Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2015
Secondary Four

INTEGRATED MATHEMATICS 1 **2 hours**

Monday **12 October 2015** **0845 – 1045**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class in the spaces at the top of this page.
2. Answer questions 1 - 11 before attempting question 12 (Bonus Question).
3. Write your answers and working on the separate writing paper provided.
4. Omission of essential working will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. MENSURATION

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

2. TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

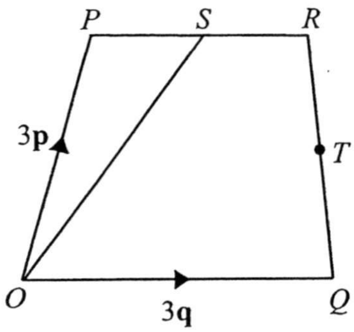
Area of triangle $ABC = \frac{1}{2}ab\sin C$

1 $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$.

(i) Find the matrix **C** such that $2\mathbf{A} + \mathbf{C} = \mathbf{B}^2$. [3]

(ii) Given that $\mathbf{ABD} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the matrix **D**. [3]

2 In the diagram, $\overrightarrow{OP} = 3\mathbf{p}$ and $\overrightarrow{OQ} = 3\mathbf{q}$.
PR is parallel to OQ and $3PR = 2OQ$.
S is the midpoint of PR and T is the midpoint of QR.
U is a point on OQ such that $OU = 2UQ$.



(i) Find $\overrightarrow{OS}$ in terms of **p** and **q**. [2]

(ii) Use vectors to determine if OS and UT are parallel to each other. [3]

3 **(a)** Simplify the expression

$$27a^3(b-c)^2 \div 18a^{-2}(b^2-c^2),$$

giving your answer in positive indices only. [2]

(b) Write the following expression as a single fraction in its simplest form:

$$\frac{4x}{(2x-1)^2} - \frac{3}{2x-1} + \frac{1}{4x-2}.$$ [4]

(c) Factorise $3p(2q-r) - (r-2q)^2$. [2]

4 On separate axes, sketch the graphs of the following functions, indicating clearly the intercepts and asymptotes where applicable.

(i) $y = (x+3)(3-2x)$, [2]

(ii) $y = \frac{2}{x-3}$. [2]

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5 A cookie factory produces cookies in three flavours and delivers them to two outlets. The number of cookies supplied in a single delivery is given by the matrix **P**.

$$\begin{array}{cc} \text{Outlet 1} & \text{Outlet 2} \end{array}$$

$$\mathbf{P} = \begin{pmatrix} 80 & 60 \\ 30 & 40 \\ 50 & 20 \end{pmatrix} \begin{array}{l} \text{Chocolate Deluxe} \\ \text{Peanut Crunch} \\ \text{Zesty Orange} \end{array}$$

(i) The cost price of a Chocolate Deluxe cookie is \$1.30.

The cost price of a Peanut Crunch cookie is \$0.80.

The cost price of a Zesty Orange cookie is \$1.10.

Represent these prices in a 1×3 row matrix **C**. [1]

(ii) Evaluate the matrix **Q**, where **Q** = **CP**. [1]

(iii) State what the elements of **Q** represent. [1]

In a particular month, there were 18 deliveries to Outlet 1 and 13 deliveries to Outlet 2.

(iv) The elements of the matrix **N**, where **N** = **PR**, represent the total number of cookies of each flavour delivered to the two outlets in that particular month. Write down the matrix **R**. [1]

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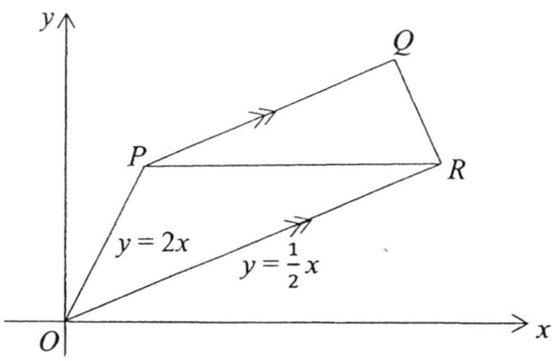

The diagram shows a trapezium *OPQR* in which *PQ* is parallel to *OR* and *O* is the origin. *QR* is perpendicular to *PQ* and the diagonal *PR* is parallel to the *x*-axis. The side *OP* has equation $y = 2x$ and the side *OR* has equation $y = \frac{1}{2}x$. The *y*-coordinate of *P* is *k*.

(i) Express the coordinates of *P* and *R* in terms of *k*. [2]

(ii) In the case where $k = 6$, find

(a) the coordinates of *Q*, [4]

(b) the coordinates of the point *S* which lies on *PR* produced such that $PR : PS = 2 : 5$. [2]

7 The table shows the number of books donated by each of 30 students in a class in a book donation drive.

1	2	1	4	3
1	2	0	3	2
0	1	6	3	2
0	3	2	1	1
2	4	1	0	3
3	0	2	1	2

(a) A student is chosen at random.

Find the probability that a student donated 2 books. [1]

(b) Two students are chosen at random.

Find the probability that

(i) one student donated two books and the other donated 4 books, [2]

(ii) both the students donated at least one book, [2]

(c) A book is chosen at random.

Find the probability that it was donated by a student who donated 3 books. [2]

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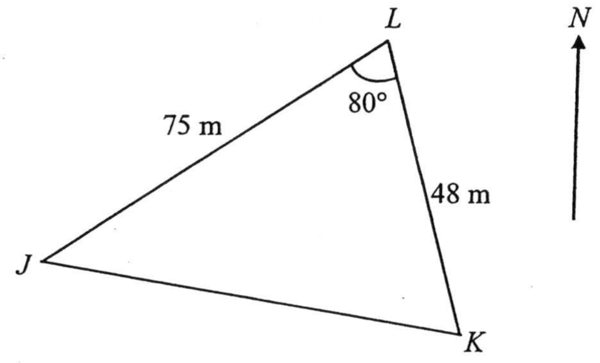


J, K and L are three points on level ground. $JL = 75$ m, $KL = 48$ m and angle $JLK = 80°$.

The bearing of K from J is 110°.

Calculate

(i) the distance JK, [2]

(ii) the area of the field JKL, [2]

(iii) the bearing of J from L. [3]

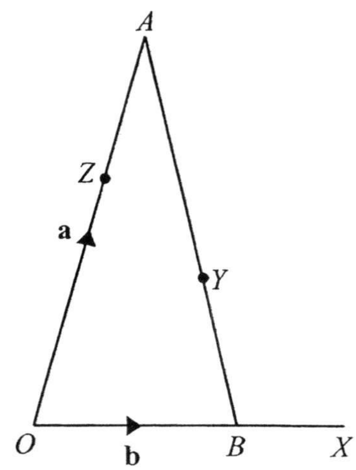
A vertical tree with height 23 m, has it base at L. A man walks from J to K. Find the greatest angle of elevation of the top of the tree when viewed from any point during his walk. [3]

9 **(a)** The points A, B, C and D are the vertices of a parallelogram $ABCD$. Given that $\overrightarrow{AB} = 6\mathbf{i} + 8\mathbf{j}$ and that $\overrightarrow{AD} = 11\mathbf{i} - 4\mathbf{j}$, find a unit vector in the direction of $\overrightarrow{BD}$. [3]

(b) In the diagram, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent vectors **a** and **b** respectively.

X, Y and Z are points such that $\overrightarrow{OX} = \frac{3}{2}\overrightarrow{OB}$,

$\overrightarrow{AY} = \frac{3}{5}\overrightarrow{AB}$ and $\overrightarrow{OZ} = \lambda\overrightarrow{OA}$.



(i) Express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of **a** and/or **b**. [2]

(ii) Express $\overrightarrow{XZ}$ in terms of λ, **a** and **b**.

Given that X, Y and Z are collinear, evaluate the value of λ and the ratio $XY:XZ$. [5]

(iii) Find $\frac{\text{area of triangle } OYZ}{\text{area of triangle } AYZ}$. [1]

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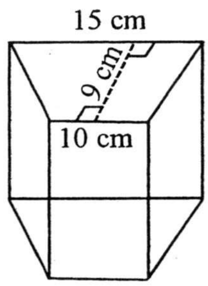

Diagram 1

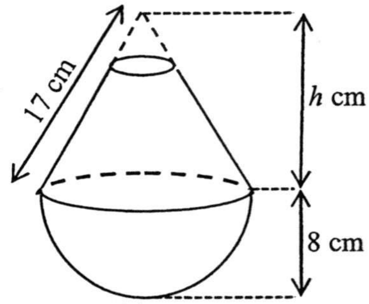

Diagram 2

In a restaurant, the fruit punch is mixed in a container with no lid as shown in Diagram 1. The container is a prism whose cross-section is a trapezium. The lengths of the parallel sides of the trapezium are 10 cm and 15 cm and the distance between the parallel sides is 9 cm. It is given that the capacity of the container is 1.8 litres.

(a) Show that the height of the container is 16 cm. [1]

(b) All the fruit punch in one full container is transferred to a jar as shown in Diagram 2. The jar is made by joining a hemisphere of radius 8 cm to a part of a right circular cone of height, h cm.

(i) Given that the slant height of the cone is 17 cm, show that $h = 15$ cm. [1]

(ii) Find the depth of the fruit punch in this jar after all the fruit punch has been transferred from the container in Diagram 1. [6]

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11 ***Answer the whole of this question on a sheet of graph paper.***

The variables x and y are connected by the equation $y = \frac{1}{10}x^2(5-x)$.

Some of the corresponding values of x and y are given in the table below.

x	−2	−1	0	1	2	3	4	5
y	a	0.6	0	0.4	1.2	1.8	1.6	0

(a) Find the value of a. [1]

(b) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = \frac{1}{10}x^2(5-x)$ for $-2 \le x \le 5$. [3]

(c) Use your graph to find

(i) the range of values of x for which $x^2(5-x) > 10$, [2]

(ii) the values of k, where k is a constant, for which the equation $\frac{1}{10}x^2(5-x) = k$ has exactly 2 solutions. [1]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$. [2]

Bonus Question

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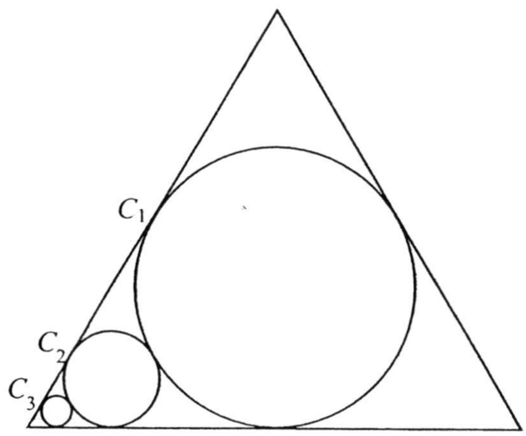

A circle C_1 is inscribed in an equilateral triangle as shown in the diagram. A second circle C_2 is tangent to the circle C_1 and to the two sides of the triangle. A third circle C_3 is tangent to the circle C_2 and to the two sides of the triangle.

Find the ratio of the radius of circle C_3 to the radius of circle C_1. [2]

END OF PAPER

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1(i)

$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} - 2\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & -2 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} -5 & 0 \\ 2 & 1 \end{pmatrix}$

1(ii)

$\mathbf{AB} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 4 \\ -1 & -2 \end{pmatrix}$

det **AB** = 2

$\mathbf{D} = (\mathbf{AB})^{-1}$

$= \frac{1}{2}\begin{pmatrix} -2 & -4 \\ 1 & 1 \end{pmatrix}$ or $\begin{pmatrix} -1 & -2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

2(i)

$\overrightarrow{PS} = \frac{1}{2}\overrightarrow{PS} = \frac{1}{2} \times \frac{2}{3}(3\mathbf{q}) = \mathbf{q}$

$\overrightarrow{OS} = 3\mathbf{p} + \mathbf{q}$

2(ii)

$\overrightarrow{UT} = \overrightarrow{UQ} + \frac{1}{2}\overrightarrow{QR}$

$= \mathbf{q} + \frac{1}{2}(3\mathbf{p} - \mathbf{q})$

$= \frac{3}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

$= \frac{1}{2}(3\mathbf{p} + \mathbf{q})$

$\overrightarrow{UT} = \frac{1}{2}\overrightarrow{OS}$

OS and UT are parallel to each other.

3(a)

$\frac{27a^3(b-c)^2}{18a^{-2}(b+c)(b-c)}$

$= \frac{3a^5(b-c)}{2(b+c)}$

3(b)

$= \frac{8x - 3(2)(2x-1) + 2x - 1}{2(2x-1)^2}$

$= \frac{8x - 12x + 6 + 2x - 1}{2(2x-1)^2}$

$= \frac{-2x+5}{2(2x-1)^2}$ or $\frac{5-2x}{2(2x-1)^2}$

3(c)

$3p(2q-r) - (2q-r)^2$

$= (2q-r)[3p - (2q-r)]$

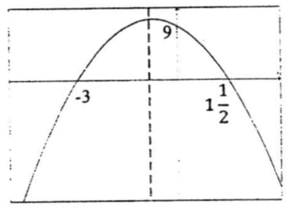
$= (2q-r)(3p - 2q + r)$

or

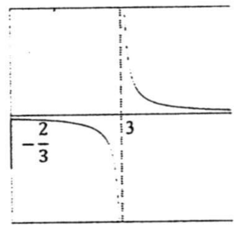
$= (r-2q)[-3p - (r-2q)]$

$= (r-2q)(-3p - r + 2q)$

4(i)



4(ii)



5(i)

$(1.30 \quad 0.80 \quad 1.10)$

5(ii)

$(1.30 \quad 0.80 \quad 1.10)\begin{pmatrix} 80 & 60 \\ 30 & 40 \\ 50 & 20 \end{pmatrix}$

$= (183 \quad 132)$

5(iii)

The **total cost** price of the cookies for **each** of the outlet

5(iv)

$\mathbf{R} = \begin{pmatrix} 18 \\ 13 \end{pmatrix}$

6(i)

Subs $y = k$

$P = \left(\frac{k}{2}, k\right)$

$R = (2k, k)$

(ii)(a)

$6 = \frac{1}{2}(3) + c$

$c = 4\frac{1}{2}$

$PQ: y = \frac{1}{2}x + 4\frac{1}{2}$

$6 = -2(12) + c$

$c = 30$

$QR: y = -2x + 30$

$PQ: y = \frac{1}{2}x + 4\frac{1}{2}$

Solve simultaneous equations.

$x = 10\frac{1}{5}$

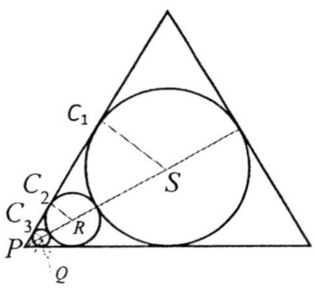
$y = 9\frac{3}{5}$

Q is $\left(10\frac{1}{5}, 9\frac{3}{5}\right)$

6(ii)(b)	$P(3,6)\quad R(12,6)$ Let S be $(x,6)$. $\frac{x-12}{12-3}=\frac{3}{2}$ $x=25\frac{1}{2}$ $S\left(25\frac{1}{2},6\right)$
7(a)	$\frac{8}{30}=\frac{4}{15}$
(b)(i)	$\frac{8}{30}\times\frac{2}{29}+\frac{2}{30}\times\frac{8}{29}$ $=\frac{16}{435}$
(b)(ii)	$\frac{25}{30}\times\frac{24}{29}$ $=\frac{20}{29}$
(c)	$\frac{18}{56}$ $=\frac{9}{28}$

8(i)	$JK=\sqrt{75^2+48^2-2(75)(48)\cos 80°}$ $=81.72=81.7$ m(3 sf)
(ii)	$\frac{1}{2}(75)(48)\sin 80°$ = 1773 sq m or 1770 sq m (3sf)
(iii)	$\frac{\sin\angle KJL}{48}=\frac{\sin 80°}{81.72}$ $\sin\angle KJL=\frac{48\sin 80°}{81.72}=0.5784$ $\angle KJL=35.34°$ $180°+110°-35.34°=254.7°$ Bearing $=254.7°$
	Let the shortest distance be d m. $\frac{d}{75}=\sin 35.34°$ $d=75\sin 35.34°=43.38$ $\tan\theta=\frac{23}{43.38}$ $\theta=27.9°$
9(a)	$\overrightarrow{BD}=-(6\mathbf{i}+8\mathbf{j})+11\mathbf{i}-4\mathbf{j}$ $=5\mathbf{i}-12\mathbf{j}$ $\lvert\overrightarrow{BD}\rvert=\sqrt{5^2+12^2}=13$ unit vector is $\frac{1}{13}(5\mathbf{i}-12\mathbf{j})$

9(b)(i)	$\overrightarrow{OX}=\frac{3}{2}\mathbf{b}$ $\overrightarrow{OY}=\frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}$
(b)(ii)	$\overrightarrow{XZ}=-\frac{3}{2}\mathbf{b}+\lambda\mathbf{a}$ $\overrightarrow{XY}=-\frac{3}{2}\mathbf{b}+\frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}$ $=-\frac{9}{10}\mathbf{b}+\frac{2}{5}\mathbf{a}$ Let $\overrightarrow{XY}=h\overrightarrow{XZ}$ $-\frac{9}{10}\mathbf{b}+\frac{2}{5}\mathbf{a}=h(-\frac{3}{2}\mathbf{b}+\lambda\mathbf{a})$ $\frac{3h}{2}=\frac{9}{10}$ $h=\frac{3}{5}$ $\lambda h=\frac{2}{5}$ $\lambda=\frac{2}{5}\times\frac{5}{3}$ $=\frac{2}{3}$ $XY:XZ=3:5$ Or alternatively, $\overrightarrow{YZ}=\frac{3}{5}(\mathbf{a}-\mathbf{b})+(1-\lambda)(-\mathbf{a})$ $=\left(\lambda-\frac{2}{5}\right)\mathbf{a}-\frac{3}{5}\mathbf{b}$ Let $\overrightarrow{XZ}=k\overrightarrow{YZ}$ $-\frac{3}{2}\mathbf{b}+\lambda\mathbf{a}=k\left[\left(\lambda-\frac{2}{5}\right)\mathbf{a}-\frac{3}{5}\mathbf{b}\right]$ $\frac{3}{5}k=\frac{3}{2}$ $k=\frac{5}{2}$ $\lambda=\frac{5}{2}\left(\lambda-\frac{2}{5}\right)$ $2\lambda=5\lambda-2$ $\lambda=\frac{2}{3}$ $XY:XZ=3:5$
(b)(iii)	$\frac{OZ}{OA}=\frac{2}{3}$ $\frac{\text{area of triangle } OYZ}{\text{area of triangle } AYZ}=\frac{OZ}{AZ}=2$
10(a)	$\left(\frac{10+15}{2}\right)(9H)=1800$ $H=16$
(c)(i)	$h^2+8^2=17^2$ Or $h=\sqrt{17^2-8^2}$ $h=15$ cm

(c)(ii)	Volume of hemisphere $= \frac{2}{3}\pi(8^3)$ $= \frac{1024\pi}{3}$ or 1072.33 cm^3 Volume of cone $= \frac{1}{3}\pi(8^2)(15)$ $= 320\pi$ or 1005.31 cm^3 Total volume $= \frac{1984\pi}{3}$ or 2077.64 cm^3 $\frac{1}{3}\pi r^2 h = 2077.64 - 1800$ $\frac{1}{3}\pi\left(\frac{8h}{15}\right)^2 h = 277.64$ $\frac{64\pi h^3}{675} = 277.64$ $h^3 = 932.086$ $h = 9.768$ depth of water = 8 + (15 − 9.768) = 13.2 cm (3 sf)
Bonus Qn 12	Let P be one vertex of the triangle and Q, R and S be the centres of the circles C_3, C_2 and C_1 respectively. Let the radii of the circles C_1, C_2 and C_3 be a, b and c respectively.  $\frac{a}{PS} = \cos 60° = \frac{1}{2} \Rightarrow PS = 2a$ $PR = 2b$ $PQ = 2c$ Length of PS: $2c + c + 2b + a = 2a$ $\Rightarrow 3c + 2b = a$____(1) Length of PS: $2b + b + a = 2a$ $\Rightarrow 3b = a$ $\Rightarrow b = \frac{a}{3}$____(2) Subs (2) in (1) $3c + 2\left(\frac{a}{3}\right) = a$ $9c + 2a = 3a$ $\frac{c}{a} = \frac{1}{9}$