BP~387

Name

Reg. No	Class

MAYFLOWER SECONDARY SCHOOL MAYFLOWER SECONDARY SCHOOL MAYFLOWER SECONDARY SCHOOL MAYFLOWER SECONDARY SCHOOL

4E/5N

ADDITIONAL MATHEMATICS

4049/01

Paper 1 [90 marks]

PRELIMINARY EXAMINATION

20 August 2024

2 hours 15 minutes

Candidates answer on the question paper.

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer ALL questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.
Write the brand and model of your calculator in the space provided below.

Brand/Model of Calculator

For Examiner's Use	
Total	90

This question paper consists of **16** printed pages and **2** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

[Turn over

1 Given that $4(5^{x+3})=20^{3-x}$, evaluate 10^x **without using a calculator**. [4]

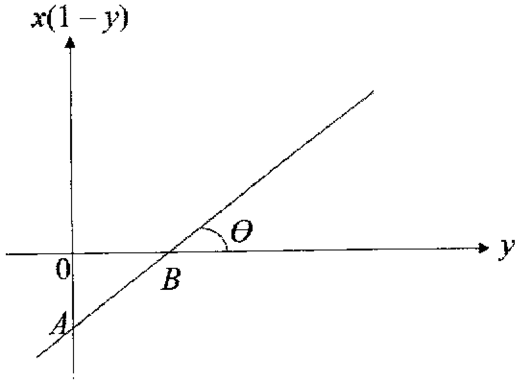
2 Solve the equations.

(a) $\log_2(x+4)=2\log_2 x-1$ [4]

(b) $10\log_y 5+3=\log_5 y$ [4]

[Turn over

3 The variables x and y are related by $y = \frac{2x+8}{2x+1}$. When values of $x(1-y)$ are plotted against y, a straight line is obtained. The straight line intersects the vertical and horizontal axes at A and B respectively.



(i) Find the coordinates of A and of B. [4]

(ii) State the value of $\tan\theta$. [1]

[Turn over

4 (i) Factorise completely $2x^3 - 3x^2 - 5x + 6$. [4]

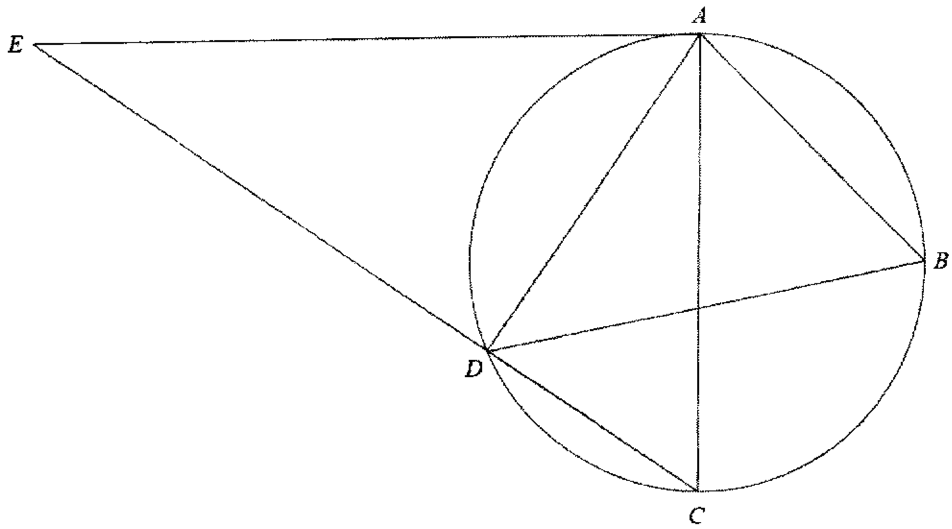
(ii) **Hence**, solve $2e^{3y} - 3e^{2y} - 5e^{y} + 6 = 0$. [3]

5 (i) Prove that $\frac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=1-\tan\theta$. [4]

(ii) Hence solve the equation $\frac{4-2\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=\sec^2\theta-2$ for $0\le\theta\le 2\pi$. [5]

6 The first two non-zero terms in the expansion of $(1+bx)(1+ax)^6$ in ascending powers of x are 1 and $-\frac{21}{4}x^2$. Find the value of each of the constants a and b, where $a<b$. [7]

7

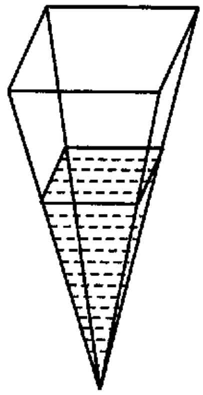


The diagram shows a circle passing through the points *A*, *B*, *C* and *D*. *AC* is a diameter of the circle. The line *EA* is a tangent to the circle and it intersects the straight line *EDC* at *E*.

(i) Show that angle *AED* = angle *DAC*. [2]

(ii) Show that $AD^2 = CD \times DE$. [4]

8 A vessel in the shape of an inverted right pyramid has a square base of side 12 cm and a height of 30 cm. Water is leaking from the vessel at a constant rate of 5 cm^3/s.



(i) Show that the volume of water in the vessel, V cm^3, is given by $V = \frac{4h^3}{75}$, where h is the depth of the water. [2]

(ii) Find the rate of change of the depth of water when the water is 6 cm deep. [3]

9 f(x) is such that $f'(x)=\sin\frac{1}{4}x-\cos 4x$. Given that $f(2\pi)=1$, show that

$16f''(x)+f(x)=a\sin 4x+b$, where a and b are constants. [6]

10 (a) (i) Find the range of values of the constant m for which the curve $y = x^2 - 5x + m$ meets the line $y = mx - 8$. [4]

(ii) Hence state the values of m for which the line is a tangent. [1]

(b) Given that $px^2 + 5x - q$ is always positive, what conditions must apply to the constants p and q? [3]

11 **(i)** Express $\dfrac{-x^2+2x+1}{(x-1)(x^2+1)}$ in the form $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$ where A, B and C are constants.

[4]

(ii) Differentiate $\ln(x^2+1)$ with respect to x. [1]

(iii) Using your results from parts **(i)** and **(ii)**, find $\int \dfrac{-2x^2+4x+2}{(x-1)(x^2+1)}\,\mathrm{d}x$. [2]

[Turn over

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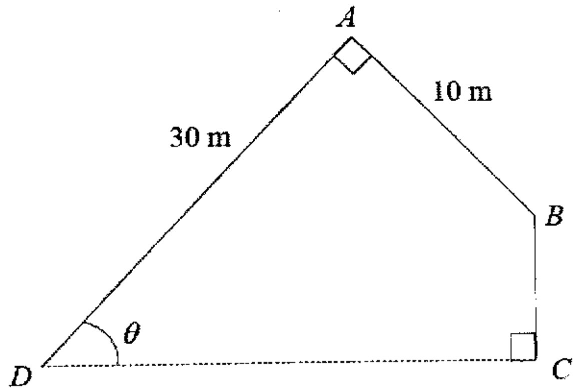


The diagram shows an area that is enclosed by glass panels at AB, BC and AD. $AB = 10$ m, $AD = 30$ m, angle DAB = angle $BCD = 90°$. The glass panel AD makes an acute angle θ with CD.

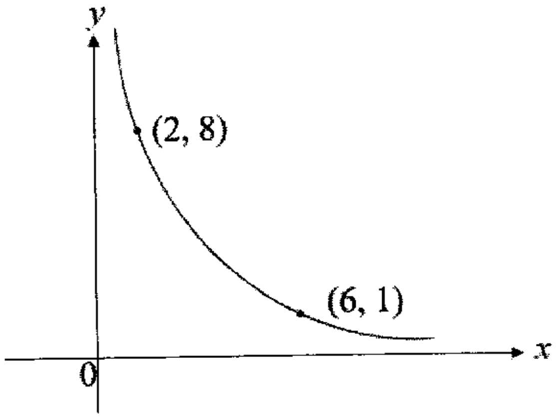
(i) Show that L m, the length of the glass panels, can be expressed as $40 + 30\sin\theta - 10\cos\theta$. [2]

(ii) Express L in the form $40 + R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) The total length of the glass panels is 65 m. Find the value of θ. [2]

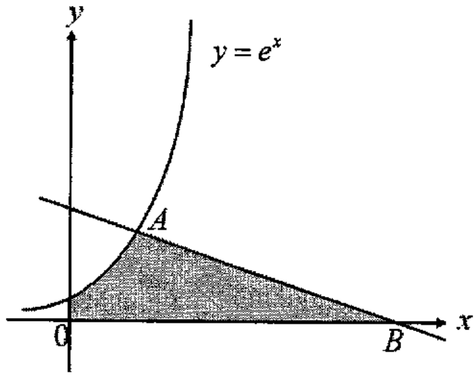
(iv) Explain whether it is possible to build an area where the length of the glass panels is 90 m. [1]

13 (a) The figure shows part of the curve $y = g(x)$. (2, 8) and (6, 1) are two points on the curve.



Given that $\int_2^6 y \, dx = 32$, find the value of $\int_1^8 x \, dy$. [2]

(b)



The diagram shows part of the curve $y = e^x$. The normal to the curve at point A where $x = 1$ intersects the x-axis at point B. Find the area of the shaded region.
Leave your answer in **exact form**. [7]

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Sec 4 Express A Math Prelim Paper 1 2024 Marking Scheme

1 Given that $4(5^{x+3}) = 20^{3-x}$, evaluate 10^x **without using a calculator**. [4]

	$4(5^{x+3}) = 20^{3-x}$ $4(5^x)(5^3) = \frac{20^3}{20^x}$ $5^x(20^x) = \frac{20^3}{4(5^3)}$ $100^x = 16$ $10^x = 4$

2 Solve the equations.

(a) $\log_2(x+4) = 2\log_2 x - 1$ [4]

(b) $10\log_y 5 + 3 = \log_5 y$ [4]

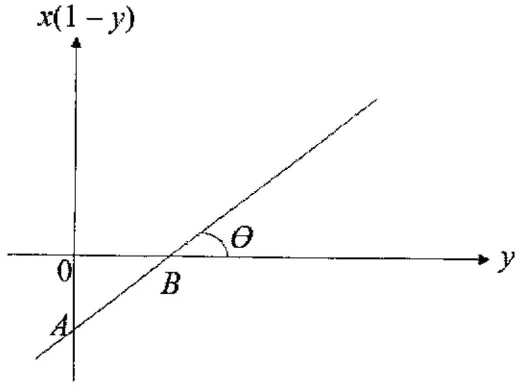
a	$\log_2(x+4) = 2\log_2 x - 1$ $2\log_2 x - \log_2(x+4) = 1$ $\log_2 x^2 - \log_2(x+4) = 1$ $\log_2 \frac{x^2}{x+4} = 1$ $\frac{x^2}{x+4} = 2^1$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4, \quad x = -2$ (rej)
b	$10\log_y 5 + 3 = \log_5 y$ $10\left(\frac{\log_5 5}{\log_5 y}\right) + 3 = \log_5 y$ $\frac{10}{\log_5 y} + 3 = \log_5 y$ Let $\log_5 y = u$ $\frac{10}{u} + 3 = u$ $10 + 3u = u^2$ $u^2 - 3u - 10 = 0$ $(u-5)(u+2) = 0$ $u = 5, \qquad u = -2$ $\log_5 y = 5 \qquad \log_5 y = -2$ $y = 5^5 \qquad y = 5^{-2}$ $y = 3125 \qquad y = \frac{1}{25}$

[Turn over

3 The variables x and y are related by $y=\frac{2x+8}{2x+1}$. When values of $x(1-y)$ are plotted against y, a straight line is obtained. The straight line intersects the vertical and horizontal axes at A and B respectively.

(i) Find the coordinates of A and of B. [4]

(ii) State the value of $\tan\theta$. [1]



i	$y=\frac{2x+8}{2x+1}$ $2xy+y=2x+8$ $2x-2xy=y-8$ $2x(1-y)=y-8$ $x(1-y)=\frac{1}{2}y-4$ $A=(0,-4)$ sub $x(1-y)=0$, $0=\frac{1}{2}y-4$ $y=8$ $B=(8,0)$
ii	$\tan\theta=\frac{1}{2}$

4 **(i)** Factorise completely $2x^3-3x^2-5x+6$. [4]

(ii) **Hence**, solve $2e^{3y}-3e^{2y}-5e^{y}+6=0$. [3]

i	$f(x)=2x^3-3x^2-5x+6$ $f(1)=2-3-5+6=0$ $(x-1)$ is a factor $x-1\overline{)2x^3-3x^2-5x+6}$ quotient $2x^2-x-6$ $-(2x^3-2x^2)$ $-x^2-5x+6$ $-(-x^2+x)$ $-6x+6$ $-(-6x+6)$ 0 $2x^3-3x^2-5x+6=(x-1)(2x^2-x-6)$ $=(x-1)(2x+3)(x-2)$
ii	$2e^{3y}-3e^{2y}-5e^{y}+6=0$ $e^y=1,\ e^y=-\frac{3}{2}(rej),\ e^y=2$ $y=0,\qquad y=\ln 2$

5 (i) Prove that $\dfrac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=1-\tan\theta$. [4]

(ii) Hence solve the equation $\dfrac{4-2\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=\sec^2\theta-2$ for $0\le\theta\le 2\pi$.

[5]

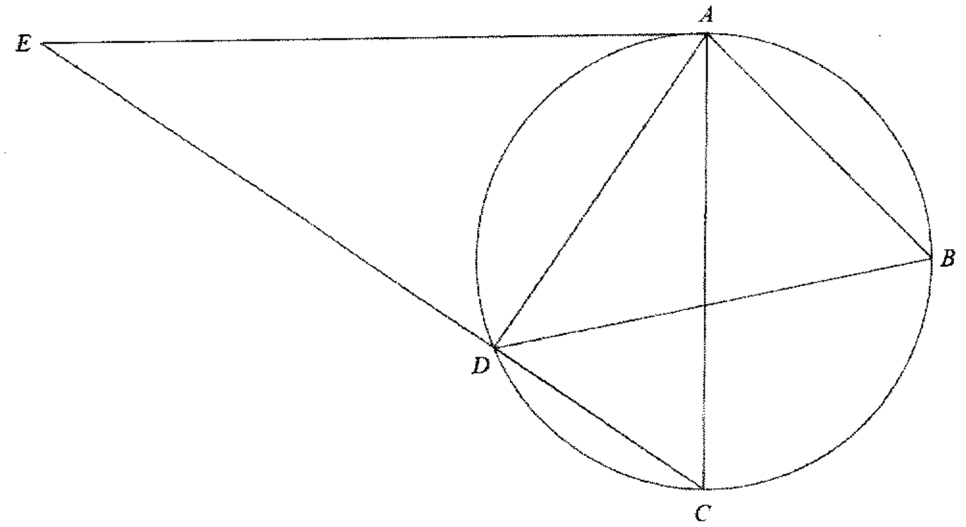
i	LHS $=\dfrac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}$ $=\dfrac{2-(1+\tan^2\theta)}{\frac{1}{\cos\theta}(\sin\theta+\cos\theta)}$ $=\dfrac{1-\tan^2\theta}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}$ $=\dfrac{1-\tan^2\theta}{\tan\theta+1}$ $=\dfrac{(1+\tan\theta)(1-\tan\theta)}{\tan\theta+1}$ $=1-\tan\theta$ $=$ RHS OR LHS $=\dfrac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}$ $=\left(2-\dfrac{1}{\cos^2\theta}\right)\div[\dfrac{1}{\cos\theta}(\sin\theta+\cos\theta)]$ $=\dfrac{2\cos^2\theta-1}{\cos^2\theta}\times\dfrac{\cos\theta}{\sin\theta+\cos\theta}$ $=\dfrac{2\cos^2\theta-1}{\cos\theta(\sin\theta+\cos\theta)}$ $=\dfrac{2\cos^2\theta-(\sin^2\theta+\cos^2\theta)}{\cos\theta(\sin\theta+\cos\theta)}$ $=\dfrac{\cos^2\theta-\sin^2\theta}{\cos\theta(\sin\theta+\cos\theta)}$ $=\dfrac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta(\sin\theta+\cos\theta)}$ $=\dfrac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta(\sin\theta+\cos\theta)}$ $=\dfrac{\cos\theta-\sin\theta}{\cos\theta}$ $=1-\dfrac{\sin\theta}{\cos\theta}$ $=1-\tan\theta$ $=$ RHS

ii	$\dfrac{4-2\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=\sec^2\theta-2$ $2(1-\tan\theta)=\sec^2\theta-2$ $2-2\tan\theta=1+\tan^2\theta-2$ $\tan^2\theta+2\tan\theta-3=0$ $(\tan\theta+3)(\tan\theta-1)=0$ $\tan\theta=-3,$ $\quad\tan\theta=1$ $\alpha=1.2490$ $\quad\alpha=\dfrac{\pi}{4}$ $\theta=\pi-1.2490, 2\pi-1.2490$ $\quad\theta=\dfrac{\pi}{4},\pi+\dfrac{\pi}{4}$ $\theta=1.8925, 5.0341$ $\quad\theta=\dfrac{\pi}{4},\dfrac{5\pi}{4}$ $\theta=1.89, 5.03$ (3sf)

6 The first two non-zero terms in the expansion of $(1+bx)(1+ax)^6$ in ascending powers of x are 1 and $-\frac{21}{4}x^2$. Find the value of each of the constants a and b, where $a<b$. [7]

	$(1+ax)^6 = 1^6 + \binom{6}{1}(1^5)(ax) + \binom{6}{2}(1^4)(ax)^2 + ...$ $= 1 + 6ax + 15a^2x^2 + ...$ $(1+bx)(1+ax)^6 = (1+bx)(1+6ax+15a^2x^2+...)$ $= 1 + 6ax + bx + 15a^2x^2 + 6abx^2 + ...$ $6a + b = 0$ $b = -6a$ ---(1) $15a^2 + 6ab = -\frac{21}{4}$ ---(2) Sub (1) into (2): $15a^2 + 6a(-6a) = -\frac{21}{4}$ $a^2 = \frac{1}{4}$ $a = -\frac{1}{2}$, $a = \frac{1}{2}$ (rej) $b = 3$, $b = -3$ (rej)

7



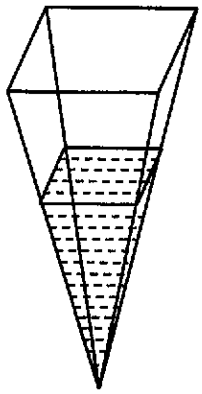
The diagram shows a circle passing through the points A, B, C and D. AC is a diameter of the circle. The line EA is a tangent to the circle and it intersects the straight line EDC at E.

(i) Show that angle AED = angle DAC. [2]

(ii) Show that $AD^2 = CD \times DE$. [4]

i	Let $\angle AED = \theta$ $\angle EAD = 180 - 90 - \theta$ $= 90 - \theta$ ($\angle$ in semicircle or sum of $\angle$s in a Δ) $\angle DAC = 90 - (90 - \theta)$ $= \theta$ (tangent $\perp$ radius) $= \angle AED$
ii	$\angle EAD = \angle ACD$ ($\angle$s in alternate segment) $\angle ADE = \angle CDA = 90°$ ($\angle$ in semicircle) $\angle DEA = \angle DAC$ (sum of $\angle$s in a Δ) ΔDEA similar to ΔDAC (AAA) $\frac{DE}{DA} = \frac{EA}{AC} = \frac{DA}{DC}$ $\frac{DE}{DA} = \frac{DA}{DC}$ $AD^2 = CD \times DE$ OR $\angle DEA = \angle DAC$ (from i) $\angle ADE = \angle CDA = 90°$ ($\angle$ in semicircle) $\angle EAD = \angle ACD$ (sum of $\angle$s in a Δ) ΔDEA similar to ΔDAC (AAA)

8 A vessel in the shape of an inverted right pyramid has a square base of side 12 cm and a height of 30 cm. Water is leaking from the vessel at a constant rate of 5 cm^3/s.



(i) Show that the volume of water in the vessel, V cm^3, is given by $V = \frac{4h^3}{75}$, where h is the depth of the water. [2]

(ii) Find the rate of change of the depth of water when the water is 6 cm deep. [3]

i	Let x be the length of the side of the water surface. $\frac{x}{12} = \frac{h}{30}$ $x = \frac{2h}{5}$ $V = \frac{1}{3}(\frac{2h}{5})^2(h)$ $V = \frac{4h^3}{75}$
ii	$V = \frac{4h^3}{75}$ $\frac{dV}{dh} = \frac{4}{25}h^2$ $\frac{dV}{dh} = \frac{dV}{dt} \div \frac{dh}{dt}$ $\frac{4}{25}(6)^2 = -5 \div \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{125}{144}$ cm/s

9 f(x) is such that $f'(x) = \sin\frac{1}{4}x - \cos 4x$. Given that $f(2\pi) = 1$, show that

$16f''(x) + f(x) = a\sin 4x + b$, where a and b are constants. [6]

	$f(x) = -4\cos\frac{1}{4}x - \frac{1}{4}\sin 4x + c$ Sub $f(2\pi) = 1$, $-4\cos\frac{2\pi}{4} - \frac{1}{4}\sin 8\pi + c = 1$ $c = 1$ $f(x) = -4\cos\frac{1}{4}x - \frac{1}{4}\sin 4x + 1$ $f''(x) = \frac{1}{4}\cos\frac{1}{4}x + 4\sin 4x$ $16f''(x) + f(x)$ $= 16(\frac{1}{4}\cos\frac{1}{4}x + 4\sin 4x) - 4\cos\frac{1}{4}x - \frac{1}{4}\sin 4x + 1$ $= 4\cos\frac{1}{4}x + 64\sin 4x - 4\cos\frac{1}{4}x - \frac{1}{4}\sin 4x + \frac{1}{4}$ $= 63\frac{3}{4}\sin 4x + 1$

10 (a) (i) Find the range of values of the constant m for which the curve $y = x^2 - 5x + m$ meets the line $y = mx - 8$. [4]

(ii) Hence state the values of m for which the line is a tangent. [1]

(b) Given that $px^2 + 5x - q$ is always positive, what conditions must apply to the constants p and q? [3]

ai	$y = x^2 - 5x + m$ --- (1) $y = mx - 8$ --- (2) $(1) = (2)$: $x^2 - 5x + m = mx - 8$ $x^2 - 5x - mx + m + 8 = 0$ $x^2 - (5+m)x + m + 8 = 0$ $b^2 - 4ac \geq 0$ $[-(5+m)]^2 - 4(1)(m+8) \geq 0$ $25 + 10m + m^2 - 4m - 32 \geq 0$ $m^2 + 6m - 7 \geq 0$ $(m-1)(m+7) \geq 0$ $m \leq -7,\quad m \geq 1$
aii	$m = -7,\quad m = 1$
b	$px^2 + 5x - q$ always positive: $p > 0$ $5^2 - 4p(-q) < 0$ $25 + 4pq < 0$ $pq < -\frac{25}{4}$

11 **(i)** Express $\dfrac{-x^2+2x+1}{(x-1)(x^2+1)}$ in the form $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$ where A, B and C are constants. [4]

(ii) Differentiate $\ln(x^2+1)$ with respect to x. [1]

(iii) Using your results from parts **(i)** and **(ii)**, find $\int \dfrac{-2x^2+4x+2}{(x-1)(x^2+1)}\,dx$. [2]

i	$\dfrac{-x^2+2x+1}{(x-1)(x^2+1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$ $-x^2+2x+1=A(x^2+1)+(Bx+C)(x-1)$ Sub $x=1$, $2=A(2)+0$ $A=1$ Sub $x=0$, $1=1+C(-1)$ $C=0$ Sub $x=2$, $1=5+(2B)(1)$ $B=-2$ $\dfrac{-x^2+2x+1}{(x-1)(x^2+1)}=\dfrac{1}{x-1}-\dfrac{2x}{x^2+1}$
ii	$\dfrac{d}{dx}\ln(x^2+1)=\dfrac{2x}{x^2+1}$
iii	$\int\dfrac{-2x^2+4x+2}{(x-1)(x^2+1)}\,dx=2\int\dfrac{1}{x-1}-\dfrac{2x}{x^2+1}\,dx$ $=2[\ln(x-1)-\ln(x^2+1)]+c$

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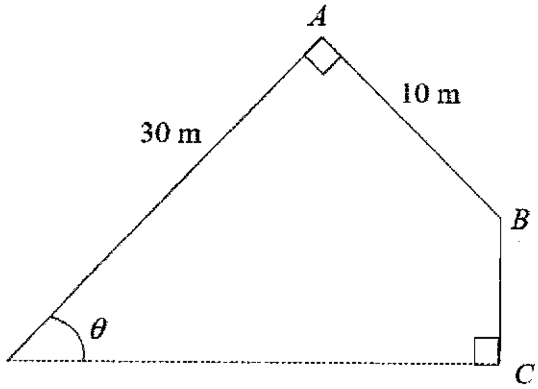


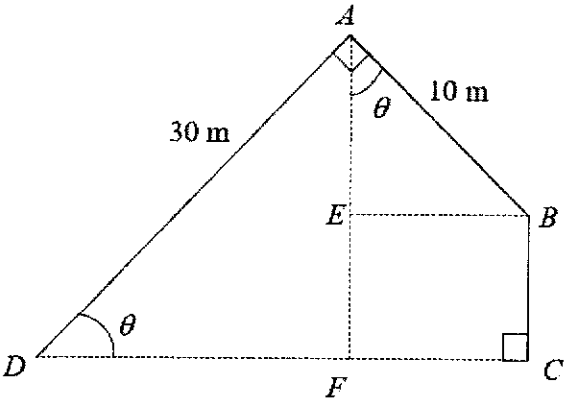
The diagram shows an area that is enclosed by glass panels at AB, BC and AD. $AB = 10$ m, $AD = 30$ m, angle DAB = angle $BCD = 90°$. The glass panel AD makes an acute angle θ with CD.

(i) Show that L m, the length of the glass panels, can be expressed as $40 + 30\sin\theta - 10\cos\theta$. [2]

(ii) Express L in the form $40 + R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

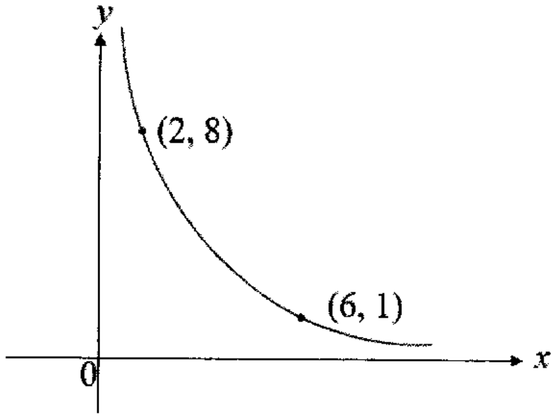
(iii) The total length of the glass panels is 65 m. Find the value of θ. [2]

(iv) Explain whether it is possible to build an area where the length of the glass panels is 90 m. [1]

i	 $\sin\theta = \dfrac{AF}{30}$ $AF = 30\sin\theta$ $\cos\theta = \dfrac{AE}{10}$ $AE = 10\cos\theta$ $L = 30 + 10 + 30\sin\theta - 10\cos\theta$ $L = 40 + 30\sin\theta - 10\cos\theta$

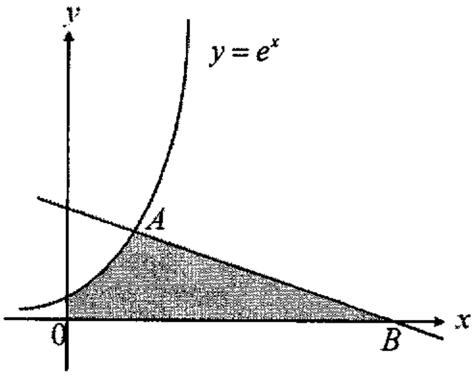
ii	$R=\sqrt{30^2+10^2}$ $R=\sqrt{1000}$ $R=10\sqrt{10}$ $\alpha=\tan^{-1}\frac{10}{30}$ $\alpha=18.434$ $L=40+10\sqrt{10}\sin(\theta-18.434)$ $L=40+10\sqrt{10}\sin(\theta-18.4)$ (1dp)
iii	$40+10\sqrt{10}\sin(\theta-18.434)=65$ $\sin(\theta-18.434)=\frac{25}{10\sqrt{10}}$ $\alpha=52.238$ $\theta-18.434=52.238$ $\theta=70.672$ $\theta=70.7°$ (1dp)
iv	Since the maximum value of $40+10\sqrt{10}\sin(\theta-18.434)=40+10\sqrt{10}<90$, it is not possible to build the area. OR $40+10\sqrt{10}\sin(\theta-18.434)=90$ $\sin(\theta-18.434)=1.5811$ Since $\sin(\theta-18.434)\leq 1$, it is not possible to build the area.

13 **(a)** The figure shows part of the curve $y = g(x)$. (2, 8) and (6, 1) are two points on the curve.



Given that $\int_2^6 y \, dx = 32$, find the value of $\int_1^8 x \, dy$. [2]

(b)



The diagram shows part of the curve $y = e^x$. The normal to the curve at point A where $x = 1$ intersects the x-axis at point B. Find the area of the shaded region.
Leave your answer in **exact form.** [7]

i	$\int_2^8 x \, dy = 32-(4\times1)+(7\times2)$ $=42$
ii	$y=e^x$ $\frac{dy}{dx}=e^x$ $m_{normal}=-\frac{1}{e}$ Subs $(1,e), m_{normal}=-\frac{1}{e}$, $e=-\frac{1}{e}(1)+c$ $c=e+\frac{1}{e}$ Normal: $y=-\frac{1}{e}x+e+\frac{1}{e}$ Sub $y=0$, $-\frac{1}{e}x+e+\frac{1}{e}=0$ $\frac{1}{e}x=e+\frac{1}{e}$ $x=e^2+1$ $\int_0^1 e^x \, dx=[e^x]_0^1$ $=e-1$ Area of triangle $=\frac{1}{2}(e^2+1-1)(e)$ $=\frac{1}{2}e^3$ Area of region $=\frac{1}{2}e^3+e-1$

END OF PAPER

BP~405

Name | Reg. No | Class

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4E/5N

ADDITIONAL MATHEMATICS

4049/02

PAPER 2 [90 marks]

PRELIMINARY EXAMINATION

23 August 2024

2 hours 15 minutes

Candidates answer in the Question Paper
No additional material required

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer ALL questions.

You are reminded of the need for clear presentation in your answers.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**.

Brand / Model of Calculator

For Examiner's Use

This question paper consists of 17 printed pages, including 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Find the remainder when $3x^3 - x^2 + 4x + 2$ is divided by $x^2 + 1$. [2]

(b) The remainder when $x^3 + ax$, where a is a constant, is divided by $x + 2$ is the same as the remainder when it is divided by $x - 1$. Find the value of a. [3]

2 The area and width of a rectangle is $\left(19 - 3\sqrt{5}\right)$ cm^2 and $\left(2 + 2\sqrt{5}\right)$ cm respectively. Express the length of the rectangle in the form of $\left(a\sqrt{5} + b\right)$ cm where a and b are rational numbers. [5]

[Turn Over

3 (i) Find the term independent of x and the $\frac{1}{x^3}$ term in the binomial expansion of

$\left(x^2 - \frac{2}{x}\right)^9$. [4]

(ii) Hence, find the term independent of x in the expansion of

$\left(3 - x^3\right)\left(x^2 - \frac{2}{x}\right)^9$. [2]

4 Given that $\sin A = -\frac{3}{5}$ and $\cos B = -\frac{8}{17}$, where angle A and angle B are in the same quadrant, find the **exact values** of

(i) $\tan(A+B)$ [3]

(ii) $\cos\frac{B}{2}$ [3]

5 **(a)** A curve has the equation $y = \frac{2x-3}{3x+4}$, where $x \neq -\frac{4}{3}$.

The normal to the point P on the curve where $x > 0$, is parallel to the line $y = -\frac{25}{17}x + 2$. Find the coordinates of P. [4]

(b) Show the function $y = \frac{4}{3}x^3 - x^2 + 3x - 10$ is always increasing for all real values of x. [3]

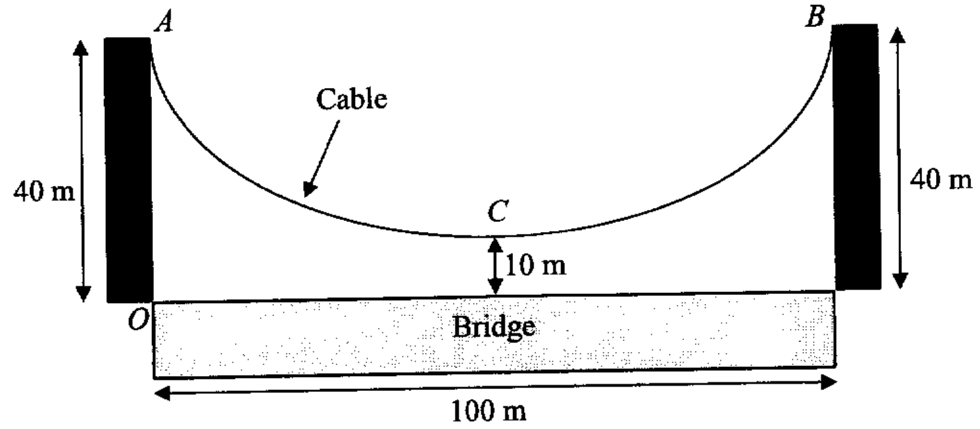
6 The line $y+2x=12$ intersects the curve $y=2x^2-6x-4$ at two distinct points A and B.

(i) Find the coordinates of A and of B. [3]

(ii) Hence, find the equation of the perpendicular bisector of AB. [4]

7 **(a)** Find the range of values of x that satisfy the inequality $5x-3<2x(5-x)$. [3]

(b)



The diagram shows a horizontal bridge of 100 metres supported by 2 pillars at the side, with a cable being suspended from point A and B. Each pillar is vertical and is 40 metres tall. The lowest point C of the cable is 10 meters above the bridge. The origin, O, is vertically below point A, where the foot of the pillar meets the bridge.

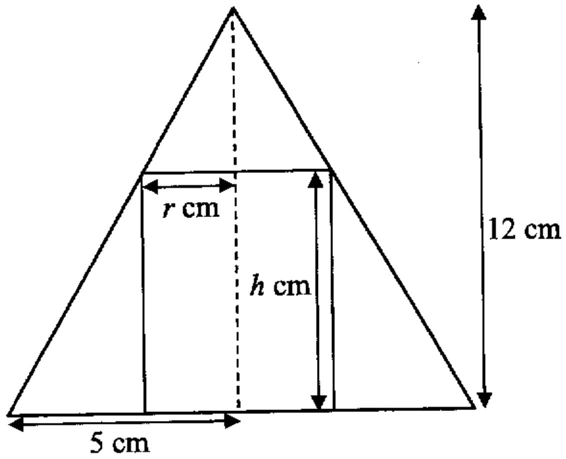
A quadratic function can be used to model the cable. Find the quadratic equation. [3]

8 The function $f(x)$, a polynomial of degree four, has stationary points at $A(1, 5)$ and $B(4, 0)$.
$f(x)$ is an increasing function when $x > 4$.
$f(x)$ is **not an increasing** function when $x < 4$.

(i) State the nature of the stationary points A and B. Explain your answer. [4]

(ii) Given that $f'(x) = a(x-1)^2(x-4)$, where a is a non-zero constant, find an expression for $f(x)$. [5]

9



A cone of height 12 cm and base radius 5 cm is placed over a cylinder of radius r cm and height h cm. The cone is in contact with the cylinder along the cylinder's upper rim. The diagram shows a vertical cross-section of the cone and the cylinder.

(i) Express h in terms of r. [2]

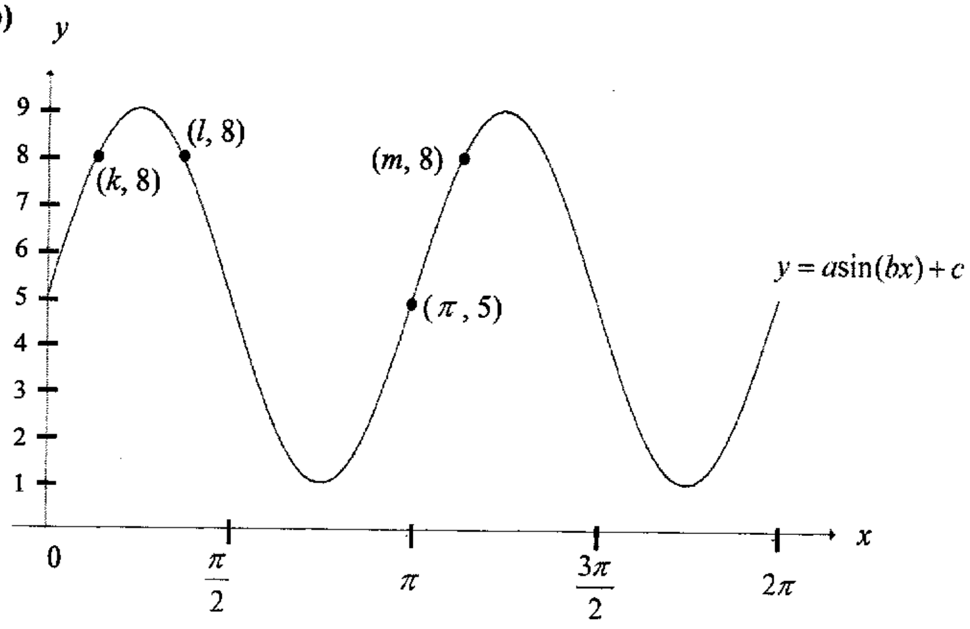
(ii) Hence show that the volume, V cm^3, of the cylinder is given by

$V = 12\pi r^2 - \frac{12}{5}\pi r^3$. [1]

(iii) Calculate the maximum value of V. [5]

10 **(a)** Calculate, in degrees, the **principal values** of x which satisfy $2\sin 2x + 3\cos x = 0$. [3]

10 (b)



The diagram above shows part of the graph of $y = a\sin(bx) + c$, passing through points (π, 5), (k, 8), (l, 8) and (m, 8), where a, b, c, k, l and m are constants.

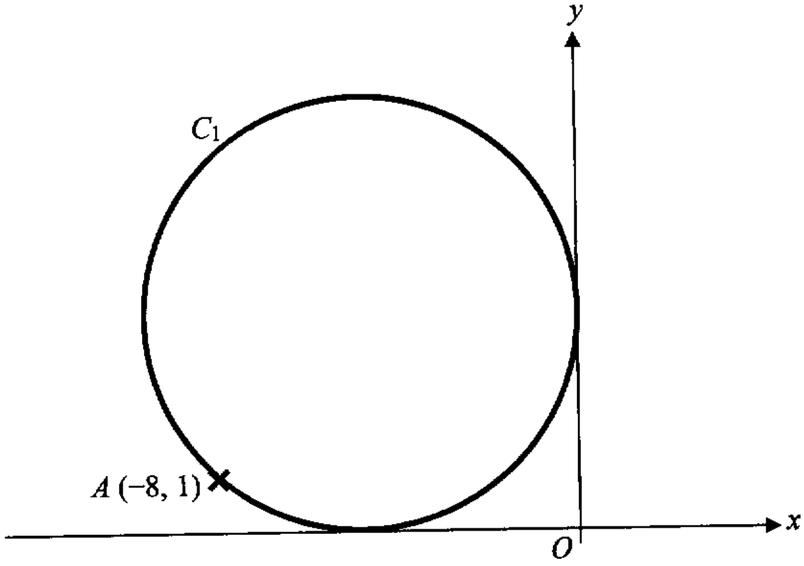
(i) Write down the values of a, of b and of c. [3]

Using the symmetry of the graph, or otherwise, find an equation connecting

(ii) π, k and m, [1]

(iii) π, k and l. [1]

11 (a)



The negative x-axis and the positive y-axis are tangents to the circle C_1 with radius r. $A(-8, 1)$ lies on C_1.

(i) What can be deduced about the coordinates of the centre of C_1? [1]

(ii) Find the equation of C_1. [5]

11 **(b)** Another circle, C_2 has equation $x^2+y^2-14x-20y=-113$.
Let P be the centre of C_2.

(i) State the coordinates of P and calculate the radius of C_2. [3]

S is a point outside C_2. Q and R are two distinct points on C_2.

(ii) Explain the relationship of SQ and SR if angle QSR is maximum. [2]

12 A particle travelling in a straight line passes through a fixed point O with a velocity of 3 m/s. The acceleration, a m/s^2, of the particle, t seconds after passing through O, is given by $a = -e^{-0.3t}$. The particle comes to instantaneous rest at the point P.

(i) Show that the particle reaches P when $t = \frac{10}{3}\ln 10$. [6]

(ii) Calculate the distance OP. [4]

(iii) Explain why the particle is again at O at some instant during the thirty-fourth second. [2]

End of Paper

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2024 4E5N AM Prelim P2 MS

Qn	Solution
1a	$\begin{array}{r} 3x-1 \\ x^2+1\overline{)3x^3-x^2+4x+2} \\ 3x^3 \quad\quad +3x \\ -x^2+x+2 \\ -x^2 \quad\quad -1 \\ x+3 \end{array}$
1b	$(-2)^3+a(-2)=1+a$ $-8-2a=1+a$ $-9=3a$ $a=-3$
2	$\dfrac{19-3\sqrt{5}}{2+2\sqrt{5}}$ $=\dfrac{19-3\sqrt{5}}{2+2\sqrt{5}}\times\dfrac{2-2\sqrt{5}}{2-2\sqrt{5}}$ $=\dfrac{38-38\sqrt{5}-6\sqrt{5}+30}{4-4(5)}$ $=\dfrac{68-44\sqrt{5}}{-16}$ $=-\dfrac{17}{4}+\dfrac{11}{4}\sqrt{5}$
3i	$T_{r+1}=\binom{9}{r}\left(x^2\right)^{9-r}\left(-\dfrac{2}{x}\right)^r$ $=\binom{9}{r}(-2)^r x^{18-3r}$ $0=18-3r$ $r=6$ $T_7=5376$ $-3=18-3r$ $r=7$ $T_8=-4608\left(\dfrac{1}{x^3}\right)$
3ii	Term indep of $x=3(5376)+(-1)(-4608)$ $=20736$

4i	$\tan A = \frac{3}{4}$ $\tan B = \frac{15}{8}$ $\tan(A+B) = \dfrac{\frac{3}{4}+\frac{15}{8}}{1-\frac{3}{4}\left(\frac{15}{8}\right)}$ $= -\frac{84}{13}$ or $-6\frac{6}{13}$
4ii	$\cos B = 2\cos^2\frac{B}{2} - 1$ $\cos^2\frac{B}{2} = \frac{1}{2}(\cos B + 1)$ $= \frac{1}{2}\left(-\frac{8}{17}+1\right)$ $= \frac{9}{34}$ $\cos\frac{B}{2} = -\frac{3}{\sqrt{34}}$ $180^\circ < B < 270^\circ$ $90^\circ < \frac{B}{2} < 135^\circ$
5a	$y = \frac{2x-3}{3x+4}$ $\frac{dy}{dx} = \frac{(3x+4)2-(2x-3)3}{(3x+4)^2}$ $= \frac{17}{(3x+4)^2}$ $\frac{17}{25} = \frac{17}{(3x+4)^2}$ $(3x+4)^2 = 25$ $x = \frac{1}{3}$ $y = -\frac{7}{15}$ $P(\frac{1}{3}, -\frac{7}{15})$

5b	$\frac{dy}{dx}=4x^2-2x+3$ $=4\left(x^2-\frac{x}{2}\right)+3$ $=4\left(\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2\right)+3$ $=4\left(x-\frac{1}{4}\right)^2+\frac{11}{4}$ $\left(x-\frac{1}{4}\right)^2\geq 0$ $4\left(x-\frac{1}{4}\right)^2+\frac{11}{4}\geq\frac{11}{4}$ >0 $\frac{dy}{dx}>0$ Therefore, y is always increasing for all real values of x
6i	$12-2x=2x^2-6x-4$ $0=2x^2-4x-16$ $0=x^2-2x-8$ $0=(x+2)(x-4)$ $x=-2$ or 4 $y=16$ or 4 (−2, 16) and (4, 4)
6ii	midpt $=(\frac{-2+4}{2},\frac{16+4}{2})$ $=(1,10)$ grad $=\frac{16-4}{-2-4}$ $=-2$ grad of perpen bisector $=\frac{1}{2}$ eqn of perpen bisector: $y-10=\frac{1}{2}(x-1)$ $y=\frac{1}{2}x+\frac{19}{2}$

7a	$5x-3<2x(5-x)$ $5x-3<10x-2x^2$ $2x^2-5x-3<0$ $(2x+1)(x-3)<0$ $-\frac{1}{2}<x<3$
7b	$y=a(x-50)^2+10$ $40=a(-50)^2+10$ $a=\frac{3}{250}$ $y=\frac{3}{250}(x-50)^2+10$
8i	see below

$x=0.9$	$x=1$	$x=1.1$
$\frac{dy}{dx}<0$	$\frac{dy}{dx}=0$	$\frac{dy}{dx}<0$

A is a point of inflexion

$x=3.9$	$x=4$	$x=4.1$
$\frac{dy}{dx}<0$	$\frac{dy}{dx}=0$	$\frac{dy}{dx}>0$

B is a minimum point

8ii	$\frac{dy}{dx}=a(x-1)^2(x-4)$ $=a(x^2-2x+1)(x-4)$ $=a(x^3-6x^2+9x-4)$ $y=a(\frac{1}{4}x^4-2x^3+\frac{9}{2}x^2-4x)+c$ $0=a(64-128+72-16)+c$ $8a=c$ $5=a(\frac{1}{4}-2+\frac{9}{2}-4)+c$ $5=-\frac{5}{4}a+8a$ $a=\frac{20}{27}$ $c=\frac{160}{27}$ $y=\frac{20}{27}(\frac{1}{4}x^4-2x^3+\frac{7}{2}x^2-4x)+\frac{160}{27}$ ⟶ $=\frac{5}{27}x^4-\frac{40}{27}x^3+\frac{70}{27}x^2-\frac{80}{27}x+\frac{160}{29}$
9i	$\frac{12-h}{12}=\frac{r}{5}$ $12-h=\frac{12r}{5}$ $h=12-\frac{12r}{5}$
9ii	$V=\pi r^2h$ $=\pi r^2(12-\frac{12r}{5})$ $=12\pi r^2-\frac{12}{5}\pi r^3$

9iii	$V = 12\pi r^2 - \frac{12}{5}\pi r^3$ $\frac{dV}{dr} = 24\pi r - \frac{36}{5}\pi r^2$ $0 = 24\pi r - \frac{36}{5}\pi r^2$ $0 = 12\pi r(2 - \frac{3}{5}r)$ $r = 0$ (rej) or $\frac{10}{3}$ $\frac{d^2V}{dr^2} = 24\pi - \frac{72}{5}\pi r$ $\left.\frac{d^2V}{dr^2}\right\|_{r=\frac{10}{3}} = -24\pi$ < 0 $\therefore V$ is max when $r = \frac{10}{3}$ $V = \frac{400}{9}\pi$ or 139.62……
10a	$2\sin 2x + 3\cos x = 0$ $4\sin x\cos x + 3\cos x = 0$ $\cos x(4\sin x + 3) = 0$ $\cos x = 0$ or $\sin x = -\frac{3}{4}$ PV of $x = 90^\circ$ or -48.6°
10bi	$a = 4$ $c = 5$ $\pi = \frac{2\pi}{b}$ $b = 2$
10bii	$m = k + \pi$
10biii	$\frac{k+l}{2} = \frac{\pi}{4}$ or $k + l = \frac{\pi}{2}$ etc
11ai	The centre of C_1 looks like $(-r, r)$

11aii	$(-r+8)^2+(r-1)^2=r^2$ $r^2-16r+64+r^2-2r+1=r^2$ $r^2-18r+65=0$ $(r-5)(r-13)=0$ $r=5$ or 13 (rej) $(x+5)^2+(y-5)^2=25$
11bi	$P\,(7, 10)$ $r=\sqrt{7^2+10^2-113}$ $=6$
11bii	$\angle QSR$ max $\Rightarrow$ SQ and SR are tangents to circle $\Rightarrow$ $SQ = SR$ since they met at an external point
12i	$v=\int -e^{-0.3t}dt$ $=\dfrac{-e^{-0.3t}}{-0.3}+c$ $=\dfrac{10}{3}e^{-0.3t}+c$ $3=\dfrac{10}{3}e^{0}+c$ $c=-\dfrac{1}{3}$ $v=\dfrac{10}{3}e^{-0.3t}-\dfrac{1}{3}$ $0=\dfrac{10}{3}e^{-0.3t}-\dfrac{1}{3}$ $\dfrac{1}{10}=e^{-0.3t}$ $\ln\dfrac{1}{10}=-0.3t$ $t=-\dfrac{10}{3}\ln\dfrac{1}{10}$ $=\dfrac{10}{3}\ln 10$

12ii	$v=\frac{10}{3}e^{-0.3t}-\frac{1}{3}$ $s=\int\frac{10}{3}e^{-0.3t}-\frac{1}{3}dt$ $=-\frac{100}{9}e^{-0.3t}-\frac{1}{3}t+c$ $0=-\frac{100}{9}e^{0}-\frac{1}{3}(0)+c$ $c=\frac{100}{9}$ $s=-\frac{100}{9}e^{-0.3t}-\frac{1}{3}t+\frac{100}{9}$ $t=\frac{10}{3}\ln 10$ $s=-\frac{100}{9}e^{-\ln 10}-\frac{1}{3}\left(\frac{10}{3}\ln 10\right)+\frac{100}{9}$ $=10-\frac{10}{9}\ln 10$ $=7.44$
12iii	When $t=33$, $s=0.111$ When $t=34$, $s=-0.223$ Since displacement changes sign from $t=33$ to $t=34$, the particle is again at O during the 34th second.