = 270^\circ$ (reject (to 1 dp) as $\tan 270^\circ$ is undefined)	A1, A1	

11 i	$y = (9-x)(x-3)$ Sub (k, k-3) into $y = (9-x)(x-3)$ $k-3 = (9-k)(k-3)$ * $k-3 = 9k-27-k^2+3k$ $k^2-11k+24=0$ $(k-3)(k-8)=0$ $k=3(N.A.)$ or $k=8$ *OR $(9-k)(k-3)-(k-3)=0$ $(k-3)(9-k-1)=0$ $(k-3)(8-k)=0$ $k=3(N.A.)$ or $k=8$	M1 substitution M1 form quadratic eqn M1 factorisation AG1 must state N.A. for $x = 3$	AO3

11 i	Let $y=0$ $\quad (9-x)(x-3)=0$ $x=3$ or $x=9$ x-coordinate of $B=9$ x-coordinate of $A=\frac{3+9}{2}=6$ **** OR use** $\frac{dy}{dx}=-2x+12$ **At turning point,** $\frac{dy}{dx}=0$ $-2x+12=0$ $x_A=6$  Area of $APCQ$ $=5\times2=10$ units^2 Area CQB $=\int_8^9(-x^2+12x-27)dx$ $=\left[-\frac{1}{3}x^3+6x^2-27x\right]_8^9$ $=\left[-\frac{1}{3}(9^3)+6(9^2)-27(9)\right]-\left[-\frac{1}{3}(8^3)+6(8^2)-27(8)\right]$ $=2\frac{2}{3}$ units^2 Shaded area $=10+2\frac{2}{3}=12\frac{2}{3}$ units^2	M1 : either x_A or x_B B1 (area of rectangle) M1: Integrate all terms correctly A1 A1	AO2

12 i	Gradient of PQ = gradient of $OR=\frac{1}{2}$ Eqn of PQ: $y-3=\frac{1}{2}(x+4)$ $y=\frac{1}{2}x+5$ ----- (1) Gradient of $QR=-2$ Eqn of QR: $y-2=-2(x-4)$ $y=-2x+10$ (1)=(2):	B1 M1: $m_1m_2=-1$ M1 (Equation of QR)	AO1

	$\frac{1}{2}x+5=-2x+10$ $x=2$ $\therefore Q(2,6)$	M1: form simultaneous eqns A1	
12 ii	In eqn (1), Let $x=0$, $y=5$ $\therefore OT=5$ units $RT=\sqrt{(4-0)^2+(2-5)^2}$ $RT=\sqrt{25}=5$ Since $OT=RT=5$ units, ΔORT is isosceles.	 B1 B1 AG1	AO3
12 iii	Let $S=(a,b)$ By inspection: $S=(0-8,0+1)=(-8,1)$ OR Midpoint of RS = Midpoint of OP $\left(\frac{a+4}{2},\frac{b+2}{2}\right)=\left(-\frac{4}{2},\frac{3}{2}\right)$ $a+4=-4$ & $b+2=3$ $a=-8$ $b=1$ Hence coordinates of $S=(-8,1)$.	 B1	AO2
12 iv	Area of trapezium OPQR $=\frac{1}{2}\begin{vmatrix}0 & -4 & 2 & 4 & 0\\ 0 & 3 & 6 & 2 & 0\end{vmatrix}$ $=\frac{1}{2}\lvert-24+4-24-6\rvert$ $=\frac{1}{2}\lvert-50\rvert$ $=25 units^2$	 M1 A1	AO1

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FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS   4049/02

Paper 2

Date: 22 August 2024   **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

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Presentation	☐ 1 ☐ 2			
Rounding off	☐ 1		Parent's/Guardian's Signature	90

This question paper consists of <u>23</u> printed pages

NAME: ______________________ (   )   CLASS: ______

Mathematical Formulae

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

NAME: ______________________ (   )   **CLASS:** _______

Answer **all** the questions.

1 Show that the equation $2(e^x - 3) = e^{\frac{1}{2}x}$ has only one solution and find this value correct to 3 significant figures. [4]

NAME: ______________________________ (   )   CLASS: ________

2 The equation of a curve is $y = ax^3 + b$, where a and b are constants. The equation of the normal to the curve at the point where $x = 1$ is $5y + 2x = 12$. Find the value of a and of b. [6]

NAME: ______________________ (   )   **CLASS:** ______

3 Given that $y = \frac{3\ln 2x}{x^2}$ for $x > 0$.

(a) Show that $\frac{dy}{dx} = \frac{3}{x^3}(1 - 2\ln 2x)$. [3]

NAME: ______________________________ (   )   CLASS: _______

(b) Hence, find $\int \frac{\ln 2x}{x^3}\,dx$. [3]

NAME: ____________________ (   ) **CLASS:** ______

4 Given that $f(x) = x^2 - ax + 3$, where a is a constant,

(a) find the range of values of a for which $f(x) > x - 1$, for all real values of x, [4]

(b) find the value(s) of a for which the line $y = a + 4$ is a tangent to the curve $y = f(x)$. [3]

NAME: ____________________________ (   )   CLASS: _______

5 **(a)** A and B are acute angles such that $\sin(A-B)=\frac{3}{8}$ and $\sin A\cos B=\frac{5}{8}$.

Without using a calculator, find the value of $\cos A\sin B$. [2]

(b) Express $2\sin 2\theta(\sec\theta-\tan\theta)$ as a quadratic expression in $\sin\theta$. [3]

NAME: ______________________________ (   )   **CLASS:** _______

(c) Use your answer to part **(b)** to find, for $0 \leq \theta \leq 2\pi$, the exact solutions of the equation $2\sin 2\theta(\sec\theta - \tan\theta) + 3 = 0$. [3]

NAME: ______________________________ (   )   CLASS: _______

6 A curve is such that $\frac{dy}{dx}=\frac{8}{x^2}-2$.

(i) Given that the curve passes through the point (1, 5), find the equation of the curve. [3]

(ii) Find the x–coordinates of the stationary points of the curve. [2]

NAME: ______________________ (   )   **CLASS:** ______

(iii) Obtain and expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of each stationary point. [3]

NAME: ______________________ (   ) CLASS: ______

7 (a) Write down the first three terms in the expansion, in descending powers of x, of $\left(x^2+\frac{m}{x}\right)^9$, where m is an integer. [2]

(b) Given that the coefficient of x^3 in the expansion of $\left(x^2+\frac{m}{x}\right)^9$ is -126,

(i) show that $m=-1$, [3]

NAME: ______________________ (   )   CLASS: _______

(ii) hence, find the term independent of x in the expansion of

$$\left(2-\frac{1}{x^3}\right)\left(x^2+\frac{m}{x}\right)^9.$$ [4]

NAME: ______________________________ (   )   CLASS: _______

8 The table shows the population, P, in thousands, of a small town decreases with time, t years.

t	6	9	12	15	18	25	33
P	274	203	151	112	83	41	18

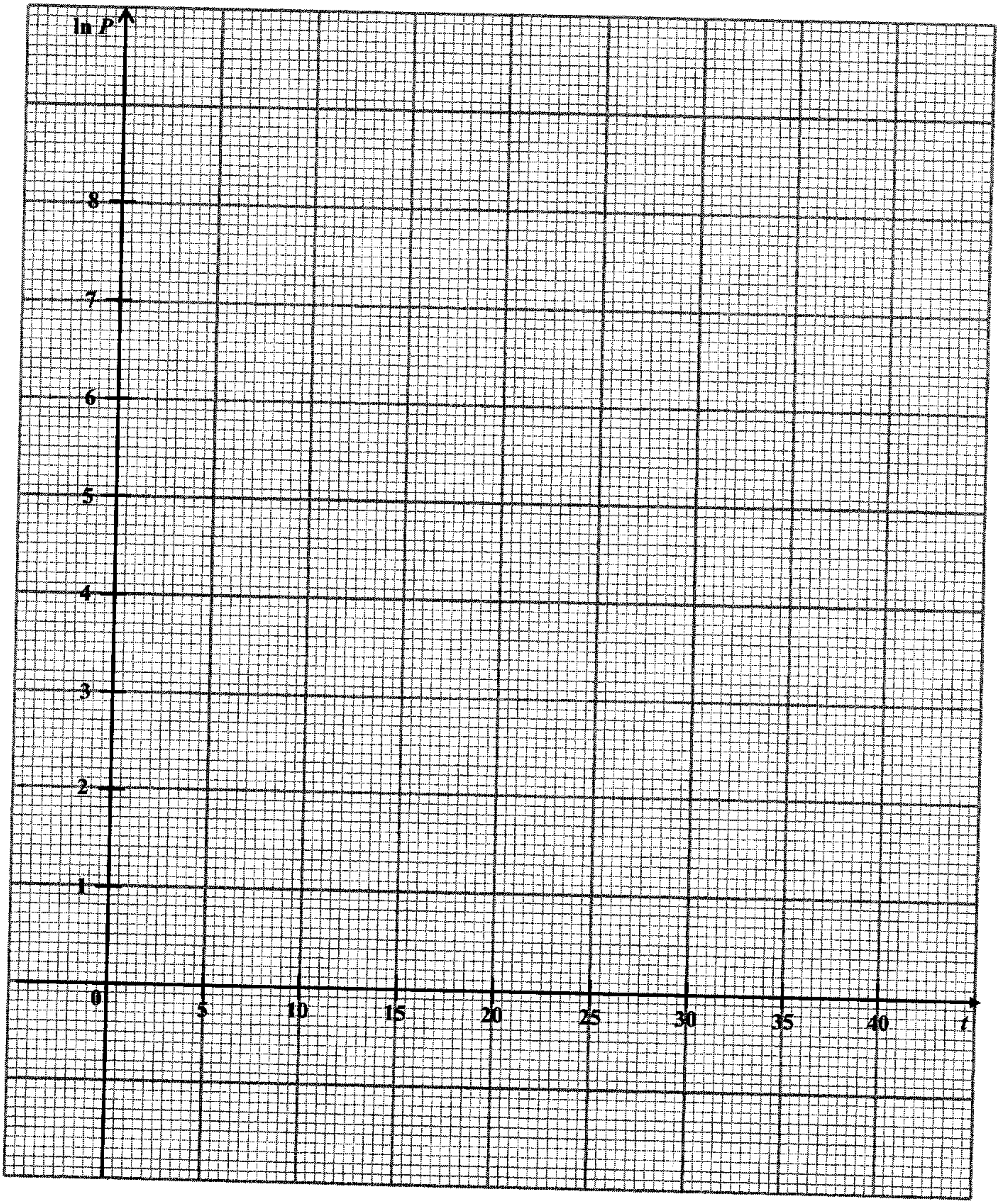
(a) Show your working clearly and draw a straight line graph of $\ln P$ against t on the grid provided. [3]

(b) Find the gradient of your straight line and hence express P in the form of Ae^{-kt}, where A and k are constants. giving your answers correct to the nearest hundred and to 1 decimal place respectively. [4]

(c) If this model for the population remains valid, find the number of years it will take for the population of the small town to drop below 100000. Give your answer correct to the nearest year. [2]

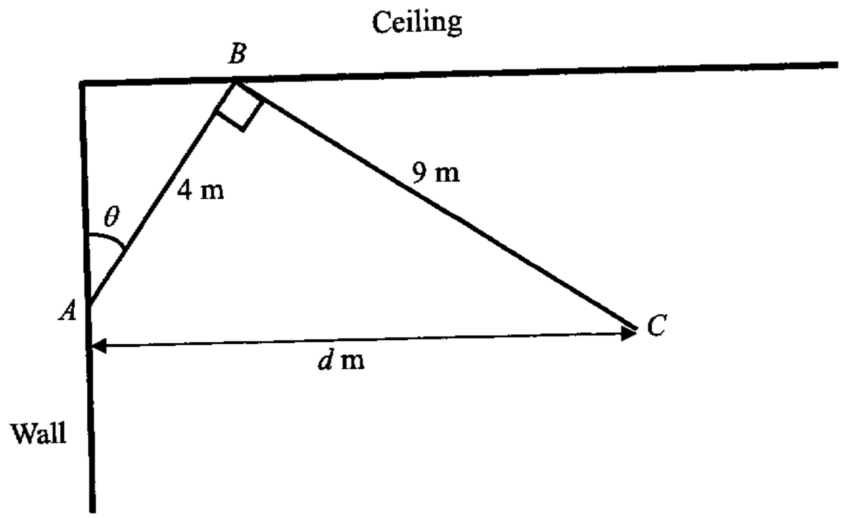
NAME: ______________________ (   )   **CLASS:** ______

8 (a)



NAME: ______________________________ (   )   CLASS: ________

9 The diagram above shows two rods AB and BC of length 4 m and 9 m respectively and $\angle ABC = 90°$. Rollers are fixed at points A and B such that A is able to move along the wall and B is able to move along the celling. The horizontal distance of C from the vertical wall is d m.



(a) Show clearly that $d = 4\sin\theta + 9\cos\theta$. [2]

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(b) Express d in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(c) Find the value of θ for which $d = 6$ m. [2]

NAME: ______________________________ (   )   CLASS: _______

(d) Find the maximum value of d and the corresponding value of θ. [2]

NAME: ______________________ (   )   **CLASS:** ______

10 A particle moves in a straight line such that, t s after passing through a fixed point O, its displacement from O is s m. The velocity v ms^{-1} of the particle is such that $v = 6\cos 4t$.

(a) State the initial velocity of the particle. [1]

(b) Find the first value of t when the acceleration of the particle is equal to 8 ms^{-2}. [2]

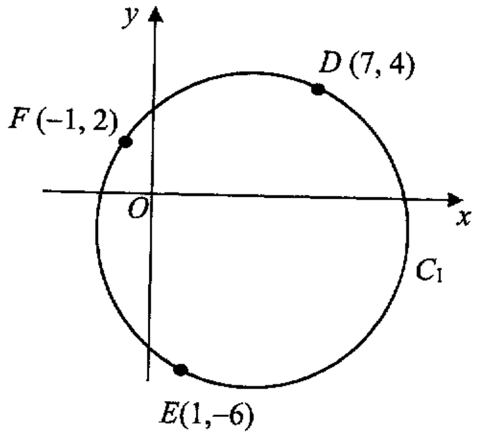
(c) Find the displacement of the particle from O when $t = 4$. [3]

NAME: ______________________ (   )   CLASS: _______

(d) Find the total distance travelled by the particle for the first $\frac{3\pi}{8}$ seconds. [5]

NAME: ______________________ (   ) CLASS: ______

11 **The diagram below is not drawn to scale.**



In the diagram, D, E and F are points on the circle C_1.

(a) Show that DE is the diameter of the circle C_1 and hence find the centre of C_1. [5]

NAME: ____________________________ (   ) CLASS: _______

(b) Find the equation of the circle C_1 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [3]

(c) Given that the circle C_2 is a reflection of the circle C_1 in the line $x = -2$, find the equation of C_2. [2]

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(d) Explain why point (3, 4) lies within only one of the circles C_1 and C_2. [2]

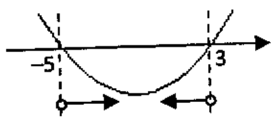
~ End of Paper ~

Sec 4 Add Math Preliminary Exam 2024 P2 Marking Scheme

Qn.	Solution	Marks	AO
1	$2(e^x - 3) = e^{\frac{1}{2}x}$ Let $y = e^{\frac{1}{2}x}$, $2(y^2 - 3) = y$ $2y^2 - y - 6 = 0$ $(2y+3)(y-2) = 0$ $y = -\frac{3}{2}$ or $y = 2$ $e^{\frac{1}{2}x} = -\frac{3}{2}$ (rej. as -ve) $e^{\frac{1}{2}x} = 2$ $\frac{1}{2}x = \ln 2$ $x = 2\ln 2$ $= 1.39$ (3 s.f) Therefore, the equation has only one solution where x=1.39	M1 (substitution) M1 (factorization) A1 (reject $-\frac{3}{2}$) (Did not award marks for students who squared both sides and could not justify why they rejected one answer when they ended up with 2 answers) AG1	3

2	$y = ax^3 + b$ $\frac{dy}{dx} = 3ax^2$ $5y + 2x = 12$ $5y = -2x + 12$ $y = -\frac{2}{5}x + \frac{12}{5}$ Grad of normal = $-\frac{2}{5}$ Grad of tangent = $\frac{5}{2}$ $\therefore \frac{dy}{dx} = \frac{5}{2}$ At $x = 1$, $y = -\frac{2}{5}(1) + \frac{12}{5}$ $y = 2$ At $x = 1, y = 2$ $2 = a(1)^3 + b$ $a + b = 2$ ------ (1) At $x = 1, \frac{dy}{dx} = \frac{5}{2}$ $\frac{5}{2} = 3a(1)^2$ ------ (2) $a = \frac{5}{6}$ Substituting $a = \frac{5}{6}$ into (1) $\frac{5}{6} + b = 2$ $b = \frac{7}{6}$	B1 (find $\frac{dy}{dx}$) B1 (grad of tangent) M1 (find $y = 2$) M1 (for either (1) or (2)) A1 (for a) A1 (for b)	2

3a	$y=\frac{3\ln(2x)}{x^2}$ $\frac{dy}{dx}=\frac{x^2\left(\frac{3(2)}{2x}\right)-6x\ln 2x}{\left(x^2\right)^2}$ $=\frac{3x-6x\ln 2x}{x^4}$ $=\frac{3}{x^3}(1-2\ln 2x)$	M1 (Apply quotient rule) M1 (Able to diff $\ln 2x=\frac{2}{2x}=\frac{1}{x}$.) A1	1
3b	$\int\left(\frac{3}{x^3}-\frac{6}{x^3}\ln 2x\right)\mathrm{d}x=\frac{3\ln 2x}{x^2}+C$ $\int\left(\frac{1}{x^3}-\frac{2}{x^3}\ln 2x\right)\mathrm{d}x=\frac{\ln 2x}{x^2}+C_1$ $\int\frac{2\ln 2x}{x^3}\mathrm{d}x=-\frac{\ln 2x}{x^2}-\frac{1}{2x^2}+C_2$ $\int\frac{\ln 2x}{x^3}\mathrm{d}x=-\frac{1}{2}\left(\frac{\ln 2x}{x^2}+\frac{1}{2x^2}\right)+C_3$	M1 (reverse differentiation – must include + *C*) M1 (integrate $\frac{1}{x^3}$) A1(must include + C) (Whole question will only deduct once if they did not put + *C*)	2

4a	$f(x) > x-1$ $x^2 - ax + 3 > x - 1$ $x^2 - (a+1)x + 4 > 0$ Since it is always positive for all real values of x, the graph of the curve $y = x^2 - (a+1)x + 4$ lies entirely above the x-axis $\Rightarrow$ the equation $x^2 - (a+1)x + 4 = 0$ has no real roots Discriminant, $D < 0$ $(a+1)^2 - 4(1)(4) < 0$ $(a+1)^2 - 4^2 < 0$ $(a+5)(a-3) < 0$  $\therefore -5 < a < 3$	M1 (form inequality) (students who equated both eqns together will not get M1 unless they explain that there are no real roots and lead to $D<0$) M1 (Discriminant less than zero) M1 (Factorization) A1	2
4b	$\begin{cases} y = f(x) \\ y = a+4 \end{cases}$ $x^2 - ax + 3 = a + 4$ $x^2 - ax - a - 1 = 0$ Since the line $y = a+4$ is a tangent to the curve, this equation has equal real roots Discriminant, $D = 0$ $(-a)^2 - 4(1)(-a-1) = 0$ $a^2 + 4a + 4 = 0$ $(a+2)(a+2) = 0$ $\therefore a = -2$	M1 (equate equations together) M1 (D = 0) A1	1
5a	$\sin(A-B) = \sin A \cos B - \cos A \sin B$ $\cos A \sin B = \sin A \cos B - \sin(A-B)$ $= \frac{5}{8} - \frac{3}{8}$ $= \frac{1}{4}$	M1 (make $\cos A \sin B$ the subject) A1	1

5b	$2\sin 2\theta(\sec\theta-\tan\theta)$ $=2(2\sin\theta\cos\theta)\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)$ $=2(2\sin\theta\cos\theta)\left(\frac{1-\sin\theta}{\cos\theta}\right)$ $=4\sin\theta-4\sin^2\theta$	M1 (double angle formula) M1 (bring $\cos\theta$ under same denominator) A1	2
5c	$2\sin 2\theta(\sec\theta-\tan\theta)+3=0$ $4\sin\theta-4\sin^2\theta+3=0$ $4\sin^2\theta-4\sin\theta-3=0$ $(2\sin\theta+1)(2\sin\theta-3)=0$ $\sin\theta=-\frac{1}{2}$ or $\sin\theta=\frac{3}{2}$ (rejected) $\alpha=\frac{\pi}{6}$ $\theta=\pi+\frac{\pi}{6},2\pi-\frac{\pi}{6}$ $\theta=\frac{7\pi}{6},\frac{11\pi}{6}$	M1 (factorize) M1 (show both and must reject one) A1 (must be in terms of π as qn asked for exact solns)	1
6(i)	$y=\int\left(\frac{8}{x^2}-2\right)dx$ $y=-\frac{8}{x}-2x+c$ At $x=1, y=5$ $5=-\frac{8}{1}-2(1)+c$ $c=15$ Equation of curve: $y=-\frac{8}{x}-2x+15$	M1 (with + c) M1 (Sub in values) A1	1
6(ii)	Let $\frac{dy}{dx}=0$ $\frac{8}{x^2}-2=0$ $\frac{8}{x^2}=2$ $x^2=4$ $x=2$ or $x=-2$	M1 A1 (both answers)	1

6(iii)	$\frac{d^2y}{dx^2}=-\frac{16}{x^3}$ At $x=2$, $\frac{d^2y}{dx^2}=-\frac{16}{2^3}=-2<0$ $\therefore$ Maximum point at $x=2$ At $x=-2$, $\frac{d^2y}{dx^2}=-\frac{16}{(-2)^3}=-\frac{16}{-8}=2>0$ $\therefore$ Minimum point at $x=-2$	M1 (2^{nd} derivative) A1 A1 (for students who got part (ii) wrong, maximum mark is 1M if M1 shown)	1
7a	$\left(x^2+\frac{m}{x}\right)^9=x^{18}+\binom{9}{1}\left(x^2\right)^8\left(\frac{m}{x}\right)^1+\binom{9}{2}\left(x^2\right)^7\left(\frac{m}{x}\right)^2+....$ $=x^{18}+9\ mx^{15}+36m^2x^{12}+....$	B2 (3 terms all correct) B1(2 terms correct)	1
7b(i)	For $\left(x^2+\frac{m}{x}\right)^9$, general term is $(r+1)^{th}$ term $=\binom{9}{r}\left(x^2\right)^{9-r}(m)^r\left(x^{-1}\right)^r$ $=\binom{9}{r}(m)^r\left(x^{18-2r}\right)\left(x^{-r}\right)$ $=\binom{9}{r}(m)^r\left(x^{18-3r}\right)$ $18-3r=3$ $r=5$ $\therefore -126=\binom{9}{5}m^5$ $-126=126m^5$ $m^5=-1$ $m=-1$	M1 (general term) A1 ($r=5$) AG1	3

<table>
<tr>
<td>7b(ii)</td>
<td>For $\left(x^2+\frac{m}{x}\right)^9$, the term independent of x:

$18-3r=0$
$r=6$

$\therefore \binom{9}{6}m^6=84(-1)^6=84$

For $\left(2-\frac{1}{x^3}\right)\left(x^2+\frac{a}{x}\right)^9$, the term independent of x:

$\left(2-\frac{1}{x^3}\right)(....-126x^3+84+....)^9$

$=2\times 84+126$

$=294$</td>
<td>M1

A1

M1

A1</td>
<td>1</td>
</tr>
<tr>
<td>8a</td>
<td>
<table>
<tr><td>t (years)</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>25</td><td>33</td></tr>
<tr><td>P</td><td>274</td><td>203</td><td>151</td><td>112</td><td>83</td><td>41</td><td>18</td></tr>
<tr><td>ln P</td><td>5.61</td><td>5.31</td><td>5.02</td><td>4.72</td><td>4.42</td><td>3.71</td><td>2.89</td></tr>
</table>
</td>
<td>P2 – Plot points accurately.

L1 – Plot straight line graph

(See graph attached.)</td>
<td>1</td>
</tr>
<tr>
<td>8b</td>
<td>$P=Ae^{-kt}$

$\ln P=\ln Ae^{-kt}$
$\ln P=\ln A+\ln e^{-kt}$
$\ln P=\ln A-kt$
$\ln P=-kt+\ln A$

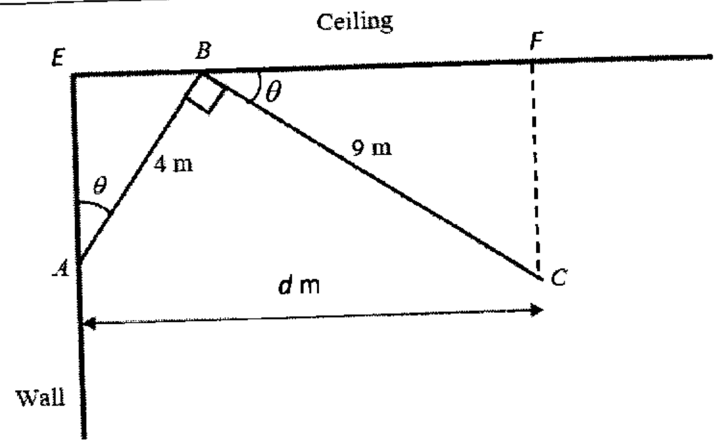
Grad $=\frac{5.61-4.72}{6-15}$
$=-0.0989$ (3 s.f)
$-k=-0.0989$
$k=0.1$(1 d.p)
$\ln A=6.2$
$A=492.75$
$A=500$ (nearest 100)

$\because P=500e^{-0.1t}$</td>
<td>M1 (product law or if evidence shown in transformation from eqn of graph to $P=Ae^{-kt}$)

M1 (gradient)

A1

B1
(If students did not use the gradient of line to solve for k and A, maximum mark is 1M as question mentioned hence.)</td>
<td>2</td>
</tr>
</table>

8c	$\ln 100 = 4.6$ When $\ln P = 4.6$ $t = 16$ (nearest year)	M1 A1	1
9a	 $EB = 4\sin\theta$ $BF = 9\cos\theta$ $d = 4\sin\theta + 9\cos\theta$	} M1 AG 1	3
9b	$R = \sqrt{81+16}$ $R = \sqrt{97}$ $9\cos\theta + 4\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $9 = R\cos\alpha$ } $4 = R\sin\alpha$ $\tan\alpha = \frac{4}{9}$ $\alpha = 23.962°(3d.p)$ $\alpha = 24.0°(1d.p)$ $9\cos\theta + 4\sin\theta = \sqrt{97}\cos(\theta - 24.0°)$	M1 (Find R) M1 (No M1 given if student do not show this) M1 (Find α) A1	1
9c	$\sqrt{97}\cos(\theta - 23.96°) = 6$ $\cos(\theta - 23.96°) = \frac{6}{\sqrt{97}}$ $\theta = 52.467° + 23.96°$ $\theta = 76.427° \approx 76.4°$	M1 A1 (no A1 if 76.5)	1

9d	Maximum value of $d = \sqrt{97}$ $\cos(\theta - 24.0°) = 1$ $\theta = 24.0$ Maximum value of $d = \sqrt{97}$ and occurs when $\theta = 24.0°$	B1 B1	1
10a	$v = 6\cos 4t$ When $t = 0, v = 6$ Initial velocity of the particle is 6m/s.	B1	1
10b	$a = \frac{dv}{dt} = -24\sin 4t$ $-24\sin 4t = 8$ $\sin 4t = -\frac{1}{3}$ $4t = 3.4814$ $t = 0.870$ (3 s.f)	M1 ($\frac{dv}{dt}$) A1	2
10c	$s = \int 6\cos 4t \; dt$ $s = \frac{6}{4}\sin 4t + c$ $s = \frac{3}{2}\sin 4t + c$ When $t = 0$, $s = 0$, $c = 0$ $s = \frac{3}{2}\sin 4t$ When $t = 4$, $s = \frac{3}{2}\sin 16$ $s = -0.432$ Displacement = -0.432m (3 sf)	M1 (integration with + c) A1 (conclude c=0) (students who used definite integral must indicate when t = 0, s=0) B1	1
10d	At instantaneous rest, $v = 0$ $6\cos 4t = 0$ $\cos 4t = 0$ $4t = \frac{\pi}{2}, \frac{3\pi}{2}$ $t = \frac{\pi}{8}, \frac{3\pi}{8}$	M1 (v= 0) M1 (values of t)	2

	When $t=\frac{\pi}{8}$, $s=\frac{3}{2}\sin\frac{\pi}{2}=1.5\text{m}$ When $t=\frac{3\pi}{8}$, $s=\frac{3}{2}\sin\frac{3\pi}{2}=-1.5\text{m}$ Total distance travelled $=(1.5\times2)+1.5$ $=4.5\text{m}$	M1 M1 A1	
11a	**<u>Solution 1</u>** Grad EF $=\frac{2-(-6)}{-1-1}=-4$ Grad DF $=\frac{4-2}{7-(-1)}=\frac{1}{4}$ Since Grad $DF\times$ Grad $EF=-4\times\frac{1}{4}=-1$ $\therefore DF\perp EF$ $\angle DFE=90°$ (Right angle in semi-circle) $\therefore DE$ is the diameter of C_1 Since PQ is the diameter of C_1 Centre of $C_1=\left(\frac{1+7}{2},\frac{-6+4}{2}\right)$ $=(4,-1)$	M1 (Find grad) B1 (Show – 1) B1(Conclude 90°) AG 1 (State reason angle in semi-circle) B1	3
11a	**<u>Solution 2</u>** $DF=\sqrt{(7+1)^2+(4-2)^2}=\sqrt{68}$ $EF=\sqrt{(1+1)^2+(-6-2)^2}=\sqrt{68}$ $DE=\sqrt{(7-1)^2+(4+6)^2}=\sqrt{136}$ $DF^2+EF^2=\left(\sqrt{68}\right)^2+\left(\sqrt{68}\right)^2=136$ $DE^2=\left(\sqrt{136}\right)^2=136$ $\because DF^2+EF^2=DE^2$ By converse of Pythagoras theorem, triangle DFE is a right angled triangle. Therefore $\angle DFE=90°$.	M1 (Find distance) B1 (Show this statement) B1(Conclude 90° - must state Pythagoras thm)	3

	Since $\angle DFE = 90°$ By converse of right angle in semi-circle $\therefore DE$ is the diameter of C_1 Since PQ is the diameter of C_1 Centre of $C_1 = \left(\frac{1+7}{2}, \frac{-6+4}{2}\right)$ $= (4, -1)$	AG 1 (State reason angle in semi-circle) B1	
11b	Radius of $C_1 = \sqrt{(4-1)^2 + (-1-(-6))^2} = \sqrt{34}$ Equation of C_1: $(x-4)^2 + (y+1)^2 = 34$ $x^2 + y^2 - 8x + 2y - 17 = 0$	B1 M1 A1	1
11c	Centre of $C_2 = (-8, -1)$ Radius $= \sqrt{34}$ Equation of C_2: $(x+8)^2 + (y+1)^2 = 34$	B1 (centre) B1	2
11d	Distance of E from centre of C_1 $= \sqrt{(3-4)^2 + (4+1)^2}$ $= \sqrt{26}$ $<$ radius of C_1 Distance of E from centre of C_2 $= \sqrt{(3+8)^2 + (4+1)^2}$ $= \sqrt{146}$ $>$ radius of C_2 $\therefore (3, 4)$ lies within C_1 only	M1 A1	2