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ST JOSEPH'S INSTITUTION

MID-YEAR EXAMINATION 2016
(SECONDARY 4)

MATHEMATICS 4048

3 May 2016

2 hours 30 minutes
(08:00 – 10:30h)

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your Class, Index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where applicable.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, hand in your answers to **Questions 1 to 6**, and **Questions 7 to 11** separately.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **14** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions

1 (a) Factorise completely (i) $16r^2 - 4s^2$ [2]

(ii) $4x^2y^2 + 14xyz - 8z^2$ [2]

(b) Express $\frac{3y}{5(x-1)} + \frac{y}{(x-1)^2}$, as a single fraction in its simplest form. [2]

(c) Solve the inequality $\frac{x-9}{2} \le \frac{x-1}{3} < \frac{2x-5}{4}$ [2]

(d) Given $S = \frac{n}{2}[2a + (n-1)d]$, make d the subject of the formula.

Hence find the value of d if $S = -495$, $n = 18$ and $a = -2$ [2]

2. Peter runs a stall in a food court. From Monday to Friday, for the morning shift, he employs 3 persons for the counter and 4 persons for the kitchen.

For the afternoon shift, he employs 5 persons for the counter and 6 persons for the kitchen.

The information can be represented by the matrix $A = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$.

On Saturday and Sunday, for the morning shift, he employs 4 persons for the counter and 5 persons for the kitchen.

For the afternoon shift, he employs 6 persons for the counter and 7 persons for the kitchen.

(i) Represent the information for the staff employed on Saturday and Sunday by the matrix B, using the same arrangement as matrix A. [1]

(ii) Evaluate $C = 5A - 2B$. What do the elements of C represent? [2]

(iii) The wages per session for each morning shift staff is $50 and that for the afternoon shift staff is $70.

Find matrix W, where $W = C\begin{pmatrix} 50 \\ 70 \end{pmatrix}$

What do the elements W represent? [2]

(iv) Evaluate the matrix T, where $T = (1 \quad 1)W$.

What do the elements of T represent? [2]

3 (a) A gardener has 357 tulip bulbs to plant.

(i) If she planted a rectangle of 15 rows with 33 bulbs in each row, how Many bulbs would be left over? [1]

(ii) If she plants x rows with y bulbs in each row, write down an expression for the number of bulbs left over. [1]

(iii) If $10 < x < 20$, and $y > 30$, find the value of x and of y such that no bulbs are left over. [2]

(b) Given $\varepsilon = \{x : x \in \mathbb{Z},\ 3 < x < 15\}$

$A = \{\text{multiples of } 3\}$
$B = \{4,\ 6,\ 12\}$

(i) List the elements of (a) A, [1]

(b) $A \cap B'$ [1]

(ii) One of the statements in the table below is false.
Identify the false statement and give the correct answer [2]

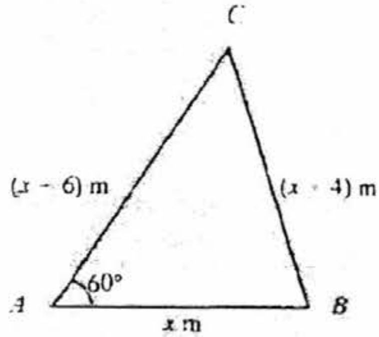
$n(A \cap B) = 1$
$n(A \cup B) = 4$
$n(A \cup B') = 6$

4 (a) The diagram below shows a triangle ABC with $AB = x$ m, $AC = (x + 6)$ m, $BC = (x + 4)$ m, and $\angle BAC = 60°$.

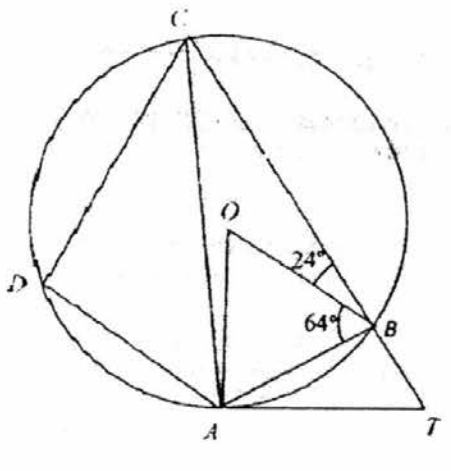
(i) Show that $x = 10$. [2]

(ii) A pole 5 m tall is erected at point C. What is the greatest angle of elevation of the top of the pole from a man walking from A to B. [2]

(iii) If B is due North of A, find the bearing of B from C. [2]



(b)



The diagram above shows a circle, centre O, and passing through the points A, B, C and D. AT is the tangent to the circle at A and CBT is a straight line. Given that $\angle ABO = 64°$ and $\angle CBO = 24°$, calculate the following, stating your reasons clearly,

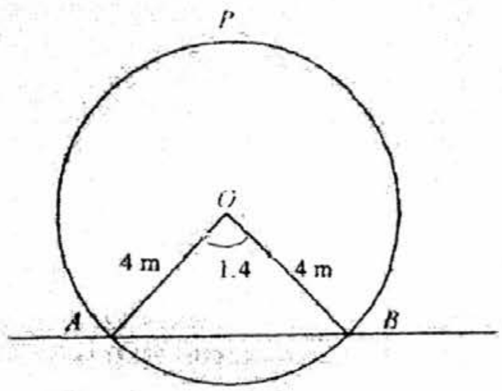
(i) $\angle ADC$, [1]

(ii) $\angle AOB$, [1]

(iii) $\angle ACB$, [1]

(iv) $\angle ATB$. [2]

5



The above diagram shows part of a circle with centre O and radius 4 m. AOB is a sector of the circle with $\angle AOB = 1.4$ radians.
Calculate

(i) the length of the arc APB. [1]

(ii) the area of the triangle AOB, [1]

(iii) the area of the segment APB. [3]

The segment APB represents the cross section of a railway tunnel which is 382 m long. Calculate the time it will take a train travelling at a speed of 200 km/h to pass completely through the tunnel if the train is 118 m long.
Give your answer in seconds. [2]

6

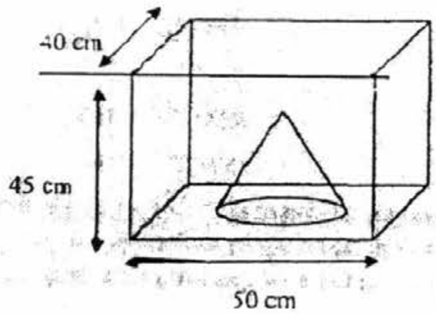


Figure I

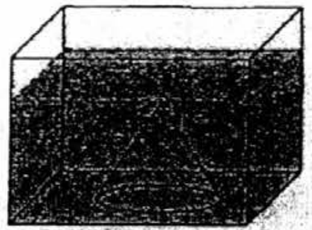


Figure II

Figure I shows an open rectangular tank which has a horizontal base of length 50 cm and breadth 40 cm. The depth of the tank is 45 cm.

A solid metal cone of volume 8000 cm^3 rests with its base on the base of the tank. 52 litres of water is then poured into the tank.

(i) Given that the water just covers the vertex of the cone as shown in the Figure II above, calculate the depth of the water. [2]

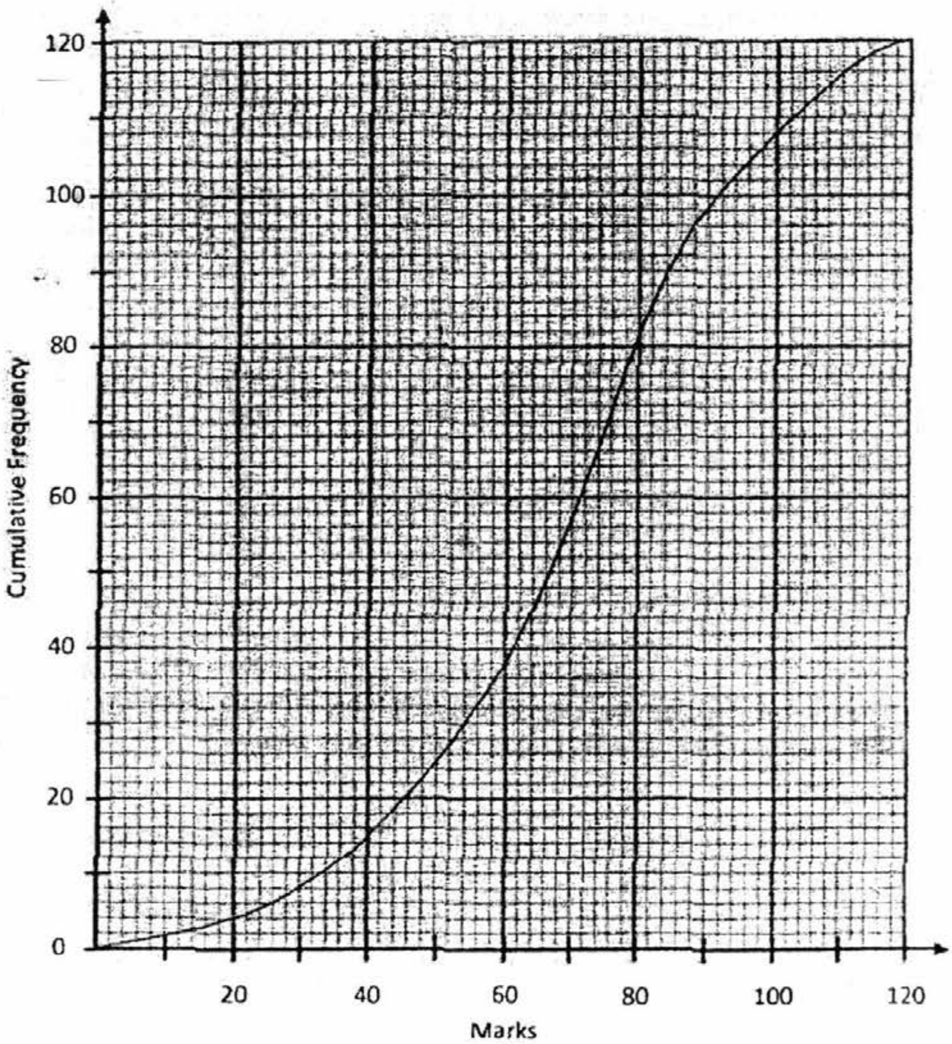
(ii) The cone is now removed from the tank. Calculate by how much the water level falls. [1]

(iii) The cone is then melted into identical smaller cones in which the radius and heights are half of the dimensions of the original cone. Find the number of smaller cones that can be obtained. [1]

(iv) The rectangular tank is now emptied of water. Cylinders of radius 2 cm and height 45 cm are then packed closely together in the tank. Calculate the maximum number of cylinders that the tank can contains. [2]

Start this section on a fresh sheet of paper.

7 The cumulative frequency curve below illustrates the marks obtained by 120 students in a Science test for Class A.



(a) Use the graph to find,

(i) the 80^{th} percentile, [1]

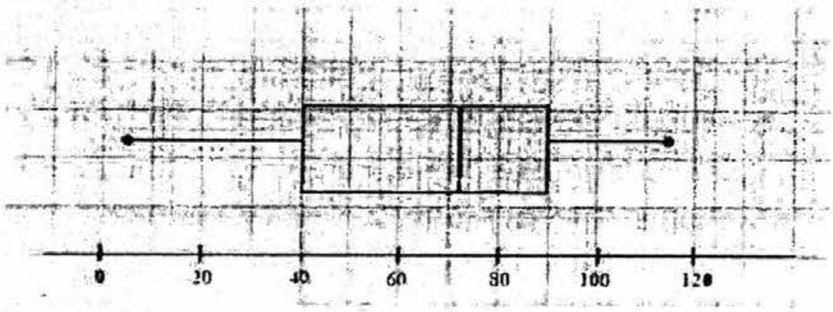
(ii) the interquartile range. [2]

(b) If only 15% of the students scored a distinction, find the least marks for a student to obtain a distinction. [1]

(c) Two students are chosen at random. Find the probability that both scored above 80 marks. [2]

(d) The box-and-whisker plot below illustrates the marks obtained by 120 students in a Science test for Class *B*.

Class *B*:



(i) How many students scored less than 40 marks in Class B? [1]

(ii) Compare the test results for the two classes and state one way in which they are different. [2]

(iii) Give one advantage of using a cumulative frequency curve compared to a box-and-whisker plot? [1]

8 (a) Janet and John compete in a chess match of 3 games.
The match ends when one of them wins two games.
In each game, the probability that John will win is 45%.
Find the probability, expressed in the same way, that John will win the chess match. [3]

(b) A container holds roses of three colours. There are 4 pink, 5 white and 6 red roses.

A customer picks 2 roses at random one after the other.

The tree diagram shows the possible outcomes and some of their probabilities.

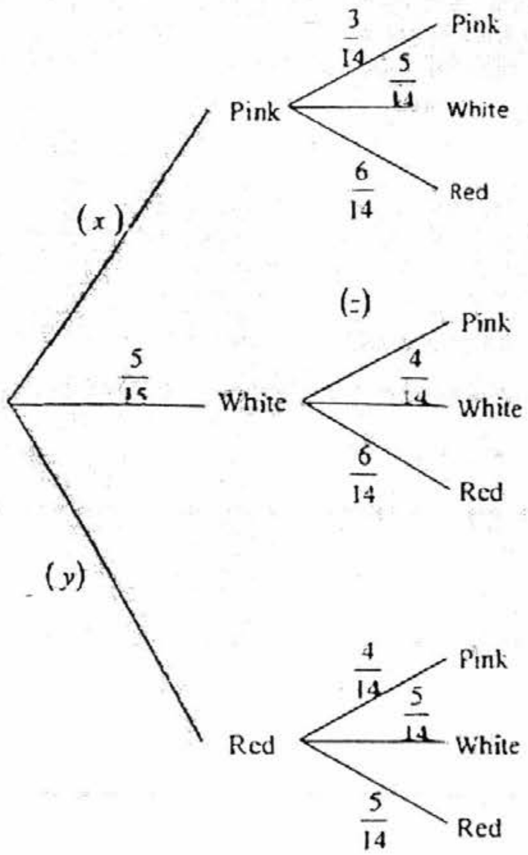
(i) State the value of x, y and z as shown in the tree diagram. [2]

Using the tree diagram, calculate the probability that

(ii) both roses are of the same colour. [2]
(iii) the second rose is white. [2]

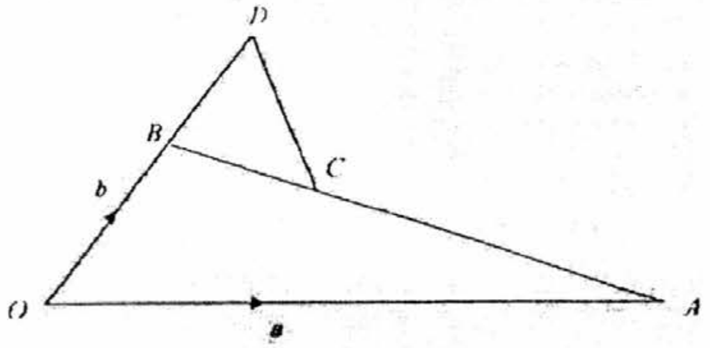
A third rose is picked.

(iv) Find the probability that none of the roses is white. [2]



3/14
Pink
5/14
White
Pink
6/14
Red
(x)
(z)
Pink
5/15
4/14
White
White
6/14
Red
(y)
4/14
Pink
5/14
Red
White
5/14
Red

9



In the diagram, OAB is a triangle. C is a point AB such that $AC : CB = 2 : 1$.

The side OB is produced to the point D such that $OB : BD = 3 : 2$.

It is given that $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

(i) Express, as simply as possible, in terms of $\boldsymbol{a}$ and /or $\boldsymbol{b}$,

(a) $\overrightarrow{AB}$ [1]

(b) $\overrightarrow{AC}$ [1]

(c) $\overrightarrow{OC}$ [1]

(d) $\overrightarrow{OD}$ [1]

(ii) Show that $\overrightarrow{CD} = \boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$. [2]

(iii) It is given that E is the point on OA such that $\overrightarrow{OE} = \frac{5}{9}\boldsymbol{a}$.

Express, as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$, the vector $\overrightarrow{ED}$. [1]

(iv) (a) Given $\overrightarrow{ED} = k\overrightarrow{CD}$, where k is a constant, find the value of k [1]

(b) Write down two facts about ED and CD. [2]

(v) Calculate the fraction $\dfrac{\text{Area of } \Delta\text{AEC}}{\text{Area of } \Delta\text{ODC}}$ [2]

10 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are related by the equation $y = x(4 - x^2)$.

The table below shows some corresponding values of x and y corrected to 1 decimal place where necessary:

x	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5
y	5.6	0	−2.6	−3	−1.9	0	1.9	3	2.6	0	−5.6

(i) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x(4 - x^2)$ for the range $-2.5 \leq x \leq 2.5$. [3]

(ii) By drawing a tangent, find the gradient of the curve when $x = 1.5$. [2]

(iii) Use your graph to find the solution of $x(4 - x^2) = 1.5$. [1]

(iv) By drawing a suitable line, use your graph to solve the equation $x^3 - x + 1 = 0$ [2]

(v) A straight line is drawn cutting the curve in three points
Two of these points are (0, 0) and (h, k).
Write down in terms of h and/or k, the coordinates of the third point. [2]

11 Paper is the most common type of waste in Singapore. Recycling paper conserves forest resources and produces fewer pollutants than conventional pulping and bleaching processes.

John is the Green Ambassador of his class. He wants to present to his classmates the number of trees that are saved each year when recycled paper is used instead of paper made from pulp. For his presentation, he would focus only on secondary school students in Singapore.

It is estimated that there are 200,000 secondary school students in Singapore. John estimates that each student uses 3 reams of paper per year. Looking at the packaging of the printing paper, he sees "*80 g/m²*" (80 grams per square metre) printed on it. Each ream of paper contains 500 sheets of printing paper.

He learns from the National Environment Agency that it takes 17 mature trees to make one tonne (1000kg) of printing paper.

John looked up the dimensions of an A4 paper. He found it to be 210 mm by 297 mm.

(i) Calculate the area of an A4 paper and show that it is approximately 0.0625 m^2. [1]

(ii) Taking the area of an A4 paper to be 0.0625 m^2, calculate the mass in grams of one sheet of A4 paper. [1]

(iii) Calculate the mass, in kg, of one ream of paper. (Exclude the mass of packaging.) [2]

(iv) John is now ready to find out how many trees are saved each year when every secondary school student uses recycled paper. Find out the answer for him. [2]

(v) If schools uses a lighter type of paper (*70 g/m²* instead of *80 g/m²*) how many more trees can be saved each year. Give your answer to the nearest hundred. [2]

END OF PAPER

Answer Key:

Q1a(i)	$4((2r+s)(2r-s)$
(ii)	$2(xy+4z)(2xy-z)$
b	$y(3x+2)$ $5(x-1)^2$
c	$5\frac{1}{2} < x \le 25$
d	$d = \frac{2S - 2na}{n(n-1)}$ $d = 3$
2 (i)	$B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$
(ii)	$C = \begin{pmatrix} 23 & 30 \\ 37 & 44 \end{pmatrix}$ The elements of C represent the number of employees for the *counter and kitchen*, for *weekdays and weekends*
(iii)	$W = \begin{pmatrix} 3250 \\ 4930 \end{pmatrix}$ The elements of W represents the total wage for weekdays, which is $3250 and the total wages for weekend , which is $4930
(iv)	$T = (8180)$ The elements of T represents the total wages for the employees for a week, which is $8180
3(a) (i)	12
(ii)	$357 - xy$
(iii)	$x = 17,\ y = 21$

5a(i)	19.5 m
(ii)	$7.88 m^2$
(iii)	$47.0\ m^2$
(b)	9 seconds.
6(i)	30 cm.
(ii)	4 cm
(iii)	8
(iv)	120
7a(i)	88 marks
(ii)	50 marks
b	93
c	703/7104
d(i)	30
(ii)	On the average, the two classes obtained similar results, but the spread of marks in Class B is Wider than that of Class A as Class B has larger interquartile range.
Q8a	0.42525
b(i)	x = 4/15, y = 6/15, z = 4/14
(ii)	31/105
(iii)	1/3
(iv)	24/91
Q9 (i)(a)	$b - a$
(b)	$\frac{2}{3}(b-a)$
(c)	$\frac{1}{3}a + \frac{2}{3}b$
(d)	$\frac{5}{3}b$
(ii)	$b - \frac{1}{3}a$
(iii)	$-\frac{5}{9}a + \frac{5}{3}b$
(iv)(a)	$k = \frac{5}{3}$

			(b)	ED is parallel to CD D is common point. Therefore E, C and D are collinear. $ED = \frac{5}{3}CD$
(b)(i)(a)	$A = \{6,9,12\}$		(v)	$\frac{8}{15}$
(b)	$A \cap B' = \{9\}$		11(ii)	−2.75.
(ii)	$n(A \cup B') = 8$		(iii)	$x = -2.4$
4a(ii)	19.8°		(iv)	$x = -1.35.$
(iii)	081.8°		(v)	$(-h, -k)$
b(i)	92°		12(i)	0.0625 m^2
(ii)	52°		(ii)	5g
(iii)	26°		(iii)	2.5.kg
(iv)	62°		(iv)	25500
			(v)	3200

1 (a) Factorise completely (i) $16r^2 - 4s^2$ [2]

(ii) $4x^2y^2 + 14xyz - 8z^2$ [2]

(b) Express $\frac{3y}{5(x-1)} + \frac{y}{(x-1)^2}$ as a single fraction in its simplest form. [2]

(c) Solve the inequality $\frac{x-9}{2} \le \frac{x-1}{3} < \frac{2x-5}{4}$ [2]

(d) Given $S = \frac{n}{2}[2a + (n-1)d]$, make d the subject of the formula.
Hence find the value of d if $S = -495$, $n = 18$ and $a = -2$. [2]

1(a) (i) $16r^2 - 4s^2 = 4(4r^2 - s^2)$
$= 4[(2r)^2 - s^2]$
$= 4((2r+s)(2r-s)$

(ii) $4x^2y^2 + 14xyz - 8z^2 = (2xy - z)(2xy + 8z)$
$= 2(xy + 4z)(2xy - z)$

(b) $\frac{3y}{5(x-1)} + \frac{y}{(x-1)^2} = \frac{3y(x-1) + 5y}{5(x-1)^2}$

$= \frac{3xy - 3y + 5y}{5(x-1)^2}$

$= \frac{3xy + 2y}{5(x-1)^2}$

$= \frac{y(3x+2)}{5(x-1)^2}$

(c) $\frac{x-9}{2} \le \frac{x-1}{3} < \frac{2x-5}{4}$

$6(x-9) \le 4(x-1) < 3(2x-5)$

$6x - 54 \le 4x - 4$

$2x \le 50$

$x \le 25$

$4x - 4 < 6x - 15$

$-2x < -11$

$x > 5\frac{1}{2}$

Ans: $5\frac{1}{2} < x \le 25$

(d) $S = \frac{n}{2}[2a + (n-1)d]$

$$2S = 2na + n(n-1)d$$

$$d = \frac{2S - 2na}{n(n-1)}$$

$S = -495$, $n = 18$ and $a = -2$.

$$d = \frac{2(-495) - 2(18)(-2)}{18(17)}$$

$$= \frac{-990 + 72}{18(17)}$$

$$= 3$$

2. . Peter runs a stall in a food court. From Monday to Friday, for the morning shift, he employs 3 persons for the counter and 4 persons for the kitchen.

For the afternoon shift, he employs 5 persons for the counter and 6 persons for the kitchen.

The information can be represented by the matrix $A = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$.

On Saturday and Sunday, for the morning shift, he employs 4 persons for the counter and 5 persons for the kitchen.

For the afternoon shift, he employs 6 persons for the counter and 7 persons for the kitchen.

(i) Represent the information for the staff employed on Saturday and Sunday by the matrix B, using the same arrangement as matrix A. [1]

(ii) Evaluate $C = 5A + 2B$. What do the elements of C represent? [2]

(iii) The wages per session for each counter staff is $50 and that for each kitchen staff is $70.

Find matrix W, where $W = C\begin{pmatrix} 50 \\ 70 \end{pmatrix}$. What do the elements W represent? [2]

(iv) Evaluate the matrix T, where $T = \begin{pmatrix} 1 & 1 \end{pmatrix} W$.
What do the elements of T represent? [2]

2(i) $B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$.

(ii) $C = 5\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} + 2\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$

$= \begin{pmatrix} 15 & 20 \\ 25 & 30 \end{pmatrix} + \begin{pmatrix} 8 & 10 \\ 12 & 14 \end{pmatrix}$

$= \begin{pmatrix} 23 & 30 \\ 37 & 44 \end{pmatrix}$

The elements of C represent the total number of employees for the counter and kitchen, for morning shift and afternoon shift, for weekdays and weekends.

(iii) $W = C\begin{pmatrix} 50 \\ 70 \end{pmatrix} = \begin{pmatrix} 23 & 30 \\ 37 & 44 \end{pmatrix}\begin{pmatrix} 50 \\ 70 \end{pmatrix} = \begin{pmatrix} 3250 \\ 4930 \end{pmatrix}$

The elements of W represents the total wage for weekdays, which is $3250 and the total wages for morning shift and $4930 for afternoon shifts.

(iv) $T = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 3250 \\ 4930 \end{pmatrix} = (8180)$

The elements of T represents the total wages for the employees for a week, which is $8180.

3 (a) A gardener has 357 tulip bulbs to plant.

(i) If she planted a rectangle of 15 rows with 23 bulbs in each row, how many bulbs would be left over? [1]

(ii) If she plants x rows with y bulbs in each row, write down an expression for the number of bulbs left over. [1]

(iii) If $10 < x < 20$, and $y > 20$, find the value of x and of y such that no bulbs are left over. [2]

(b) Given $\varepsilon = \{x : x \in Z, 3 < x < 13\}$

$A = \{\text{multiples of } 3\}$
$B = \{4,\ 6,\ 12\}$

(i) List the elements of (a) A. [1]
(b) $A \cap B'$ [1]

(ii) One of the statements in the table below is false.
Identify the false statement and give correct answer. [2]

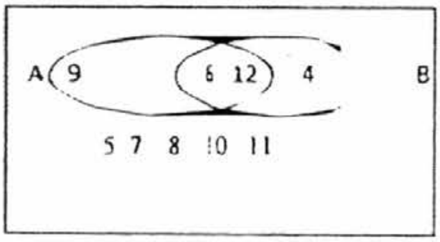
$n(A' \cap B) = 1$
$n(A \cup B) = 4$
$n(A \cup B') = 6$

---

3 (a) (i) $15 \times 23 = 345$.
Leftover $= 357 - 345 = 12$.
(ii) Leftover $= 357 - xy$
(iii) $357 = 3 \times 7 \times 17$

Since $10 < x < 20$, and $y > 20$,
$x = 17$,
$y = 21$.

(b) (i) (a) $A = \{6,9,12\}$
(b) $B' = \{5,7,8,9,10,11\}$
$A \cap B' = \{9\}$



$n(A' \cap B) = 1$ True
$n(A \cup B) = 4$ True
$n(A \cup B') = 6$ False

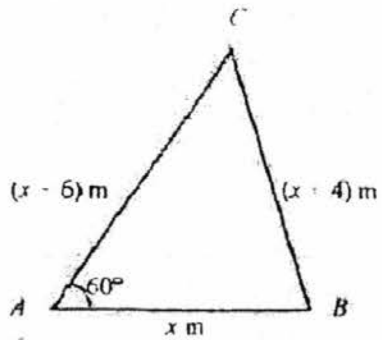
$n(A \cup B') = 8$

4 (a) The diagram below shows a triangle ABC with $AB = x$ m, $AC = (x + 6)$ m, $BC = (x + 4)$ m, and $\angle BAC = 60°$.

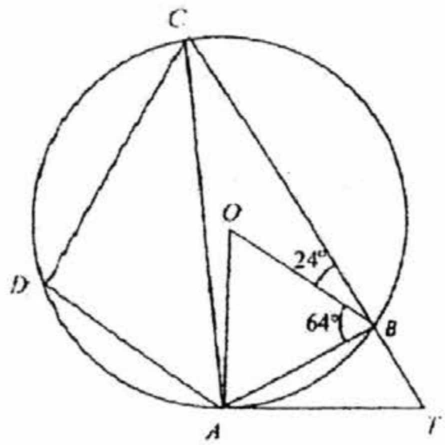
(i) Show that $x = 10$ [2]

(ii) A pole 5 m tall is erected at point C. What is the greatest angle of elevation of the top of the pole from a man walking from A to B. [2]

(iii) If B is due North of A, find the bearing of B from C. [2]



(b)



The diagram above shows a circle, centre O, and passing through the points A, B, C and D. AT is the tangent to the circle at A and CBT is a straight line. Given that $\angle ABO = 64°$ and $\angle CBO = 24°$, calculate the following, stating your reasons clearly.

(i) $\angle ADC$, [1]

(ii) $\angle AOB$, [1]

(iii) $\angle ACB$, [1]

(iv) $\angle ATB$. [2]

4(a) (i) $(x + 4)^2 = (x + 6)^2 + x^2 - 2x(x + 6)\cos 60°$

$x^2 + 8x + 16 = x^2 + 12x + 36 + x^2 - 2x(x + 6)(\frac{1}{2})$

$x^2 + 8x + 16 = 2x^2 + 12x - 6x + 36$

$x^2 + 8x + 16 = x^2 + 12x - 6x + 36$

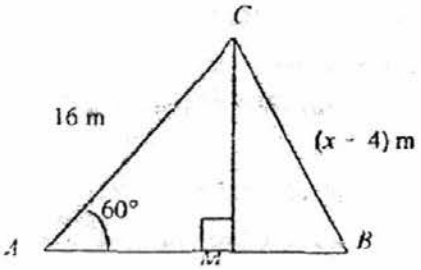
$2x = 20$

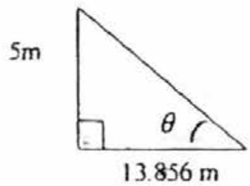
$x = 10$. (Shown.)

(ii) $\sin 60° = \frac{CM}{16}$

$CM = 16\sin 60°$

$= 13.856$ m





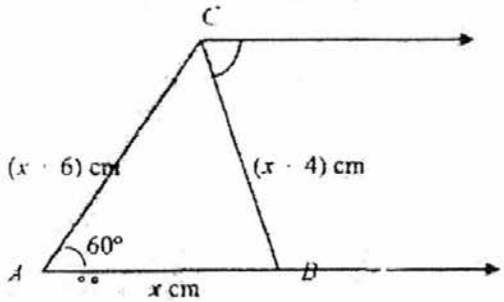
$\tan \theta = \frac{5}{13.856}$

$\theta = 19.84°$

$= 19.8°$ *(to 1 dec place)*

Greatest angle of elevation is 19.8°

(iii)



$\frac{\sin \angle ABC}{16} = \frac{\sin 60°}{14}$

$\sin \angle ABC = \frac{16\sin 60°}{14}$

$\angle ABC = \frac{16\sin 60°}{14}$

$\angle ABC = 81.8°$

$\alpha = 81.8°$ (alternate angles)

Bearing of B from C is 081.8°.

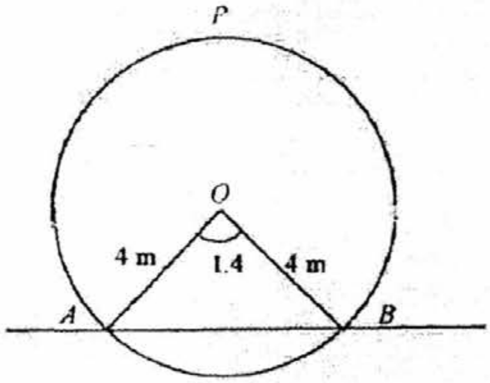
(b) (i) $\angle ADC = 180° - (24° + 64°)$ (sum of angles in opp segments)
$= 92°$

(ii) $\angle OAB = 64°$ (base angles of isosceles triangle)
$\angle AOB = 180° - 2(64)°$ (angle sum of triangle)
$= 52°$

(iii) $\angle ACB = \frac{1}{2}(52°)$ (angle at centre = 2 angle at circumference)
$= 26°$

(iv) $\angle ABT = 92°$ (ext angle of cyclic quad)
$\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle BAT = 90° - 64° = 26°$
$\angle ATB = 180° - 26° - 92°$ (angle sum of triangle)
$= 62°$

5



The above diagram shows part of a circle with centre O and radius 4 m. AOB is a sector of the circle with $\angle AOB = 1.4$ radians.
Calculate

(i) the length of the arc APB, [1]

(ii) the area of the triangle AOB, [1]

(iii) the area of the segment APB. [3]

If the segment APB represents the cross section of a railway tunnel which is 382 m long, calculate the time it will take a train 118 m long to pass completely through the tunnel if it travels at a speed of 200 km/h. Give your answer in seconds. [2]

5(a)(i) length of the arc APB

$= 4\,(2\pi - 1.4)$

$= 19.536$ m

$= 19.5$ m. (to 3 sig.fig.)

(ii) area of the triangle AOB

$= \frac{1}{2}(4)^2(\sin 1.4) = 7.8839784 = 7.88\text{m}^2$ (to 3 sig.fig.)

(iii) area of the segment APB

= area of circle − (area of sector AOB − area of triangle AOB)

$= \pi(4)^2 - \frac{1}{2}(4)^2(1.4) + 7.884$

$= 50.272 - 11.2 + 7.884$

$= 46.956$

$= 47.0\ \text{m}^2$ (to 3 sig fig.)

(b) Total length that train must travel

$= 382 + 118$

$= 500$ m

Speed of train $= \frac{200000}{60 \times 60}$

$= \frac{2000}{36}$ m/s

Time taken $= \frac{500}{2000} \times 36$

$= 9$ seconds.

6

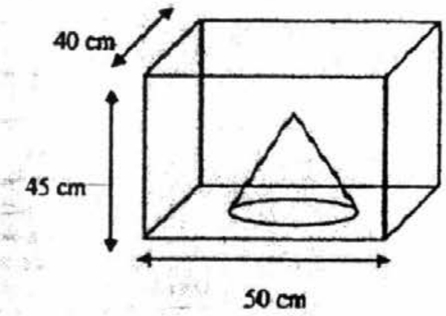


Figure I

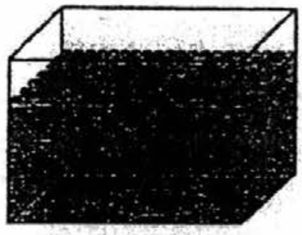


Figure II

Figure I shows an open rectangular tank which has a horizontal base of length 50 cm and breadth 40 cm. The depth of the tank is 45 cm.
A solid metal cone of volume 8000 cm^3 rests with its base on the base of the tank.
52 litres of water is then poured into the tank.

(i) Given that the water just covers the vertex of the cone as shown in the Figure II above, calculate the depth of the water. [2]

(ii) The cone is now removed from the tank. Calculate by how much the water level falls. [1]

(iii) The cone is then melted into identical smaller cones in which the radius and heights are half of the dimensions of the original cone.
Find the number of smaller cones that can be obtained. [1]

(iv) The rectangular tank is now emptied of water. Cylinders of radius 2 cm and height 45 cm are then packed closely together in the tank.
Calculate the maximum number of cylinders that the tank can contains. [2]

(i) Volume of water + cone = 52,000 + 8,000 = 60,000 cm^3.

Area of base of tank = $50 \times 40 = 2000$ cm^2.

Depth of water = $\frac{60000}{2000} = 30$ cm.

(ii) Height of fall of water level = $\frac{8000}{2000} = 4$ cm.

(iii) $\frac{\text{Volume of smaller cone}}{\text{Volume of cone}} = (\frac{1}{2})^3 = \frac{1}{8}$

Volume of each smaller cones = 1000
Number of smaller cones = 8.

(iv) Height of cylinder = 45 cm.

Diameter of cylinder = 4 cm.

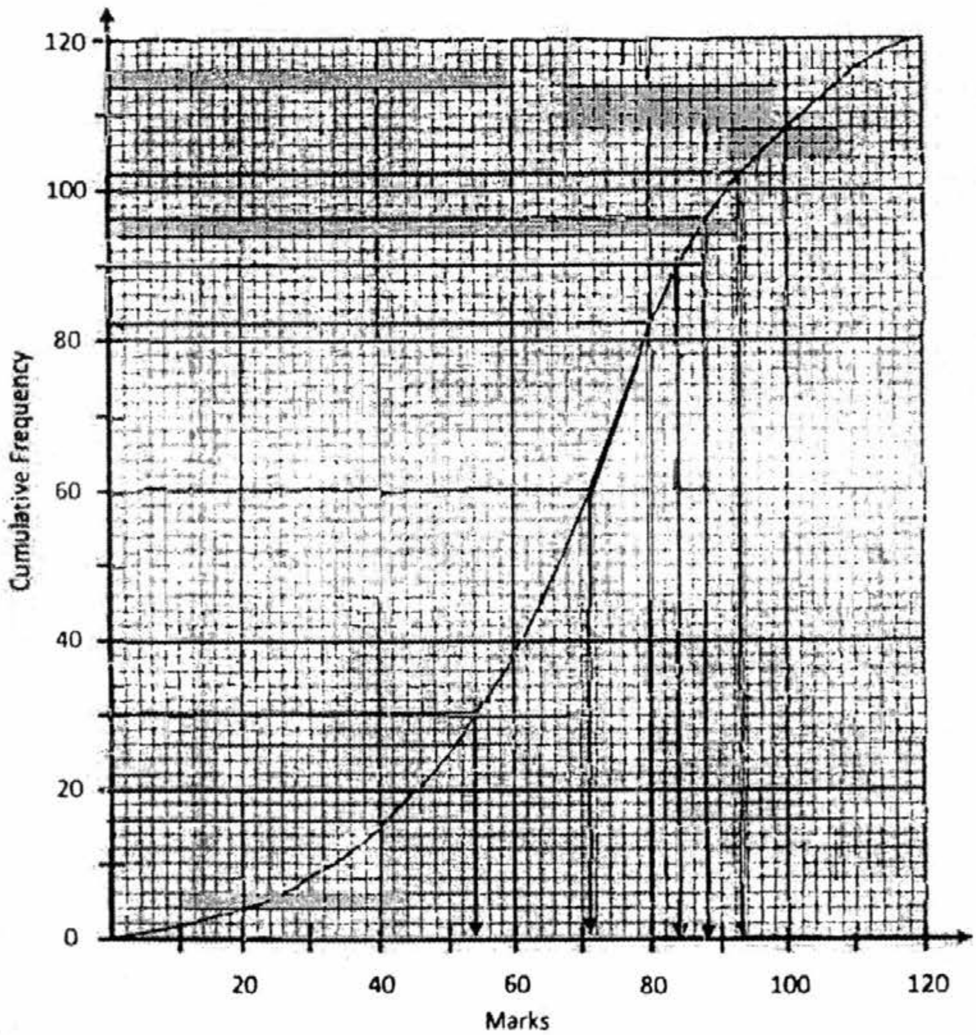
Cylinders are placed vertically.

Number of cylinders that tank can take $\leq (40 \div 4) \times (50 \div 4)$
$= 10 \times 12.5$

Max number if cylinders = 120.

Start this section on a fresh sheet of paper.

7 The cumulative frequency curve below illustrates the marks obtained by 120 students in a Science test for Class A.



Use the graph to find

(a) (i) the 80^{th} percentile. [1]

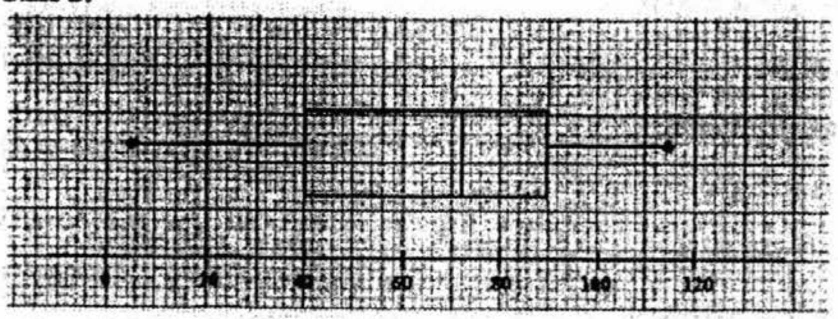
(ii) the interquartile range. [2]

(b) If only 15% of the students scored a distinction, find the least marks for a student to obtain a distinction. [1]

(c) Two students are chosen at random. Find the probability that both scored above 80 marks. [2]

(d) The box-and-whisker diagram below illustrates the marks obtained by 120 students in a Science test for Class *B*.

Class *B*:



(i) How many students scored less than 40 marks in Class B? [1]

(ii) Compare the test results for the two classes and state one way in which they are different. [2]

(iii) Give one advantage of using a cumulative frequency curve compared to a box-and-whisker plot? [1]

(a) (i) 88 marks.

(ii) Lower Quartile = 54, Upper Quartile = 84
Interquartile Range = 30 marks.

(b) 15% of 120 = 18.
Number who did not get Distinction = 120 - 18 = 102.
Least mark = 93.

(c) Number scoring 80 marks = 38.

$$\text{Probability} = \frac{38}{120} \times \frac{37}{119}$$

$$= \frac{703}{7140}$$

(d) (i) 30 students. B1

(ii) Class A has median of 71 marks and interquartile range of 30 marks.
Class B has median of 72 marks and interquartile range of 50 marks.
On the average, the two classes obtained similar results, but the spread of marks in Class B is Wider than that of Class A as Class B has larger interquartile range.

(iii) A cumulative frequency curve shows original data and is useful for deciding on cut-off points.
A box-and-whisker plot summarizes the data into 5 data points and hence original data is lost.

8 (a) Janet and John compete in a chess match of up to 3 games.
The match ends when one of them wins two games.
In each game, the probability that John will win is 45%.
Find the probability, expressed in the same way, that John will win the chess match. [3]

(b) A container holds roses of three colours. There are 4 pink, 5 white and 6 red roses.

A customer picks 2 roses at random one after the other.

The tree diagram shows the possible outcomes and some of their probabilities.

(i) State the values of x, y and z as shown in the tree diagram. [2]

Find the probability that

(ii) both roses are of the same colour. [2]

(iii) the second rose is white. [2]

A third rose is picked.

(iv) Find the probability that none of the roses is white. [2]

(a) p(John wins match) = p(win 1st , win 2nd)
+ p(win 1st, not win 2nd, win 3rd) + p(not win 1st, win 2nd, win 3rd)

$= (0.45 \times 0.45) + (0.45 \times 0.55 \times 0.45) + (0.55 \times 0.45 \times 0.45)$

$= 0.2025 + 0.111375 + 0.111375$

$= 0.42525$

Probability of John winning match = 42.525%.

(b) (i) $x = \frac{4}{15}$, $y = \frac{6}{15}$, $z = \frac{4}{14}$

(ii) p(same colour) = p(pink, pink) + p(white , white) + p(red, red)

$= (\frac{4}{15} \times \frac{3}{14}) + (\frac{5}{15} \times \frac{4}{14}) + (\frac{6}{15} \times \frac{5}{14})$

$= \frac{31}{105}$

(iii)p(2nd is white) = p(1st any colour, 2nd white)

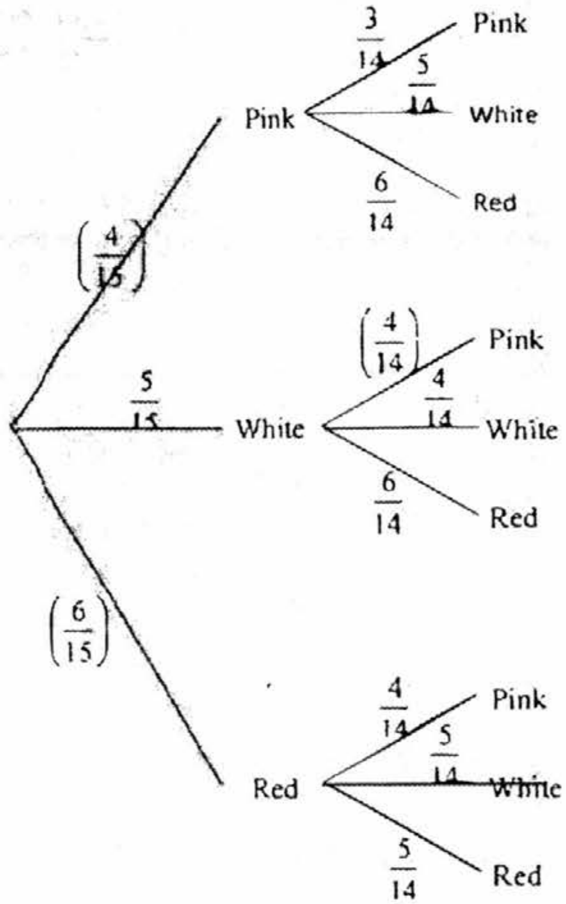
$= (\frac{4}{15} \times \frac{5}{14}) + (\frac{5}{15} \times \frac{4}{14}) + (\frac{6}{15} \times \frac{5}{14})$

$= \frac{1}{3}$

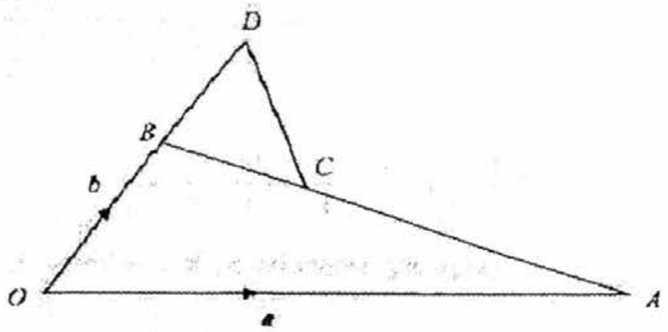
(iv) p(none white) = p(not white, not white, not white)

$= \frac{10}{15} \times \frac{9}{14} \times \frac{8}{13}$ M1

$= \frac{24}{91}$ A1



9



In the diagram, OAB is a triangle. C is a point on AB such that $AC : CB = 2 : 1$.

The side OB is produced to the point D such that $OB : BD = 3 : 2$.

It is given that $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

(i) Express, as simply as possible, in terms of $\boldsymbol{a}$ and /or $\boldsymbol{b}$,

(a) $\overrightarrow{AB}$ [1]

(b) $\overrightarrow{AC}$ [1]

(c) $\overrightarrow{OC}$ [1]

(d) $\overrightarrow{OD}$ [1]

(ii) Show that $\overrightarrow{CD} = \boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$. [2]

(iii) It is given that E is the point on OA such that $\overrightarrow{OE} = \frac{5}{9}\boldsymbol{a}$.

Express, as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$, the vector $\overrightarrow{ED}$. [1]

(iv) (a) Given $\overrightarrow{ED} = k\overrightarrow{CD}$, where k is a constant, find the value of k. [1]

(b) What two facts can be deduced about E, C and D. [2]

(v) Calculate the fraction $\dfrac{\text{Area of } \Delta AEC}{\text{Area of } \Delta ODC}$ [2]

9

(i) (a) $\overline{AB} = b - a$

(b) $\overline{AC} = \frac{2}{3}(b - a)$

(c) $\overline{OC} = \overline{OA} + \overline{AC}$

$= a + \frac{2}{3}(b - a)$

$= \frac{1}{3}a + \frac{2}{3}b$

(d) $\overline{OD} = \frac{5}{3}b$

(ii) $\overline{CD} = \overline{CB} + \overline{BD}$

$= \frac{1}{3}(b - a) + \frac{2}{3}b$

$= \frac{1}{3}b - \frac{1}{3}a + \frac{2}{3}b$

$= b - \frac{1}{3}a$

(iii) $\overline{ED} = \overline{EO} + \overline{OD}$

$= -\frac{5}{9}a + \frac{5}{3}b$

(iv) (a) $-\frac{5}{9}a + \frac{5}{3}b = k(b - \frac{1}{3}a)$

$\frac{5}{3}(b - \frac{1}{3}a) = k(b - \frac{1}{3}a)$

$k = \frac{5}{3}$

(b) $\overline{ED} = \frac{5}{3}\overline{CD}$

$\Rightarrow$ ED is parallel to CD.

Since D is the common point, E, C, and D are collinear points.

Also, ED = $\frac{5}{3}$ CD

(e) $\frac{\text{Area of triangle AEC}}{\text{Area of traingle ODC}}$

$= \frac{\text{Area of triangle AEC}}{\text{Area of traingle OEC}} \times \frac{\text{Area of triangle OEC}}{\text{Area of triangle ODC}}$

$= \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

10 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are related by the equation $y = x(4 - x^2)$.

The table below shows some corresponding values of x and y corrected to 1 decimal place where necessary:

x	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5
y	5.6	0	−2.6	−3	−1.9	0	1.9	3	2.6	0	−5.6

(i) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x(4 - x^2)$ for the range $-2.5 \leq x \leq 2.5$. [3]

(ii) By drawing a tangent, find the gradient of the curve when $x = 1.5$. [2]

(iii) Use your graph to find the solution of $\frac{1}{4}x(4 - x^2) = 1$. [1]

(iv) By drawing a suitable line, use your graph to solve the equation $x^3 - x + 1 = 0$ [2]

(v) A straight line is drawn cutting the curve in three points.
Two of these points are $(0, 0)$ and (h, k).
Write down in terms of h and/or k, the coordinates of the third point. [2]

(i) Graph.

(ii)

Gradient $= \frac{6.8-(-2)}{0-3.2} = \frac{8.8}{3.2} = -2.75$.

(iii) $\frac{1}{4}x(4-x^2) = 1$

$x(4-x^2) = 4$

Draw line $y = 4$.

From graph, $x = -2.4$

(iv) $x^3 - x + 1 = 0$

$-x^3 + x + 3x - 1 = 3x$

$-x^3 + 4x = 3x + 1$

$x(4 - x^2) = 3x + 1$

Draw line $y = 3x + 1$

From graph, $x = -1.35$

(v) Draw line.

$(-h, -k)$

11 Paper is the most common type of waste in Singapore. Recycling paper conserves forest resources and produces fewer pollutants than conventional pulping and bleaching processes.

John is the Green Ambassador of his class. He wants to present to his classmates the number of trees that are saved each year when recycled paper is used instead of paper made from pulp. For his presentation, he would focus only on secondary school students in Singapore.

It is estimated that there are 200,000 secondary school students in Singapore. John estimates that each student uses 3 reams of paper per year. Looking at the packaging of the printing paper, he sees "$80\ g/m^2$" (80 grams per square metre) printed on it. Each ream of paper contains 500 sheets of printing paper.

He learns from the National Environment Agency that it takes 17 mature trees to make one tonne (1000kg) of printing paper.

John looked up the dimensions of an A4 paper. He found it to be 210 mm by 297 mm.

(i) Calculate the area of an A4 paper and show that it is approximately $0.0625\ m^2$.

[1]

(ii) Taking the area of an A4 paper to be $0.0625\ m^2$, calculate the mass in grams of one sheet of A4 paper. [1]

(iii) Calculate the mass, in kg, of one ream of paper. (Exclude the mass of packaging.)

[2]

(iv) John is now ready to find out how many trees are saved each year when every secondary school student uses recycled paper. Find out the answer for him.

[2]

(v) If schools uses a lighter type of paper ($70\ g/m^2$ instead of $80\ g/m^2$) how many more trees can be saved each year. Give your answer to the nearest hundred.

[2]

(i) Area = $210 \times 297 \times 10^{-6} = 62370 \times 10^{-6} \approx 0.0625 \text{ m}^2$.

(ii) Mass of one sheet of A4 paper $= 0.0625 \times 80$

$= 5\text{g}$

(iii) Mass of one ream of paper $= 5 \times 500$

$= 2500 \text{ g}$

$= 2.5 \text{ kg}$

(iv) Amount of paper used $= 3 \times 200{,}000 \times 2.5 = 1.5 \times 10^6 \text{ kg}$

To make 1000 kg of printing paper, it takes 17 mature trees

$$\text{Number of trees saved} = \frac{1.5 \times 10^6}{1000} \times 17$$

$$= 25500$$

(v)

$$\text{Reduction in weight} = \frac{10}{80}$$

$$\text{Reduction in number of trees} = 25500 \times \frac{10}{80} = 3187.5 \approx 3200 \text{ (to nearest hundred.)}$$

Alternative:

Mass of 1 ream of paper $= 0.0625 \times 500 \times 70 = 2187.5 = 2.1875$ kg

Amount of paper used $= 3 \times 200{,}000 \times 2.1875 = 1{,}312{,}500$ kg

To make 1000 kg of printing paper, it takes 17 mature trees

$$\text{Number of trees saved} = \frac{1.3125 \times 10^6}{1000} \times 17$$

$$= 22312.5$$

Reduction in number of trees saved $= 25500 - 22312.5$

$= 3187.5$

≈ 3200 (to nearest hundred.)

END OF PAPER