XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2024

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/01

Paper 1
Secondary 4 Express

23 August 2024
2 hour 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

For Examiner's Use
90

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This document consists of **19** printed pages and 1 blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A triangle has a base of length $\left(6+2\sqrt{7}\right)$ cm and an area of $\left(17+7\sqrt{7}\right)$ cm^2.

Find, **without using a calculator**, the perpendicular height to the base of the triangle, in cm, in the form $\left(a+b\sqrt{7}\right)$, where a and b are integers. [3]

2 Solve the equation $\sqrt{5-\sqrt{x+1}}=\sqrt{x}$. [4]

3 The equation of a curve is $y=\frac{4x+2}{\sqrt{x+1}}$, where $x>-1$.

(a) Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, leaving your answer in the form $\frac{ax+b}{\sqrt{(x+1)^n}}$, where a, b and n are constants. [2]

(b) Explain why the curve is a increasing function. [2]

4 The line $2x+3y=12$ intersects the curve $y^2=4x-8$ at points A and B.
Find the value of p and q for which the length of AB can be expressed as $p\sqrt{q}$. [6]

5 The height, h m, of a baseball above ground t seconds after it has been hit is given by $h = c + 24t - 4t^2$, where c is a constant.

(a) If $c = 1.65$, express h in the form $h = p + q(t + r)^2$ where p, q and r are constants to be determined. Hence, state the maximum height attained by the baseball and the time at which this occurs. [4]

(b) Find the range of values of c if the baseball did not reach a height of 40 m. [2]

6 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 12e^{6x} + 15e^{-3x}$.

The point $P(0,-2)$ lies on the curve and the normal to the curve at P is parallel to the y-axis. Find the equation of the curve. [6]

7 The equation of a curve is $y = kx^2 + kx + p$, where p and k are constants.

(a) Show that $p > \frac{k}{4}$ for which the curve lies completely above the x-axis. [3]

(b) In the case where $k = 2$ and $p = 4$, find the values of m for which the line $y = mx - 4$ is a tangent to the curve. [4]

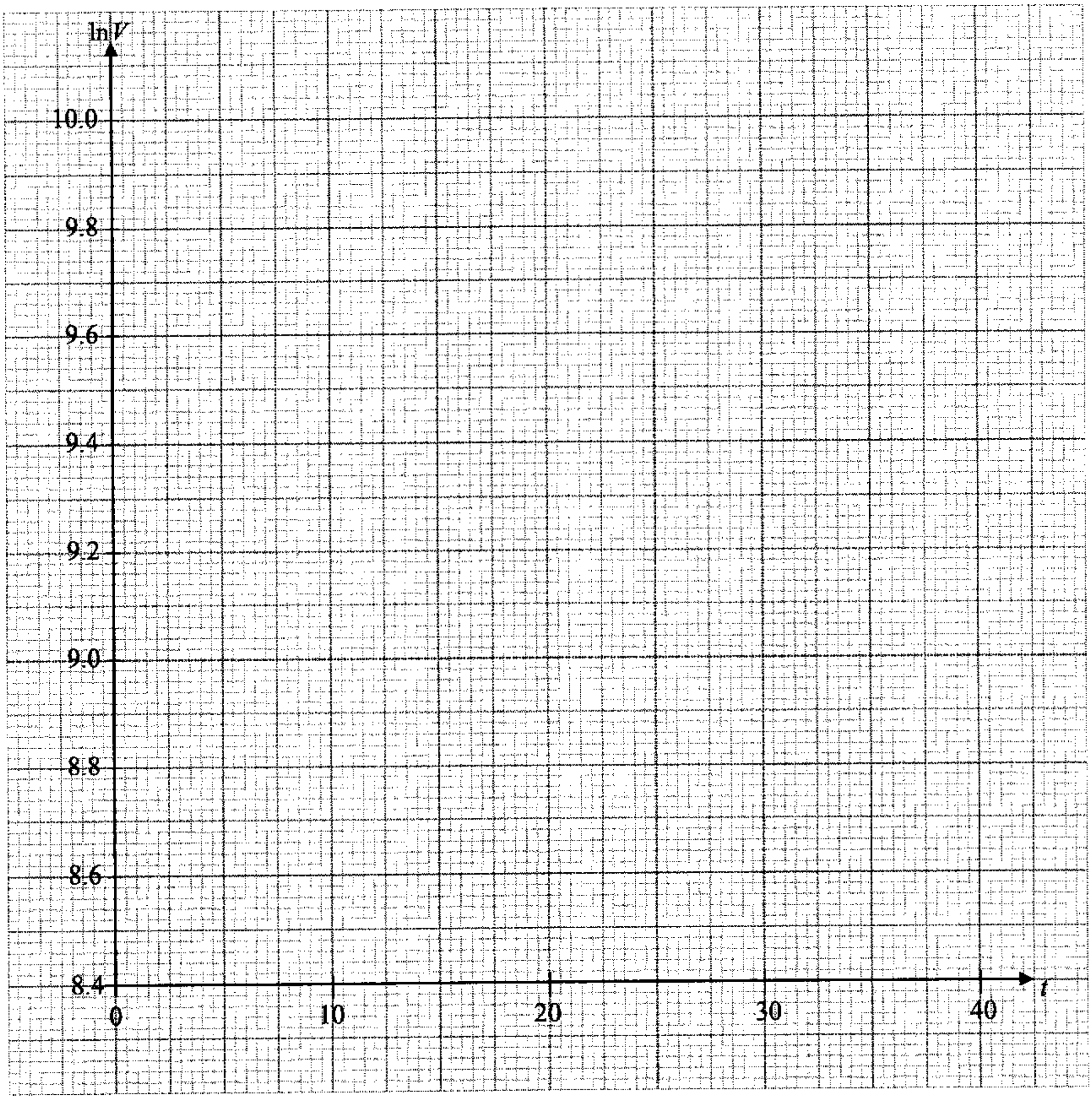
8 **(a)** Divide $2x^3+5x^2+5x+9$ by x^3+3x. [1]

(b) Express $\frac{2x^3+5x^2+5x+9}{x^3+3x}$ in partial fractions. [5]

9 The value, $\$V$, of a watch is related to t, the number of years since 1980.
The table below gives the value of the watch in 1990, 2000, 2010, 2020.

Year	1990	2000	2010	2020
t (years)	10	20	30	40
V (\$)	7200	9600	12 800	17 200

(a) Plot $\ln V$ against t and draw a straight line graph to illustrate the information. [2]



(b) Find the gradient and the intercept of the vertical axis of your straight line graph. Hence, express V in the form Ae^{kt}, where A and k are constants. [4]

(c) Explain what the constant A represents. [1]

10 **(a)** **(i)** Write down the first four terms in the expansion of $(3-2x)^7$. [2]

(ii) Find the coefficient of x^3 in the expansion of $\left(1-7x^2\right)(3-2x)^7$. [2]

(b) In the binomial expansion of $\left(x+\frac{k}{x}\right)^{11}$, where k is a positive constant, the coefficient of $\frac{1}{x^5}$ is 8 times the coefficient of x^3. Find the value of k. [5]

11 The triangle ABC is such that A is $(9,9)$, B is $(1,-3)$ and C is (p,q) where $q > p$. C lies on the perpendicular bisector of AB and area of triangle ABC is 26 units^2.

(a) Find the equation of the perpendicular bisector of AB. [4]

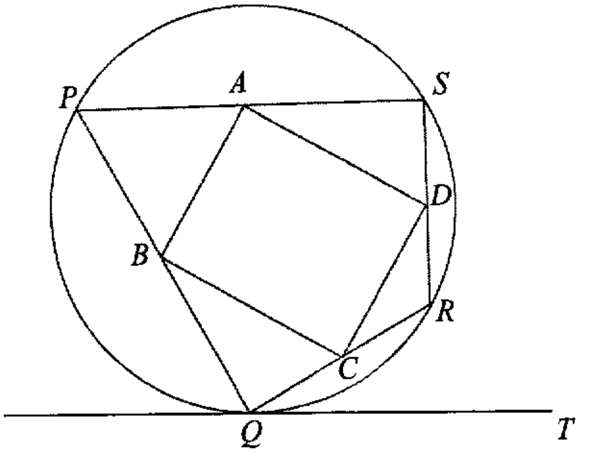
(b) Find the coordinates of C. [3]

12 **(a)** The graph of $y = a\sin bx + 2$ has one maximum point at $\left(\frac{5\pi}{4}, 7\right)$ and the next maximum point after this has coordinates $\left(\frac{9\pi}{4}, 7\right)$. Find the values of a and b. [2]

(b) **(i)** Sketch, on the same diagram, the graph of $y = -4\cos 4x$ and $y = -\frac{4}{\pi}x + 4$ for $0 \leq x \leq \pi$ radians. [4]

(ii) Hence, state the number of solutions to the equation $\frac{x}{\pi} - \cos 4x - 1 = 0$ for $0 \leq x \leq \frac{\pi}{2}$. [2]

13



In the diagram, *PQRS* is a kite, where all four points lie on the circumference of the circle. $RS = RQ$ and $PQ = PS$. *QT* is a tangent to the circle at *Q*.

(a) Show that $\angle QPS = 2 \times \angle RQT$. [3]

(b) A circle can be drawn passing through A, B, C and D, with BD as the diameter.
Given that A, B, C and D are midpoints of PS, PQ, QR and SR respectively, what can you deduce about quadrilateral $ABCD$? [4]

14 A particle travelling in a straight line, has a velocity, v m/s, at time t seconds, $t \geq 0$, given by $v = 3\sin 2t - 4\cos 2t$.

(a) Find the initial acceleration of the particle. [2]

(b) Find the total distance travelled by the particle in the first 1.5 seconds. [8]

Continuation of working space for question **14(b).**

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Find, **without using a calculator**, the perpendicular height to the base of the triangle, in cm, in the form $\left(a+b\sqrt{7}\right)$, where a and b are integers. [3]

$$\frac{1}{2}\left(6+2\sqrt{7}\right)\times h=\left(17+7\sqrt{7}\right) \text{ --- [M1]}$$

$$h=\frac{2\left(17+7\sqrt{7}\right)}{6+2\sqrt{7}}$$

$$h=\frac{\left(17+7\sqrt{7}\right)\left(3-\sqrt{7}\right)}{\left(3+\sqrt{7}\right)\left(3-\sqrt{7}\right)} \text{ --- [M1]}$$

$$h=\frac{51-17\sqrt{7}+21\sqrt{7}-49}{3^2-7}$$

$$h=\frac{2+4\sqrt{7}}{2}$$

$$h=1+2\sqrt{7} \text{ ---[A1]}$$

Alternative

$$h=\frac{\left(34+14\sqrt{7}\right)\left(6-2\sqrt{7}\right)}{\left(6+2\sqrt{7}\right)\left(6-2\sqrt{7}\right)} \text{ --- [M1]}$$

$$h=\frac{204-68\sqrt{7}+84\sqrt{7}-196}{36-4(7)}$$

$$h=\frac{8+16\sqrt{7}}{8}$$

$$h=1+2\sqrt{7} \text{ ---[A1]}$$

2 Solve the equation $\sqrt{5-\sqrt{x+1}}=\sqrt{x}$. [4]

$$5-\sqrt{x+1}=x \text{ --- [M1]}$$

$$5-x=\sqrt{x+1}$$

$$(5-x)^2=x+1$$

$$25-10x+x^2=x+1$$

$$\left.\begin{aligned}x^2-11x+24&=0\\(x-3)(x-8)&=0\end{aligned}\right\}$$

Students must show the relevant method in solving quad. equation (either factorisation or quad. formula). Failure to do so would result in the loss of M1; A1 would still be awarded accordingly.

$$\underbrace{x=3}_{\text{[A1]}} \text{ or } \underbrace{x=8}_{\text{[A1: no A1 if students do not reject]}} \text{ (rej)}$$

3 The equation of a curve is $y = \frac{4x+2}{\sqrt{x+1}}$, where $x > -1$.

(a) Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, leaving your answer in the form $\frac{ax+b}{\sqrt{(x+1)^n}}$. [2]

Method 1: Quotient Rule	**Method 2: Product Rule**
$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{4\sqrt{x+1} - (4x+2)\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}}{x+1}$ --- [M1] $= \frac{4\sqrt{x+1} - \frac{2x+1}{\sqrt{x+1}}}{x+1}$ $= \frac{4(x+1)-(2x+1)}{(x+1)^{\frac{3}{2}}}$ $= \frac{2x+3}{\sqrt{(x+1)^3}}$ ---[A1]	$\frac{\mathrm{dy}}{\mathrm{dx}} = (4x+2)\left(-\frac{1}{2}\right)(x+1)^{-\frac{3}{2}}(1) + (x+1)^{-\frac{1}{2}}(4)$ --- [M1] $= -\frac{2x+1}{\sqrt{(x+1)^3}} + \frac{4}{\sqrt{x+1}}$ $= \frac{-2x-1+4(x+1)}{\sqrt{(x+1)^3}}$ $= \frac{-2x-1+4x+4}{\sqrt{(x+1)^3}}$ $= \frac{2x+3}{\sqrt{(x+1)^3}}$ --- [A1]

(b) Explain why the curve is an increasing function. [2]

$$\left.\begin{array}{l}\text{Since } x > -1, \\ \sqrt{(x+1)^3} > 0, \text{ and } 2x+3 > 0\end{array}\right\} \text{M1}$$

$$\left.\begin{array}{l}\therefore \frac{2x+3}{\sqrt{(x+1)^3}} > 0, \text{ for } x > -1 \rightarrow \frac{dy}{dx} > 0 \\ \\ \text{Since } \frac{dy}{dx} > 0, y \text{ is an increasing function.}\end{array}\right\} \text{A1}$$

4 The line $2x+3y=12$ intersects the curve $y^2=4x-8$ at points A and B.
Find the value of p and q for which the length of AB can be expressed as $p\sqrt{q}$. [6]

$2x+3y=12$ --- (1)
$y^2=4x-8$ --- (2)
From (1):
$2x=12-3y$ ---(3)
Subst (3) into (2):
$y^2=2(12-3y)-8$ --- [M1]
$y^2=24-6y-8$
$y^2+6y-16=0$ --- [M1]
$(y-2)(y+8)=0$
$y=2$ or $y=-8$

No M1 if students do not show working for factorisation/ quadratic formula. Award the remaining marks accordingly.

when $y=2$,
$2x=12-3(2)$
$x=3$
$\therefore A(3,2)$ --- [A1]

when $y=-8$,
$2x=12-3(-8)$
$x=18$
$\therefore B(18,-8)$ --- [A1]

Length of AB: $\sqrt{(18-3)^2+(-8-2)^2}$ --- [M1, allow ecf]
$=\sqrt{325}$
$=5\sqrt{13}$
$\therefore [p=5, q=13]$--- [A1]

Alternative

$2x+3y=12$ --- (1)
$y^2=4x-8$ --- (2)
From (1):
$y=\dfrac{12-2x}{3}$ ---(3)
Subst (3) into (2):
$\left(\dfrac{12-2x}{3}\right)^2=4x-8$ --- [M1]
$\dfrac{(12-2x)^2}{9}=4x-8$
$144-48x+4x^2=36x-72$
$4x^2-84x+216=0$ ---[M1]
$x^2-21x+54=0$
$(x-3)(x-18)=0$
$x=3$ or $x=18$

when $x=3$,
$y=\dfrac{12-2(3)}{3}$
$y=2$
$\therefore A(3,2)$ --- [A1]

when $x=18$,
$y=\dfrac{12-2(18)}{3}$
$y=-8$
$\therefore B(18,-8)$ --- [A1]

Length of AB: $\sqrt{(18-3)^2+(-8-2)^2}$ --- [M1]
$=\sqrt{325}$
$=5\sqrt{13}$
$\therefore [p=5, q=13]$--- [A1]

5 The height, h m, of a baseball above ground t seconds after it has been hit is given by $h = c + 24t - 4t^2$, where c is a constant.

(a) If $c = 1.65$, express h in the form $h = p + q(t+r)^2$ where p, q and r are constants to be determined. Hence, state the maximum height attained by the baseball and the time at which this occurs. [4]

$h = 1.65 + 24t - 4t^2$

$= 1.65 - 4(t^2 - 6t)$

$= 1.65 - 4(t^2 - 6t \underline{+3^2}) + \underline{4(3^2)}$ --- [M1 for completing the square. Awd if wrong c val subst.]

Note: M1 is not awarded if students wrote $(-3)^2$

$= 37.65 - 4(t-3)^2$ --- [A1]

max. height: 37.65 ---[A1]

time: 3 seconds or $t = 3$ ---[A1]

Awarded even if completed square form is partially correct.

(b) Find the range of values of c if the baseball did not reach a height of 40 m. [2]

$-4t^2 + 24t + c = 40$

$-4t^2 + 24t + c - 40 = 0$

Since ball did not reach 40m, there is no solution to the equation.

$\therefore\ b^2 - 4ac < 0$

$a = -4,\ b = 24,\ c = c - 40$

$24^2 - 4(-4)(c-40) < 0$ --- [M1]

$576 - 640 + 16c < 0$

$16c < 64$

$c < 4$ --- [A1]

Alternatively

$h = c + 24t - 4t^2$

$= -4\left(t^2 - 6t - \frac{c}{4}\right)$

$= -4[(t-3)^2 - (3)^2 - \frac{c}{4}]$

$= -4[(t-3)^2 - 9 - \frac{c}{4}]$

$= -4(t-3)^2 + 36 + c$ --- [M1]

$36 + c < 40$

$c < 4$ --- [A1]

6 A curve is such that $\frac{d^2y}{dx^2} = 12e^{6x} + 15e^{-3x}$.

The point $P(0,-2)$ lies on the curve and the normal to the curve at P is parallel to the y-axis. Find the equation of the curve. [6]

$$\frac{dy}{dx} = \int 12e^{6x} + 15e^{-3x}\ dx$$
$$= \frac{12e^{6x}}{6} + \frac{15e^{-3x}}{-3} + c \text{ --- [M1]}$$
$$= 2e^{6x} - 5e^{-3x} + c$$

at $x = 0$, $\frac{dy}{dx} = 2 - 5 + c$ --- [M1 for subst. $x = 0$ into $\frac{dy}{dx}$]

$$= c - 3$$

$m_{\text{tangent at P}} : 0$

$\therefore c - 3 = 0$ --- [M1]

$c = 3$

$$\therefore \frac{dy}{dx} = 2e^{6x} - 5e^{-3x} + 3$$

$$y = \int 2e^{6x} - 5e^{-3x} + 3\ dx$$
$$= \frac{2e^{6x}}{6} - \frac{5e^{-3x}}{-3} + 3x + d \text{ --- [M1, allow ecf]}$$
$$= \frac{e^{6x}}{3} + \frac{5e^{-3x}}{3} + 3x + d$$

at $x = 0$, $y = -2$,

$\frac{1}{3} + \frac{5}{3} + d = -2$ ---[M1, allow ecf]

$2 + d = -2$

$d = -4$

$$\therefore y = \frac{e^{6x}}{3} + \frac{5e^{-3x}}{3} + 3x - 4 \text{ ---[A1]}$$

7 The equation of a curve is $y = kx^2 + kx + p$, where p and k are constants.

(a) Show that $p > \frac{k}{4}$ for which the curve lies completely above the x-axis. [3]

$a = k, b = k, c = p$

$k^2 - 4(k)(p) < 0$ ---[M1]

$k(k - 4p) < 0$

Since $k > 0, k - 4p < 0$ ---[M1, need to see both inequalities formed]

$\therefore \quad 4p > k$

$p > \frac{k}{4}$ ---[A1]

(b) In the case where $k = 2$ and $p = 4$, find the values of m for which the line $y = mx - 4$ is a tangent to the curve. [4]

at $k = 2, p = 4$: $y = 2x^2 + 2x + 4$

$2x^2 + 2x + 4 = mx - 4$ --- [M1]

$2x^2 + (2 - m)x + 8 = 0$

Since $y = mx - 4$ is a tangent to the curve,

$(2 - m)^2 - 4(2)(8) = 0$ --- [M1]

$4 - 4m + m^2 - 64 = 0$

$m^2 - 4m - 60 = 0$

$(m + 6)(m - 10) = 0$

Students must show the relevant method in solving quad. equation (either factorisation or quad. formula). Failure to do so would result in the loss of M1; A1 would still be awarded accordingly.

$m = -6$ or $m = 10$ ---[A1]

8 **(a)** Divide $2x^3+5x^2+5x+9$ by x^3+3x. [1]

$$2+\frac{5x^2-x+9}{x^3+3x} \text{ --- [B1]}$$

(b) Express $\dfrac{2x^3+5x^2+5x+9}{x^3+3x}$ in partial fractions. [5]

$$\frac{5x^2-x+9}{x^3+3x}=\frac{5x^2-x+9}{x(x^2+3)}$$

$$\frac{5x^2-x+9}{x(x^2+3)}=\frac{A}{x}+\frac{Bx+C}{x^2+3} \text{ --- [M1]}$$

$$5x^2-x+9=A(x^2+3)+(Bx+C)x$$

when $x=0,\ 9=3A \Rightarrow A=3$ —[M1]

By comparing coefficients: $-1=C$ ---[M1]

$5=A+B \Rightarrow B=2$ ---[M1]

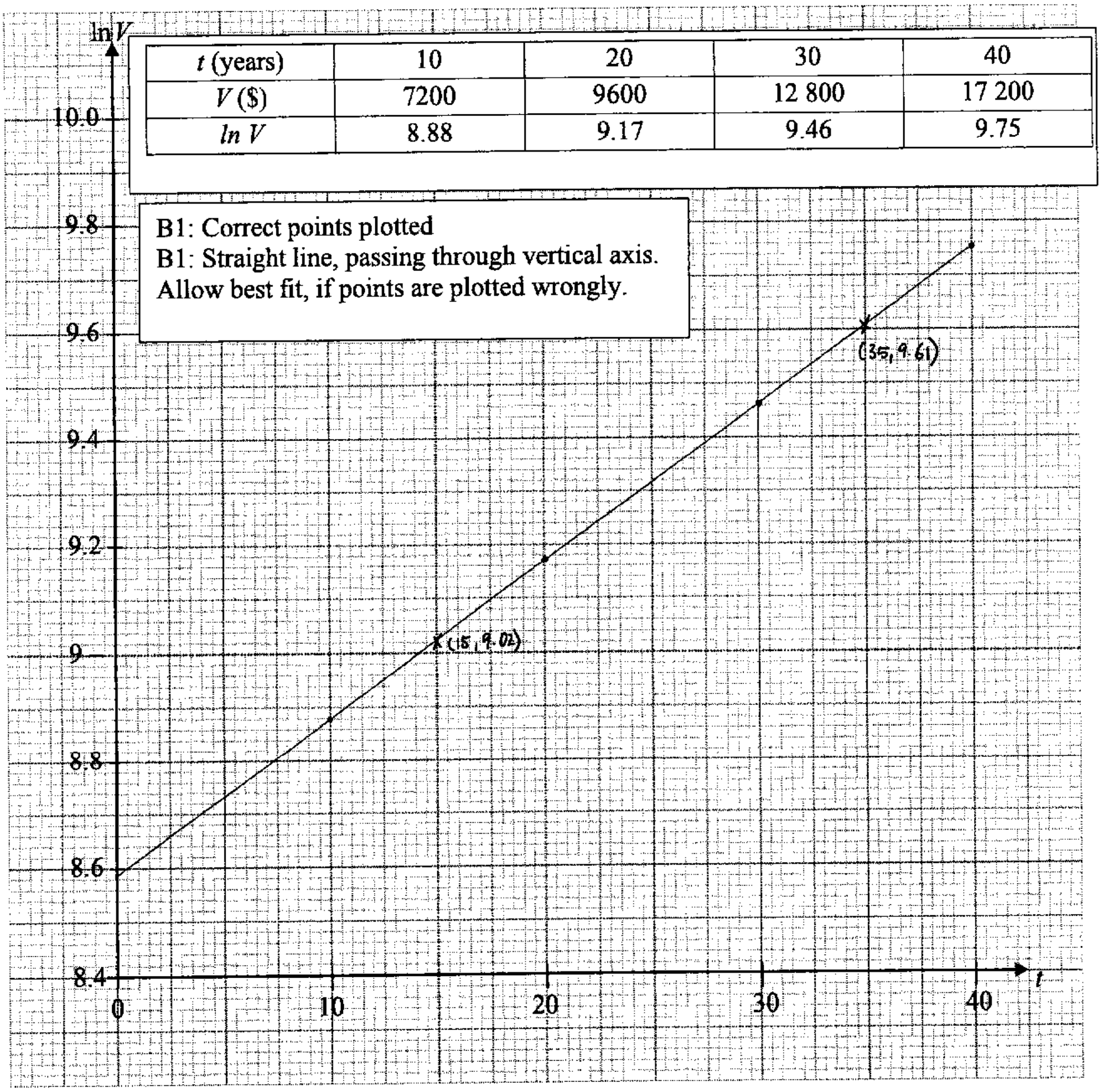
If students made error in partial fraction decomposition or long division, award M1 (ecf) for correctly applying substitution/ comparing coefficients to find numerators. Max. 1mark.

$$\therefore \frac{2x^3+5x^2+5x}{x^3+3x}=2+\frac{3}{x}+\frac{2x-1}{x^2+3} \text{ ---[A1]}$$

9 The value, \$$V$, of a watch is related to t, the number of years since 1980.
The table below gives the value of the watch in 1990, 2000, 2010, 2020.

Year	1990	2000	2010	2020
t (years)	10	20	30	40
V (\$)	7200	9600	12 800	17 200

(a) Plot $\ln V$ against t and draw a straight line graph to illustrate the information. [2]



t (years)	10	20	30	40
V (\$)	7200	9600	12 800	17 200
ln V	8.88	9.17	9.46	9.75

B1: Correct points plotted
B1: Straight line, passing through vertical axis.
Allow best fit, if points are plotted wrongly.

(b) Find the gradient and the intercept of the vertical axis of your straight line graph. Hence, express V in the form Ae^{kt}, where A and k are constants. [4]

Using 2 points on straight line graph plotted:
(15,9.02) and (35,9.61)

$$m = \frac{9.61 - 9.02}{35 - 15}$$
$$= 0.0295$$ --- [M1]

(Acceptable range: 0.0275 to 0.0315)

$C = 8.59$ (Accept: 8.57, 8.58, 8.6, 8.61) ---[A1]

$\ln V = 0.0295t + 8.59$ ---[M1]

$V = e^{0.0295t+8.59}$

$V = e^{0.0295t} \cdot e^{8.59}$

$V = 5380e^{0.0295t}$ (3 s.f.) —[A1]

(c) Explain what the constant A represents. [1]

A represents the **value** of the watch in **1980**. --- [B1]

10 **(a)** **(i)** Write down the first four terms in the expansion of $(3-2x)^7$. [2]

$$(3-2x)^7 = 3^7 + \binom{7}{1}(3)^6(-2x)^1 + \binom{7}{2}(3)^5(-2x)^2 + \binom{7}{3}(3)^4(-2x)^3 + \ldots \text{ ---[M1]}$$

$$= 2187 + 5103(-2x) + 5103(4x^2) + 2835(-8x^3) + \ldots$$

$$= 2187 - 10206x + 20412x^2 - 22680x^3 + \ldots$$

[A1: do not award A1 if students did not give four terms]

(ii) Find the coefficient of x^3 in the expansion of $(1-7x^2)(3-2x)^7$. [2]

$(1-7x^2)(2187-10206x+20412x^2-22680x^3+\ldots)$

Coefficient of x^3 : $1(-22680)-7(-10206)$ ---[M1: award if seen/implied; allow ecf]

$= 48762$ ---[A1]

10 **(b)** In the binomial expansion of $\left(x+\frac{k}{x}\right)^{11}$, where k is a positive constant, the coefficient of $\frac{1}{x^5}$ is 8 times the coefficient of x^3. Find the value of k. [5]

$$T_{r+1}=\binom{11}{r}x^{11-r}\left(\frac{k}{x}\right)^r \text{ --- [M1]}$$

$$=\binom{11}{r}x^{11-2r}k^r$$

For x^3 term:

$11-2r=3$

$2r=8$

$r=4$

Coefficient of x^3: $\binom{11}{4}k^4=330k^4$

M1 for either correct

M1 for either correct

For x^{-5} term:

$11-2r=-5$

$2r=16$

$r=8$

Coefficient of x^{-5}: $\binom{11}{8}k^8=165k^8$

$$\therefore \frac{165k^8}{330k^4}=8 \text{ --- [M1]}$$

$$k^4=16$$

$$k=2 \text{ --- [A1]}$$

11 The triangle ABC is such that A is $(9,9)$, B is $(1,-3)$ and C is (p,q) where $q>p$. C lies on the perpendicular bisector of AB and area of triangle ABC is 26 units^2.

(a) Find the equation of the perpendicular bisector of AB. [4]

$$m_{AB}=\frac{9-(-3)}{9-1}=\frac{3}{2}$$

$$m_{\text{perpendicular bisector}}=-\frac{2}{3}\text{ ---[M1]}$$

$$\text{midpoint of AB}:\left(\frac{9+1}{2},\frac{9-3}{2}\right)=(5,3)\text{ ---[M1]}$$

Equation of perpendicular bisector:

$$y-3=-\frac{2}{3}(x-5)\text{ ---[M1]}$$

$$y=-\frac{2}{3}x+6\frac{1}{3}\text{ ---[A1]}$$

(b) Find the coordinates of C. [3]

coordinates of $C:(p,q)$

$$\frac{1}{2}\begin{vmatrix}9 & p & 1 & 9\\ 9 & q & -3 & 9\end{vmatrix}=26\text{ --- [M1]}$$

$$\frac{1}{2}\left[(9q-3p+9)-(9p+q-27)\right]=26$$

$$9q-3p+9-9p-q+27=52$$

$$8q-12p=16$$

$$2q-3p=4\ ---(1)$$

$$q=-\frac{2}{3}p+6\frac{1}{3}\ ---(2)$$

Sub (2) into (1)

$$2(-\frac{2}{3}p+6\frac{1}{3})-3p=4$$

$$2(-2p+19)-9p=12$$

$$-4p+38-9p=12$$

$$-13p=-26$$

$$p=2\text{ --- [M1]}$$

$$\text{when }p=2, q=-\frac{2}{3}(2)+6\frac{1}{3}$$

$$q=5$$

$$\therefore C(2,5)\text{ --- [A1]}$$

Alternative

$$AB=\sqrt{(9-1)^2+(9+3)^2}$$

$$=\sqrt{208}$$

$$\frac{1}{2}\times\sqrt{208}\times h=26\text{ --- [M1]}$$

$$h=\frac{52}{\sqrt{208}}$$

$$=\frac{13}{\sqrt{13}}$$

$$\sqrt{(x-5)^2+\left(-\frac{2}{3}x+6\frac{1}{3}-3\right)^2}=\frac{13}{\sqrt{13}}\text{---[M1]}$$

$$x^2-10x+25+\left(-\frac{2}{3}x+3\frac{1}{3}\right)^2=13$$

$$x^2-10x+25+\frac{4}{9}x^2-\frac{40}{9}x+\frac{100}{9}=13$$

$$13x^2-130x+208=0$$

$$x^2-10x+16=0$$

$$(x-8)(x-2)=0$$

$$x=8\text{ or }x=2$$

$$\text{when }x=8, y=1\text{ --- (rej)}$$

$$\text{when }x=2, y=5$$

$$\therefore C(2,5)\text{---[A1]}$$

12 (a) The graph of $y = a\sin bx + 2$ has one maximum point at $\left(\frac{5\pi}{4}, 7\right)$ and the next maximum point after this has coordinates $\left(\frac{9\pi}{4}, 7\right)$. Find the values of a and b. [2]

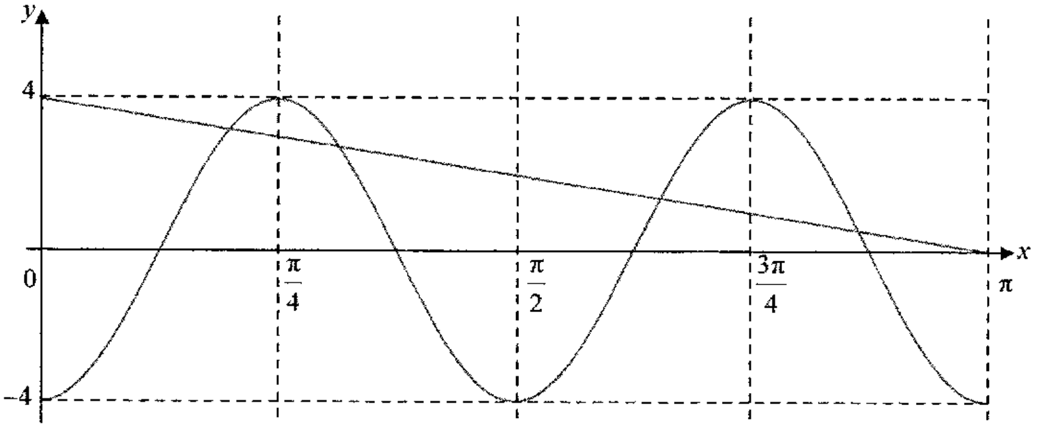
amplitude of graph: $7 - 2 = 5$

$\therefore a = 5$ ---[B1]

period of graph: $\frac{9\pi}{4} - \frac{5\pi}{4} = \pi$

$\frac{2\pi}{b} = \pi \rightarrow b = 2$ ---[B1]

(b) (i) Sketch, on the same diagram, the graph of $y = -4\cos 4x$ and $y = -\frac{4}{\pi}x + 4$ for $0 \le x \le \pi$ radians. [4]



B1: Correct shape and max/min. y value of cosine graph; B1: number of cycles and labelling on x-axis for cosine graph

B1: Correct slope and y-int of linear graph; B1: correct x-int of linear graph.

(ii) Hence, state the number of solutions to the equation $\frac{x}{\pi} - \cos 4x - 1 = 0$ for $0 \le x \le \frac{\pi}{2}$. [2]

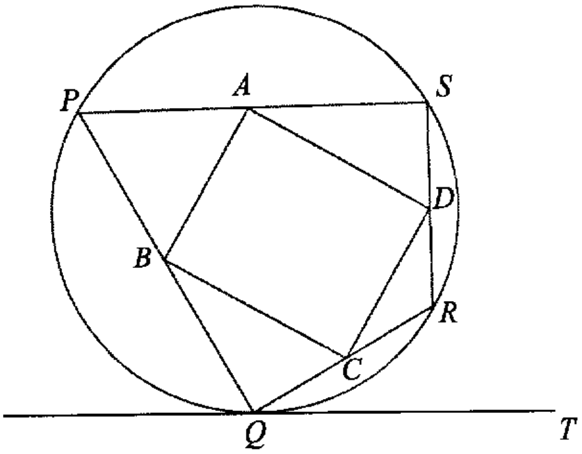
$$\left.\begin{aligned} &\frac{x}{\pi} - \cos 4x - 1 = 0 \\ &-\cos 4x = -\frac{x}{\pi} + 1 \\ &-4\cos 4x = -\frac{4}{\pi}x + 4 \end{aligned}\right\} \text{M1}$$

Students must show the manipulation of equation to contain the expression of the graphs on each side. Do not award any mark if students do not show any manipulation of the equation but wrote down the correct final answer.

From graph, there are 2 intersections for $0 \le x \le \frac{\pi}{2}$.

$\therefore$ no. of solutions: 2 — [A1]

13



In the diagram, $PQRS$ is a kite, where all four points lie on the circumference of the circle. $RS = RQ$ and $PQ = PS$. QT is a tangent to the circle at Q.

(a) Show that $\angle QPS = 2\times\angle RQT$. [3]

Let $\angle RQT = \alpha$

$\angle QPR = \angle RQT$ (alt. segment theorem / tan. chord theorem) --- [M1]

$= \alpha$

$\angle QPR = \angle SPR$ --- [M1]

$= \alpha$

$$\left.\begin{aligned}\angle QPS &= \angle QPR + \angle SPR\\ &= \alpha+\alpha\\ &= 2\alpha\\ &= 2\times\angle RQT \text{ (shown)}\end{aligned}\right\}\text{[A1]}$$

Alternative

Let $\angle RQT = \alpha$

$\angle RQT = \angle QSR$ (alt. segment theorem / tan. chord theorem) --- [M1]

$\angle QSR = \angle SQR$ (base $\angle$, isos. triangles)

$\angle SRQ = 180° - \angle QSR - \angle SQR$ --- [M1]

$= 180° - 2\alpha$

$$\left.\begin{aligned}\angle QPS &= 180° - (180° - 2\alpha) \text{ ($\angle$s in opp. segment)}\\ &= 2\alpha\\ &= 2\times\angle RQT \text{ (shown)}\end{aligned}\right\}\text{[A1]}$$

-1m from the whole of question 13 for missing/wrong geometrical reason(s).

(b) A circle can be drawn passing through A, B, C and D, with BD as the diameter. Given that A, B, C and D are midpoints of PS, PQ, QR and SR respectively, what can you deduce about quadrilateral $ABCD$? [4]

Since A and B are midpoints of PS and PQ,

by midpoint theorem, $AB = \frac{1}{2}QS$ and $AB//QS$.

Since C and D are midpoints of RQ and RS,

by midpoint theorem, $CD = \frac{1}{2}QS$ and $CD//QS$.

[M1] for either.

Since $AB = \frac{1}{2}QS$ and $CD = \frac{1}{2}QS \rightarrow AB = CD$

Since $AB//QS$ and $CD//QS \rightarrow AB//CD$

Since $AB = CD$ and $AB//CD$, $ABCD$ is a parallelogram. --- [M1]

$\angle BAD = 90°$ (rt. angle in semicircle) --- [M1]

Since $ABCD$ is a parallelogram with $\angle BAD = 90°$, $ABCD$ is a rectangle. --- [A1]

Alternative

Since B and C are midpoints of PQ and QR respectively, by midpoint theorem,

$BC = \frac{1}{2}PR$ and $BC // PR$.

Since A and D are midpoints of PS and SR respectively, by midpoint theorem,

$AD = \frac{1}{2}PR$ and $AD // PR$.

[M1] for either.

$\therefore BC = AD$ and $BC // AD$ --- [M1]
Hence, $ABCD$ is a parallelogram.

$\angle BAD$ (or $\angle BCD$) $= 90°$ (rt. angle in semi circle). --- [M1]
Since $ABCD$ is a parallelogram with $\angle BAD = 90°$, $ABCD$ is a rectangle. --- [A1]

-1m from the whole of question 13 for missing/wrong geometrical reason(s).

14 A particle travelling in a straight line, has a velocity, v m/s, at time t seconds, $t \geq 0$, given by $v = 3\sin 2t - 4\cos 2t$.

(a) Find the initial acceleration of the particle. [2]

$$a = 3(\cos 2t)(2) - 4(-\sin 2t)(2)$$
$$= 6\cos 2t + 8\sin 2t \text{ --- [M1]}$$

$$\text{at } t = 0,\ a = 6\cos 0 + 8\sin 0$$
$$= 6 \text{ --- [A1]}$$

(b) Find the total distance travelled by the particle in the first 1.5 seconds. [8]

When particle comes to rest, $v = 0$,

$3\sin 2t - 4\cos 2t = 0$ --- [M1]

$3\sin 2t = 4\cos 2t$

$\dfrac{\sin 2t}{\cos 2t} = \dfrac{4}{3}$

$\tan 2t = \dfrac{4}{3}$ — [M1]

Basic Angle: $\tan^{-1}\left(\dfrac{4}{3}\right) = 0.92729$ (5 s.f.)

$2t = 0.92729,\ \pi + 0.9272,\ \ldots$

$t = 0.46364$ (5 s.f.), ... --- [A1]

Continuation of working space for question **13(b).**

$s=\int 3\sin 2t-4\cos 2t\ \mathrm{d}t$

$=-\frac{3}{2}\cos 2t-2\sin 2t+c$ — [M1 for correct integration of trigo functions]

at t = 0, s = 0:

$0=-\frac{3}{2}\cos 0-2\sin 0+c$ --- [M1]

$c=\frac{3}{2}$

$\therefore s=-\frac{3}{2}\cos 2t-2\sin 2t+\frac{3}{2}$

at t = 0.46364, $s=-\frac{3}{2}\cos(2\times 0.46364)-2\sin(2\times 0.46364)+\frac{3}{2}$ --- [M1, allow ecf]

$= -0.99999$ (5 s.f.)

at t = 1.5, $s=-\frac{3}{2}\cos(2\times 1.5)-2\sin(2\times 1.5)+\frac{3}{2}$ --- [M1, allow ecf]

$=2.7027$ (5 s.f.)

Total distance: $2\times 0.99999+2.7027=4.70$ m (3 s.f.) — [A1]

END OF PAPER

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XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2024

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **26 August 2024**

Secondary 4 Express

2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**

Errors	**Qn No.**	**Errors**	**Qn No.**
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

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/ 90

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[Turn over

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A company purchased a colour copier machine at a cost of \$8500. The value of this machine decreases with time such that its value, \$$V$, after t months of usage is given by $V = 8500e^{-kt}$, where k is a constant.

(a) The value of the copier machine is expected to fall to \$6400 after 8 months of usage. Estimate the value, to the nearest dollar, of the machine after 2 years of usage. [4]

(b) Copier machines are to be replaced when its value reaches $\frac{1}{7}$ of its initial value.

The company's manager, Mrs Lee, claims that the machine will last for at least 5 years before a replacement is due. Showing all necessary working, explain whether you agree with Mrs Lee. [2]

2 **(a)** Differentiate $2x\sin\frac{x}{2}$. [2]

(b) Use the result in part **(a)** to evaluate $\int_0^{\pi} 3x\cos\frac{x}{2}\,dx$, leaving your answer as an exact value in the form $a\pi - b$, where a and b are constants. [4]

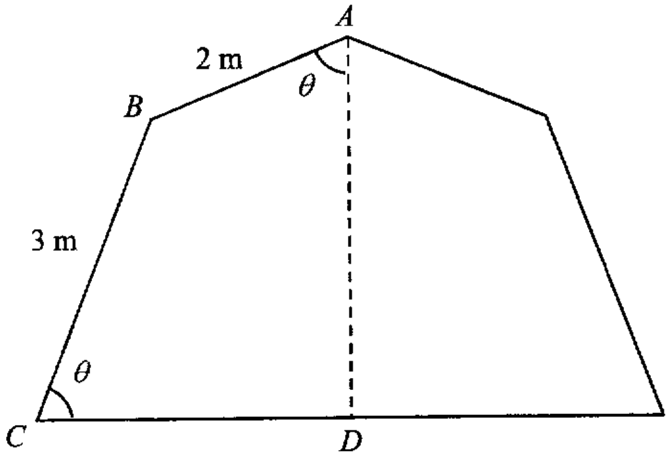
3 It is given that $\mathrm{f}(x)=2x^3+px^2+qx+3$, where p and q are constants, has a factor of $2x-1$ and leaves a remainder of -75 when divided by $x+2$.

(a) Show that $p=-15$ and $q=1$. [4]

(b) Solve the equation $\mathrm{f}(x)=0$. [4]

(c) Hence, solve the equation $2k\sqrt{k}+pk+q\sqrt{k}+3=0$. [2]

4 The diagram shows a vertical cross section of a tent in which $AB = 2$ m, $BC = 3$ m and angle $BAD =$ angle $BCD = \theta$. The tent is symmetrical about its vertical height AD and it is set up on horizontal ground.



(a) Show that $AD = 3\sin\theta + 2\cos\theta$. [2]

(b) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

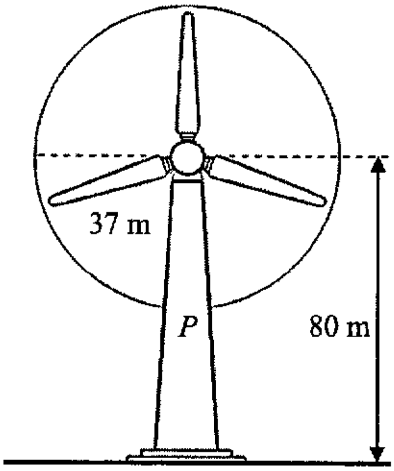
(c) Given that the vertical height of the tent is 3.45 m, calculate the value of θ. [2]

(d) Find the value of θ for which AD is a maximum. [2]

5 **(a)** Given that $\log_p A^2 = 10$ and $\log_p B = 2$, find the value of $\log_A pB$. [3]

(b) Solve $3^x = 6 - 5\left(3^{-x}\right)$. [4]

6 The diagram shows a wind turbine with propeller-like blades that have a length of 37 m each. Wind turns the blades that spin around a rotor in the centre to generate electricity. The height, h m, of the tip of each blade above the ground, t seconds after leaving a particular point P, can be modelled by $h = a - 37\cos bt$, where a and b are constants.



The centre of the wind turbine's rotor is 80 m from the ground and on average, the blades rotate in an anti-clockwise direction at a rate of 1 revolution every 8π seconds.

(a) Show that $a = 80$ and $b = \frac{1}{4}$. [2]

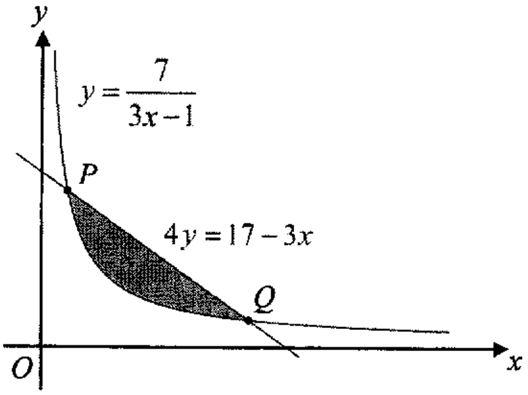
(b) Find the time taken, in seconds, for the blade to first reach a height of 89 m above ground after leaving P. [3]

7 The equation of a curve is $y = 5\ln x$. The tangent to the curve at $x = e^2$ intersects the x-axis at A.

(a) Show that the coordinates of A are $\left(-e^2,\ 0\right)$. [5]

(b) Find the area bounded by the tangent, the line $x = e^2$ and the x-axis. [2]

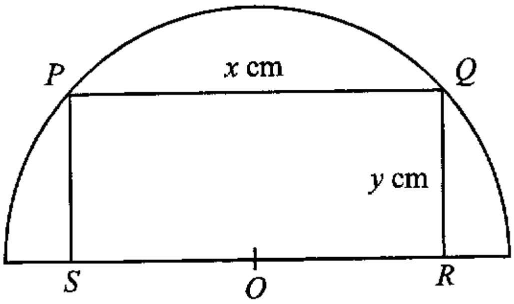
8



The diagram shows part of the curve $y = \dfrac{7}{3x-1}$ and the line $4y = 17 - 3x$, where the curve intersects the line at points P and Q.

Find, showing all necessary working, the area of the shaded region that can be expressed in the form $a - b\ln 7$, where a and b are constants. [6]

9 *PQRS* is a rectangle with $PQ = x$ cm and $QR = y$ cm. It is inscribed in a semicircle with centre *O* and radius 10 cm.



(a) Show that the area of the rectangle, *A* cm², is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [2]

(b) Given that *x* can vary, find the value of *x* for which the area of the rectangle is stationary. Leave your answer in the form $a\sqrt{b}$, where *a* and *b* are constants. [4]

(c) Explain why the value of x in **part (b)** gives the largest possible value of A and hence, find the maximum area of rectangle *PQRS*. [3]

10 AB is a chord of the circle C_1, where the coordinates of A and B are $(2,\ 5)$ and $(6,\ 3)$ respectively. The line $y = 5 - x$ passes through the centre of the circle.

(a) Find the coordinates of the centre of C_1. [4]

(b) Find the equation of the circle in the form $x^2+y^2+px+qy+r=0$, where p, q and r are integers. [3]

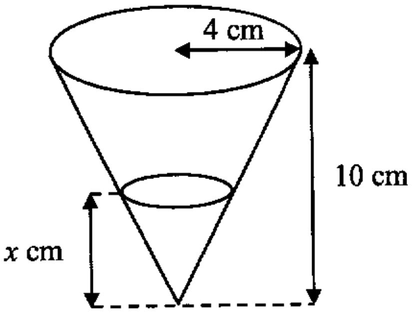
(c) Another circle C_2 with centre $(2,\ 3)$ passes through the centre of C_1.

Explain if the C_2 lies entirely within C_1. [2]

11 **(a)** Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$. [4]

(b) Hence, solve the equation $\frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = 5 - 2\sec^2 x$ for $0° < x < 360°$. [4]

12 Water is dispensed at a constant rate into an empty paper cup in the form of an inverted cone of height 10 cm and radius 4 cm. After t seconds, the depth of the water in the conical cup is x cm.



(a) Show that the volume, V cm^3, of water in the cup is given by $\frac{4\pi x^3}{75}$. [2]

(b) The water dispenser is a cylindrical container with radius 12 cm.
Given that the depth of water in the cylinder dispenser decreases at a constant rate of 0.0035 cm/s, find the rate of increase in the volume of water dispensed into the conical cup, leaving your answer in terms of π. [2]

(c) Hence, find the rate of increase in the depth of water in the conical cup when the volume of water dispensed is $\frac{5\pi}{6}$ cm^3. [4]

END OF PAPER

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Preliminary Examination 2024

CANDIDATE NAME Mark Scheme

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/02

Secondary 4 Express

26 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

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Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Errors	**Qn No.**	**Errors**	**Qn No.**
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \text{ ,}$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

Answer **all** the questions.

1 A company purchased a colour copier machine at a cost of \$8500. The value of this machine decreases with time such that its value, \$$V$, after t months of usage is given by $V = 8500e^{-kt}$, where k is a constant.

(a) The value of the copier machine is expected to fall to \$6400 after 8 months of usage. Estimate the value, to the nearest dollar, of the machine after 2 years of usage. [4]

At $t=8$,

$6400 = 8500e^{-8k}$ [M1] sub in values

$e^{-8k} = \frac{64}{85}$

$-8k = \ln\frac{64}{85}$

$k = -\frac{1}{8}\ln\frac{64}{85}$

$= 0.035471$

either [M1] value of k

At $t=24$,

$V = 8500e^{-0.035471\times24}$ [M1] sub in values, allow ecf

$= \$3628.29$

$= \underline{\$3628}$ [A1] nearest dollar

(b) Copier machines are to be replaced when its value reaches $\frac{1}{7}$ of its initial value.

The company's manager, Mrs Lee, claims that the machine will last for at least 5 years before a replacement is due. Showing all necessary working, explain whether you agree with Mrs Lee. [2]

$\frac{1}{7}(\cancel{8500}) = \cancel{8500}e^{-0.035471t}$

$\ln\frac{1}{7} = -0.035471t$

$t = \frac{\ln\frac{1}{7}}{-0.035471}$

$= 54.859$ months

either (allow ecf) [M1] value of t in months

≈ 4.57 years $\underline{< 5 \text{ years}}$

* must show comparison w respect to context

[A1] elaboration/reference to 5years explained.

Disagree, it will require a replacement in less than 5 years.

[Alternative]

$\text{Value}_o = \frac{1}{7}\times8500$

$= \$1214.29$

$\text{Value}_n = 8500e^{(-0.035471)(60)}$

$= \$1011.88$

[M1] allow ecf

Disagree,

since $\underline{\$1011.88 < \$1214.29}$,

it will require replacement in less than 5 years.

[Turn over

2 **(a)** Differentiate $2x\sin\frac{x}{2}$. [2]

Penalise under [Presentation] if '$\frac{dy}{dx}$' is written without defining Let $y = 2x\sin\frac{x}{2}$.

$$\frac{d}{dx}\left(2x\sin\frac{x}{2}\right) = 2x\cdot\frac{1}{2}\cos\frac{x}{2} + 2\sin\frac{x}{2} \quad \text{[M1] product rule}$$

$$= x\cos\frac{x}{2} + 2\sin\frac{x}{2} \quad \text{[A1]}$$

(b) Use the result in part **(a)** to evaluate $\int_0^{\pi} 3x\cos\frac{x}{2}\,dx$, leaving your answer as an exact value in the form $a\pi - b$, where a and b are constants. [4]

From (a),

$$\int_0^{\pi} x\cos\frac{x}{2} + 2\sin\frac{x}{2}\,dx = \left[2x\sin\frac{x}{2}\right]_0^{\pi}$$

$$\int_0^{\pi} x\cos\frac{x}{2}\,dx = \left[2x\sin\frac{x}{2}\right]_0^{\pi} - \int_0^{\pi} 2\sin\frac{x}{2}\,dx \quad \text{— [M1] —}$$

$$\therefore \int_0^{\pi} 3x\cos\frac{x}{2}\,dx = 6\left[x\sin\frac{x}{2}\right]_0^{\pi} - \underline{6\left[-2\cos\frac{x}{2}\right]_0^{\pi}} \quad \text{— [M1] —}$$

for $\int_0^{\pi}\sin\frac{x}{2}\,dx$ or $\int_0^{\pi} 2\sin\frac{x}{2}\,dx$

$$= 6\left[x\sin\frac{x}{2} + 2\cos\frac{x}{2}\right]_0^{\pi}$$

$$= 6\left(\underline{\pi\overset{1}{\sin\frac{\pi}{2}} + 2\overset{0}{\cos\frac{\pi}{2}} - 2\overset{1}{\cos 0}}\right) \quad \text{— [M1]}$$

definite integral value

$$= 6(\pi - 2)$$

$$= \underline{6\pi - 12} \quad \text{[A1]}$$

[Alternative]

$$\int_0^{\pi} x\cos\frac{x}{2}\,dx = \left[2x\sin\frac{\pi}{2}\right]_0^{\pi} - \int_0^{\pi} 2\sin\frac{x}{2}\,dx$$

$$= \left(2\pi\sin\frac{\pi}{2} - 0\right) - \left[\underline{-4\cos\frac{x}{2}}\right]_0^{\pi}$$

$$= \underline{2\pi\overset{1}{\sin\frac{\pi}{2}} + \left(4\overset{0}{\cos\frac{\pi}{2}} - 4\overset{1}{\cos 0}\right)}$$

$$= 2\pi - 4$$

$$\therefore \int_0^{\pi} 3x\cos\frac{x}{2}\,dx = 3(2\pi - 4)$$

$$= \underline{6\pi - 12}$$

3 It is given that $f(x) = 2x^3 + px^2 + qx + 3$, where p and q are constants, has a factor of $2x - 1$ and leaves a remainder of -75 when divided by $x + 2$.

(a) Show that $p = -15$ and $q = 1$. [4]

(-1m) from q2 if students do not show "= 0" at all.

$f(\frac{1}{2}) = 0$

$2(\frac{1}{2})^3 + p(\frac{1}{2})^2 + q(\frac{1}{2}) + 3 \underline{= 0}$ [M1] factor theorem

$\frac{1}{4}p + \frac{1}{2}q + \frac{13}{4} = 0$

$p + 2q + 13 = 0$

$p = -2q - 13$ —①

$f(-2) = -75$

$2(-2)^3 + p(-2)^2 + q(-2) + 3 = -75$ [M1] Remainder theorem

$4p - 2q - 13 = -75$

$2p = q - 31$ —②

sub ① into ②:

$2(-2q-13) = q - 31$

$-4q - 26 = q - 31$

$5q = 5$

$q = 1$

[A1] for 1st unknown shown through substitution or elimination method.

$\therefore p = -2(1) - 13$

$= -15$ [A1]

(b) Solve the equation $f(x) = 0$. [4]

(-1m) from q2 if no "= 0" at all.

Note: if both [M1] not awarded, no [A2]

$f(x) = 2x^3 - 15x^2 + x + 3$

Method #1: ⟷ either [M1] Method #2

Method #1:

$$\begin{array}{r} x^2 - 7x - 3 \\ 2x-1 \enclose{longdiv}{2x^3 - 15x^2 + x + 3} \\ -(2x^3 - x^2) \\ \hline -14x^2 + x \\ -(-14x^2 + 7x) \\ \hline -6x + 3 \\ -(-6x + 3) \\ \hline \underline{0} \end{array}$$

Method #2

Let $f(x) = (2x-1)(Ax^2 + Bx + C)$

comparing coeff. of x^3 and constant:

$\Rightarrow A = 1, \quad C = -3$

comparing coeff. of x:

$1 = 2(-3) - B$

$B = -6 - 1$

$= -7$

$\Rightarrow f(x) = (2x-1)(x^2 - 7x - 3)$

$\Rightarrow (2x-1)(x^2 - 7x - 3) \underline{= 0}$ [M1]

$2x - 1 = 0$ or $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-3)}}{2(1)}$

$\underline{x = \frac{1}{2}}$ [A1]

$= 7.4051$ or -0.40512

$= \underline{7.41 \text{ or } -0.405}$ [A1] only awarded when quadratic formula shown.

OR $x = \frac{7 \pm \sqrt{61}}{2}$

(c) Hence, solve the equation $2k\sqrt{k} + pk + q\sqrt{k} + 3 = 0$. [2]

Let $\underline{x = \sqrt{k}}$,

$\sqrt{k} = \frac{1}{2}$ OR $\sqrt{k} = 7.4051$ OR $\sqrt{k} = -0.40512$

$\underline{k = \frac{1}{4}}$ [B1]

$k = 7.4051^2$

$= \underline{54.8}$

$\underline{(\text{rej.})}$

[A1] both must show rejected value. (since $\sqrt{k} > 0$)

[Alternative]

Let $\underline{k = x^2}$

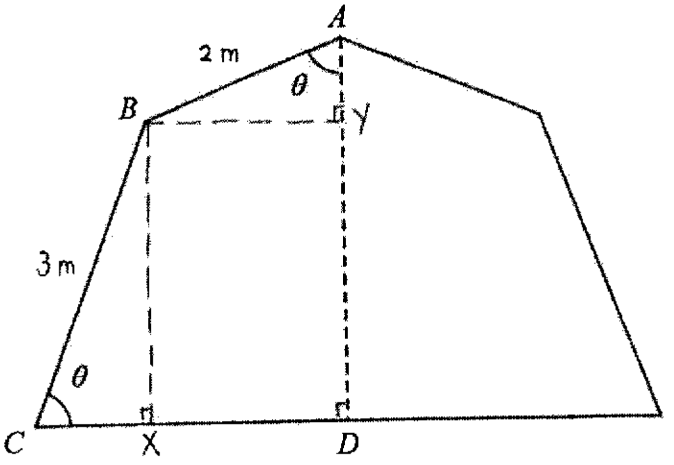
⋮

$\underline{k = \frac{1}{4}, 54.8, 0.164}$

[Turn over

angle BAD = angle BCD = θ.

4 The diagram shows a vertical cross section of a tent in which $AB = 2$ m, BC = 3m and ∧
It is symmetrical about its vertical height AD and it is set up on horizontal ground.



(a) Show that $AD = 3\sin\theta + 2\cos\theta$. [2]

must be shown.

$*\cos\theta = \frac{AY}{2}$

$AY = 2\cos\theta$

either [M1]

$*\sin\theta = \frac{BX}{3}$

$BX = 3\sin\theta$

$AD = AY + YD$

$= AY + BX$

$= 2\cos\theta + 3\sin\theta$ [A1] shown

(b) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

Let $R\cos(\theta - \alpha) = 2\cos\theta + 3\sin\theta$

$R = \sqrt{2^2 + 3^2}$ [M1]

$= \sqrt{13}$

$\Rightarrow AD = \underline{\sqrt{13}\cos(\theta - 56.3°)}$, [A1]

OR $3.61\cos(\theta - 56.3°)$

$\tan\alpha = \frac{3}{2}$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right)$ [M1]

$= 56.309°$

$= 56.3°$

(c) Given that the vertical height of the tent is 3.45 m, calculate the value of θ. [2]

$$\sqrt{13}\cos(\theta - 56.309°) = 3.45$$

$$\cos(\theta - 56.309°) = \frac{3.45}{\sqrt{13}}$$ or 0.95685 [M1] allow ecf (max 1m) for error from (b)

$$\text{ref.}\measuredangle = \cos^{-1}\left(\frac{3.45}{\sqrt{13}}\right)$$

$$= 16.891°$$

since θ is acute,

$$\theta - 56.309° = 16.891°$$

$$\theta = 16.891° + 56.309°$$

$$= \underline{73.2°}$$ [A1]

(d) Find the value of θ for which AD is a maximum. [2]

Maximum value of AD occurs at:

$$\cos(\theta - 56.309°) = 1$$ [M1] allow ecf from (b)

$$\theta - 56.309° = 0°$$

$$\therefore \theta = \underline{56.3°}$$ [A1]

[Alternative]

$$\left.\begin{aligned} &\frac{d}{dx}(3\sin\theta + 2\cos\theta) \\ &= 3\cos\theta - 2\sin\theta \\ &3\cos\theta - 2\sin\theta = 0 \\ &2\tan\theta = 3 \end{aligned}\right\}$$

[M1] allow ecf
need to show $\frac{d}{dx}\cdots = 0$
for maximum.

$$\vdots$$

$$\underline{\theta = 56.3°}$$ (acute) [A1]

5 **(a)** Given that $\log_p A^2 = 10$ and $\log_p B = 2$, find the value of $\log_A pB$. [3]

$2\log_p A = 10$

$\log_p A = 5$

$\log_p B = 2$

$p^2 = B$

either [M1]

$\log_A pB = \dfrac{\log_p pB}{\log_p A}$ [M1] change base

$= \dfrac{\log_p (p \times p^2)}{5}$

$= \dfrac{\log_p (p^3)}{5}$

$= \underline{\dfrac{3}{5}}$ [A1]

[Alternative ①]

$2\log_p A = 10$

$\log_p A = 5$ [M1]

$\log_A pB = \dfrac{\log_p pB}{\log_p A}$ [M1]

$= \dfrac{\log_p p + \log_p B}{5}$

$= \dfrac{1+2}{5}$

$= \dfrac{3}{5}$ [A1]

[Alternative ②]

$2\log_p A = 10$

$\log_p A = 5$

$\log_A p = \dfrac{\log_p p}{\log_p A} = \dfrac{1}{5}$

either [M1]

$\log_A B = \dfrac{\log_p B}{\log_p A} = \dfrac{2}{5}$ [M1]

$\therefore \log_A pB = \log_A p + \log_A B$

$= \dfrac{1}{5} + \dfrac{2}{5}$

$= \dfrac{3}{5}$ [A1]

[Alternative ③]
change to base A

$\log_p A = 5$

$\log_A p = \dfrac{1}{5}$

either [M1]

$\dfrac{\log_A B}{\log_A p} = 2$ [M1]

$\log_A B = 2 \times \dfrac{1}{5} = \dfrac{2}{5}$

$\log_A p + \log_A B = \dfrac{1}{5} + \dfrac{2}{5} = \dfrac{3}{5}$ [A1]

(b) Solve $3^x = 6 - 5\left(3^{-x}\right)$. [4]

$3^x = 6 - \dfrac{5}{3^x}$

$3^{2x} = 6(3^x) - 5$ [M1] multiply by 3^x throughout eqⁿ

$3^{2x} - 6(3^x) + 5 = 0$

Let $u = 3^x$,

$u^2 - 6u + 5 = 0$

$(u-1)(u-5) = 0$

[M1] both (substitution & factorise)

$u = 1$ or $u = 5$

$\Rightarrow 3^x = 3^0$

$\therefore \underline{x = 0}$

[A1]

$\Rightarrow 3^x = 5$

$\lg 3^x = \lg 5$

$x = \dfrac{\lg 5}{\lg 3}$

$= \underline{1.46}$

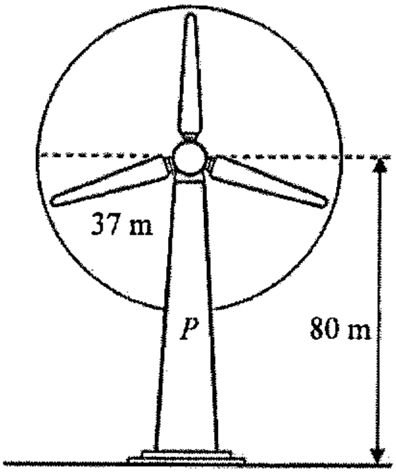
[A1]

(accept $x = \log_3 5$)

(-1) deduct [A1] if any x values rejected.

Ⓟ^{1} lg written as log / $\log_{10}$

6 The diagram shows a wind turbine with propeller-like blades that have a length of 37 m each. Wind turns the blades that spin around a rotor in the centre to generate electricity. The height, h m, of the tip of each blade above the ground, t seconds after leaving a particular point P, can be modelled by $h = a - 37\cos bt$, where a and b are constants.



The centre of the wind turbine's rotor is 80 m from the ground and on average, the blades rotate in an anti-clockwise direction at a rate of 1 revolution every 8π seconds.

(a) Explain why $a = 80$ and show that $b = \frac{1}{4}$. [2]

$\boxed{\text{For } a = 80}$

$a \neq$ amplitude

at point P, $t = 0$

$h = 80 - 37$

$= 43$ m

$\Rightarrow 43 = a - 37\overset{1}{\cos(0)}$

$\therefore a = 80$ [A1]

[Alternatives]

Accept all other equivalent answers: if $b = \frac{1}{4}$ found first.

- when $t = 2\pi$, $h = 80$
- when $t = 4\pi$, $h = 117$
- when $t = 6\pi$, $h = 80$

$\boxed{\text{OR}}$ max. $h = 117$
min. $h = 43$
$a = \frac{117 + 43}{2} = 80$ [A1]

$\boxed{\text{For } b = \frac{1}{4}}$

Period $= 8\pi$

$\frac{2\pi}{b} = 8\pi$

$b = \frac{2\pi}{8\pi}$

$= \frac{1}{4}$ [A1]

$\boxed{\text{OR}}$ $\cos(2\pi b) = 0$

$b = \frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4}$

(b) Find the time taken, in seconds, for the blade to first reach a height of 89 m above ground after leaving P. [3]

$h = 80 - 37\cos\frac{t}{4}$

$80 - 37\cos\frac{t}{4} = 89$

$\cos\frac{t}{4} = \frac{89 - 80}{-37}$

$\cos\frac{t}{4} = -\frac{9}{37}$ [M1]

$\underset{(\text{ref} \angle)}{\alpha} = \cos^{-1}\left(\frac{9}{37}\right)$

$= 1.3250$ rad.

$\therefore \frac{t}{4} = \pi - 1.3250,\ \pi + 1.3250$

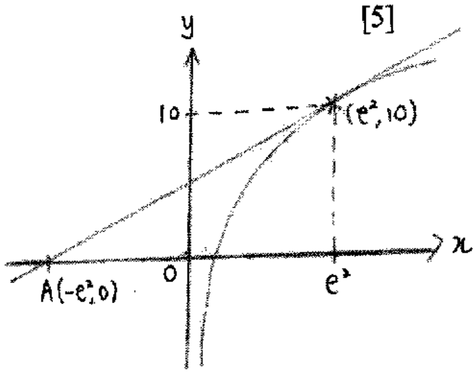
$= 1.8165,\ 4.4665$ [M1]

$\underbrace{t = 7.27}_{[\text{A1}]},\ \underset{(\text{rej.})}{17.9}$

[✱ since first t value,
benefit of doubt:
accept $\frac{t}{4} = \cos^{-1}\left(-\frac{9}{37}\right)$
To take note of α (ref ∠) misconception.]

7 The equation of a curve is $y = 5\ln x$. The tangent to the curve at $x = e^2$ intersects the x-axis at A.

(a) Show that the coordinates of A are $\left(-e^2,\ 0\right)$. [5]



at $x = e^2$, $y = 5 \ln e^2$

$= 5(2 \ln e)$

$= 10$ [M1]

$\frac{dy}{dx} = 5\left(\frac{1}{x}\right)$

$= \frac{5}{x}$ [M1]

$m_{(\text{at } x = e^2)} = \frac{5}{e^2}$

$\therefore$ Eqⁿ of tangent : $y - 10 = \frac{5}{e^2}(x - e^2)$

$y = \frac{5}{e^2}x + 5$ [M1] allow ecf

$\Rightarrow$ Intersects x-axis, $y = 0$.

$\therefore \frac{5}{e^2}x + 5 = 0$ [M1] allow ecf.

$\frac{5}{e^2}x = -5$

$\frac{x}{e^2} = -1$

$\therefore x = -e^2$

Solve for x [A1]

$\Rightarrow$ coordinates of $A(-e^2, 0)$. (shown)

OR Let $A(x, 0)$,

$\frac{10 - 0}{e^2 - x} = \frac{5}{e^2}$ [M1]

$10e^2 = 5(e^2 - x)$ [M1]

$-5x = 5e^2$

$x = \frac{5e^2}{-5}$

$= -e^2$

$\therefore A(-e^2, 0)$

[A1]

(b) Find the area bounded by the tangent, the line $x = e^2$ and the x-axis. [2]

Area of bounded region

$= \frac{1}{2} \times 2e^2 \times 10$ [M1]

$= 10e^2$ units² [A1]

OR 73.9 units²

[Alternative]

Area $= \int_{-e^2}^{e^2} \frac{5}{e^2}x + 5 \, dx$

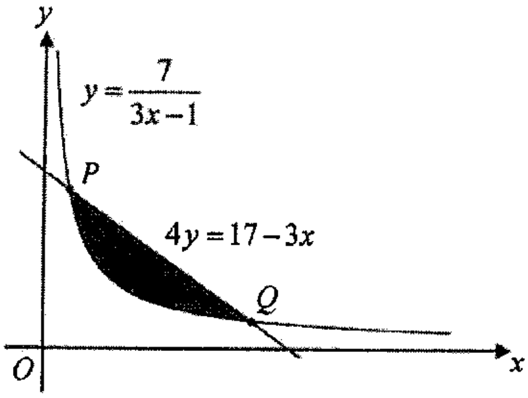
$= \left[\frac{5}{2e^2}x^2 + 5x\right]_{-e^2}^{e^2}$ [M1]

$= \left(\frac{5}{2}e^2 + 5e^2\right) - \left(\frac{5}{2}e^2 - 5e^2\right)$

$= 5e^2 + 5e^2$

$= 10e^2$ units² [A1]

8



The diagram shows part of the curve $y=\frac{7}{3x-1}$ and the line $4y=17-3x$, where the curve intersects the line at points P and Q.

Find, showing all necessary working, the area of the shaded region that can be expressed in the form $a-b\ln 7$, where a and b are constants. [6]

$y=\frac{7}{3x-1}$ —①

$4y=17-3x$

$y=\frac{17-3x}{4}$ —②

sub② into①:

$\frac{17-3x}{4}=\frac{7}{3x-1}$ [M1]

$(17-3x)(3x-1)=28$

$51x-17-9x^2+3x=28$

$9x^2-54x+45=0$

$x^2-6x+5=0$

$(x-1)(x-5)=0$

overall -1m for GB for lack of equivalence or not factorised (eg. skip steps for ÷9)

$x=1$ or 5 [M1]

Area of Shaded region

$=\int_1^5 \frac{17-3x}{4}-\frac{7}{3x-1}\,dx$

$=\Big[\underbrace{\tfrac{17}{4}x-\tfrac{3}{8}x^2}_{\substack{\text{[M1]}\\ \text{area under line}}}\Big]_1^5 \underbrace{-\tfrac{7}{3}\big[\ln(3x-1)\big]_1^5}_{\substack{\text{[M1]}\\ \text{area under curve}}}$

$=\underline{\left[\tfrac{17}{4}(5)-\tfrac{3}{8}(5)^2\right]-\left(\tfrac{17}{4}-\tfrac{3}{8}\right)}$ − OR $\underline{\tfrac{7}{3}\left[\ln 14-\ln 2\right]}$

[M1] either definite integral value

$=\frac{95}{8}-\frac{31}{8}-\frac{7}{3}\ln\left(\frac{14}{2}\right)$

$=\underline{8-\frac{7}{3}\ln 7}$ [A1]

[Alternative]

Area $=\underbrace{\overset{\text{area of trapezium}}{\tfrac{1}{2}(3.5+0.5)(5-1)}}_{\text{[M1]}}-\int_1^5\frac{7}{3x-1}\,dx$

$=\frac{1}{2}(4)(4)\underbrace{-\frac{7}{3}\left[\ln(3x-1)\right]_1^5}_{}$ [M1] area under curve

$=8-\underline{\frac{7}{3}(\ln 14-\ln 2)}$ [M1]

$=8-\frac{7}{3}\ln\left(\frac{14}{2}\right)$

$=8-\frac{7}{3}\ln 7$

[Alternative] against the y-axis

Area $=\int_{0.5}^{3.5}\frac{17-4y}{3}-\frac{7+y}{3y}\,dy$

$=\underbrace{\left[\frac{17}{3}-\frac{4}{3}y\right]_{0.5}^{3.5}}_{\text{[M1]}}-\Big[\underbrace{\tfrac{7}{3}\ln y+\tfrac{1}{3}y}_{\text{[M1]}}\Big]_{0.5}^{3.5}$

$=\underline{\left(\frac{119}{6}-\frac{49}{6}\right)-\left(\frac{17}{6}-\frac{1}{6}\right)}-\left[\underline{\frac{7}{3}\ln 3.5+\frac{7}{6}-\frac{7}{3}\ln 0.5-\frac{1}{6}}\right]$

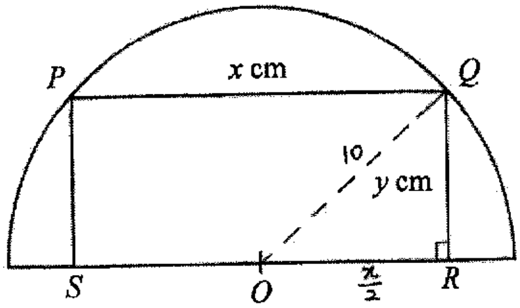
[M1] either

$=9-\left(1+\frac{7}{3}\ln 7\right)$

$=8-\frac{7}{3}\ln 7$ [A1]

[Turn over

9 *PQRS* is a rectangle with $PQ = x$ cm and $QR = y$ cm. It is inscribed in a semicircle with centre *O* and radius 10 cm.



(a) Show that the area of the rectangle, A cm^2, is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [2]

$$y^2 + \left(\frac{x}{2}\right)^2 = 10^2$$

$$y = \sqrt{100 - \frac{x^2}{4}} \quad \text{[M1]}$$

(since $y > 0$)

$$\therefore \text{Area} = xy$$

$$= x\sqrt{100 - \frac{x^2}{4}}$$

$$= \underbrace{x\sqrt{\frac{400 - x^2}{4}}}$$

$$= \frac{x}{2}\sqrt{400 - x^2}$$

manipulation must be shown

[A1] shown

OR

$$y^2 = 100 - \frac{x^2}{4}$$

$$4y^2 = 400 - x^2$$

$$2y = \sqrt{400 - x^2}$$

[M1]

$$y = \frac{1}{2}\sqrt{400 - x^2}$$

sub

$$\text{Area} = x\,y$$

$$= x\left(\frac{1}{2}\sqrt{400 - x^2}\right)$$

$$= \frac{x}{2}\sqrt{400 - x^2}$$

[A1]

OR

$$\text{Area} = x\,y$$

$$= x\sqrt{100 - \frac{x^2}{4}}$$

$$= \underline{\frac{x}{2}(2)}\sqrt{100 - \frac{x^2}{4}}$$

$$= \underline{\frac{x}{2}\sqrt{4}}\sqrt{100 - \frac{x^2}{4}}$$

$$= \frac{x}{2}\sqrt{400 - x^2}$$

must be shown.

[A1]

(b) Given that x can vary, find the value of x for which the area of the rectangle is stationary. Leave your answer in the form $a\sqrt{b}$, where a and b are constants. [4]

$$\frac{dA}{dx} = \frac{1}{2}x \cdot \frac{1}{2}(400 - x^2)^{-\frac{1}{2}}(-2x) + \frac{1}{2}(400 - x^2)^{\frac{1}{2}}$$

[M1] OR Quotient rule

$$= -\frac{x^2}{2\sqrt{400 - x^2}} + \frac{\sqrt{400 - x^2}}{2}$$

$$= \frac{-x^2 + 400 - x^2}{2\sqrt{400 - x^2}}$$

$$= \frac{200 - x^2}{\sqrt{400 - x^2}}$$

any acceptable [M1]

When $\frac{dA}{dx} = 0$,

$$\frac{200 - x^2}{\sqrt{400 - x^2}} = 0$$

[M1] for equating $\frac{dA}{dx} = 0$, using their $\frac{dA}{dx}$ above.
e.c.f allowed.

$$200 = x^2$$

$$x = \sqrt{200} \quad (\text{since } x > 0)$$

$$= \underbrace{10\sqrt{2}} \quad \text{[A1]}$$

(accept $5\sqrt{8}$ as per $a\sqrt{b}$ form)

Ⓟ$^{-1}$ for any errors in expressions for $\frac{dA}{dx}$

Not accepted:

$$\frac{dA}{dx}\left(\frac{x}{2}\sqrt{400 - x^2}\right)$$

or $\frac{dy}{dx}$ ← etc ...

(c) Explain why the value of x in (b) gives the largest possible value of A and hence, find the maximum area of the rectangle. [3]

⇒ Proving/finding maximum area

Ⓟ$^{-1}$ $\frac{dy}{dx}$ stated instead of $\frac{dA}{dx}$

Method #1 (2nd derivative test)

$$\frac{d^2A}{dx^2} = (200-x^2)\cdot -\frac{1}{2}(400-x^2)^{-\frac{3}{2}}(-2x) + (-2x)(400-x^2)^{-\frac{1}{2}}$$

$$= \frac{x(200-x^2)}{(400-x^2)^{\frac{3}{2}}} - \frac{2x}{(400-x^2)^{\frac{1}{2}}}$$

[M1] any equivalent $\frac{d^2A}{dx^2}$ expression ecf allowed from (b)

$$= \frac{200x - x^3 - 800x + 2x^3}{(400-x^2)^{\frac{3}{2}}}$$

$$= \frac{x^3 - 600x}{(400-x^2)^{\frac{3}{2}}}$$

at $x = 10\sqrt{2}$,

$$\frac{d^2A}{dx^2} = \frac{(10\sqrt{2})^3 - 600(10\sqrt{2})}{(400-200)^{\frac{3}{2}}}$$

$$= -2 < 0$$

Since $\frac{d^2A}{dx^2} < 0$, maximum area occurs at $x = 10\sqrt{2}$.

[A1] $\frac{d^2A}{dx^2}$ value must be shown

$$\therefore \text{Max area} = \frac{10\sqrt{2}}{2}\sqrt{400-(10\sqrt{2})^2}$$

$$= 5\sqrt{2}\cdot\sqrt{200}$$

$$= 5\sqrt{400}$$

$$= 100 \text{ cm}^2$$ [A1]

Method #2 (1st derivative test)

x	$(10\sqrt{2})^-$	$10\sqrt{2}$	$(10\sqrt{2})^+$
$\frac{dA}{dx}$	>0	$=0$	<0
sketch of tangent	/	—	\

[M1]

By the 1st derivative test,
⇒ maximum area occurs at $x = 10\sqrt{2}$ [A1]

$$\therefore \text{Max area} = \frac{10\sqrt{2}}{2}\sqrt{400-(10\sqrt{2})^2}$$

$$= 100 \text{ cm}^2$$ [A1]

Explanations NOT accepted:

'$x = 10\sqrt{2}$ is a maximum point'.

'$x = 10\sqrt{2}$ is maximum' ← any unclear/inaccurate indication of the meaning of x.

10 *AB* is a chord of the circle C₁, where the coordinates of *A* and *B* are $(2, 5)$ and $(6, 3)$ respectively. The line $y = 5 - x$ passes through the centre of the circle.

(a) Find the coordinates of the centre of C₁. [4]

Method#1

Mid-point of AB $= \left(\frac{2+6}{2}, \frac{5+3}{2}\right)$

$= (4, 4)$ [M1]

$m_{AB} = \frac{5-3}{2-6}$

$= -\frac{1}{2}$

$\therefore m_{\perp} = 2$ [M1]

Eqⁿ of ⊥ bisector:

$y - 4 = 2(x-4)$

$y = 2x - 4$ —①

$y = 5 - x$ —②

Sub ① into ②:

$2x - 4 = 5 - x$ [M1] intersection of ⊥ bisector and line

allow ecf

$3x = 9$

$x = 3$

$\therefore y = 5 - 3$

$= 2$

⇒ centre: $(3, 2)$ [A1]

Method #2

Let centre of C₁ be (a, b)

Length of AC₁ = Length of BC₁

$\sqrt{(5-b)^2 + (2-a)^2} = \sqrt{(3-b)^2 + (6-a)^2}$ [M1]

$25 - 10b + \cancel{b^2} + 4 - 4a + \cancel{a^2} = 9 - 6b + \cancel{b^2} + 36 - 12a + \cancel{a^2}$

$29 - 10b - 4a = 45 - 6b - 12a$

$0 = 16 + 4b - 8a$

$b - 2a + 4 = 0$

$b = 2a - 4$ —①

For line $y = 5 - x$,

at (a, b),

$b = 5 - a$ —② [M1]

Sub ① into ②:

$2a - 4 = 5 - a$ [M1]

$3a = 9$

$a = 3$

$\therefore b = 5 - 3$

$= 2$

⇒ centre $(3, 2)$ [A1]

(b) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [3]

Method #1

radius $= \sqrt{3^2+1^2}$

$= \sqrt{10}$ [M1]

allow ecf

$(x-3)^2 + (y-2)^2 = (\sqrt{10})^2$ [M1]

$x^2 - 6x + 9 + y^2 - 4y + 4 = 10$

$x^2 + y^2 - 6x - 4y + 3 = 0$ [A1]

Method #2

radius $= \sqrt{3^2+1^2}$

$= \sqrt{10}$ [M1]

using $x^2+y^2+2gx+2fy+c=0$,

centre of circle : $C(-g, -f)$

$\Rightarrow -g = 3$ $\Rightarrow -f = 2$

$g = -3$ $f = -2$

$\underline{2g = -6 \quad 2f = -4}$

radius $= \sqrt{f^2+g^2-c}$

$\sqrt{10} = \sqrt{(-2)^2+(-3)^2-c}$

$10 = 13 - c$

$\underline{c = 3}$

either values of both p and q [M1] or value of r.

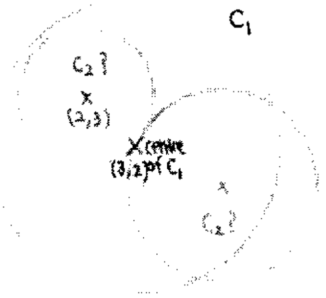
allow ecf

$\therefore x^2+y^2-6x-4y+3=0$

[A1]

(c) Another circle C_2 with centre $(2, 3)$ passes through the centre of C_1.

Explain if the C_2 lies entirely within C_1. [2]



for C_2 to lie entirely within C_1,

diameter of C_2 < radius of C_1

radius of C_2 $= \sqrt{1^2+1^2}$

$= \sqrt{2}$ } [M1] radius of C_2

$\therefore$ diameter of C_2 $= \underline{2\sqrt{2} < \sqrt{10}}$

Since the diameter of circle C_2 is shorter than the radius of circle C_1, it lies entirely within C_1. } [A1] explained with comparison.

11 **(a)** Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$ [4]

$$\text{LHS} = \frac{2\sin x\cos x - (1-2\sin^2 x) + 1}{2\sin x\cos x + (2\cos^2 x - 1) + 1}$$

[M1] double angle formula for sin 2x

[M1] either (2cos²x−1) selected in numerator or (1−2sin²x) selected in denominator

$$= \frac{2\sin x\cos x + 2\sin^2 x}{2\sin x\cos x + 2\cos^2 x}$$

$$= \frac{2\sin x(\cancel{\cos x + \sin x})}{2\cos x(\cancel{\sin x + \cos x})}$$

[M1] factorisation

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

[A1] shown.

Alternative

$$\frac{\overset{[M1]}{\underline{2\sin x\cos x}} - (\cos^2 x - \sin^2 x) + 1}{2\sin x\cos x + (\cos^2 x - \sin^2 x) + 1}$$

$$= \frac{2\sin x\cos x \underline{- \cos^2 x + \overset{\text{either [M1]}}{\sin^2 x}} + \underline{(\sin^2 x + \cancel{\cos^2 x})}}{2\sin x\cos x \underline{+ \cos^2 x - \sin^2 x} + \underline{(\sin^2 x + \cos^2 x)}}$$

$$= \frac{2\sin x\cos x + 2\sin^2 x}{2\sin x\cos x + 2\cos^2 x} = \frac{\overset{[M1]}{2\sin x(\cos x + \sin x)}}{2\cos x(\sin x + \cos x)}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

[A1]

Alternative:

$$\text{LHS} = \frac{2\sin x\cos x - (2\cos^2 x - 1) + 1}{2\sin x\cos x + (1-2\sin^2 x) + 1}$$

[M1] double angle formula for sin 2x

$$= \frac{2\sin x\cos x - 2\cos^2 x + 2}{2\sin x\cos x - 2\sin^2 x + 2}$$

$$= \frac{\cancel{2}(\sin x\cos x - \cos^2 x + 1)}{\cancel{2}(\sin x\cos x - \sin^2 x + 1)}$$

$$= \frac{\sin x\cos x - (1-\sin^2 x) + 1}{\sin x\cos x - (1-\cos^2 x) + 1}$$

[M1] either numerator: cos 2x = 2cos²x−1 selected first, then cos²x = 1−sin²x

or denominator: cos 2x = 1−2sin²x selected first, then sin²x = 1−cos²x.

$$= \frac{\sin x\cos x + \sin^2 x}{\sin x\cos x + \cos^2 x}$$

$$= \frac{\sin x(\cancel{\cos x + \sin x})}{\cos x(\cancel{\sin x + \cos x})}$$

[M1] factorisation

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

[A1] shown

(b) Hence, solve the equation $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = 5 - 2\sec^2 x$ for $0° < x < 360°$. [4]

Hence, $\tan x = 5 - 2\sec^2 x$

$\tan x = 5 - 2(1 + \tan^2 x)$ [M1] $\sec^2 x = 1 + \tan^2 x$

award M1 even if solution from (a) is applied incorrectly

$2\tan^2 x + \tan x - 3 = 0$

$(2\tan x + 3)(\tan x - 1) = 0$ [M1]

$\tan x = -\frac{3}{2}$ or $\tan x = 1$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right)$

$= 56.309°$

$\alpha = 45°$

$\therefore x = \underbrace{45°,\ 225°}_{[A1]}$

$\therefore x = 180° - \alpha,\ 360° - \alpha$

$= 180° - 56.309°,\ 360° - 56.309°$

$= \underline{123.7°,\ 303.7°}$ [A1]

$\Rightarrow x = 45°, 123.7°, 225°, 303.7°$.

ⓤ$^{-1}$ degrees must be stated.

[A1] deducted for <1dp answers.
(eg. 123°/124° or 303°/304°)

[Alternative]

$\tan x = 5 - 2\sec^2 x$

$\dfrac{\sin x}{\cos x} = 5 - \dfrac{2}{\cos^2 x}$

$\sin x \cos x = 5\cos^2 x - 2$

$5\cos^2 x - \sin x \cos x - 2 = 0$

$5\cos^2 x - \sin x \cos x - 2(\sin^2 x + \cos^2 x) = 0$ [M1]

$3\cos^2 x - \sin x \cos x - 2\sin^2 x = 0$

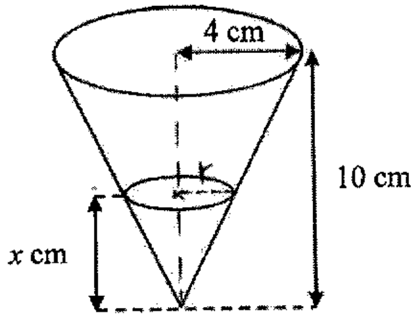
$(3\cos x + 2\sin x)(\cos x - \sin x) = 0$ [M1]

$3\cos x = -2\sin x$ or $\cos x = \sin x$

$\tan x = -\frac{3}{2}$ $\qquad$ $\tan x = 1$

12 Water is dispensed at a constant rate into an empty paper cup in the form of an inverted cone of height 10 cm and radius 4 cm. The water dispenser is a cylindrical container with radius 12 cm. After t seconds, the depth of the water in the conical cup is x cm.

* Penalise 1m overall for Q11 for absence of units in (b), (c)



(a) Show that the volume of water in the cup is $\frac{4\pi x^3}{75}$ cm^3. [2]

$$\frac{r}{4} = \frac{x}{10}$$

$$r = \frac{2}{5}x \quad [M1]$$

$$\text{volume} = \frac{1}{3}\pi\left(\frac{2}{5}x\right)^2(x)$$

$$= \frac{2^2\pi x^3}{3\times5^2}$$

$$= \frac{4\pi x^3}{75}$$

[A1] shown with substitution of r into vol. of cone formula.

$\boxed{OR}$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{V_1}{\frac{1}{3}\pi(4)^2(10)} = \left(\frac{x}{10}\right)^3 \quad [M1]$$

$$\left.\begin{aligned} V_1 &= \frac{x^3}{1000} \times \frac{160\pi}{3} \\ &= \frac{160\pi x^3}{3000} \\ &= \frac{4\pi x^3}{75} \end{aligned}\right\} [A1]$$

(b) Given that the depth of water in the cylinder dispenser decreases by 0.0035 cm/s, find the rate of increase in the volume of water dispensed in the conical cup, in terms of π. [2]

Common misconception:

misinterpretation of variables in qn, values happen to give same final answer but NO marks awarded.

$\frac{dx}{dt} \neq 0.0035$

rate of increase in the height of water in cone is not a constant.

Let h be the depth of water in cylinder.

$$\frac{dh}{dt} = -0.0035 \text{ cm/s}$$

rate of increase in cup (in vol.) = rate of decrease in cylinder (in vol.)

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

Volume of cylinder $= \pi(12^2)h$

$= 144\pi h$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$= \underbrace{144\pi \times -0.0035}\ [M1]$$

$$= -0.504\pi \text{ cm}^3/\text{s}$$

$\Rightarrow$ rate of increase in cup

$= \underline{0.504\pi \text{ cm}^3/\text{s}}$ [A1]

(or $\frac{63\pi}{125}$ cm^3/s)

(u)$^{-1}$ cm³/s

accept passive $\frac{dV}{dt}$ directly. $\frac{dV}{dt} = 144\pi \times 0.0035$

$= 0.504\pi$ cm^3/s

[Alternative]

$$(\text{cylinder}) \frac{dV}{dt} = \pi r^2(\Delta h)$$

$$= \pi(12)^2(-0.0035) \quad [M1]$$

$$= -0.504\pi \text{ cm}^3/\text{s}$$

$$\therefore \underset{(\text{cone})}{\frac{dV}{dt}} = 0.504\pi \text{ cm}^3/\text{s} \quad [A1]$$

(c) Hence, find the rate of increase in the depth of water in the conical cup when the volume of water dispensed is $\frac{5\pi}{6}$ cm^3. [4]

$$\underset{(\text{cup})}{\frac{dV}{dt}} = \frac{dV}{dx} \times \frac{dx}{dt}$$

When $\text{vol}_{\text{cup}} = \frac{5\pi}{6}$,

$$\frac{4\pi x^3}{75} = \frac{5\pi}{6}$$

$$x^3 = \frac{5}{6}\left(\frac{75}{4}\right)$$

$$x = \sqrt[3]{\frac{125}{8}}$$

$$= 2.5 \text{ cm} \quad [M1]$$

$$\frac{dV}{dx} = \frac{3(4\pi)x^2}{75}$$

$$= \frac{4\pi x^2}{25} \quad [M1]$$

$$\therefore \underline{0.504\pi} = \frac{4\pi(2.5)^2}{25} \times \frac{dx}{dt} \quad [M1]$$ allow ecf from part(b) or incorrect x-value above.

$\underline{\text{OR}}$ $\frac{dx}{dt} = \frac{63}{20x^2}$ [M1]

$$\therefore \frac{dx}{dt} = \frac{0.504\pi(25)}{4\pi(2.5)^2}$$

$$= 0.504 \text{ cm/s} \quad [A1] \quad \left(\text{or } \frac{63}{125} \text{ cm/s}\right)$$

Ⓤ$^{-1}$ cm/s

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Answer Key

Qn No.		**Answers**
1	(a) (b)	\$3628 Disagree.
2	(a) (b)	$x\cos\frac{x}{2}+2\sin\frac{x}{2}$ $6\pi-12$
3	(b) (c)	$x=7.41$ or -0.405 $k=0.25$ or 54.8
4	(b) (c) (d)	$\sqrt{13}\cos(\theta-56.3°)$ $73.2°$ $56.3°$
5	(a) (b)	$\frac{3}{5}$ $x=0$ or 1.46
6	(b)	$t=7.27$ s
7	(b)	$10e^2$ or 73.9 units^2
8		$8-\frac{7}{3}\ln 7$
9	(b) (c)	$10\sqrt{2}$ Maximum area 100 cm^2
10	(a) (b) (c)	$(3,2)$ $x^2+y^2-6x-4y+3=0$ C_2 lies entirely within C_1
11	(b)	$x=45°,123.7°,225°,303.7°$
12	(b) (c)	0.504π cm^3/s or $\frac{63\pi}{125}$cm^3/s 0.504 cm/s