ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS /
5 NORMAL ACADEMIC

Candidate Name	Class	Register Number

ADDITIONAL MATHEMATICS — 4049/01

Paper 1

28 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, or correction fluid.

Answer **all** the questions.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90**.

	90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1. **(a)** Solve the simultaneous equations [3]

$$x-2y+4=0$$
$$x^2+y^2=2x+4$$

(b) Explain the geometrical meaning of your answer in **(a)**. [1]

2. **(a)** Express $y = 3 - 8x - 2x^2$ in the form $y = a(x+b)^2 + c$ and hence state the maximum value of y. [3]

(b) Show that there are no values of p for which the curve $y = (p-3)\,x^2 + 2px + (p+1)$ is always positive. [3]

(c) A quadratic equation is given by $hx^2 - 2kx + 6k - 9h = 0$, where h and k are constants and $h \neq 0$.

(i) Show that the equation has real roots for all values of h and k. [3]

(ii) In the case where the equation has two real and equal roots, express h in terms of k. [2]

3. Given that $\cos A = \sqrt{\frac{2}{11}}$ where $180^\circ < A < 360^\circ$, find, without the use of a calculator, the value of

(a) $\tan A$, [2]

(b) $\sin(A - 90^\circ)$, [2]

(c) $\dfrac{1}{\sec 2A}$. [2]

4. **(a)** Factorise $27x^3 - \frac{y^3}{8}$ completely. [2]

(b) Express $\frac{8x^3 - 7x^2 + 4x - 3}{(2x^2 - x)(2x - 1)}$ in partial fractions. [6]

5. A curve is such that $\frac{dy}{dx}=\frac{2x-ax^2}{3}$, where a is a constant.

(a) Given that the curve has a turning point at $(3,7)$, show that the value of a is $\frac{2}{3}$. [1]

(b) Find the range of values of x for which y decreases as x increases. [3]

(c) Find the equation of the curve. [4]

6. (a) Prove the identity $(\cot x - \operatorname{cosec} x)^2 = \dfrac{1-\cos x}{1+\cos x}$. [4]

(b) Hence, solve the equation $2(\cot x - \operatorname{cosec} x)^2 = 3\cos x$ for $0 \le x \le 2\pi$. [3]

(c) State the number of solutions of the equation $2(\cot 2x - \operatorname{cosec} 2x)^2 = 3\cos 2x$ in the range $-2\pi \le 2x \le 2\pi$. [1]

7 **(a)** The graph of $y = \log_a x$ passes through the points with coordinates (125, 3) and (1, b).

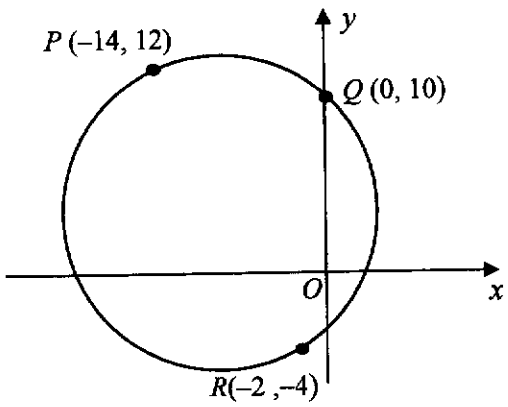
(i) Determine the values of a and b. [2]

(ii) Sketch the graph of $y = \log_{a^{-1}} x$ indicating clearly any intercept on the axes. [2]

(b) Find the values of a and b such that $\lg\left(\frac{8}{y}\right)+4\lg y=a\lg(by)$. [4]

[Turn over

8 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram which is not drawn to scale, P, Q and R are points on the circle.

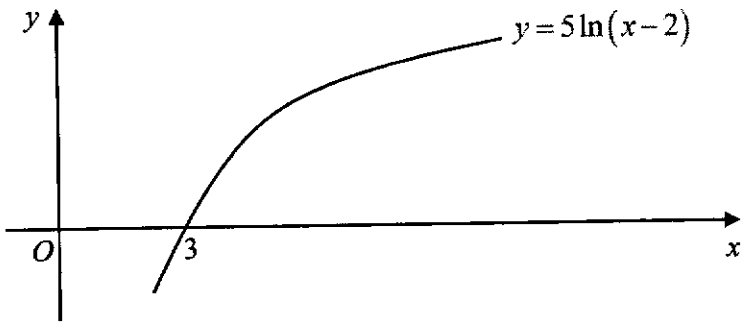
(a) Show that PR is the diameter of the circle and hence find the centre of the circle. [5]

(b) Find the equation of the circle that passes through the points P, Q and R. [2]

(c) Determine whether the point $S(-14,-2)$ lies inside or outside the circle. [2]

[Turn over

9 The diagram shows part of the curve $y = 5\ln(x-2)$.



(a) Find the exact value of $\int_0^5 x \, dy$. [3]

(b) On the diagram above, shade the region whose area is $\int_0^5 x \, dy$, showing your upper limit clearly. [1]

(c) Hence find $\int_3^{e+2} 5\ln(x-2)\ dx$. [3]

10 Water is being added at a constant rate of 4 cm^3/s to an inverted right cone. The height of the cone is twice the radius of the cone.

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

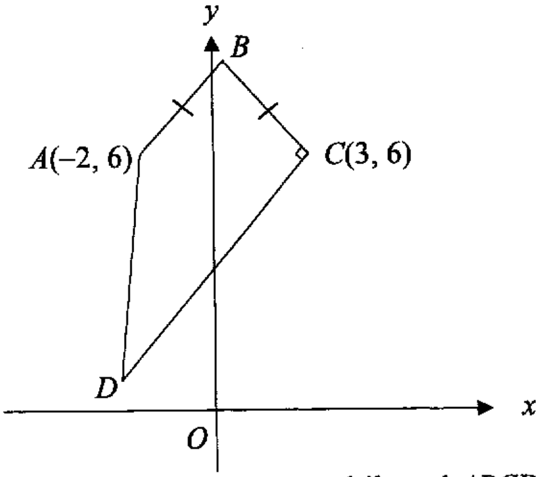
(a) Show that the height of the water level in the cone is 6 cm when the volume of water in the cone is 18π cm^3. [2]

(b) Calculate the rate of change of height of the water level when the volume of water is 18π cm^3. Leave your answer in its exact form. [3]

11 **(a)** Given that $y=(x+5)\sqrt{2x-5}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{kx}{\sqrt{2x-5}}$, where k is a constant. [2]

(b) Hence, find $\int \frac{x-4}{\sqrt{2x-5}}\, dx$. [4]

12 Solutions to this question by accurate drawing will not be accepted.



The diagram (not drawn to scale) shows a quadrilateral $ABCD$ such that $AB = BC$ and angle $BCD = 90°$. Point A is $(-2, 6)$ and point C is $(3, 6)$. Given that the area of triangle ABC is 7.5 square units and point D lies on the line $y + x + 2 = 0$,

(a) show that the coordinates of B is $\left(\frac{1}{2}, 9\right)$. [2]

(b) Find the coordinates of D. [4]

(c) Find the area of *ABCD*. [2]

(d) If *ABCT* is a parallelogram, find the coordinates of *T*. [2]

-End of paper-

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1. **(a)** Solve the simultaneous equations

$$x-2y+4=0$$
$$x^2+y^2=2x+4$$ [3]

$x=2y-4$ (1)
$x^2+y^2=2x+4$ (2)

Sub (1) into (2),
$(2y-4)^2+y^2=2x+4$
$4y^2-16y+16+y^2=4y-8+4$
$5y^2-20y+20=0$
$y^2-4y+4=0$
$(y-2)^2=0$
$y=2$

Sub $y=2$ into (1),
$x=2(2)-4$
$x=0$

$\therefore x=0,\ y=2$

(b) Explain the geometrical meaning of your answer in **(a)**. [1]

The line $x-2y+4=0$ is a tangent to the circle at (0, 2).

2. **(a)** Express $y = 3 - 8x - 2x^2$ in the form $y = a(x+b)^2 + c$ and hence state the maximum value of y. [3]

$y = -2x^2 - 8x + 3$
$y = -2(x^2 + 4x) + 3$
$y = -2[(x+2)^2 - 2^2] + 3$
$y = -2(x+2)^2 + 8 + 3$
$y = -2(x+2)^2 + 11$

Maximum value of $y = 11$

(b) Show that there are no values of p for which the curve $y = (p-3)x^2 + 2px + (p+1)$ is always positive. [3]

Always positive,
$p - 3 > 0$ and $b^2 - 4ac < 0$

$p > 3$ and $(2p)^2 - 4(p-3)(p+1) < 0$
$4p^2 - 4(p^2 - 2p - 3) < 0$
$4p^2 - 4p^2 + 8p + 12 < 0$
$8p + 12 < 0$
$p < -\frac{3}{2}$

For the curve y to be always positive, $p > 3$ and $p < -\frac{3}{2}$.
There are no values of p for which y is always positive.

(c) A quadratic equation is given by $hx^2 - 2kx + 6k - 9h = 0$, where h and k are constants and $h \neq 0$.

(i) Show that the equation has real roots for all values of h and k. [3]

$b^2 - 4ac$

$= (-2k)^2 - 4(h)(6k - 9h)$

$= 4k^2 - 24hk + 36h^2$

$= 4(k^2 - 6hk + 9h^2)$

$= 4(k - 3h)^2 \geq 0$

for all values of h and k.

Therefore, the roots are real (shown).

(ii) In the case where the equation has two real and equal roots, express h in terms of k. [2]

$b^2 - 4ac = 0$

$4(k - 3h)^2 = 0$

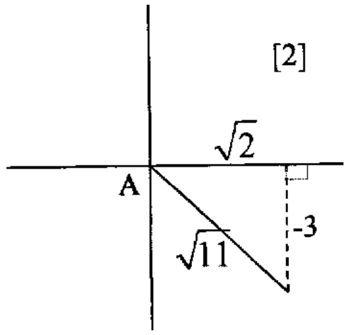
$(k - 3h)^2 = 0$

$k = 3h$

$h = \dfrac{k}{3}$

3. Given that $\cos A = \sqrt{\frac{2}{11}}$ where $180^\circ < A < 360^\circ$, find, without the use of a calculator, the value of

(a) $\tan A$, [2]



A lies in the 4th quadrant.

$(opp)^2 = (\sqrt{11})^2 - (\sqrt{2})^2$

$opp = -3 \ or \ 3 \ (rej)$

$\tan A = -\frac{3}{\sqrt{2}} \ or \ -\frac{3\sqrt{2}}{2}$

(b) $\sin(A - 90^\circ)$, [2]

$\sin(A - 90^\circ)$

$= \sin A \cos 90^\circ - \cos A \sin 90^\circ$

$= \sin A(0) - \cos A(1)$

$= -\cos A$

$= -\sqrt{\frac{2}{11}}$

OR

$\sin(A - 90^\circ)$

$= \sin[-(90^\circ - A)]$

$= -\sin(90^\circ - A)$

$= -\cos A$

$= -\sqrt{\frac{2}{11}}$

(c) $\frac{1}{\sec 2A}$. [2]

$$\frac{1}{\sec 2A}$$
$$= \cos 2A$$
$$= 2\cos^2 A - 1$$
$$= 2\left(\sqrt{\frac{2}{11}}\right)^2 - 1$$
$$= \frac{4}{11} - 1$$
$$= -\frac{7}{11}$$

4. **(a)** Factorise $27x^3 - \frac{y^3}{8}$ completely. [2]

$$27x^3 - \frac{y^3}{8}$$
$$= (3x)^3 - \left(\frac{y}{2}\right)^3$$
$$= \left(3x - \frac{y}{2}\right)\left(9x^2 + \frac{3xy}{2} + \frac{y^2}{4}\right)$$

OR

$$27x^3 - \frac{y^3}{8}$$
$$= \frac{1}{8}\left(216x^3 - y^3\right)$$
$$= \frac{1}{8}\left[(6x)^3 - y^3\right]$$
$$= \frac{1}{8}(6x - y)\left(36x^2 + 6xy + y^2\right)$$

(b) Express $\dfrac{8x^3-7x^2+4x-3}{(2x^2-x)(2x-1)}$ in partial fractions. [6]

$$(2x^2-x)(2x-1)=x(2x-1)^2$$

$$\begin{array}{r} 2 \\ 4x^3-4x^2+x\overline{)8x^3-7x^2+4x-3} \\ \underline{-(8x^3-8x^2+2x)} \\ x^2+2x-3 \end{array}$$

$$\frac{8x^3-7x^2+4x-3}{(2x^2-x)(2x-1)}=2+\frac{x^2+2x-3}{x(2x-1)^2}$$

$$\frac{x^2+2x-3}{x(2x-1)^2}=\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{(2x-1)^2}$$

$$x^2+2x-3=A(2x-1)^2+Bx(2x-1)+Cx$$

When $x=\frac{1}{2}$, $\quad \frac{1}{4}+1-3=\frac{1}{2}c$

$$-\frac{7}{4}=\frac{1}{2}C$$

$$C=-\frac{7}{2}$$

When $x=0$, $\quad A=-3$

When $x=1$,

$$1+2-3=-3+B-\frac{7}{2}$$

$$0=-\frac{13}{2}+B$$

$$B=\frac{13}{2}$$

$$2-\frac{3}{x}+\frac{13}{2(2x-1)}-\frac{7}{2(2x-1)^2}$$

5. A curve is such that $\frac{dy}{dx} = \frac{2x - ax^2}{3}$, where a is a constant.

(a) Given that the curve has a turning point at $(3, 7)$, show that the value of a is $\frac{2}{3}$. [1]

At turning point, $\frac{dy}{dx} = 0$

$$\frac{2x - ax^2}{3} = 0$$

$$2x - ax^2 = 0$$

When $x = 3$,

$$2(3) - a(3)^2 = 0$$

$$6 = 9a$$

$$a = \frac{2}{3} \quad \text{(shown)}$$

(b) Find the range of values of x for which y decreases as x increases. [3]

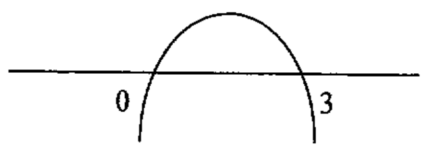
y decreases as x increases, $\frac{dy}{dx} < 0$

$$\frac{2x - \frac{2}{3}x^2}{3} < 0$$

$$2x - \frac{2}{3}x^2 < 0$$

$$6x - 2x^2 < 0$$

$$2x(3 - x) < 0$$



$x < 0$ *or* $x > 3$

(c) Find the equation of the curve. [4]

$$y=\int\frac{2x-\frac{2}{3}x^2}{3}dx$$

$$y=\frac{1}{3}\int\left(2x-\frac{2}{3}x^2\right)dx$$

$$y=\frac{1}{3}\left[\frac{2x^2}{2}-\frac{2x^3}{3(3)}\right]+c$$

$$y=\frac{x^2}{3}-\frac{2x^3}{27}+c$$

When $x=3, y=7,$

$$7=\frac{3^2}{3}-\frac{2(3)^3}{27}+c$$

$$7=3-2+c$$

$$c=6$$

$$y=\frac{x^2}{3}-\frac{2x^3}{27}+6$$

6. **(a)** Prove the identity $(\cot x - \operatorname{cosec} x)^2 = \dfrac{1-\cos x}{1+\cos x}$. [4]

LHS

$= (\cot x - \operatorname{cosec} x)^2$

$= \left(\dfrac{\cos x - 1}{\sin x}\right)^2$

$= \dfrac{(\cos x - 1)^2}{\sin^2 x}$

$= \dfrac{[-(1-\cos x)]^2}{(1-\cos^2 x)}$

$= \dfrac{(1-\cos x)^2}{(1+\cos x)(1-\cos x)}$

$= \dfrac{1-\cos x}{1+\cos x}$ (proven)

(b) Hence, solve the equation $2(\cot x - \operatorname{cosec} x)^2 = 3\cos x$ for $0 \le x \le 2\pi$. [3]

$2\left(\dfrac{1-\cos x}{1+\cos x}\right) = 3\cos x$

$2 - 2\cos x = 3\cos x + 3\cos^2 x$

$3\cos^2 x + 5\cos x - 2 = 0$

$(3\cos x - 1)(\cos x + 2) = 0$

$\cos x = \dfrac{1}{3}$ *or* $\cos x = -2$ (*no solution*)

basic angle, α

$= \cos^{-1}\left(\dfrac{1}{3}\right)$

$= 1.230959$

$x = 1.23$ *or* $2\pi - 1.230959$

$x = 1.23$ *or* 5.05 ($3 s.f.$)

(c) State the number of solutions of the equation $2(\cot 2x - \operatorname{cosec} 2x)^2 = 3\cos 2x$ in the range $-2\pi \le 2x \le 2\pi$. [1]

Angle changes from x to $2x$.

$2x = 1.23$ *or* $2\pi - 1.230959$ *or* -1.23 *or* $-(2\pi - 1.230959)$

$2x = 1.23$ *or* 5.05 *or* -1.23 *or* -5.05

$x = 0.615$ *or* 2.52 *or* -1.23 *or* -2.52 ($3 s.f.$)

There are 4 solutions to the equation.

7 **(a)** The graph of $y = \log_a x$ passes through the points with coordinates (125, 3) and (1, b).

(i) Determine the values of a and b. [2]

Sub $x = 125, y = 3$

$3 = \log_a 125$

$a^3 = 125$

$a = 5$

Sub $x = 1, y = b$

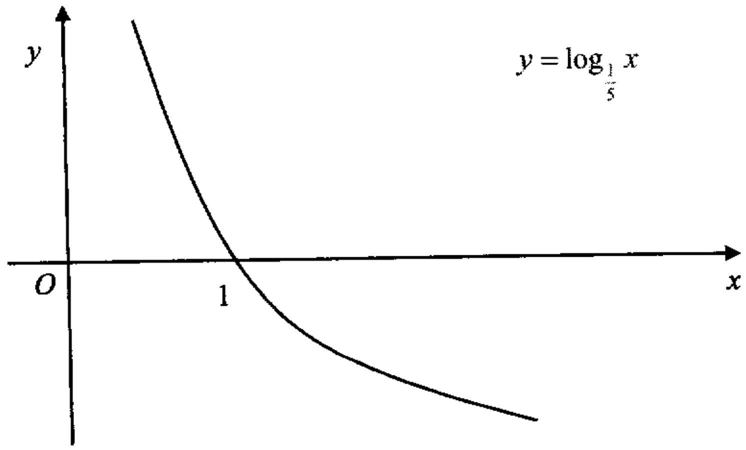
$b = \log_5 1$

$5^b = 1$

$5^b = 5^0$

$b = 0$

(ii) Sketch the graph of $y = \log_{a^{-1}} x$ indicating clearly any intercept on the axes. [2]



(b) Find the values of a and b such that $\lg\left(\frac{8}{y}\right) + 4\lg y = a\lg(by)$. [4]

$\lg\left(\frac{8}{y}\right) + \lg y^4 = \lg(by)^a$

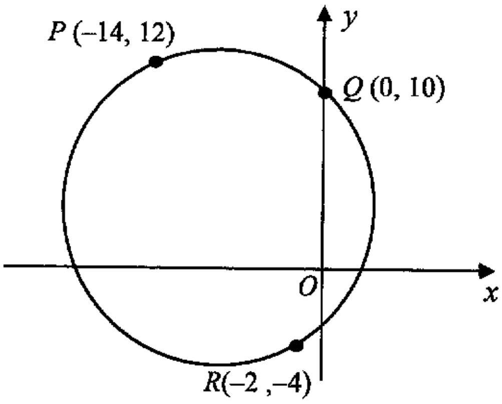
$\lg\left(\frac{8}{y} \times y^4\right) = \lg(by)^a$

$\lg\left(8y^3\right) = \lg(by)^a$

$\lg\left(2y\right)^3 = \lg(by)^a$

$\therefore a = 3,\ b = 2$

8 Solutions to this question by accurate drawing will not be accepted.



In the diagram which is not drawn to scale, P, Q and R are points on the circle.

(a) Show that PR is the diameter of the circle and hence find the centre of the circle. [5]

Gradient of PQ

$$= \frac{12-10}{-14-0}$$

$$= -\frac{1}{7}$$

Gradient of QR

$$= \frac{-4-10}{-2-0}$$

$$= 7$$

Since gradient of PQ x gradient of $QR = -1$,

line PQ is perpendicular to line QR.

Angle PQR is $90°$ (angle in a semicircle).

$\therefore PR$ is the diameter of the circle. (shown)

$$\text{Centre} = \left(\frac{-14-2}{2}, \frac{12-4}{2}\right)$$

$$= (-8, 4)$$

[Turn over

(b) Find the equation of the circle that passes through the points P, Q and R. [2]

Radius
$= \sqrt{(-8+2)^2 + (4+4)^2}$
$= \sqrt{100}$
$= 10\,units$

Equation of circle:
$(x+8)^2 + (y-4)^2 = 100$

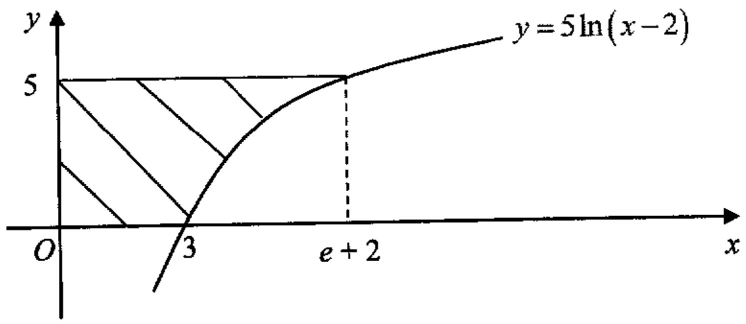
(c) Determine whether the point $S(-14,-2)$ lies inside or outside the circle. [2]

Centre $(-8,4)$

Distance of S from centre
$= \sqrt{(-8+14)^2 + (4+2)^2}$ [M1] FT their centre to point S
$= \sqrt{72}$
$= 8.4852 < 10$

Since the distance of point S from centre C is less than the radius, S lies inside the circle. [A1] no FT

9 The diagram shows part of the curve $y = 5\ln(x-2)$.



(a) Find the exact value of $\int_0^5 x \, dy$. [3]

$y = 5\ln(x-2)$

$\frac{y}{5} = \ln(x-2)$

$e^{\frac{y}{5}} = x-2$

$x = e^{\frac{y}{5}} + 2$

$$\int_0^5 (e^{\frac{y}{5}} + 2)\ dy$$
$$= \left[5e^{\frac{y}{5}} + 2y \right]_0^5$$
$$= (5e + 10) - 5$$
$$= 5e + 5$$

(b) On the diagram above, shade the region whose area is $\int_0^5 x\ dy$, showing your upper limit clearly. [1]

Must see 5, correct shading and horizontal line on the above diagram.

Need not see $e + 2$

(c) Hence find $\int_3^{e+2} 5\ln(x-2)\ dx$. [3]

When $x = e + 2$,

$$e + 2 = e^{\frac{y}{5}} + 2$$
$$e = e^{\frac{y}{5}}$$
$$\frac{y}{5} = 1$$
$$y = 5$$

$$\int_3^{e+2} 5\ln(x-2)\ dx$$
$$= \text{rectangle } - (5e + 5)$$
$$= 5(e + 2) - (5e + 5)$$
$$= 5e + 10 - 5e - 5$$
$$= 5$$

10 Water is being added at a constant rate of 4 cm^3/s to an inverted right cone. The height of the cone is twice the radius of the cone.

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

(a) Show that the height of the water level in the cone is 6 cm when the volume of water in the cone is 18π cm^3. [2]

$h = 2r$

$r = \frac{h}{2}$

$\frac{1}{3}\pi r^2 h = 18\pi$

$\frac{1}{3}\left(\frac{h}{2}\right)^2 h = 18$

$\frac{h^3}{12} = 18$

$h^3 = 216$

$h = 6cm$ (shown)

OR

$\frac{1}{3}\pi r^2(2r) = 18\pi$

$\frac{2}{3}\pi r^3 = 18\pi$

$r^3 = 27$

$r = 3$

$h = 3(20$

$h = 6$

(b) Calculate the rate of change of height of the water level when the volume of water is 18π cm^3. Leave your answer in its exact form. [3]

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dh}$

$= \frac{\pi}{12}(3)h^2$

$= \frac{\pi}{4}h^2$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$4 = \frac{\pi}{4}h^2 \times \frac{dh}{dt}$

$\frac{dh}{dt} = 4 \div \frac{\pi h^2}{4}$

$\frac{dh}{dt} = \frac{16}{\pi(6)^2}$

$\frac{dh}{dt} = \frac{4}{9\pi} cm/s$

11 (a) Given that $y=(x+5)\sqrt{2x-5}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\frac{kx}{\sqrt{2x-5}}$, where k is a constant. [2]

$$\frac{dy}{dx}$$

$$=(x+5)\left(\frac{1}{2}\right)(2x-5)^{-\frac{1}{2}}(2)+(2x-5)^{\frac{1}{2}}$$

$$=(x+5)(2x-5)^{-\frac{1}{2}}+(2x-5)^{\frac{1}{2}}$$

$$=(2x-5)^{-\frac{1}{2}}(x+5+2x-5)$$

$$=\frac{3x}{\sqrt{2x-5}}$$

(b) Hence, find $\int\frac{x-4}{\sqrt{2x-5}}\,\mathrm{d}x$. [4]

$$\int\frac{x-4}{\sqrt{2x-5}}\,\mathrm{d}x$$

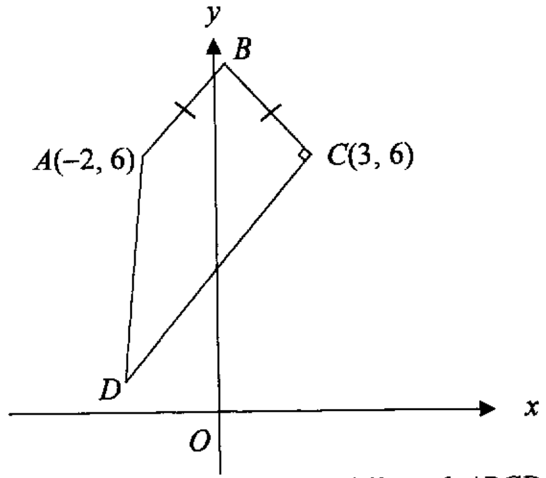
$$=\int\frac{x}{\sqrt{2x-5}}\,dx-4\int\frac{1}{\sqrt{2x-5}}\,dx$$

$$=\frac{1}{3}\int\frac{3x}{\sqrt{2x-5}}\,dx-4\int(2x-5)^{-\frac{1}{2}}\,dx$$

$$=\frac{1}{3}(x+5)\sqrt{2x-5}-\frac{4(2x-5)^{\frac{1}{2}}}{\frac{1}{2}(2)}+c$$

$$=\frac{1}{3}(x+5)\sqrt{2x-5}-4\sqrt{2x-5}+c$$

12 Solutions to this question by accurate drawing will not be accepted.



The diagram (not drawn to scale) shows a quadrilateral $ABCD$ such that $AB = BC$ and angle $BCD = 90°$. Point A is $(-2, 6)$ and point C is $(3, 6)$. Given that the area of triangle ABC is 7.5 square units and point D lies on the line $y + x + 2 = 0$,

(a) show that the coordinates of B is $\left(\frac{1}{2}, 9\right)$. [2]

Given that $AB = BC$, triangle ABC is an isosceles triangle.

$x_B = \frac{-2+3}{2}$

$x_B = \frac{1}{2}$

Area of triangle $ABC = 7.5$

$\frac{1}{2}(5)(h) = 7.5$

$h = 3$

$y_B = 6 + 3$

$y_B = 9$

$B\left(\frac{1}{2}, 9\right)$ (shown)

(b) Find the coordinates of D. [4]

Gradient of BC

$$= \frac{9-6}{\frac{1}{2}-3}$$

$$= -\frac{6}{5}$$

Gradient of CD

$$= -1 \div \left(-\frac{6}{5}\right)$$

$$= \frac{5}{6}$$

Equation of CD:

$$y-6 = \frac{5}{6}(x-3)$$

$$y = \frac{5}{6}x - \frac{15}{6} + 6$$

$$y = \frac{5}{6}x + \frac{7}{2} \quad (1)$$

$$y + x + 2 = 0 \quad (2)$$

Sub (1) into (2),

$$\frac{5}{6}x + \frac{7}{2} + x + 2 = 0$$

$$\frac{11}{6}x = -\frac{11}{2}$$

$$x = -3$$

$$y = \frac{5}{6}(-3) + \frac{7}{2}$$

$$y = 1$$

$D(-3,1)$

(c) Find the area of $ABCD$. [2]

Area of $ABCD$

$$=\frac{1}{2}\begin{vmatrix} 3 & \frac{1}{2} & -2 & -3 & 3 \\ 6 & 9 & 6 & 1 & 6 \end{vmatrix}$$

$$=\frac{1}{2}[(27+3-2-18)-(3-18-18+3)]$$

$$=\frac{1}{2}(40)$$

$$=20 units^2$$

(d) If $ABCT$ is a parallelogram, find the coordinates of T. [2]

Midpoint of AC = midpoint of BD

$$\left(\frac{-2+3}{2},\frac{6+6}{2}\right)=\left(\frac{\frac{1}{2}+x}{2},\frac{9+y}{2}\right)$$

$$\left(\frac{1}{2},6\right)=\left(\frac{1}{4}+\frac{x}{2},\frac{9+y}{2}\right)$$

$$\frac{1}{4}+\frac{x}{2}=\frac{1}{2}$$

$$1+2x=2$$

$$2x=1$$

$$x=\frac{1}{2}$$

$$\frac{9+y}{2}=6$$

$$9+y=12$$

$$y=3$$

$$T\left(\frac{1}{2},3\right)$$

-End of paper-

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS — 4049/02

PAPER 2

9 September 2024

Candidates answer on the Question Paper.
No Additional Materials are required.

2 hours 15 minutes

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For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 In Singapore, car owners need to secure a Certificate of Entitlement (COE) before they can register their vehicle and use it for 10 years.

David bought a Toyota Prius for $170 000 in January 2022. The value of the car, C, can be modelled by the equation $C = k + 150000e^{nt}$, where k and n are constants, t is the number of years after 2022 and $0 \le t \le 10$.

(a) Show that $k = 20000$. [2]

It is given that the value of the car is worth $140 000 in 2023.

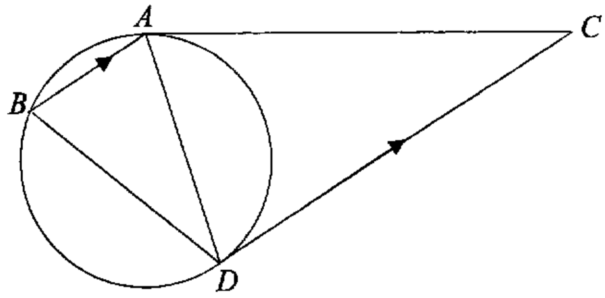
(b) Find the value of n. [3]

(c) David intends to sell the car before it depreciates below $70 000. Which is the latest year that David has to sell the car? [4]

2 **(a)** By considering the general term in the binomial expansion of $\left(px+\frac{2}{x}\right)^7$, where p is a constant, explain why every term is dependent on x. [3]

(b) In the expansion of $\left(px+\frac{2}{x}\right)^7(5-2x)$, the term independent of x is -241920. Find the value of p. [4]

3



In the diagram, points A, B and D lie on a circle. The tangents at A and D meet at C and BA is parallel to DC.

(a) Prove that the triangle ABD is isosceles. [3]

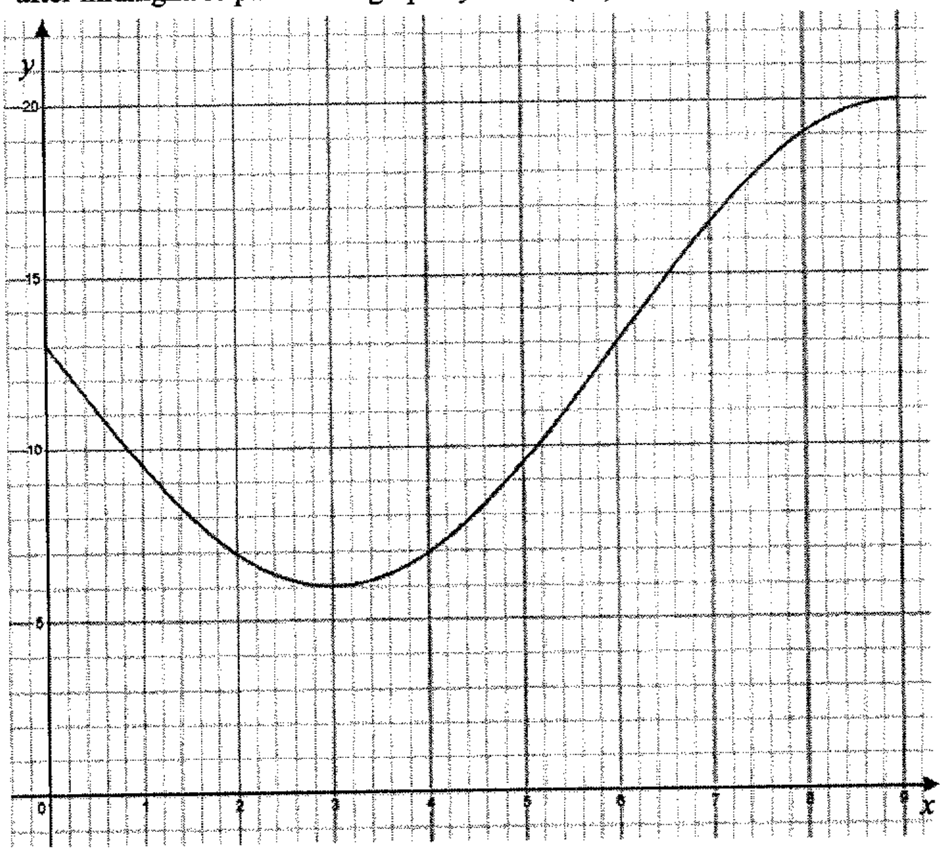
(b) Prove that angle BDA = angle DCA. [3]

4 (a) State the values between which each of the following must lie:

(i) The principal value of $\cos^{-1} x$. [1]

(ii) The principal value of $\tan^{-1} x$. [1]

(b) The depth of the water, y m, at a harbour on a particular day is given by $y = a\sin(bx) + c$, where a, b and c are constants and x is the time in hours after midnight. A part of the graph $y = a\sin(bx) + c$ is shown below.



(i) Find the value of each of the constants a, b and c. [3]

(ii) A cargo ship requires the depth of the water to be at least 7m in order to sail in to dock at the harbour. The cargo ship is expected to arrive after 7am. What would be the next time interval in which the ship would be unable to sail in to dock? Leave your answer to the nearest hour. [2]

5 **A calculator must not be used in the question.**

(a) Show that $\sec 105° = -\sqrt{2} - \sqrt{6}$. [4]

(b) The equation of a curve is $f(x)=\frac{e^{3x}}{x-1}$, where $x \neq 1$.

(i) Find an expression for $f'(x)$ in the form $\frac{e^{3x}(ax+b)}{(x-1)^2}$, where a and b are integers. [3]

(ii) The curve, $f(x)$, cuts the y axis at S, and the normal to the curve at S cuts the x axis at P. Find the coordinates of P. [4]

6 **(a)** Show that $2x+1$ is a factor of $10x^3-9x^2-3x+2$ and hence factorise $10x^3-9x^2-3x+2$ completely. [5]

(b) Solve the equation $5\left(9^{y}\right)+3^{-y}=\frac{3}{2}\left(3^{y+1}+1\right)$ and explain why there are only 2 real solutions. [7]

7 A particle, A, travelling along a straight road, passes a point O. The velocity of A, v m/s, t seconds after passing through O, is given by $v = 10\cos(5 - 2t) + 50$.

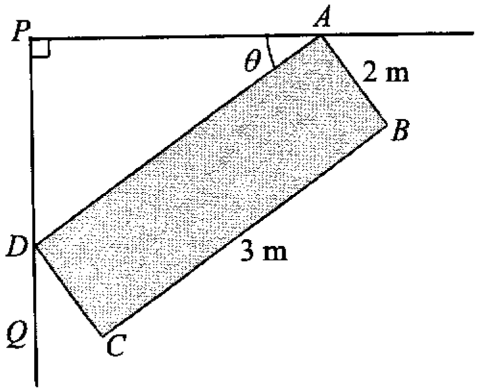
(a) Find the initial acceleration of A. [4]

Another particle, B, also travelling along the straight road, passes the point O with a velocity of 5 m/s. The acceleration of B, a m/s^2, t seconds after passing through O, is given by $a = \frac{-24}{(t+2)^2}$.

(b) Find the value(s) of t at which B is at instantaneous rest. [4]

(c) Find the total distance travelled by B for the first 10 seconds of its journey. [6]

8 The diagram shows a rectangular table, $ABCD$, placed at the corner of the hall. It is given that the table has length $BC = 3\,\text{m}$, width $AB = 2\,\text{m}$, $\angle APD = \angle DQC = 90^\circ$ and $\angle PAD = \theta^\circ$.

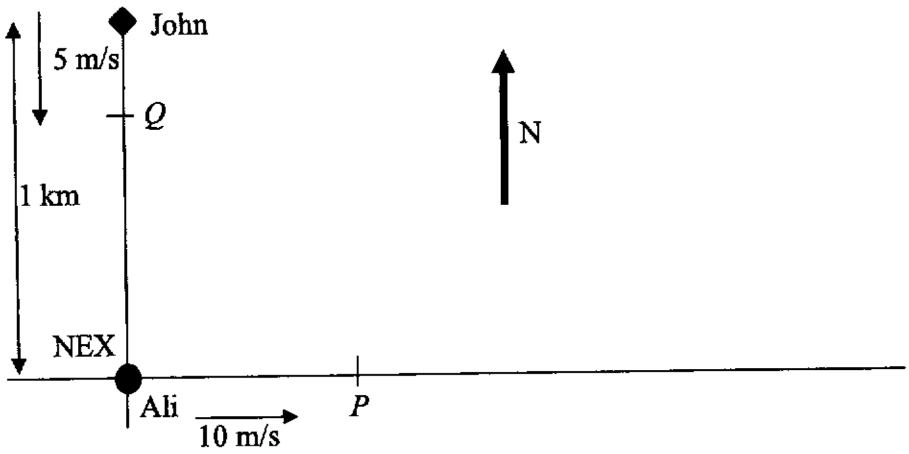


(a) Show that PQ can be expressed as $a\sin\theta + b\cos\theta$, where a and b are constants. [2]

(b) Express PQ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(c) Find the maximum value of PQ and the corresponding value of θ. [3]

9 On a typical day at the local fast food outlet in NEX, a delivery rider, Ali, is heading East, away from NEX, at a constant speed of 10 m/s after collecting his order. Another delivery rider, John, 1km away from NEX, is heading South towards NEX at a constant speed of 5 m/s, to collect his order.



At time t s, Ali would have reached P and John would have reached the point Q.

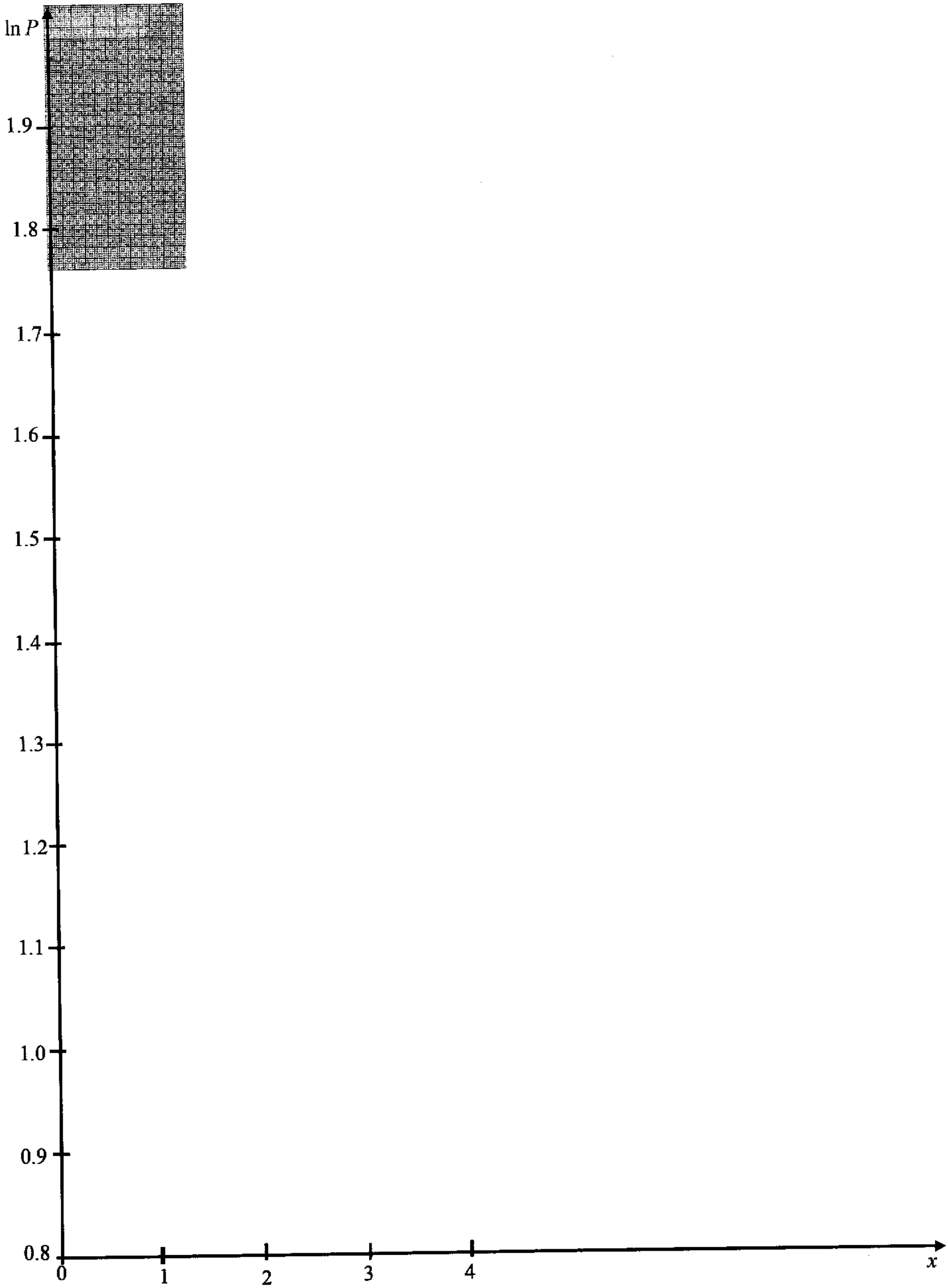
(a) Show that the distance between the two delivery riders, PQ, at time t is given by $s = \sqrt{1000000 - 10000t + 125t^2}$. [3]

(b) Find the least distance between Ali and John. You are **not** required to justify that this time is the least. [4]

10 The table below shows, to 3 significant figures, the population, P, in millions of a country on January 1st at intervals of 10 years from 1980 to 2020. The variable x is measured in units of 10 years.

Year	1980	1990	2000	2010	2020
x	0	1	2	3	4
P	2.45	3.09	3.94	4.95	6.3

(a) On the grid below plot $\ln P$ (correct to 2 decimal places) against x and draw a straight line. [2]


ln P
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0
1
2
3
4
x

(b) Find the gradient of your straight line and hence express P in the form Ae^{kx}, where A and k are constants. [4]

(c) If this model for the population remains valid, find the first year of the interval in which the population exceeds 13 million. [2]

End of paper

ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

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STUDENT SOLUTIONS		

ADDITIONAL MATHEMATICS 4049/02

PAPER 2

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2 hours 15 minutes

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[Turn over

1	(a)	When $t = 0$, $170000 = k + 150000e^0$ $170000 = k + 150000$ $k = 20000$	
	(b)	$140000 = 20000 + 150000e^n$ $120000 = 150000e^n$ $\frac{4}{5} = e^n$ $\ln\frac{4}{5} = n$ $n = -0.223144$ $n = -0.223$	
	(c)	$70000 < 20000 + 150000e^{-0.223t}$ $50000 < 150000e^{-0.223t}$ $\frac{1}{3} < e^{-0.223t}$ $\ln\frac{1}{3} < -0.223t$ $t < 4.92$ $t = 4$ David must sell the car in 2026	

2	(a)	General term $\left(px+\frac{2}{x}\right)^7$ $=\binom{7}{r}(px)^{7-r}\left(\frac{2}{x}\right)^r$ Consider the power of x $=7-r-r$ $=7-2r$ If there is an independent term, $7-2r=0$ $r=\frac{7}{2}$ Since r is not a whole number, there is no term independent of x and as such, every term is dependent on x
	(b)	$\left(px+\frac{2}{x}\right)^7(5-2x)$ Since there is no independent term in $\left(px+\frac{2}{x}\right)^7$, the only term to form the independent term is the $\frac{1}{x}$ in the expansion of $\left(px+\frac{2}{x}\right)^7(5-2x)$. Find the $\frac{1}{x}$ term Power of x: $7-2r=-1$ $r=4$ $\frac{1}{x}$ term $=\binom{7}{4}(px)^3\left(\frac{2}{x}\right)^4$ $=\frac{35p^3\times16}{x}$ $=\frac{560p^3}{x}$

			Thus, the term independent of x $\frac{560p^3}{x}\times -2x = -241920$ $p^3 = 216$ $p = 6$
3	(a)		Consider $\angle ABD = \angle ADC$ (by tangent chord theorem) $\angle ADC = \angle BAD$ (alternate angles) Thus, $\angle ABD = \angle BAD$. Since there are 2 equal angles in triangle ABD, triangle ABD is isosceles.
	(b)		Let $\angle ABD = x$, Since triangle ABD is isosceles (part(a)), $\angle BDA = 180 - 2x$ (angle sum of triangle) by tangent chord theorem, $\angle ABD = \angle ADC = x$ Or $\angle ADC = 180 - [x + 180 - 2x]$ $= x$ (interior angles) Since tangents from an external point are equal, triangle CDA is also isosceles. Thus, $\angle ADC = \angle CAD = x$ And $\angle DCA = 180 - 2x = \angle BDA$ (shown)
4	(a)	(i)	$0^\circ \le \cos^{-1} x \le 180^\circ$ or $0 \le \cos^{-1} x \le \pi$
		(ii)	$-90^\circ < \tan^{-1} x < 90^\circ$ or $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$
	(b)	(i)	$a = -\left(\frac{20-6}{2}\right) = -7$ $b = \frac{2\pi}{12} = \frac{\pi}{6}$ $c = 13$
		(ii)	$14 < x < 16$

5	(a)		$\cos 105$ $= \cos(60+45)$ $= \cos 60 \cos 45 - \sin 60 \sin 45$ $= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ $= \frac{\sqrt{2}-\sqrt{6}}{4}$ $\sec 105°$ $= \frac{1}{\cos 105}$ $= \frac{4}{\sqrt{2}-\sqrt{6}}$ $= \frac{4}{\sqrt{2}-\sqrt{6}} \times \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}$ $= -\sqrt{2}-\sqrt{6}$
	(b)	(i)	$f(x) = \frac{e^{3x}}{x-1}$ $f'(x) = \frac{3e^{3x}(x-1) - e^{3x}}{(x-1)^2}$ $f'(x) = \frac{e^{3x}(3x-4)}{(x-1)^2}$ $a = 3$ $b = -4$
		(ii)	At y axis, $x = 0$. When x=0, $y = -1$ $f'(0) = -4$ Thus, gradient of normal $= \frac{1}{4}$ Equation of normal: $y = \frac{1}{4}x + c$ Sub x=0, $y = -1$ $c = -1$ $y = \frac{1}{4}x - 1$ When $y = 0$, $x = 4$ $P(4,0)$

6 (a)

Let $f(x)=10x^3-9x^2-3x+2$

$f(-\frac{1}{2})=10\left(-\frac{1}{2}\right)^3-9\left(-\frac{1}{2}\right)^2-3\left(-\frac{1}{2}\right)+2$

$f(-\frac{1}{2})=0$

By factor theorem, $2x+1$ is a factor.

$f(x)=10x^3-9x^2-3x+2$

$$\begin{array}{r l}
 & 5x^2-7x+2 \\
2x+1 \,\big) & 10x^3-9x^2-3x+2 \\
- & 10x^3+5x^2 \\
\hline
 & -14x^2-3x+2 \\
- & -14x^2-7x \\
\hline
 & 4x+2 \\
- & 4x+2 \\
\hline
 & 0
\end{array}$$

$f(x)=(2x+1)(5x^2-7x+2)$

	x	-1
$5x$	$5x^2$	$-5x$
-2	$-2x$	2

$f(x)=(2x+1)(5x-2)(x-1)$

(b)

Let $3^y=a$.

$5(a^2)+\frac{1}{a}=\frac{3}{2}(3a+1)$

$10a^2+\frac{2}{a}=9a+3$

$10a^3+2=9a^2+3a$

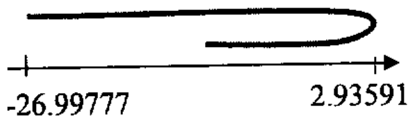
$10a^3-9a^2-3a+2=0$

Comparing with (a)

$(2a+1)(5a-2)(a-1)=0$

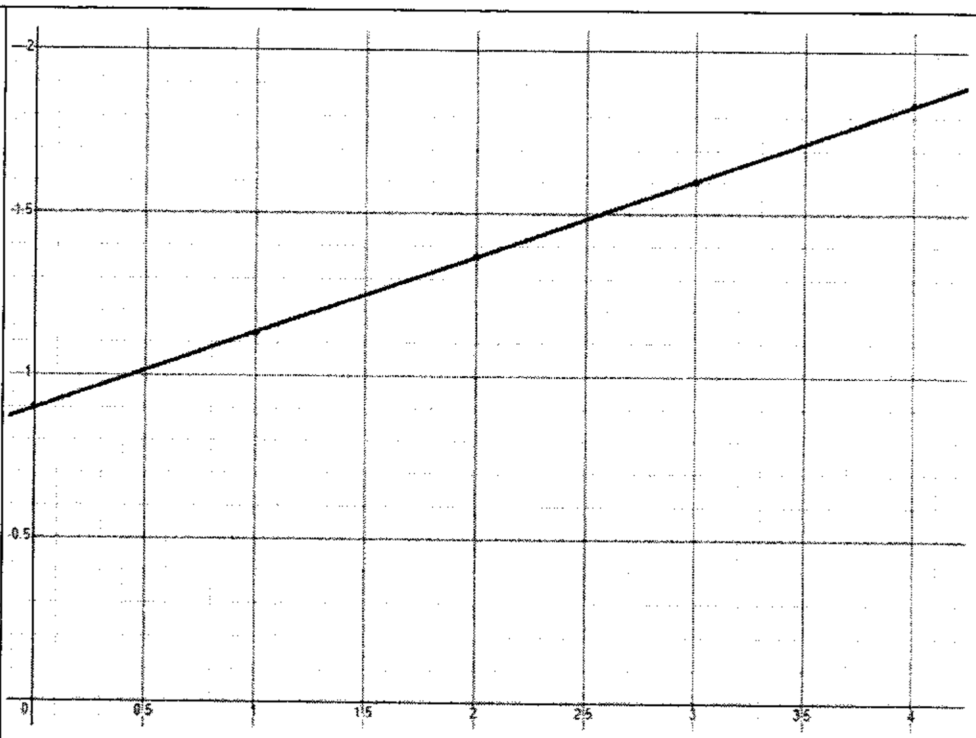
$a=-\frac{1}{2}$ or $a=\frac{2}{5}$ or $a=1$

		$3^y = -\frac{1}{2}$ or $3^y = \frac{2}{5}$ or $3^y = 1$ (reject as $3^y > 0$) or $y\ln 3 = \ln\frac{2}{5}$ $y = 0$ $y = -0.834$	
7	(a)	$v = 10\cos(5-2t)+50$ Acceleration = $\frac{dv}{dt}$. $\frac{dv}{dt} = 10(-\sin(5-2t)\times -2)$ $\frac{dv}{dt} = 20\sin(5-2t)$ When $t=0$, $20\sin(5)$ $= -19.2\text{ m/s}^2$	
7	(b)	$v_c = \int \frac{-24}{(t+2)^2}\,dt$ $v_c = \int -24(t+2)^{-2}\,dt$ $v_c = 24(t+2)^{-1} + c$ When $t=0$, $v_c = 5$ Thus, $5 = 12 + c$ $c = -7$ $v_c = 24(t+2)^{-1} - 7$ cyclist is instantaneously at rest $\Rightarrow v_c = 0$ $24(t+2)^{-1} - 7 = 0$ $\frac{24}{(t+2)} = 7$ $24 = 7(t+2)$ $24 = 7t + 14$ $t = \frac{10}{7}$	
	(c)	Displacement, $s = \int v_c\,dt$ $s = \int 24(t+2)^{-1} - 7\,dt$ $s = 24\ln(t+2) - 7t + c$	

		When $t=0$, $s=0$ $c=-24\ln 2$ $s=24\ln(t+2)-7t-24\ln 2$ At $t=\frac{10}{7}$, $s=24\ln\left(\frac{10}{7}+2\right)-7\left(\frac{10}{7}\right)-24\ln 2$ $s=2.93591\,\text{m}$ At $t=10$, $s=24\ln(10+2)-7(10)-24\ln 2$ $s=-26.99777\,\text{m}$  Total distance = 2.93591 + (2.93591+26.99777) =32.9m
8	**(a)**	$PQ=PD+DQ$ Consider triangle APD, $\frac{PD}{3}=\sin\theta$ $PD=3\sin\theta$ Consider triangle DCQ, $\frac{DQ}{2}=\cos\theta$ $DQ=2\cos\theta$ Thus, $PQ=3\sin\theta+2\cos\theta$
	(c)	$PQ=\sqrt{13}\sin(\theta+33.7)$ Maximum value = $\sqrt{13}$ $\sqrt{13}\sin(\theta+33.7)=\sqrt{13}$ $\sin(\theta+33.7)=1$ $\theta+33.7=90$ $\theta=56.3^\circ$
9	**(a)**	At time t, Distance from NEX to P = $10t$ Distance from NEX to Q = $1000-5t$ $PQ^2=(10t)^2+(1000-5t)^2$

		$PQ^2 = 100t^2 + 1000000 - 10000t + 25t^2$ $PQ = \sqrt{125t^2 - 10000t + 1000000}$ $s = \sqrt{1000000 - 10000t + 125t^2}$
	(b)	Least distance occurs at minimum point i.e. $\frac{ds}{dt} = 0$ $\frac{ds}{dt} = \frac{1}{2}\left(1000000 - 10000t + 125t^2\right)^{-\frac{1}{2}}(250t - 10000)$ $\frac{1}{2}\left(1000000 - 10000t + 125t^2\right)^{-\frac{1}{2}}(250t - 10000) = 0$ $250t - 10000 = 0$ $t = 40$ When $t = 40$, $s = \sqrt{1000000 - 10000(40) + 125(40)^2}$ $s = 894.4$ $s = 894\text{ m}$

10 **(a)**



Year	1980	1990	2000	2010	2020
x	0	1	2	3	4
P	2.45	3.09	3.94	4.95	6.30
$\ln P$	0.90	1.13	1.37	1.60	1.84

(b) From the graph,

		$m = 0.235\ (\pm 0.2)$ $c = 0.90\ (\pm 0.2)$ $\ln P = 0.235x + 0.9$ $P = e^{0.235x+0.9}$ $P = 2.45e^{0.235x}$	
	(c)	When $P = 13$ $13 = 2.45e^{0.235x}$ $t = 7.10$ First year in the interval would be 2050.	

End of paper