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ORCHID PARK SECONDARY SCHOOL
Preliminary Examination 2024

CANDIDATE NAME

CLASS

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INDEX NUMBER

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ADDITIONAL MATHEMATICS**4049/01**

Paper 1

20 August 2024

Secondary 4 Express / 5 Normal (Academic)

2 hours 15 minutes**90 Marks**

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

Use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.If **working** is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks of this paper is 90.

For Examiner's Use	
Total	

This document consists of **19** printed pages.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle = \frac{1}{2} bc \sin A$$

3

- 1 Find the range of values of p for which the line $y = px - 5$ meets the curve $y = 3x^2 + 4x - 2$. [4]

- 2 Express $\frac{5x^2 - 6x + 13}{(x-1)(x^2+3)}$ in partial fractions. [5]

4

3 Prove that $2 \cot 2\theta = \operatorname{cosec} \theta \sec \theta - 2 \tan \theta$.

[5]

5

4 (a) Find $\frac{d}{dx}(5xe^{2x+1})$.

[2]

(b) Hence find $\int xe^{2x+1} dx$.

[4]

6

5 Solve the equation $1 + 3 \sin^2 \theta = 4 \cos \theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

[6]

6 The function f is given by $f(x) = \frac{x^2}{x-2k}$, for $x > 2k$, where k is a positive constant.

(a) Find $f'(x)$.

[2]

The function g , defined for $x > 2k$, has the property that $g'(x) = (x - 2k)^2 f'(x)$.
 g decreases for $k < x < 6$.

(b) Show that a possible value of k is 3.

[4]

- 7 Prove that there are no values of k for which $kx^2 + 2x - 2k - 3$ is always positive. [6]

9

- 8 The line $y - x = 2$ intersects the curve $y^2 = 4(2x + 1)$ at two points.
Find the coordinates of these two points.

[5]

9 A curve has equation $y = x^3 + mx - 15$. It has a stationary point A where $x = 2$.

(a) Show that the value of the constant m is -12 .

[2]

(b) Find the coordinates of the other stationary point B .

[2]

11

It is given that P is a point on the curve where the **gradient is a minimum**.

(c) Find the coordinates of the point P .

[3]

(d) Prove that the gradient is a minimum at P .

[2]

10 The equation of a circle C is $x^2 + y^2 - 4x - 6y - 12 = 0$.

(a) Find the coordinates of the centre of C , and the radius of C .

[4]

(b) Find the coordinates of the points at which the circle intersects the x -axis.

[3]

(c) State an equation of the circle which is a reflection of C in the y -axis. [2]

(d) Explain whether the circle in **part (c)** lies entirely in the 2nd quadrant. [1]

- 11 An open cylinder has radius r cm and total surface area A cm².
It is given that $\frac{dA}{dr} = 2\pi(r + k)$.

(a) Find an expression for A in terms of r .

[2]

(b) Express the height of the cylinder in terms of k .

[1]

15

The radius of the cylinder is increasing with the height remaining constant.

It is given that, at time t seconds, $\frac{d^2r}{dt^2} = \frac{5}{2t+1}$.

It is also known that initially, the radius was increasing at 3 cm/s.

(c) Find an expression for $\frac{dr}{dt}$. [2]

(d) Hence find the rate of increase of the total surface area of the cylinder after 4 seconds, given that the radius is 15 cm and $k = 10$ at this instant. [3]

- 12 A piece of wire, 100 cm in length, is divided into two parts. One part is bent to form a square of side x cm, and the other square of side y cm.

(a) Express y in terms of x . [2]

(b) Find the total area, A cm², of the two squares, leaving your answer in the form $p(x + q)^2 + r$, where p , q and r are constants. [4]

- (c) Hence state the minimum total area of the two squares, and the value of x at which this occurs.

[2]

- (d) When $x = x_1$, the total area of the two squares is A_1 , where $0 < x_1 < 12\frac{1}{2}$. State another value of x , in terms of x_1 , which also gives a total area of A_1 .

[1]

- 13 The perpendicular bisector of the line joining the points $A(3, 2h)$ and $B(-7, -10)$ passes through the point $X(h, 3)$, where h is a constant.

(a) Find the mid-point M of AB .

[2]

(b) Find the gradient of AB .

[2]

(c) Hence find the possible values of h .

[7]

Paper-1

Answers

BP-425

Qn	Solution	Mk	RHS	Remarks
1	$y = px - 5 \quad \text{--- (1)}$ $y = 3x^2 + 4x - 2 \quad \text{--- (2)}$ <p>Sub (1) into (2):</p> $3x^2 + 4x - 2 = px - 5$ $3x^2 + (4 - p)x + 3 = 0$ <p>Line meets curve \Rightarrow Discriminant ≥ 0:</p> $(4 - p)^2 - 4(3)(3) \geq 0$ $16 - 8p + p^2 - 36 \geq 0$ $p^2 - 8p - 20 \geq 0$ $(p - 10)(p + 2) \geq 0$ $p \leq -2 \text{ or } p \geq 10$	M1		
2	<p>Let $\frac{5x^2 - 6x + 13}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3}$</p> $5x^2 - 6x + 13 = A(x^2 + 3) + (Bx + C)(x - 1)$ <p>Method 1:</p> <p>When $x = 1, 12 = 4A$ $A = 3$</p> <p>When $x = 0, 13 = 3(3) + C(-1)$ $C = -4$</p> <p>When $x = -1, 24 = 3(4) + (-B - 4)(-2)$ $B = 2$</p> <p>Hence, $\frac{5x^2 - 6x + 13}{(x-1)(x^2+3)} = \frac{3}{x-1} + \frac{2x-4}{x^2+3}$</p>	M1		
3	<p>RHS</p> $= \operatorname{cosec} \theta \sec \theta - 2 \tan \theta$ $= \frac{1}{\sin \theta} \left(\frac{1}{\cos \theta} \right) - \frac{2 \sin \theta}{\cos \theta}$ $= \frac{1}{\sin \theta \cos \theta} - \frac{2 \sin^2 \theta}{\sin \theta \cos \theta}$ $= \frac{1 - 2 \sin^2 \theta}{\sin \theta \cos \theta}$ $= \frac{\cos 2\theta}{\frac{1}{2} \sin 2\theta}$ $= \frac{2 \cos 2\theta}{\sin 2\theta} = 2 \cot 2\theta = LHS$	M1	M1	<p>for changing either cosec, sec or tan correctly</p> <p>for common denominator</p> <p>1 mark for $\cos 2\theta$, 1 mark for $\sin 2\theta$</p>
		M2		
		A1		

4	(a)	$5e^{2x+1} + 10xe^{2x+1}$	B2	1 mark for each term
	(b)	$5xe^{2x+1} = \frac{5}{2}e^{2x+1} + 10 \int x e^{2x+1} dx + c_1$ M1 M1 where c_1 is an arbitrary constant $5xe^{2x+1} - \frac{5}{2}e^{2x+1} - c_1 = 10 \int x e^{2x+1} dx$ $\int x e^{2x+1} dx = \frac{1}{2}xe^{2x+1} - \frac{1}{4}e^{2x+1} + c$ where $c = -\frac{1}{10}c_1$	M1 A1	
5		$1 + 3(1 - \cos^2 \theta) = 4 \cos \theta$ $-3 \cos^2 \theta - 4 \cos \theta + 4 = 0$ Let $y = \cos \theta$ $-3y^2 - 4y + 4 = 0$ $3y^2 + 4y - 4 = 0$ $(3y - 2)(y + 2) = 0$ $y = \frac{2}{3} \text{ or } y = -2$ $\cos \theta = \frac{2}{3} \text{ or } \cos \theta = -2 \text{ (rej)}$ $\theta = 0.841 \text{ or } -0.841 \text{ (3 s.f.)}$	M1 M1 M2 A2	
6	(a)	$\frac{f'(x)}{(x-2k)(2x) - x^2(1)}$ $= \frac{(x-2k)^2}{(x-2k)^2}$ $= \frac{x^2 - 4kx}{(x-2k)^2}$	M1 A1	

6	(b)	$g'(x)$ $= x^2 - 4kx$ $= x(x - 4k)$ <p>g decreases $\Rightarrow g'(x) < 0$</p> $x(x - 4k) < 0$ $0 < x < 4k$ <p>Since $x > 2k$, then $2k < x < 4k$ _____ (1)</p> <p>But it is also given that $k < x < 6$, i.e. $2k < x < 12$ _____ (2)</p> <p>By (1) and (2), $4k = 12$ $k = 3$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	
7		<p>For the function to be always positive, $k > 0$ (so that the graph is U-shaped) _____ (1)</p> <p>We also require discriminant < 0 (so that graph never cuts x-axis)</p> $2^2 - 4(k)(-2k - 3) < 0$ $2k^2 + 3k + 1 < 0$ $(k + 1)(2k + 1) < 0$ $-1 < k < -\frac{1}{2}$ _____ (2) <p>But (1) and (2) cannot happen at the same time (k cannot be positive but yet also be between -1 and $-\frac{1}{2}$)</p> <p>\therefore There is no value of k for which the function is positive (proven)</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	

8		$y - x = 2$ $y = x + 2 \text{ _____ (1)}$ $y^2 = 4(2x + 1) \text{ _____ (2)}$ <p>Sub (1) into (2):</p> $(x + 2)^2 = 4(2x + 1)$ $x^2 + 4x + 4 = 8x + 4$ $x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0 \text{ or } x = 4$ <p>Sub into (1):</p> $y = 2 \text{ or } y = 6$ <p>The two points are (0, 2) and (4, 6)</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	
9	(a)	$\frac{dy}{dx} = 3x^2 + m$ <p>When $x = 2, \frac{dy}{dx} = 0$ (given):</p> $0 = 3(2)^2 + m$ $m = -12$	<p>M1</p> <p>A1</p>	
	(b)	$\frac{dy}{dx} = 3x^2 - 12$ <p>For stationary points, $\frac{dy}{dx} = 0$</p> $3x^2 - 12 = 0$ $x^2 = 4$ $x = 2 \text{ or } -2$ <p>When $x = -2,$</p> $y = (-2)^3 - 12(-2) - 15$ $y = 1$ <p>B is (-2, 1)</p>	<p>M1</p> <p>A1</p>	

	(c)	For gradient to be a min, $\frac{d^2y}{dx^2} = 0$ $6x = 0$ $x = 0$ When $x = 0$, $y = -15$ P is (0, -15)	M1 M1 A1	
	(d)	$\frac{d^3y}{dx^3} = 6$ Since $\frac{d^3y}{dx^3}$ is positive, the gradient is a minimum.	M1 A1	
10	(a)	Method 1: $(x - 2)^2 + (y - 3)^2 - 2^2 - 3^2 - 12 = 0$ $(x - 2)^2 + (y - 3)^2 = 25$ Method 2: $2g = -4$ $2f = -6$ $g = -2$ $f = -3$ $g^2 + f^2 - c = 4 + 9 + 12 = 25$ Centre = (2, 3) Radius = 5 units	M1 M1 M1 M1 A1 A1	
	(b)	When $y = 0$, $x^2 - 4x - 12 = 0$ $(x - 6)(x + 2) = 0$ $x = 6$ or $x = -2$ The points are (6, 0) and (-2, 0)	M1 A2	
	(c)	Centre = (-2, 3), radius = 5 units Eqn: $(x + 2)^2 + (y - 3)^2 = 25$	M1 A1 / B2	
	(d)	Since the centre is only 2 units away from the y-axis and the radius is 5 units, the circle will cut the y-axis and does not lie entirely in the 2 nd quadrant.	M1 A1	Accept any similar answer (e.g. centre is 3 units away from x-axis)

11	(a)	$A = \pi r^2 + 2\pi kr + c$ where c is an arbitrary constant When $r = 0, A = 0$: (when there is no radius, there is no area) $c = 0$ Hence, $A = \pi r^2 + 2\pi kr$	M1 A1	
	(b)	Curved surface area: $2\pi rh = 2\pi kr$ $h = k$	B1	
	(c)	$\frac{dr}{dt} = \frac{5}{2} \ln(2t + 1) + c_1$ where c_1 is an arbitrary constant When $t = 0, \frac{dr}{dt} = 3$: $3 = c_1$ $\frac{dr}{dt} = \frac{5}{2} \ln(2t + 1) + 3$	M1 A1	
	(d)	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $\frac{dA}{dt} = 2\pi(r + k) \times \left[\frac{5}{2} \ln(2t + 1) + 3 \right]$ When $r = 15, t = 4, k = 10$: $\frac{dA}{dt} = 2\pi(15 + 10) \times \left(\frac{5}{2} \ln 9 + 3 \right)$ $\frac{dA}{dt} =$	M1 M1 A1	
12	(a)	$4x + 4y = 100$ $y = 25 - x$	M1 A1	
	(b)	A $= x^2 + y^2$ $= x^2 + (25 - x)^2$ $= 2x^2 - 50x + 625$ $= 2 \left[\left(x - \frac{25}{2} \right)^2 - \left(\frac{25}{2} \right)^2 + \frac{625}{2} \right]$ $= 2 \left(x - \frac{25}{2} \right)^2 + \frac{625}{2}$	M1 M1 M1 A1	

	(c)	$\text{Min area} = \frac{625}{2}$ $\text{when } x = \frac{25}{2}$	B1 B1	
	(d)	$25 - x_1$ <p>(E.g. if $x_1 = 10$, then $25 - x_1 = 15$ will give the same area)</p>	B1	
13	(a)	$\left(\frac{3-7}{2}, \frac{2h-10}{2} \right)$ $= (-2, h-5)$	M1 A1	
	(b)	$\frac{2h+10}{\frac{3+7}{h+5}}$ $= \frac{5}{5}$	M1 A1	
	(c)	<p>Let the equation of the perpendicular bisector be</p> $\frac{y-(h-5)}{x-(-2)} = -\frac{5}{h+5}$ <p>Sub (h, 3):</p> $\frac{3-(h-5)}{h-(-2)} = -\frac{5}{h+5}$ $h^2 - 8h - 50 = 0$ $h = \frac{8 \pm \sqrt{8^2 - 4(1)(-50)}}{2(1)}$ $h = 12.1 \text{ or } -4.12 \text{ (3 s.f.)}$	M2 M1 M1 M1 A2	1 mark for substituting M 1 mark for gradient



ORCHID PARK SECONDARY SCHOOL

Preliminary Examination 2024

CANDIDATE NAME

CLASS

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INDEX NUMBER

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ADDITIONAL MATHEMATICS**4049/02**

Paper 2

23 August 2024

Secondary 4 Express / 5 Normal (Academic)

2 hours 15 minutes**90 Marks**

Additional Materials: NIL

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle = \frac{1}{2}bc \sin A$$

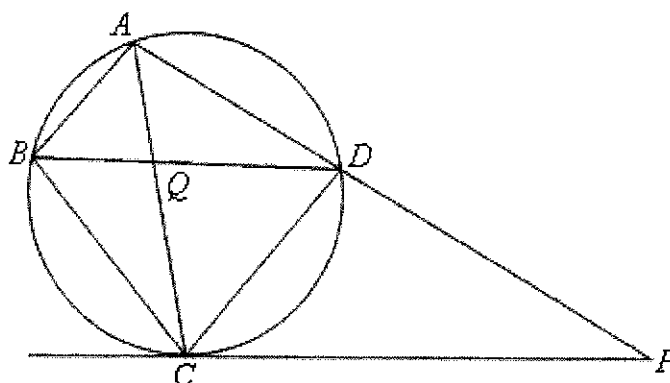
- 1 (a) Expand $\left(x + \frac{1}{x}\right)^4$ in descending powers of x . [2]

- (b) Hence, given that $\left(x + \frac{1}{x}\right)^4 - \left(x - \frac{1}{x}\right)^4 = ax^2 + \frac{b}{x^2}$, find the value of a and of b . [3]

(c) Given that there is no x term in the expansion of $\left(\frac{4}{3}x + \frac{k}{x} + \frac{x^3}{k}\right)\left(x + \frac{1}{x}\right)^4$, find the value of k .

[3]

- 2 In a diagram, A, B, C and D are points on the circle. The tangent at C meets AD produced at P . The chords AC and BD intersect at Q . The line BQD bisects angle ABC .



Prove that

(a) $\angle DCP = \angle ACD$, [3]

(b) $\triangle PCD$ is similar to $\triangle PAC$, [2]

(c) $PC^2 = PA \times PD$. [1]

3 In recent years, the release of greenhouse gases has accelerated the melting of glaciers and thus, resulting in a rise of global temperatures. With minimal actions taken to prevent global warming, the average temperature, $T^{\circ}\text{C}$, projected to rise after x years from 2024, is given by $T = 31(1.5)^{kx}$, where k is a constant.

- (a) Given that the projected average temperature in Yishun in 2027 is 35°C , find the value of k correct to 1 decimal place. [2]
- (b) Find the average temperature in Yishun in 2024. [1]
- (c) In which year will the average temperature in Yishun first be at least 15% higher than its temperature in 2024? [4]
- (d) Sketch the graph of T against x . [2]

4 (a) The function f is given by $f(x) = a\sin 3x + b$, where a and b are positive integers and $0 \leq x \leq \pi$. The maximum and minimum value of f are 6 and -2 respectively.

(i) State the period of f . [1]

(ii) Find the value of a and of b . [2]

(iii) Sketch the graph of $y = -f(x)$. [3]

4(b)

Prove the identity $\frac{\sin \theta - \sin 2\theta + \sin 3\theta}{\cos \theta - \cos 2\theta + \cos 3\theta} = \tan 2\theta$.

[4]

5 A particle moving in a straight line is such that its displacement, s metres, from a fixed point O , is given by $s = 4 - 2e^{-t} - t$ where t is the time in seconds after passing through a point B on the line. Find

(a) the distance OB , [1]

(b) the initial velocity of the particle, [2]

(c) the value of t when the particle is instantaneously at rest, [2]

(d) the total distance travelled by the particle in the first two seconds.

[3]

6 The equation of a polynomial is given by $P(x) = 2x^3 + mx^2 + x + n$, where m and n are constants.

(a) Given that $2x^2 + x - 1$ is a factor of $P(x)$, find the value of m and of n .

[4]

(b) Hence solve the equation $P(x) = 0$.

[3]

(c) Using the results in parts (a) and (b), solve the equation $\frac{1}{4}y^3 + \frac{m}{4}y^2 + \frac{1}{2}y + n = 0$. [3]

7 Two simultaneous equations are given:

$$64^x \times 4^y = 1,$$

$$\log_{\frac{1}{2}}(x - 3y - 6) + 2 = \log_2 x.$$

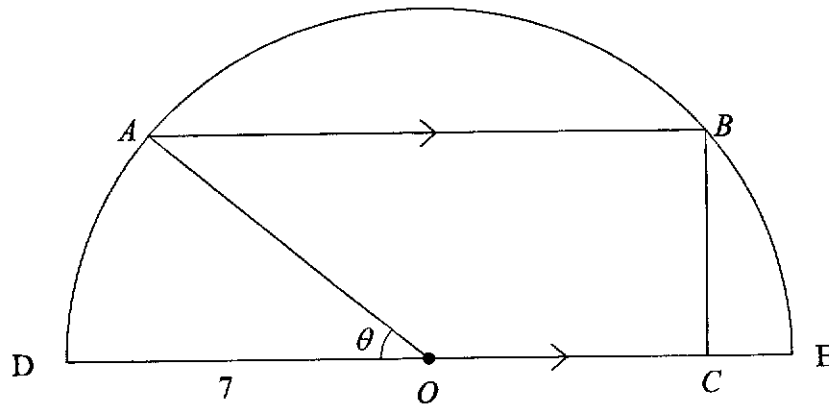
(a) Show that $\log_{\frac{1}{2}}(x - 3y - 6) + 2 = \log_2 x$ can be written as

$$\log_2(x - 3y - 6) = \log_2 \frac{4}{x} \quad [3]$$

(b) Solve, for x and y .

[7]

8



At the Singapore Book of Records event, the students taking Additional Maths need to paste their cranes in the shape of a trapezium $OABC$ inscribed in a semicircle with centre O , and radius 7 cm. OA makes an angle θ with the diameter DE . BC is perpendicular to both AB and CO .

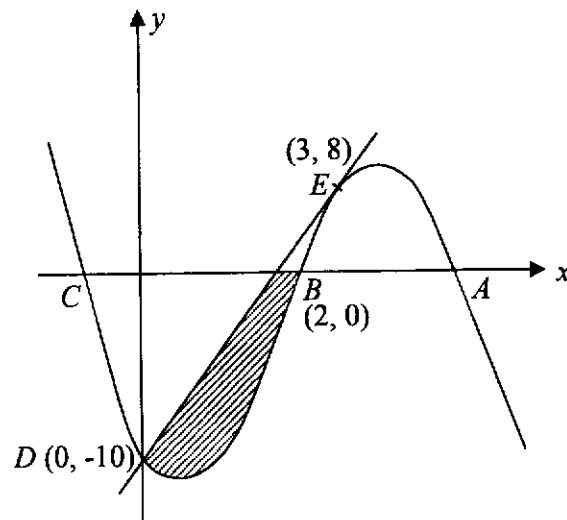
(a) State the property that shows that AB is twice the length of OC . [1]

(b) Show that P cm, the perimeter of the trapezium, can be expressed in the form $m + 21 \cos \theta + 7 \sin \theta$, where m is a constant to be found. [2]

- (c) Express P in the form $m + R \cos(\theta - \alpha)$, where $R > 0$ and θ is an acute angle in degree mode. [3]

- (d) Hence, find the maximum value of P and the corresponding value of θ at which this occurs. [3]

9



The diagram shows part of the curve $y = -x^3 + 6x^2 - 3x - 10$. The curve cuts the x -axis at A , $B(2, 0)$ and C , and cuts the y -axis at $D(0, -10)$.

(a) Show that the tangent to the curve at $E(3, 8)$ passes through D .

[5]

(b) Calculate the area of the shaded region.

[7]

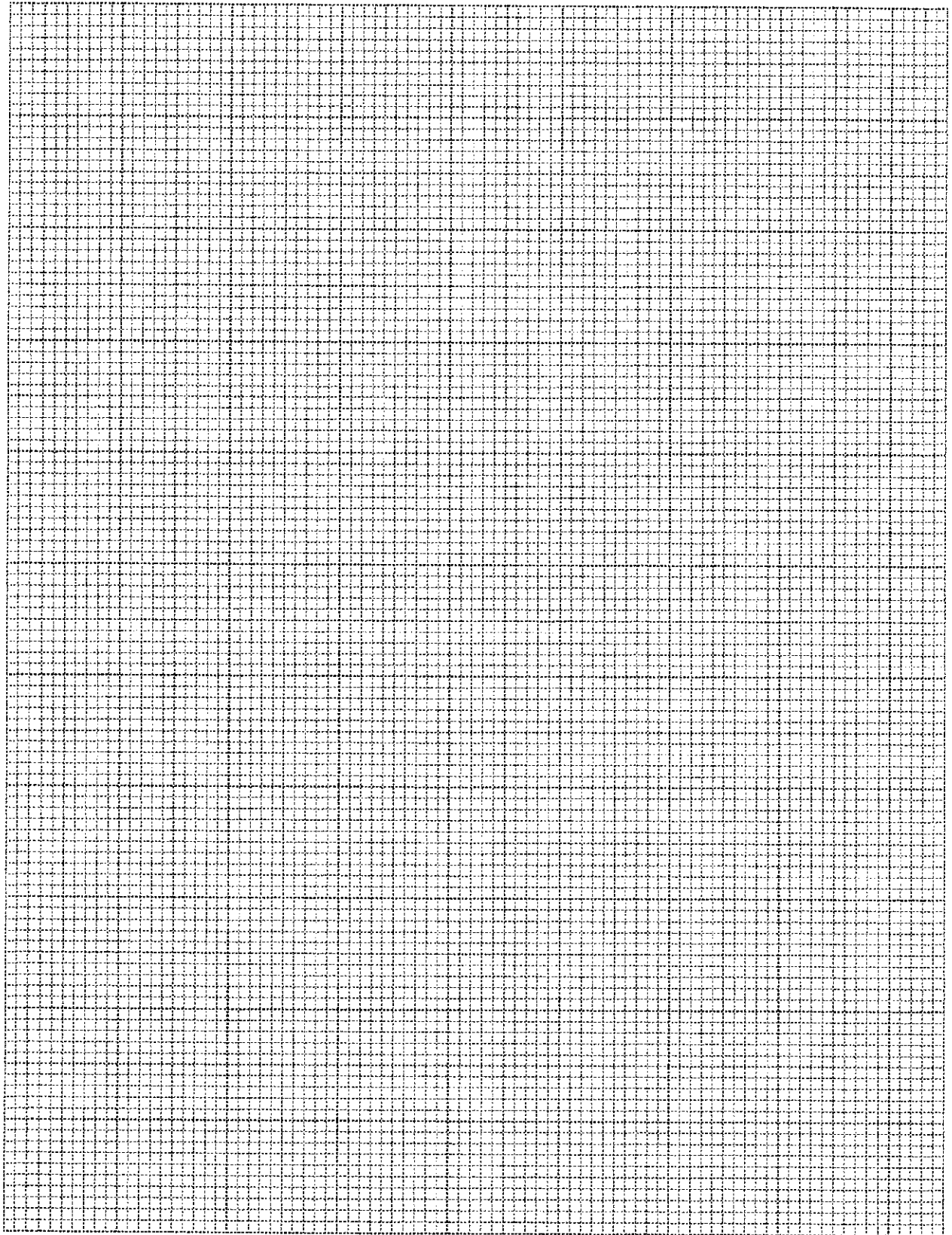
- 10 The table shows experimental values of two variables x and y when Arne Mo Salim was doing his O Level Physics Practical Exam.

x	0.2	0.6	1.0	1.4
y	0.5	1.4	2.5	4.0

It is known that x and y are related by the equation $y = ax\sqrt{x} + b\sqrt{x}$, where a and b are constants.

- (a) Explain how a straight line graph can be obtained using the above data. [2]

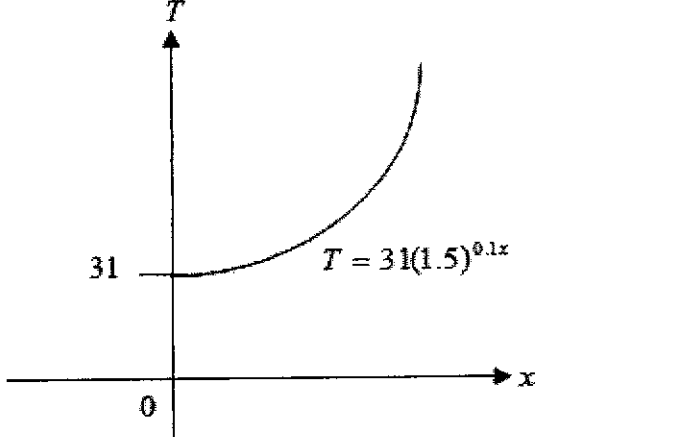
- (b) Draw this graph on the next page for the given data and use it to estimate the value of a and of b . [6]



End of Paper

AMP2 Prelims 2024

No	Answer	Marks
1(a)	$\left(x + \frac{1}{x}\right)^4 = x^4 + {}^4C_1(x^3)\left(\frac{1}{x}\right) + {}^4C_2(x^2)\left(\frac{1}{x}\right)^2 + {}^4C_3(x^1)\left(\frac{1}{x}\right)^3 + {}^4C_4(x^0)\left(\frac{1}{x}\right)^4$ $= x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4}$	M1 A1
1(b)	$\left(x + \frac{1}{x}\right)^4 - \left(x - \frac{1}{x}\right)^4$ $= x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4} - (x^4 - 4x^2 + 6 - 4x^{-2} + x^{-4})$ $= 8x^2 + 8x^{-2}$ $\therefore a = 8, b = 8$	M1 A2
1(c)	$\left(\frac{4}{3}x + \frac{k}{x} + \frac{x^3}{k}\right)\left(x + \frac{1}{x}\right)^4 = \left(\frac{4}{3}x + \frac{k}{x} + \frac{x^3}{k}\right)(x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4})$ $= \dots \frac{4}{3}x(6) + \frac{k}{x}(4x^2) + \frac{x^3}{k}(4x^{-2}) + \dots = \dots 8x + 4kx + \frac{4}{k}x + \dots \text{ (find coefficient of } x)$ <p>Since there is no x term, $8x + 4kx + \frac{4}{k}x = 0$</p> $x\left(8 + 4k + \frac{4}{k}\right) = 0$ $8 + 4k + \frac{4}{k} = 0 \quad \text{or} \quad x = 0 \text{ (NA)}$ $4k^2 + 8k + 4 = 0$ $k^2 + 2k + 1 = 0$ $(k + 1)^2 = 0$ $\therefore k = -1$	M1 M1 A1
2(a)	$\angle DCP = \angle CBD$ (tangent chord theorem) $\angle ABD = \angle CBD$ (bisected angles of triangle ABC) $\angle ABD = \angle ACD$ (angles in same segment) So $\angle DCP = \angle ACD$ (shown)	M1 M1 M1
2(b)	$\angle CPD = \angle APC$ (common angle) $\angle PCD = \angle PAC$ (tangent chord theorem) So by AA similarity test, $\triangle PCD$ similar to $\triangle PAC$	M1 A1
2(c)	Since $\triangle PCD$ similar to $\triangle PAC$ from ii), $\frac{PC}{PA} = \frac{PD}{PC}$ $PC^2 = PA \times PD \text{ (shown)}$	B1
3(a)	When $x = 3$, $T = 35$, $35 = 31(1.5)^{k(3)}$ $k = 0.1$ (1dp)	M1 A1
3(b)	When $t = 0$, $T = 31(1.5)^0 = 31^\circ\text{C}$	B1

3(c)	$115\% \text{ of initial temp} = \frac{115}{100} \times 31$ $= 35.65^\circ\text{C}$ <p>When $T = 35.65^\circ\text{C}$,</p> $35.65 = 31(1.5)^{0.1x}$ $\frac{35.65}{31} = (1.5)^{0.1x}$ $\lg\left(\frac{35.65}{31}\right) = \lg(1.5)^{0.1x}$ $\lg(1.15) = 0.1x \lg(1.5)$ $0.1x = \frac{\lg 1.15}{\lg 1.5}$ $x = \frac{\lg 1.15}{0.1}$ $= 3.45(3.s.f)$ <p>So the temperature will increase by at least 15% in 2028.</p>	M1 M1 M1 A1
3(d)	 <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> G1 Shape G1 y-intercept </div>	
4(a)(i)	Period = $\frac{2\pi}{3}$	B1
4(a)(ii)	$a + b = 6$ $-a + b = -2$ Solve simultaneously, $a = 4, b = 2$	M1A1

<p>4(a)(iii)</p>		<p>1 mark for correct number of cycles 1 mark for correct endpoints, max points 1 mark for correct shape</p>	
<p>4(b)</p>	$\frac{\sin \theta - \sin 2\theta + \sin 3\theta}{\cos \theta - \cos 2\theta + \cos 3\theta} = \tan 2\theta$ $\text{LHS} = \frac{\sin(2\theta - \theta) - \sin 2\theta + \sin(2\theta + \theta)}{\cos(2\theta - \theta) - \cos 2\theta + \cos(2\theta + \theta)}$ $= \frac{\sin 2\theta \cos \theta - \cos 2\theta \sin \theta - \sin 2\theta + \sin 2\theta \cos \theta + \cos 2\theta \sin \theta}{\cos 2\theta \cos \theta + \sin 2\theta \sin \theta - \cos 2\theta + \cos 2\theta \cos \theta - \sin 2\theta \sin \theta}$ $= \frac{2\sin 2\theta \cos \theta - \sin 2\theta}{2\cos 2\theta \cos \theta - \cos 2\theta}$ $= \frac{\sin 2\theta(2\cos \theta - 1)}{\cos 2\theta(2\cos \theta - 1)} = \tan 2\theta = \text{RHS}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p>	
<p>5(a)</p>	<p>When $t = 0, s = 4 - 2e^0 = 2 \text{ m}$</p>	<p>B1</p>	
<p>5(b)</p>	<p>$s = 4 - 2e^{-t} - t$ $V = 2e^{-t} - 1$ When $t = 0, V = 2e^{-0} - 1 = 1 \text{ m/s}$</p>	<p>M1</p> <p>A1</p>	
<p>5(c)</p>	<p>When $V = 0, 0 = 2e^{-t} - 1$ $e^{-t} = 0.5$ $e^t = 2$ $t = \ln 2 = 0.693 \text{ s (3 s.f.) (Accept } t = \ln 2)$</p>	<p>M1</p> <p>A1</p>	
<p>5(d)</p>	<p>When $t = 0, S = 2 \text{ m}$ When $t = 0.693 \text{ s}, S = 2.30685 \text{ m}$ When $t = 2 \text{ s}, S = 1.729329 \text{ m}$ } Total distance travelled $= (2.30685 - 2) + (2.30685 - 1.729329)$ $= 0.884 \text{ m (3 s.f.)}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>Correct S values found</p> <p>Accept integration method</p>	
<p>6(a)</p>	<p>$P(-1) = 0 \Rightarrow m + n = 3$ ----- (1) $P\left(\frac{1}{2}\right) = 0 \Rightarrow m + 4n = -3$ ----- (2) $m = 5$ and $n = -2$</p>	<p>M1</p> <p>M1</p> <p>A2</p>	
<p>6(b)</p>	<p>$P(x) = 2x^3 + 5x^2 + x - 2 = (2x^2 + x - 1)(ax + b)$ By using the comparison method of the powers of x, $a = 1$ and $b = 2$ (or long division method)</p>	<p>M1</p> <p>M1</p>	

	$P(x) = 0$ $\Rightarrow x = -2, x = -1, x = \frac{1}{2}$	A1 (all 3 values)
6(c)	$\frac{1}{4}y^3 + \frac{m}{4}y^2 + \frac{1}{2}y + n = 0$ $2\left(\frac{1}{2}y\right)^3 + m\left(\frac{1}{2}y\right)^2 + \frac{1}{2}y + n = 0$ <p>Replace x by $\frac{1}{2}y$</p> $\therefore y = -4, y = -2, y = 1$	M1 A2 (all 3 values)
7(a)	$64^x \times 4^y = 1$ $\log_1(x - 3y - 6) + 2 = \log_2 x$ $\frac{\log_2(x - 3y - 6)}{\log_2 \frac{1}{2}} = \log_2 x - \log_2 4$ $-\log_2(x - 3y - 6) = \log_2 x - \log_2 4$ $\log_2(x - 3y - 6) = \log_2 4 - \log_2 x$ $\log_2(x - 3y - 6) = \log_2 \frac{4}{x}$	M1 M1 A1
7(b)	$64^x \times 4^y = 1$ $2^{6x} \times 2^{2y} = 2^0$ $6x + 2y = 0$ $3x + y = 0$ $y = -3x \text{ --- (1)}$ $x - 6 - 3y = \frac{4}{x} \text{ --- (2)}$ <p>Sub (1) into (2)</p> $x - 6 + 9x = \frac{4}{x}$ $10x - 6 = \frac{4}{x}$ $10x^2 - 6x = 4$ $5x^2 - 3x - 2 = 0$ $(5x + 2)(x - 1) = 0$ $x = -\frac{2}{5} \text{ or } 1$ $y = \frac{6}{5} \text{ or } -3$	M1 M1 M1 M1 A1 A1
8(a)	\perp from centre bisects chord	B1
8(b)	$\angle BAO = x$ (alt \angle) $\sin \theta = \frac{OX}{7} \Rightarrow OX = 7 \sin \theta$	M1 M1

	$\cos \theta = \frac{AX}{7} \Rightarrow AX = 7 \cos \theta$ $P = 7 + 14 \cos \theta + 7 \sin \theta + 7 \cos \theta$ $P = 7 + 21 \cos \theta + 7 \sin \theta$	A1
8(c)	$R = \sqrt{21^2 + 7^2}$ $R = 7\sqrt{10}$ $\theta = \tan^{-1}\left(\frac{7}{21}\right) = 18.435^\circ$ $\therefore P = 7 + 7\sqrt{10} \cos(\theta - 18.4^\circ) \text{ cm}$	M1 M1 A1
8(d)	$\text{Max } P = 7 + 7\sqrt{10} \text{ or } 29.1$ $\theta - 18.435^\circ = \cos^{-1}(1)$ $\theta = 18.4^\circ$	M1 M1 A1
9(a)	$\frac{dy}{dx} = -3x^2 + 12x - 3$ $x = 3, m_{\tan} = 6$ $y = 6x + c$ $c = -10$ $y = 6x - 10 \text{ equation of tangent}$ $\text{sub } x = 0 \text{ in } y = 6(0) - 10 = -10$ <p>Since D(0, -10) satisfy the equation of tangent at E, therefore tangent passes through D.</p>	M1 M1 M1 M1 A1
9(b)	$\text{Area } BCD = - \int_0^2 -x^3 + 6x^2 - 3x - 10 \, dx$ $= \left[\frac{-1}{4}x^4 + 2x^3 - \frac{3}{2}x^2 - 10x \right]_0^2$ $= 14 \text{ units}^2$ <p>Equation of line $y = 6x - 10$</p> <p>When $y = 0$, $x = \frac{5}{3}$</p> $\text{Area of triangle } BCD = \frac{1}{2} \times 10 \times \frac{5}{3} = \frac{25}{3}$ $\text{Shaded area} = 14 - \frac{25}{3} = 5\frac{2}{3} \text{ units}^2$	M1 A1 M1 A1 M1A1 A1
Or 9(b)	<p>area A =</p> $\int_0^3 (6x - 10) - (-x^3 + 6x^2 - 3x - 10) \, dx$ $\int_0^3 (9x + x^3 - 6x^2) \, dx$ $= \left[\frac{9x^2}{2} + \frac{x^4}{4} - \frac{6x^3}{3} \right]_0^3$ $= 6.75$	Equation of line M1 M1 A1

	<p>Area of B =</p> $\frac{1}{2} \times 8 \times \left(3 - \frac{5}{3}\right) - \int_2^3 (-x^3 + 6x^2 - 3x - 10) dx$ $5 \frac{1}{3} - \frac{3}{2} \left[\frac{-x^4}{4} + 2x^3 - \frac{3}{2}x^2 - 10x \right]$ $= \frac{13}{12}$ <p>Answer: $6 \frac{3}{4} - \left(\frac{13}{12}\right) = 5 \frac{2}{3} \text{ units}^2$</p>	<p>M1</p> <p>M1A1</p> <p>A1</p>
10(a)	$y = ax\sqrt{x} + b\sqrt{x}$ $\frac{y}{\sqrt{x}} = ax + b$ <p>Plot $\frac{y}{\sqrt{x}}$ against x</p>	<p>M1</p> <p>A1</p>
10(b)	<p>Graph</p> $a = \text{gradient} \approx \frac{3.38 - 1.12}{1.4 - 0.2} \approx 1.57$ $b = \frac{y}{\sqrt{x}} - \text{intercept} \approx 0.75$	<p>P1C1S1</p> <p>M1A1</p> <p>A1</p>
	