For Examiner's use

Answer all questions

1 (a) Calculate $\frac{\pi - 56\%}{2 - 2.31^2}$, giving your answer correct to 4 significant figures.

(b) Write your answer to part (a) correct to 3 significant figures.

Answer (a) _____ [1]

(b) _____ [1]

2 The first five terms of a sequence are 19, 15, 11, 7, 3.

(a) Write down the 8^{th} term of this sequence.

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer (a) _____ [1]

(b) _____ [1]

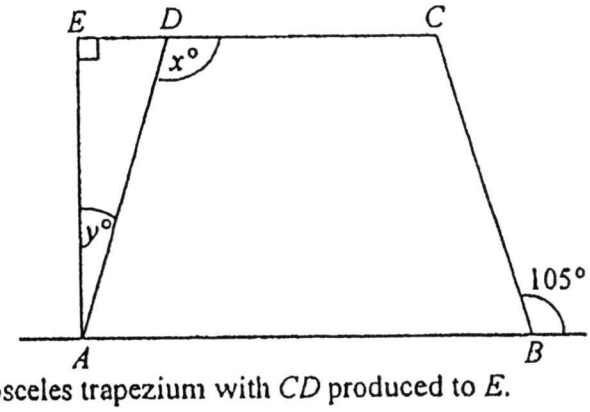
3 (a) Given that $9^3 \div 9^{-2n} = 9^7$, find the value of n.

(b) Calculate $3.4 \times 10^7 - 5 \times 10^6$, giving your answer in standard form.

Answer (a) $n =$ _____ [1]

(b) _____ [1]

4



ABCD is an isosceles trapezium with *CD* produced to *E*.
Find

(a) x,

(b) y.

Answer (a) $x =$ _____ [1]

(b) $y =$ _____ [1]

For Examiner's use

5 Simplify $(3x-5)^2-(-3x-5)^2$.

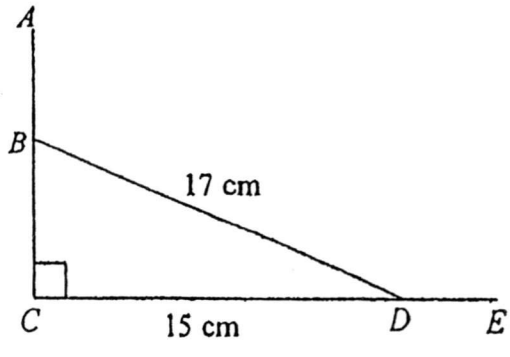
Answer _____ [2]

6 BCD is a right-angled triangle. $CD = 15$ cm and $BD = 17$ cm. ABC and CDE are straight lines. Express, as a fraction, the value of

(a) sin $\angle CBD$,

(b) cos $\angle BDE$,

(c) tan $(90° - \angle CBD)$.



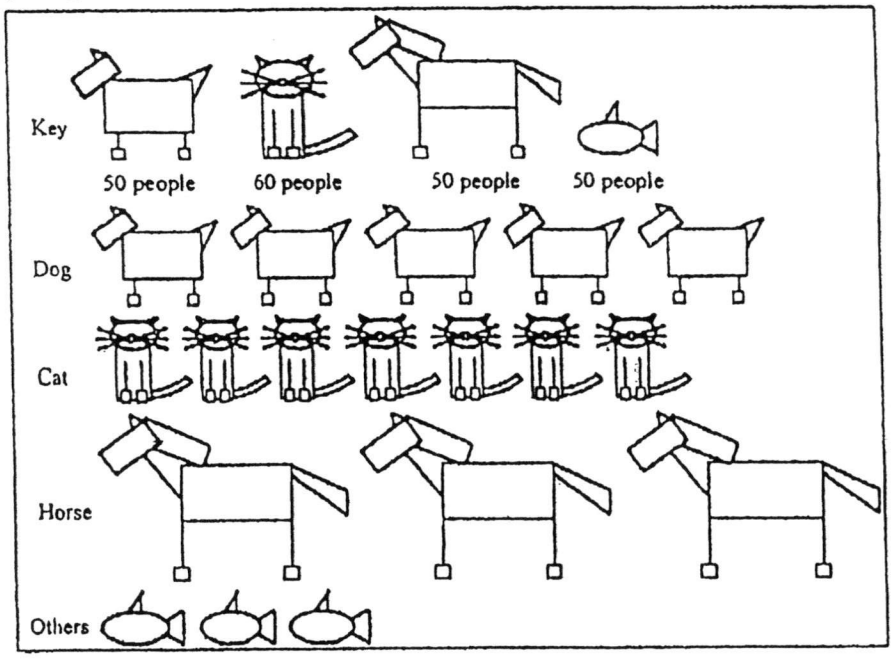
Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [1]

For Examiner's use

7 The pictogram shows the number of people who own different types of pets in a town.



List two things misleading with this pictogram. [2]

Answer 1. _____

2. _____

8 A bag contains 1 red, 3 blue and 6 white balls. Two balls are taken from the bag, at random, without replacement.
Find the probability that

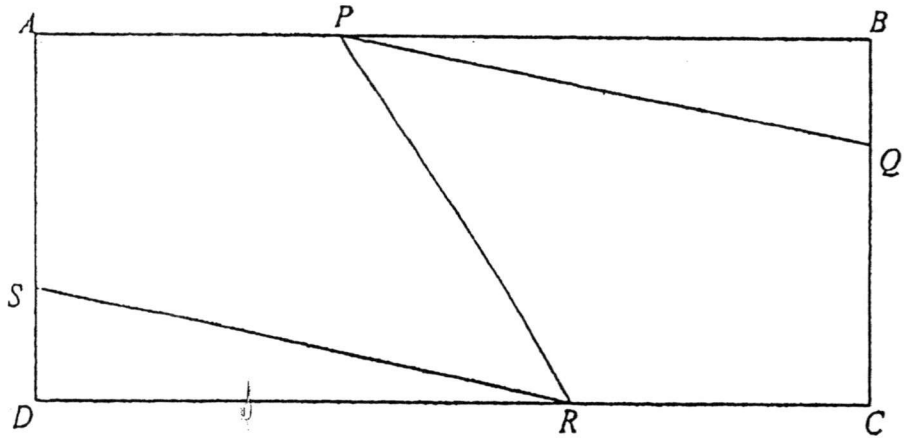
(a) the first ball is red,

(b) none of the balls are blue.

Answer (a) _____ [1]

(b) _____ [2]

9



$ABCD$ is a rectangle.
Points P, Q, R and S lie on AB, BC, CD and DA respectively such that triangle PBQ is congruent to triangle RDS.

(a) Show that $\angle RPQ = \angle PRS$. [3]

Answer _____

_____

_____

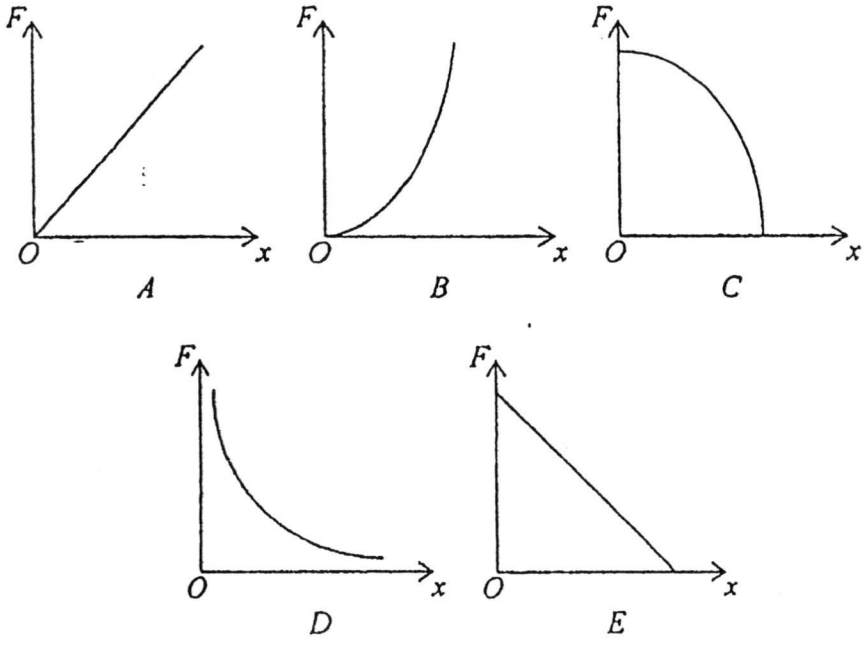
_____

(b) Given further triangle ASP is congruent to triangle CQR.
State the name of the quadrilateral $PQRS$.

Answer (b) _____ [1]

10 The force (F units) between two particles is inversely proportional to the square of the distance (x units) between them.

(a) Select one of the graphs below that could represent the relation between the force and the distance.



Answer (a) _____ [1]

(b) When the distance between the two particles is x, the force is F.
Find the percentage change in the value of the force when the distance is reduced to 50%.

Answer (b) _____ % [2]

For Examiner's use

11

A company sells cereals in boxes which measure 18 cm by 12 cm by 30 cm.

(a) Calculate the volume of the box.

(b) They make a special edition box which is geometrically similar to the original box. The volume of the special edition box is 1920 cm^3.
Find the dimensions of this box.

Answer *(a)* _____ cm^3 [1]

(b) _____ cm by _____ cm by _____ cm [3]

12 (a) Factorise $(8x+13)^2 - 12(8x+13) + 36$ completely.

(b) Hence find the value of
$(48+13)^2 - 12(48+13) + 36$.

Answer *(a)* _____ [2]

(b) _____ [2]

294

For Examiner's use

13 20 students from Class 3A and 20 students from 3B took a Mathematics Test. The marks are shown in the stem-and-leaf diagram.

Class 3A						Class 3B					
		9	7	6	0	5	7				
9	8	6	5	4	1	1	8	8	9		
5	5	4	2	2	2	5	6	7	7	7	7
			8	3	3	2	5	8	9		
	7	4	4	4	4	5	5	5			
				0	5	0					

Key (Class 3A)
0 | 5 means 50

Key (Class 3B)
5 | 0 means 50

(a) (i) Find the median marks for Class 3A.

(ii) Find the range for the top 25% students marks in Class 3A.

Answer *(a) (i)* _____ marks [1]

(ii) _____ marks [1]

(b) Here are two statements comparing the performance for the two classes. For each one, write whether you agree or disagree. Give a reason for each answer, stating clearly which statistic you use to make your decision.

(i) On the average, Class 3A performed better than Class 3B. [1]

Answer _____ because _____

_____

_____

_____

(ii) The top 25% of Class 3A results are closer together. [1]

Answer _____ because _____

_____

_____

_____

14 (a) The value of Ms Tan's car is $120 000. By the end of each year, the value of the car decreases by 10 % of its value at the start of the year.
Find the value of the car at the end of 2 years.

Answer (a) $ _____ [2]

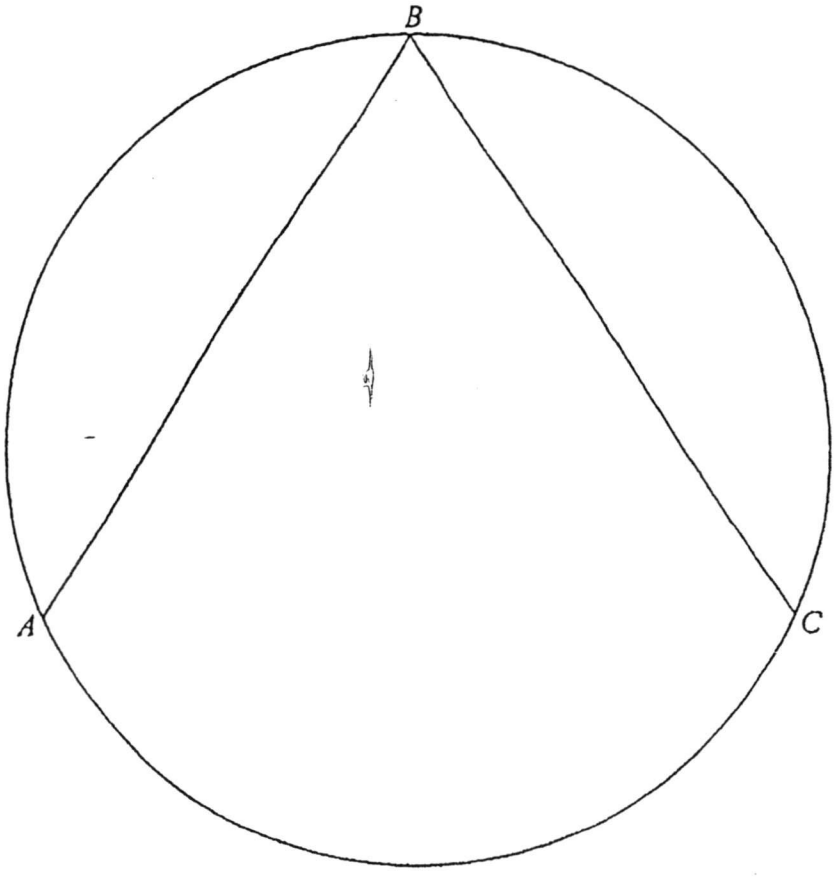
(b) A car is priced at $200 000. It can be bought on hire purchase with a down payment of $110 000, interest rate of 2% per annum over 7 years and equal monthly repayment. Find the amount of each monthly installment.

Answer (b) $ _____ [2]

295

For Examiner's use

15 The points A, B, and C lie on the circle and $AB = BC$.



(a) Construct

(i) the bisector of angle ABC, [1]

(ii) the perpendicular bisector of AB. [1]

(b) (i) Mark clearly a point, P, on the circle along minor arc AC such that $\triangle BPC$ is a right-angled triangle. [1]

(ii) State the name of the line BP.

Answer (ii) _____ [1]

(iii) Hence explain why $\triangle BPC$ is a right–angle triangle. [1]

Answer (iii) _____

_____

_____

16 (a) A race is x km long. A runner ran $12\frac{1}{2}$ km/h for 50% of the race, 10 km/h for $\frac{1}{5}$ of it and 5 km/h for the remaining distance.

(i) Find, in terms of x, the total time taken for the race.

(ii) Calculate the average speed, in kilometres/hour of the runner for the whole journey.

Answer (i) _____ h [2]

(ii) _____ km/h [2]

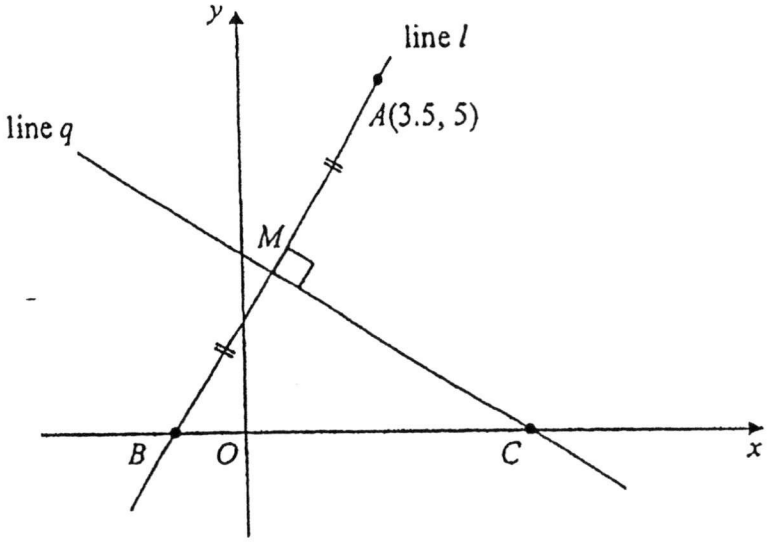
(b) The Dead Sea is 380 m below the sea level.
A helicopter hovers above the Dead Sea at 70 m above the sea level before lowering 150 m.
How far is the helicopter above the Dead Sea?

Answer (b) _____ m [2]

296

For Examiner's use

17 The diagram shows line l with the equation $2y = 3 + 2x$ passing through point $A(3.5, 5)$ and the x-axis at B.
Another line q is perpendicular to line l. It passes through M, the mid-point of AB and cuts the x-axis at C.



Find

(a) the coordinates of B,

(b) the mid-point of AB,

(c) the equation of the line q,

(d) the ratio of area of ΔBMC to area of ΔABC.

Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [2]

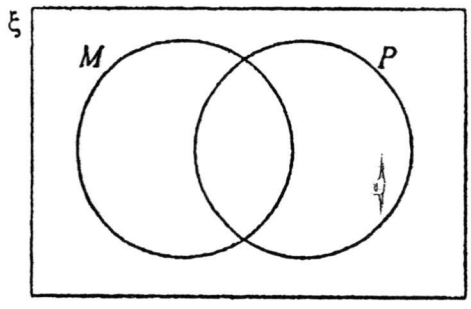
(d) _____ : _____ [2]

For Examiner's use

18 ξ = {Secondary 4 students who study Physics, Mathematics or both}
P = {Students who study Physics}
M = {Students who study Mathematics}

(a) On the Venn Diagram below, shade the region representing $M \cap P'$.

Answer [1]



(b) Describe the elements of $M \cap P'$. [1]

Answer _____

_____

_____

(c) There are 138 students who study Physics out of 220 students in Secondary 4 and $n(P) = n(M)$.

(i) Find the number of students who study both Physics and Mathematics.

(ii) If one student is chosen at random from the Secondary 4 students, find the probability that the student does not study Mathematics.

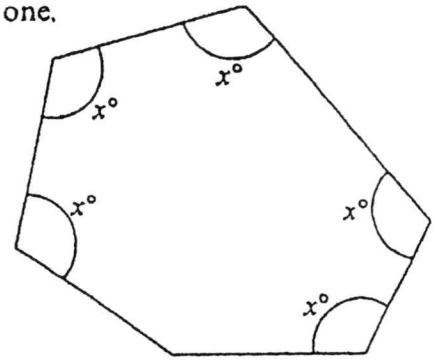
Answer (i) _____ [2]

(ii) _____ [1]

297

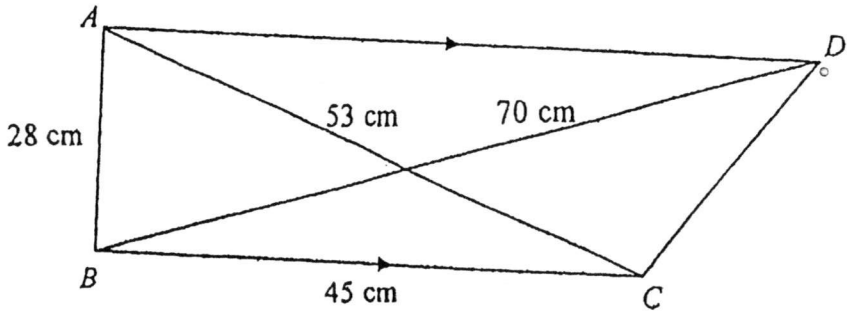
For Examiner's use

19 In a hexagon, all the interior angles are $x°$ except one.
One of the exterior angles is 36°.
Find the value of x.



Answer _____ [3]

20 The diagram shows a trapezium $ABCD$ with AD parallel to BC.



Given $AB = 28$ cm, $BC = 45$ cm, $AC = 53$ cm and $BD = 70$ cm,
Find

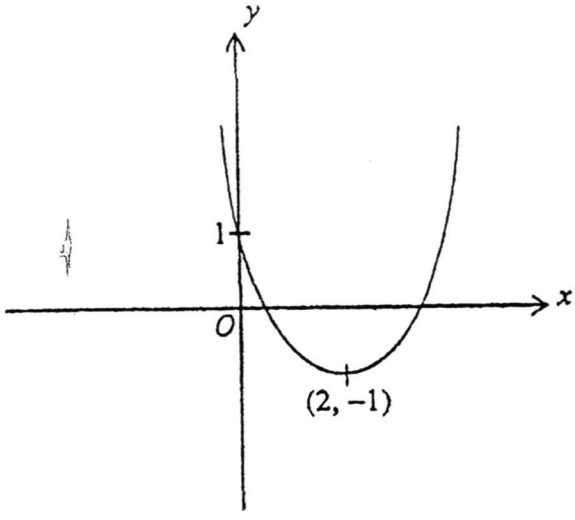
(a) angle ABC,

(b) the shortest distance of C to BD.

Answer (a) _____ ° [1]

(b) _____ cm [3]

21 (a) The diagram shows a quadratic curve which can be expressed in the form $y = a(x + b)^2 + c$.
Find the values of a, b and c.



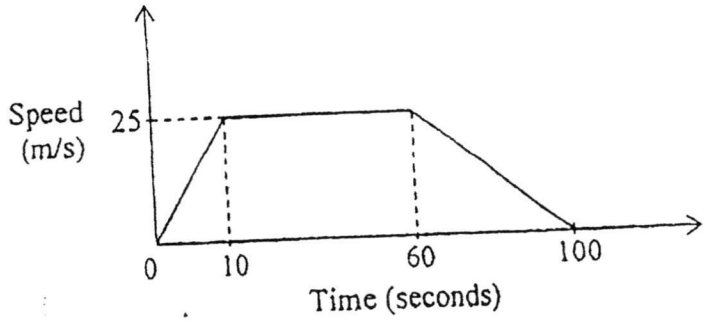
Answer (a) $a =$ ____ $b =$ ____ $c =$ ____ [2]

(b) Hence find the x-intercepts, giving your answers correct to two decimal places.

Answer (b) $x =$ _____ or _____ [3]

For Examiner's use

22 The diagram shows the speed-time graph for a cyclist's journey that started from Point *A* to Point *B*.



(a) Calculate the acceleration during the first 10 seconds.

(b) Calculate the total distance travelled for the journey.

Answer (a) _____ m/s^2 [1]

(b) _____ m [2]

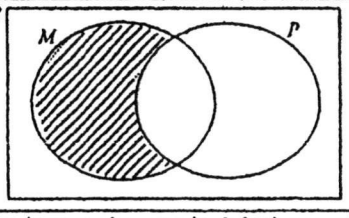
(c) A rider on an electric scooter started at the same time as the cyclist. It travelled at a constant speed of 20 m/s and was overtaken by the cyclist some time later. Find the time taken for the cyclist to overtake the rider.

Answer (c) _____ seconds [2]

END OF PAPER

Answer

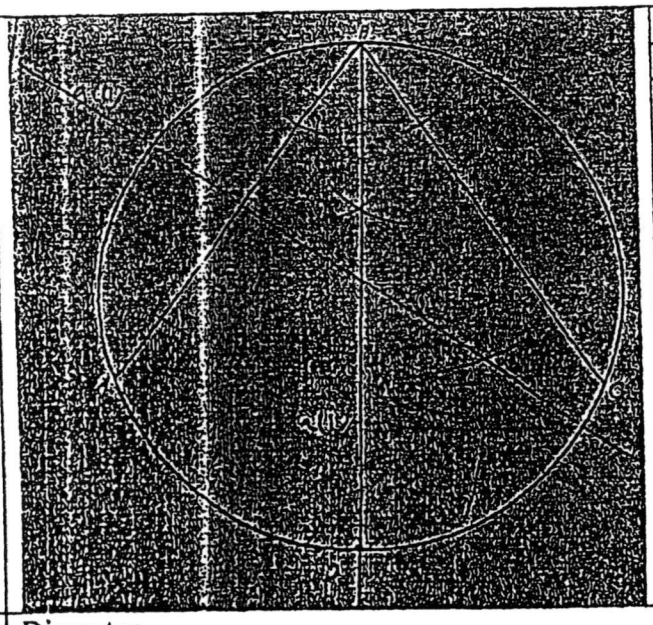
Qn	Solution
1(a)	–0.7738
(b)	–0.774
2(a)	–9
(b)	$23 - 4n$
3(a)	2
(b)	2.9×10^7
4(a)	105
(b)	15
5	$= -60x$
6(a)	$\frac{15}{17}$
(b)	$-\frac{15}{17}$
(c)	$\frac{8}{15}$
8(a)	$\frac{1}{10}$
(b)	$\frac{7}{15}$
9(b)	Parallelogram
10(a)	D
(b)	% changes = 300
T1(a)	6480
(b)	12 by 8 by 20 cm
12(a)	$(8x+7)^2$
(b)	3025
13(ai)	23
(aii)	50 – 44 = 6
14(a)	97 200
(b)	Monthly instalment = 1220
15(bii)	Diameter

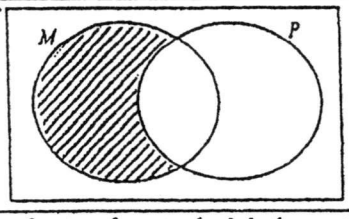
Qn	Solution
16(ai)	$\frac{3x}{25}$
(aii)	$8\frac{1}{3}$
(b)	300
17(a)	$\left(-1\frac{1}{2},0\right)$
(b)	$\left(1,2\frac{1}{2}\right)$
(c)	$y=-x+3\frac{1}{2}$
(d)	1 : 2
18(a)	
(b)	Sec 4 students who study Mathematics only
(ci)	$x=56$
(cii)	$\frac{41}{110}$
19	115.2
20(a)	$\angle ABC=90°$
(b)	18
21(a)	$a=\frac{1}{2}$ $b=-2$ $c=-1$
(b)	$x=0.59, 3.41$
22(a)	2.5
(b)	Dist = 1875
(c)	$x=25$

2015 Prelim 1 Maths Paper 1 Marking Scheme

Qn	Solution	Marks
1(a)	-0.7738	A1
(b)	-0.774	A1
2(a)	-9	A1
(b)	$23-4n$	A1
3(a)	2	A1
(b)	2.9×10^{7}	A1
4(a)	105	A1
(b)	15	A1
5	$(3x-5)^2-(-3x-5)^2$ $=(3x-5+3x+5)(3x-5-3x-5)$ $=-60x$	M1 $(a+b)(a-b)$ or correct expansion A1
6(a)	$\frac{15}{17}$	A1
(b)	$-\frac{15}{17}$	A1
(c)	$\frac{8}{15}$	A1
7	Inconsistent pictorial representation. The picture of the cat should represent 50 people instead of 60. Size of each type of pictorial representation should be consistent. For example, the horses are bigger than the rest of the pictures	B1 o.e scale B1 o.e size
8(a)	$\frac{1}{10}$	A1
(b)	$\frac{7}{10}\times\frac{6}{9}$ $=\frac{7}{15}$	M1 $\frac{6}{9}$ or o.e seen A1
9(a)	$\angle BPR=\angle DRP$ (alt $\angle$s) $\angle DRS=\angle BPQ$ ($\Delta PBQ\equiv\Delta RDS$) $\angle BPR-\angle BPQ=\angle DRP-\angle DRS$ $\angle RPQ=\angle PRS$	B1 B1 B1 Answer Given
(b)	Parallelogram	B1

10(a)	D	A1
(b)	$F = \frac{k}{x^2}$ $k = Fx^2$ $F_1 = \frac{Fx^2}{\frac{1}{4}x^2}$ $= 4F$ % changes = 300	M1 $\frac{1}{4}$ seen A1
11(a)	6480	A1
(b)	$\left(\frac{1920}{6480}\right)^{\frac{1}{3}} = \frac{2}{3}$ $\frac{2}{3} \times 18 = 12$ $\frac{2}{3} \times 12 = 8$ $\frac{2}{3} \times 30 = 20$	M1 $\left(\frac{1920}{6480}\right)^{\frac{1}{3}}$ SOI. Using similarity ratio of volume A2 if All 3 answers correct, A1 if 2 answers correct, A1 correct but wrong order.
12(a)	$(8x+13)^2 - 12(8x+13) + 36$ $= (8x+13)^2 - 2(8x+13)(6) + (6)^2$ $= (8x+13-6)^2$ $= (8x+7)^2$	M1 $a^2 - 2ab + b^2$ seen or implied or expansion A1
(b)	$x = 6$ $(8(6) + 7)^2$ $= 3025$	M1 sub $x = 6$ into answer of 12(a) A1
13(ai)	23	A1
(aii)	$50 - 44 = 6$	A1
(bi)	Disagree because it has a lower median than Class 3B	B1
(bii)	Agree because spread of the top 25% of Class 3A is smaller than Class 3B	B1 spread or range
14(a)	120 000 ×0.9 ×0.9 = 97 200	M1 ×0.9×0.9 seen A1
(b)	Monthly instalment = $\frac{90000 \times 2\% \times 7 + 90000}{7(12)}$ = 1220	B1 Interest and principal seen A1 at least 3 s.f.

15(ai)		C1
(aii)		C1
(bi)		P1
(bii)	Diameter	B1
(biii)	Since BP is a diameter, ΔBPC is a right–angle triangle given the property of angle in semi-circle.	B1 Not Pythagoras Thm
16(ai)	$\frac{\frac{1}{2}x}{12.5} + \frac{\frac{1}{5}x}{10} + \frac{\frac{3}{10}x}{5}$ $= \frac{2x}{50} + \frac{x}{50} + \frac{3x}{50}$ $= \frac{3x}{25}$	M1 eqn, o.e use T=D/S A1
(aii)	$\frac{x}{\frac{3x}{25}}$ $= 8\frac{1}{3}$	M1 $\frac{x}{\text{their answer}}$ seen A1 reject 8.33
(b)	$380 + 70 - 150$ $= 300$	M1 $+70-150$ seen or use of diagram A1
17(a)	$\left(-1\frac{1}{2}, 0\right)$	A1
(b)	$\left(1, 2\frac{1}{2}\right)$	A1
(c)	$y - 2\frac{1}{2} = -(x-1)$ $y = -x + 3\frac{1}{2}$	M1 $y - y_1 = m(x - x_1)$ seen A1
(d)	Ratio $= \frac{BM}{AM}$ $= \frac{1}{2}$ 1 : 2	M1 ratio of bases seen. A1 3 01

18(a)		A1
(b)	Sec 4 students who study Mathematics only	A1 o.e students ...but not physics
(ci)	$138-x+x+138-x=220$ $x=56$	M1 A1
(cii)	$\frac{138-56}{220}$ $=\frac{41}{110}$	A1 No decimals
19	Int $\angle = 180-36$ $=144$ $x=\frac{4(180)-144}{5}$ $=115.2$	M1 correct deduction M1 Using int or ext $\angle$ formula A1 Ignore ° symbol
20(a)	$28^2+45^2=2808$ $=53^2$ $\angle ABC=90°$	A1
(b)	$\frac{1}{2}\times 45\times 28$ $=630$ Shortest Dist $=\frac{630}{\frac{1}{2}\times 70}$ $=18$	M1 Area of triangle Using $\Delta ABC=\Delta PBC$ M1 $\frac{1}{2}\times 70\times$ shortest dist seen (Form Eqn) A1
21(a)	$a=\frac{1}{2}$ $b=-2$ $c=-1$	A2 A2 all answers correct, A1 2 answers correct
(b)	$0=\frac{1}{2}(x-2)^2-1$ $(x-2)^2=2$ $x-2=\pm\sqrt{2}$ $x=0.59, 3.41$	M1 Eqn = 0 A1 A1
22(a)	2.5	A1
(b)	Dist $=\frac{1}{2}\times 25\times(50+100)$ $=1875$	M1 area of trapezium or o.e A1

(c)	$125 + 25(x - 10) = 20x$ $x = 25$	M1 Dist of cyc = Dist of rider A1

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 1 2015
Secondary 4

CANDIDATE NAME

CLASS | | | INDEX NUMBER | | |

MATHEMATICS

4016/02

Paper 2

Thursday 2 July 2015

2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

22

303

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r\ell$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

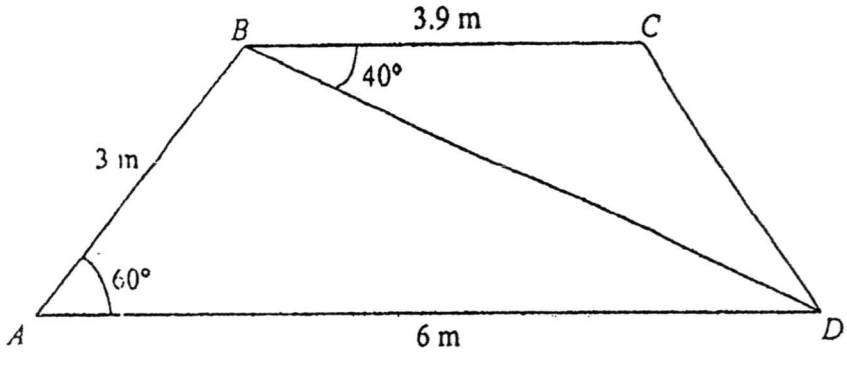
$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Bob has $x and Kim has half as much as Bob. Bob receives $10 each month and Kim receives 4 times as much as Bob. In two months' time, Kim will have 3 times as much as Bob. How much money does Bob have now? [3]

(b) Given that $\frac{x+3y}{5x-4y}=\frac{1}{3}$, find the ratio of $x : y$. [3]

(c) Solve the equation $2x^2=-3(2x-11)$, giving your answers to two decimal places. [3]

2



The diagram shows the points A, B, C and D on level ground. It is given that $AB = 3$ m, $AD = 6$ m, $BC = 3.9$ m, $\angle BAD = 60°$ and $\angle CBD = 40°$.

(a) Calculate

(i) the distance from B to D, [2]

(ii) the shortest distance from C to BD, [2]

(iii) the area of $ABCD$. [3]

(b) A vertical pole stands at point A and the angles of elevation of the top of the pole from both points B and D are measured. Given that the smaller of these two angles is 18°, find the height of the pole. [2]

304

3 (a) A shop sells two brands of mattresses, Hoiland and Sleeping King. Each brand of mattress comes in three different sizes, King, Queen and Single. The sales of the two brands of mattresses over a period of two months are given in the table below.

	November			December		
	King	Queen	Single	King	Queen	Single
Hoiland	10	15	25	12	20	40
Sleeping King	8	5	12	10	10	25

Given that the above sales table can be represented by matrix $P = \begin{pmatrix} 10 & 15 & 25 \\ 8 & 5 & 12 \end{pmatrix}$

and matrix $Q = \begin{pmatrix} 12 & 20 & 40 \\ 10 & 10 & 25 \end{pmatrix}$.

(i) Calculate $P + Q$. [1]

(ii) Describe what is represented by the elements of $P + Q$. [1]

(iii) Find matrix R such that $S = RP$ and S represents the total number of King, Queen and Single sizes mattresses sold by both Hoiland and Sleeping King in November. [1]

(iv) Evaluate matrix S. [1]

(v) The profit made for each mattress regardless of the brands are \$350, \$500 and \$250 for King, Queen and Single sizes respectively. Using matrix multiplication involving two matrices only, find the profit made by the shop in November. [2]

(b) On a map depicting part of a country, the scale is 1 : 250 000.

(i) Calculate the distance between the Town A to Town B, given that the distance on the map is 1.5 cm. Leave your answer in kilometres. [2]

(ii) Ali walks from Town B to Town A at the speed of 1 m/s. Calculate the time Ali takes for the walk, giving your answer to the nearest minute. [2]

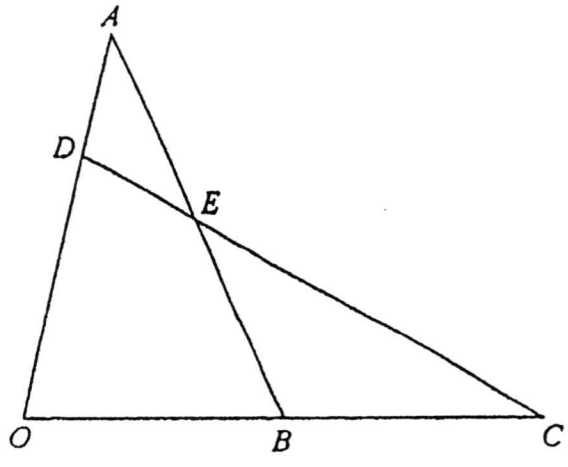
(iii) Kim walks from Town B to Town A at the speed of 4 km/h. She started her journey at the same time as Ali.
Who will reach Town A first? Justify your answer. [2]

4 (a) Solve the inequality $\frac{2x-1}{5} \geq \frac{x-3}{9}$. [2]

(b) Simplify $\frac{20x-15y}{32x^2-18y^2}$. [3]

(c) Express $\frac{5x+6}{4x^2-9} + \frac{2}{3-2x}$ as a single fraction in its simplest form. [3]

5



(a) In the diagram, B is the midpoint of OC and $OA = 3AD$. It is also given that $\overrightarrow{OA} = 3\mathbf{a} + \mathbf{b}$ and $\overrightarrow{OB} = 6\mathbf{a} - \mathbf{b}$. E is the point of intersection of AB and CD.

(i) Express as simply as possible, in terms of a and/or b,

(a) $\overrightarrow{AB}$, [1]

(b) $\overrightarrow{DC}$. [2]

(ii) Find the ratio of $\frac{\text{area of triangle } ADE}{\text{area of triangle } DEO}$. [1]

(b) The vector $\overrightarrow{OP}$ has a magnitude of 55 units and has the same direction as $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Express $\overrightarrow{OP}$ as a column vector. [2]

305

6 The variables x and y are connected by the equation

$$y = 2x - x^2 + 6.$$

Some corresponding values of x and y are given in the following table.

x	-2	-1	-0.5	0.5	2	3	4
y	-2	3	4.75	p	6	3	-2

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \le x \le 4$.
Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $-2 \le y \le 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

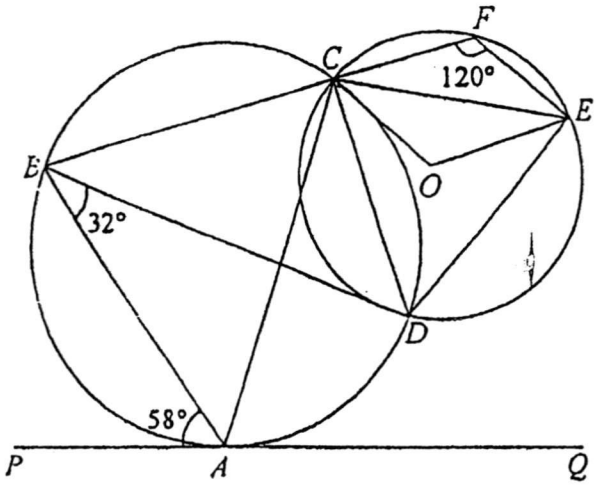
(c) Find the equation of the line of symmetry. [1]

(d) Use your graph to solve $4x + 7 = 2x^2$. [3]

(e) By drawing a tangent, find the gradient of the curve at the point where $x = 2$. [2]

(f) A straight line has the equation $y = ax + b$. Given that $ax + b = 2x - x^2 + 6$ has solutions at $x = 2$ and $x = -1$, find the value of a and of b. [2]

7 In the diagram, PAQ is a tangent to the circle $ABCD$ at A. O is the centre of the circle $CDEF$ and BCF is a straight line. It is given that $\angle PAB = 58°$, $\angle ABD = 32°$ and $\angle CFE = 120°$.



(a) Find

(i) angle ACD, [1]

(ii) angle ACB. [1]

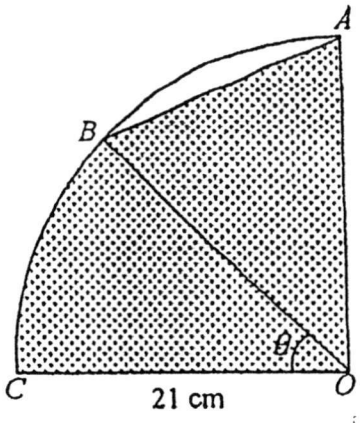
(b) Explain why BD is a diameter of circle $ABCD$. [2]

(c) Given that $FC = FE$, show that triangle CDE is equilateral. [3]

(d) What is the special name given to quadrilateral $COEF$? Explain. [3]

306

8

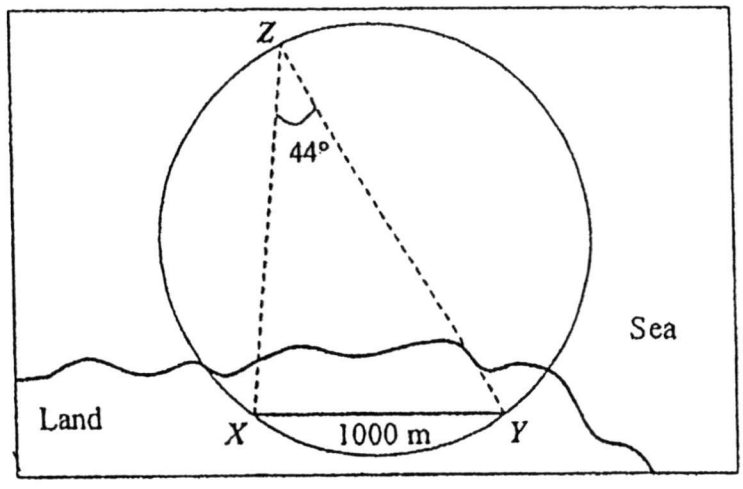


(a) $OABC$ is a quadrant of a circle with centre O and radius 21 cm. Given that $\angle BOC = \theta$ rad and Arc AB : Arc BC = 4 : 3, calculate

(i) the value of θ, [3]

(ii) the area of the shaded region. [3]

(b) A ship at sea is represented by point Z and it is travelling in a circular arc as shown in the diagram such that $\angle XZY = 44°$. The two landmarks X and Y are such that Y is 1000 m due east of X.

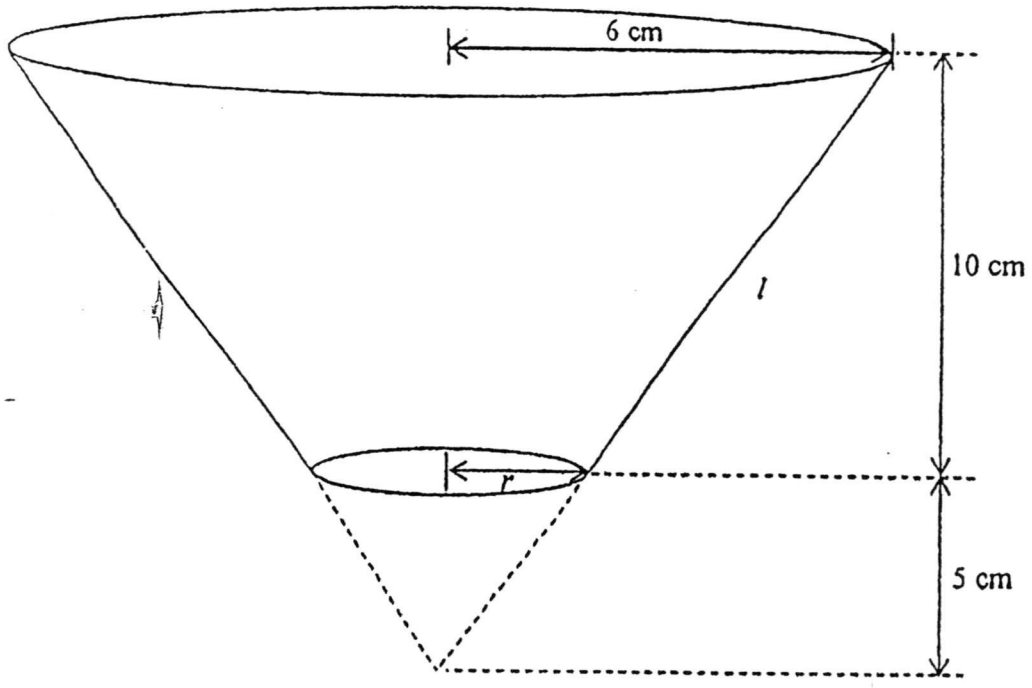


(i) Find the bearing of X from Z, when Z is due north of Y. [1]

(ii) Find the bearing of Z from Y, when Z is equidistant from X and Y. [2]

(iii) Calculate the distance from Z to X when the ship is 1000 m from Y. [3]

9 The bulb end of a torchlight consists of an open frustum as shown in the diagram. The frustum is formed by cutting off the bottom 5 cm from an inverted cone of radius 6 cm and height 15 cm.



(a) Find the base radius r of the frustum. [2]

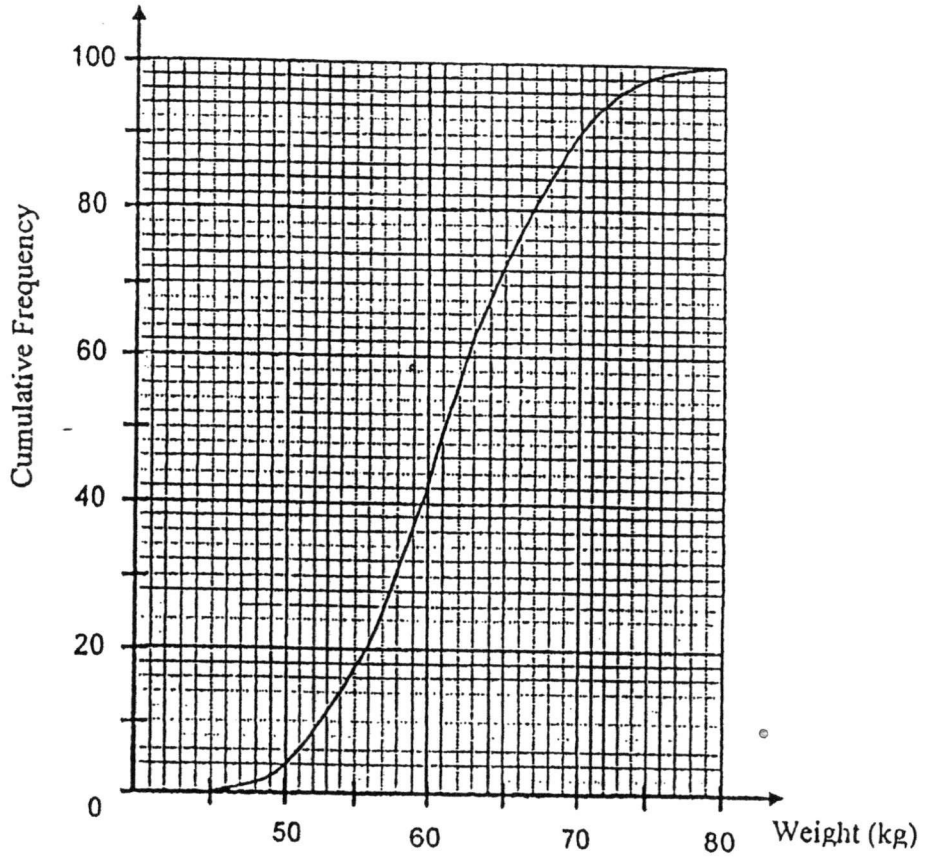
(b) Find l, the slant height of the frustum. [2]

(c) Calculate the volume of the frustum. [2]

(d) The inner surface of the frustum, which will eventually contain the bulb, has to be lined with a reflective material. Find the amount of the material required. [2]

307

10 The cumulative frequency curve below illustrates the weights of 100 students in Shine Secondary School.



(a) Use the graph to find

(i) the median weight of the students, [1]

(ii) the interquartile range. [2]

(b) The grouped frequency table of the weights of the same 100 students in Shine Secondary School is shown below. Find p. [1]

Weight (x kg)	$45 < x \leq 50$	$50 < x \leq 55$	$55 < x \leq 60$	$60 < x \leq 65$	$65 < x \leq 70$	$70 < x \leq 75$	$75 < x \leq 80$
No. of students	4	14	24	30	p	8	2

Using the table, calculate an estimate of

(i) the mean weight of students, [2]

(ii) the standard deviation. [2]

The students are classified into three categories:
Students who weigh 50 kg or less are in the ***underweight*** category
Students who are heavier than 70 kg are in the ***overweight*** category
The rest of the students are in the ***normal*** weight category

(c) A student is selected at random from the 100 students.

(i) Find the probability that the student is underweight. [1]

(ii) If the probability that the student weighs more than y kg is $\frac{1}{10}$, find y. [1]

Two students are selected at random from the group of 100 students.

(iii) Find the probability that at least one of the students is of normal weight. [2]

(d) The weights of 100 students in Oxford Secondary School have the same median but a smaller standard deviation.
Describe how the cumulative frequency curve will differ from the given curve. [2]

-----End of Paper -----

308

Answers:

1(a) \$8 (b) $x : y = 13 : 2$ (c) $x = 2.83$ or $x = -5.83$

2(a)(i) 5.20 m (ii) 2.51 m (iii) 14.3 m^2 (b) $h = 1.95$ m

3(a)(i) $\begin{pmatrix} 22 & 35 & 65 \\ 18 & 15 & 37 \end{pmatrix}$ (iii) $R = \begin{pmatrix} 1 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 18 & 20 & 37 \end{pmatrix}$ (v) \$25 550

(iii) It represents the **total** number of the King, Queen and Single sizes mattresses **sold over the two months** for **both brands** respectively.

(b)(i) 3.75 km (ii) $T = 63$ mins (iii) $1\frac{1}{9}$ m/s . Kim will reach Town A first as her speed is faster.

4(a) $x \geq -\frac{6}{13}$ (b) $\frac{5}{2(4x+3y)}$ (c) $\frac{x}{(2x+3)(2x-3)}$

5(a)(i)(a) $3a - 2b$ (b) $10a - \frac{8}{3}b$ (ii) $\frac{\text{area of triangle } ADE}{\text{area of triangle } DEO} = \frac{1}{2}$ (b) $\overrightarrow{OP} = \begin{pmatrix} 33 \\ -44 \end{pmatrix}$

6(a) $p = 6.75$ (c) $x = 1$ (d) $y = 2.5$ (to be plotted), $x = -1.1$ or $x = -3.1$ (e) $= -2.4$ (f) $a = 1$, $b = 4$

7(a)(i) $\angle ACD = 32°$ (ii) $\angle ACB = 58°$ (b) $\angle BCD = 32° + 58° = 90°$

8(a)(i) $\theta = 0.673$ (ii) 32 cm^2 (b)(i) 224° (ii) 338° (iii) $XZ = 1440$ m

9(a) $r = 2$ cm (b) 10.8 (c) 545 cm^3 (d) 271 cm^2

10(a)(i) 61 (ii) 8.5 (b) 18 (i) 61.3 (ii) 6.68 (c)(i) $\frac{1}{25}$

(ii) 70 (iii) $\frac{4859}{4950}$ (d) The graph will be **steeper** but they will **intersect at the median mark**.

Marking Scheme 2015 Prelims 1 Sec 4 Mathematics P2

No	Solution	Marks	Remarks
1(a)	Bob = $\$x$ Kim = $\$\frac{1}{2}x$ In 2 months, Bob = $\$(x+20)$ Kim = $\$\left(\frac{1}{2}x+80\right)$ $\frac{1}{2}x+80=3(x+20)$ $x+160=6x+120$ $x=8$ Bob = \$8	B1 soi (Kim) M1 equation A1	3 m
(b)	$\frac{x+3y}{5x-4y}=\frac{1}{3}$ $3(x+3y)=5x-4y$ $13y=2x$ $\frac{x}{y}=\frac{13}{2}$ $\therefore x:y=13:2$	M1 cross-multiply M1 grouping x & y A1 required form	3 m
(c)	$2x^2=-3(2x-11)$ $2x^2+6x-33=0$ $x=\frac{-6\pm\sqrt{(6)^2-4(2)(-33)}}{2(2)}$ $x=2.83$ (2 dp) or $x=-5.83$ (2 dp)	B1 equation (=0) M1 formula + sub A1 must indicate x somewhere	3 m
			9 marks
2 (a)(i)	$BD=\sqrt{3^2+6^2-2(3)(6)\cos 60°}$ $=5.19615$ m $=5.20$ m (3sf)	M1 cosine rule A1	2 m
(ii)	Let the shortest distance be CX. $\sin 40°=\frac{CX}{3.9}$ $CX=2.50687$ m $CX=2.51$ m (3 sf)	M1 trigo ratios A1	2 m

309

No	Solution	Marks	Remarks
(iii)	$A_{\Delta ABD} = \frac{1}{2}(3)(6)\sin 60°$ $= 7.7942 \text{ m}^2$ $A_{\Delta BCD} = \frac{1}{2}(3.9)(5.19615)\sin 40°$ $= 6.51304 \text{ m}^2$ $A_{ABCD} = 7.7942 + 6.51304$ $= 14.307 \text{ m}^2$ $= 14.3 \text{ m}^2$ (3 sf)	M1 area $A_{\Delta ABD}$ M1 area $A_{\Delta BCD}$ (5.20 not acceptable although ans is correct) A1	3 m
(b)	Let the height of the pole be h. $\tan 18° = \frac{h}{6}$ $h = 1.9495$ m $h = 1.95$ m (3 sf)	M1 identifying AD A1	2 m
			9 marks
3(a)(i)	$P+Q = \begin{pmatrix} 10 & 15 & 25 \\ 8 & 5 & 12 \end{pmatrix} + \begin{pmatrix} 12 & 20 & 40 \\ 10 & 10 & 25 \end{pmatrix}$ $= \begin{pmatrix} 22 & 35 & 65 \\ 18 & 15 & 37 \end{pmatrix}$	A1 correct order	1 m
(ii)	It represents the <u>total</u> number of the King, Queen and Single sizes mattresses <u>sold over the two months</u> for <u>both brands</u> respectively.	B1 oe (use diff)	1 m
(iii)	$R = (1 \quad 1)$	B1 correct order	1 m
(iv)	$S = (1 \quad 1)\begin{pmatrix} 10 & 15 & 25 \\ 8 & 5 & 12 \end{pmatrix}$ $= (18 \quad 20 \quad 37)$	A1	1 m
(v)	$(18 \quad 20 \quad 37)\begin{pmatrix} 350 \\ 500 \\ 250 \end{pmatrix}$ $= (25550)$ $\therefore$ Profit = \$25 550	M1 2 correct matrices No marks if matrix has unit B1 statement seen	2 m
(b)(i)	Distance $= 1.5 \times 2.5$ km $= 3.75$ km	M1 x scale A1	2 m
(ii)	$T = \frac{3750}{1}$ $T = 3750$ secs $T = 63$ mins (nearest mins)	B1 convert (b)(i) to metres A1	2 m
(iii)	$\frac{4 \times 1000}{3600} = 1.11$ m/s or $1\frac{1}{9}$ m/s $\therefore$ Kim will reach Town A first as her speed is faster.	M1 conversion for both num and den B1	2 m
			12 marks

No	Solution	Marks	Remarks
4(a)	$\frac{2x-1}{5} \geq \frac{x-3}{9}$ $9(2x-1) \geq 5(x-3)$ $13x \geq -6$ $x \geq -\frac{6}{13}$	M1 cross-multiply A1	2 m
(b)	$\frac{20x-15y}{32x^2-18y^2}$ $= \frac{5(4x-3y)}{2(16x^2-9y^2)}$ $= \frac{5(4x-3y)}{2(4x-3y)(4x+3y)}$ $= \frac{5}{2(4x+3y)}$	M1 taking out both common factors M1 diff of 2 sq A1	3 m
(c)	$\frac{5x+6}{4x^2-9} + \frac{2}{3-2x}$ $= \frac{5x+6}{(2x+3)(2x-3)} - \frac{2}{2x-3}$ $= \frac{5x+6-2(2x+3)}{(2x+3)(2x-3)}$ $= \frac{x}{(2x+3)(2x-3)}$	M1 one factor common (den $2x-3$) M1 combining term A1 no marks if den expanded	3 m
			8 marks
5(a)(i) (a)	$\overrightarrow{AB} = (6a-b)-(3a+b)$ $= 3a-2b$	No vector symbol [-1m] B1	1 m
(b)	$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$ $= 2(6a-b) - \frac{2}{3}(3a+b)$ $= 10a - \frac{8}{3}b$	M1 $\overrightarrow{OC}$ soi A1	2 m
(ii)	$\frac{\text{area of triangle } ADE}{\text{area of triangle } DEO} = \frac{1}{2}$	A1	1 m
(b)	Let $OP = \begin{pmatrix} 3a \\ -4a \end{pmatrix}$. $\sqrt{(3a)^2+(-4a)^2} = 55$ $a = 11$ (same direction) $\therefore \overrightarrow{OP} = \begin{pmatrix} 33 \\ -44 \end{pmatrix}$	M1 using magnitude A1	2m
			6 marks

No	Solution	Marks	Remarks
5(a)	$p = 6.75$	A1	1 m
(b)	Refer to graph.	P2 pts. correct G1 smoothness (diff scale -1m)	3 m
(c)	$x = 1$	A1	1 m
(d)	$4x + 7 = 2x^2$ $4x - 2x^2 + 7 = 0$ $2x - x^2 + \frac{7}{2} = 0$ $2x - x^2 + 6 = -\frac{7}{2} + 6$ $\therefore y = 2.5$ (to be plotted) $x = -1.1$ or $x = -3.1$	M1 attempt to make y the subject L1 line to be Plotted A1	3 m
(e)	Gradient $= \frac{8.5 - 1.3}{1 - 4}$ $= -2.4$	M1 tangent correct A1	2 m
(f)	$a = \frac{6 - 3}{2 - (-1)}$ $= 1$ $b = 4$	A1 A1	2 m
			12 marks
7(a)(i)	$\angle ACD = 32°$	A1 $\angle$ in same seg.	1 m
(ii)	$\angle ACB = 58°$	A1 $\angle$ in alt. seg.	1m
(b)	$\angle BCD = 32° + 58°$ $= 90°$ Since it obeys $\angle$s in semicircle property, $\Rightarrow BD$ is a diameter. (shown)	B1 proving 90° B1	2 m
(c)	$\angle FCE = \angle FEC = \frac{180° - 120°}{2}$ (base $\angle$s of isos. Δ) $= 30°$ $\angle ECD = 180° - 90° - 30°$ (adj. $\angle$s) $= 60°$ $\angle FED = 90°$ ($\angle$s in opp. seg.) $\angle CED = 90° - 30°$ $= 60°$ $\therefore \angle CDE = 60°$ $\Rightarrow \Delta CDE$ is an equilateral Δ. (shown)	M1 attempt to show $\angle ECD = 60°$ M1 $\angle$s in opp. seg. M1 attempt to show $\angle CED = 60°$	3 m
(d)	$\angle COE = 120°$ ($\angle$ at centre = 2 $\angle$s at circumference) $\Rightarrow \angle OCE = \angle OEC = 30°$ CE is common $\therefore \Delta OCE \equiv \Delta FCE$ (AAS) $\angle OCF + \angle CFE = 180°$ (int. $\angle$s) $\Rightarrow COEF$ is a rhombus.	M1 proving congruency M1 int. $\angle$s /alt $\angle$s B1	3 m
			10 marks

No	Solution	Marks	Remarks
8(a)(i)	$\frac{Arc_{AB}}{Arc_{BC}} = \frac{4}{3}$ $\frac{r\left(\frac{\pi}{2}-\theta\right)}{r\theta} = \frac{4}{3}$ $3\left(\frac{\pi}{2}-\theta\right) = 4\theta$ $7\theta = \frac{3\pi}{2}$ $\theta = \frac{3}{14}\pi$ $\theta = 0.673198$ $\theta = 0.673$ (3 sf)	M1 $\frac{\pi}{2}$ seen M1 $s = r\theta$ soi A1 oe	3 m
(ii)	Area of sector$_{BOC} = \frac{1}{2}(21)^2\left(\frac{3}{14}\pi\right)$ $= 47\frac{1}{4}\pi$ cm^2 Area of $\Delta_{OAB} = \frac{1}{2}(21)^2 \sin\left(\frac{\pi}{2}-\frac{3}{14}\pi\right)$ $= \frac{1}{2}(21)^2 \sin\frac{2}{7}\pi$ $= 172.3938$ cm^2 $\therefore$ Shaded Area $= 172.3938 + 47\frac{1}{4}\pi$ $= 320.834$ cm^2 $= 321$ cm^2 (3 sf)	M1 Area of sector M1 Area of Δ A1	3 m
(b)(i)	Bearing $= 180° + 44°$ $= 224°$	A1	1 m
(ii)	$\angle ZYX = \frac{180° - 44°}{2}$ (base $\angle$s of isos. Δ) $= 68°$ $\therefore$ Bearing $= 270° + 68°$ $= 338°$	M1 Isos. Δ A1	2 m
(iii)	$\angle ZXY = 44°$ (base $\angle$s of isos. Δ) $\angle ZYX = 180° - 2(44°)$ ($\angle$s sum in Δ) $= 92°$ $\frac{XZ}{\sin 92°} = \frac{1000}{\sin 44°}$ $XZ = 1438.68$ m $XZ = 1440$ m (3 sf)	B1 Isos. Triangle / finding $\angle ZYX$ M1 sine rule A1 311	3 m
			12 marks

No	Solution	Marks	Remarks
9(a)	By similar Δs, $\frac{r}{6}=\frac{5}{15}$ $r=2$ cm	M1 similar Δ A1	2 m
(b)	$l=\sqrt{15^2+6^2}-\sqrt{5^2+2^2}$ $=10.7703$ $=10.8$ (3 sf)	M1 pythagoras' thm A1	2 m
(c)	Volume of frustum $=\frac{1}{3}\pi(6)^2(15)-\frac{1}{3}\pi(2)^2(5)$ $=544.5427$ cm^3 $=545$ cm^3 (3 sf)	M1 vol of cone A1	2 m
(d)	Amount of material required $=\pi(6)\left(\sqrt{15^2+6^2}\right)-\pi(2)\left(\sqrt{5^2+2^2}\right)$ $=270.688$ cm^2 $=271$ cm^2 (3 sf)	M1 surface area pf cone A1	2 m
			8 marks
10(a)(i)	Median weight = 61	A1	1 m
(ii)	Interquartile range $=65.5-57$ $=8.5$	M1 UQ - LQ A1	2 m
(b)	$p=90-72$ $=18$	B1	1 m
(i)	Mean $=\frac{47.5\times4+\cdots+77.5\times2}{100}$ $=61.3$ (3 sf)	M1 Formulae A1	2 m
(ii)	Standard deviation $=\sqrt{\frac{4(47.5-61.3)^2+\cdots+2(77.5-61.3)^2}{100}}$ $=6.68$ (3 sf)	M1 Formulae A1	2 m
(c)(i)	P (student is underweight) $=\frac{4}{100}$ $=\frac{1}{25}=0.04$	A1 oe	1 m

No	Solution	Marks	Remarks
(ii)	P (student weighs more than y kg) $= \frac{1}{10} = \frac{10}{100}$ From graph, $y = 70$	A1	1 m
(iii)	P (out of 2 students, at least one of the student is of normal weight) $= \frac{86}{100} \times \frac{14}{99} \times 2 + \frac{86}{100} \times \frac{85}{99}$ $= \frac{4859}{4950}$	M1 A1	2 m
(d)	The graph will be <u>steeper</u> but they will <u>intersect at the median mark</u>.	B2 or graphs drawn [B1] Label [B1]	2m
			14 marks

312
END