5

Answer **all** the questions.

1 Write the following numbers in order of size, starting with the **smallest**. [1]

$-\frac{4}{7}, -\frac{4}{5}, -0.8^2, -0.\dot{8}$

2 During a children's day celebration, a charity organization distributed 825 files, 495 pens and 660 pencils equally among the children in a children's home. Each child received the same number of files, pens and pencils.

(a) Find the largest possible number of children. [2]

(b) Hence, find the number of files, pens and pencils each child received. [1]

3 It is given that $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

(a) Find f when $u = 1.2$ and $v = 0.4$. [1]

(b) Express u in terms of f and v. [2]

4 A restaurant charges \$27.80 per person for a buffet lunch. On a particular day, 114 people dined in the restaurant.
By approximating both the charge and the number of diners to 2 significant figures, **estimate** the total amount received by the restaurant on that particular day.

Show your working and give your answer to a reasonable degree of accuracy. [2]

5 A piece of metal is heated to 375°C and then left to cool for 15 minutes.
The temperature of the metal decreases at a rate of 18°C/min for the first 5 minutes and then decreases at a rate of 7°C/min for the next 10 minutes.

Find the time taken for the metal to cool to a temperature of 250°C. [2]

6 **(a)** Solve the inequality $1 - x \leq 4 + x < 13 - 2x$. [2]

(b) Write down all the integers which satisfy $1 - x \leq 4 + x < 13 - 2x$. [1]

7 The current, I amperes, passing through a circuit is inversely proportional to its resistance, R ohms. When the resistance of the circuit is 3 ohms, the current passing through it is 2 amperes.

(a) Find an equation connecting I and R. [2]

(b) Calculate the resistance of the circuit when 1.5 amperes of current passes through it. [1]

(c) Sketch the graph of I against R. [1]

8 Two containers are geometrically similar.
The surface area of the larger container is 63 cm^2 and the surface area of the smaller container is 28 cm^2.
The height of the smaller container is 5 cm.
Calculate the height of the larger container. [2]

9 Between 2014 and 2015, the number of pupils who applied for a particular school as their first choice school increased by 25%.
In 2015, the number of applicants for that school was 425.

Calculate the number of applicants in 2014. [2]

10 The probability that it will rain on any particular day is 0.3.

Calculate the probability that on two consecutive days, it will rain on only one of the days.

[2]

---

11 The table below shows the number of internet-connected devices in some households.

Number of devices	1	2	3	4	5	6
Number of households	2	4	x	7	5	3

(a) If the modal number of devices is 4, state the maximum possible value of x. [1]

(b) If the mean number of devices is 3.6, calculate the value of x. [2]

(c) If the median number of devices is 4, write down all the possible values of x. [1]

12 Peter drove from Town X to Town Z, passing by Town Y along the way.
He took 40 minutes to drive from Town X to Town Y at an average speed of 72 km/h.
He rested in Town Y for 10 minutes before continuing his journey to Town Z.
The distance between Town Y and Town Z is 52 km.
His average speed for the whole journey is 60 km/h.

Calculate

(a) the distance between Town X and Town Y, [1]

(b) the average speed for the journey between Town Y and Town Z. [3]

13 The point (1, 1) is marked on the diagram.

Sketch the graph of $y = 8 - x^3$ in the answer space below. [1]

14 David wants to invest \$500 for 3 years.
Company A offers 8% simple interest per year.
Company B offers 6% interest per year compounded quarterly.

In which company should David invest his money? Justify your answer. [3]

15 $\xi = \{x: x \text{ is an integer}, 1 \leq x \leq 100\}$

$A = \{x: x \text{ is divisible by } 11\}$

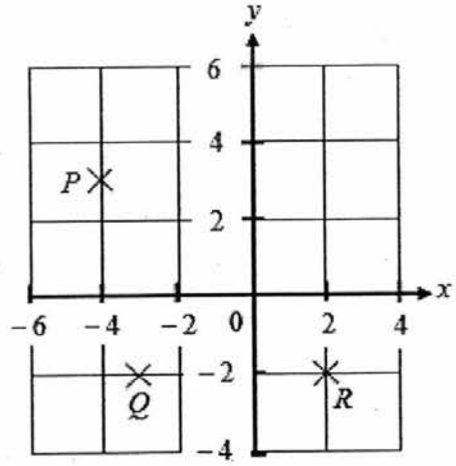
$B = \{x: x \text{ is divisible by } 22\}$

$C = \{x: x \text{ is divisible by } 33\}$

(a) List the elements of $A \cap (B \cup C)'$. [1]

(b) Draw, in the answer space, a clearly labelled Venn diagram to illustrate the three sets A, B and C.
[2]

16 On the axes shown, P is $(-4, 3)$, Q is $(-3, -2)$ and R is $(2, -2)$.



Find

(a) the gradient of PQ, [1]

(b) $\tan P\hat{R}Q$, [1]

(c) the equation of the line PR, [2]

(d) the area of triangle PQR, [1]

(e) the coordinates of two possible points S, such that the four points P, Q, R and S are the four vertices of a parallelogram. [2]

17

$T_1 T_2$ T_3 T_4

The figures T_1, T_2, T_3 ..... are made up of squares.
N is the number of rows of squares in each shape.
S is the number of squares in each shape.
D is the number of dots in each shape.
The values of N, S and D in T_1, T_2, T_3 and T_4 are recorded in the table below.

Figure	T_1	T_2	T_3	T_4
N	1	2	3	4
S	1	4	p	16
D	4	10	q	28
$D-N^2$	3	6	r	s

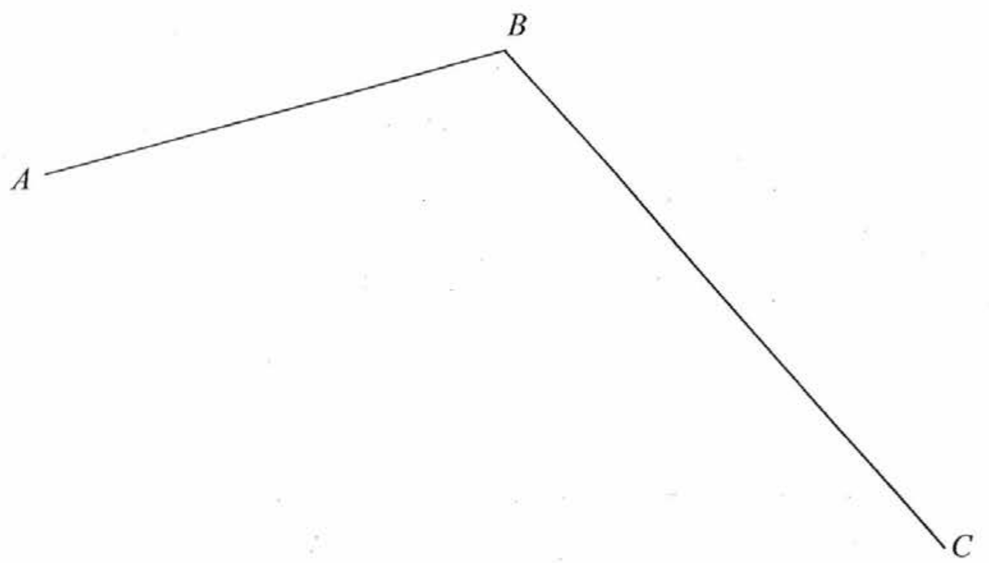
(a) Find the values of p, q, r and s. [2]

(b) Express S in terms of N. [1]

(c) Express D in terms of N. [1]

(d) Explain why the number of dots cannot be 42. [1]

18 Three points A, B and C are shown below.



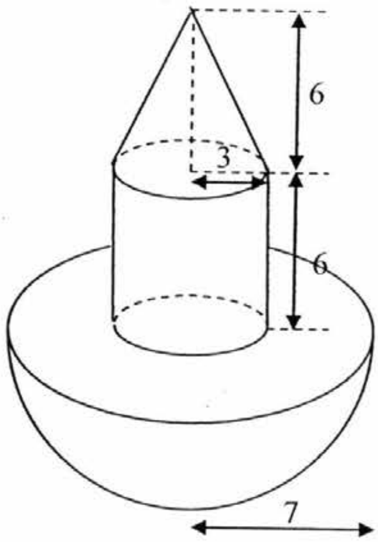
(a) Construct the perpendicular bisector of BC. [1]

(b) Construct the bisector of angle ABC. [1]

(c) Mark clearly a possible point, X, which is equidistant from the lines AB and BC, and equidistant from the points B than$C$. [1]

(d) The point D is such that $ABCD$ is a parallelogram. Find the position of D.[1]

19 A gold solid is formed by joining the plane faces of a cone, a cylinder and a hemisphere.
The cone and cylinder have a base radius of 3 cm and height 6 cm.
The hemisphere has a radius of 7 cm.



Calculate

(a) the length of the slant height of the cone, [2]

(b) the surface area of the gold solid, [3]

(c) the surface area of the gold solid. [2]

The density of gold is 19.32 g/cm^3.
A gold bar has length 25 cm, width 7 cm and height 3.5 cm.
Five gold bars were melted down and all the gold was used to make a large number of these gold solids.

(d) Calculate the mass of gold that remains after the gold solids are made, giving your answer correct to two significant figures. [4]

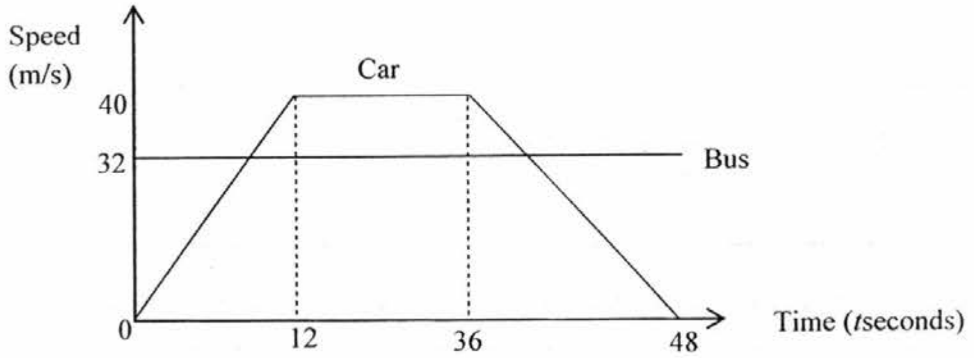
20 O is the origin. A is the point $(3, p)$. B is the point $(-4, 5)$. $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$.

(a) If $\overrightarrow{BC}$ is parallel to $\overrightarrow{OA}$, find the value of p. [2]

(b) Find the ratio $OA : BC$. [1]

(c) Find the position vector of M such that $OAMB$ is a parallelogram. [2]

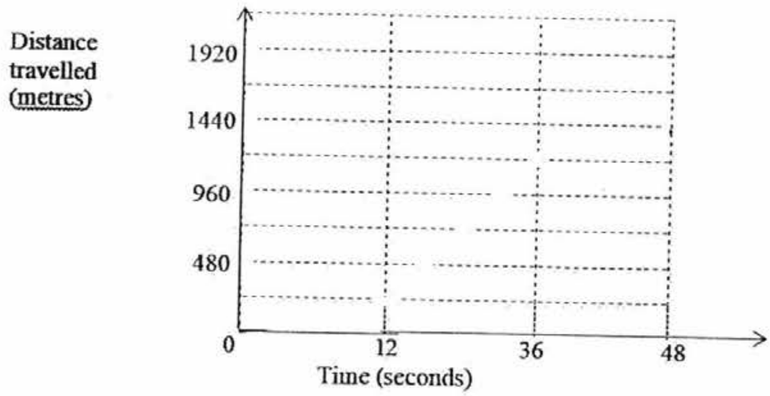
21 The diagram, not drawn to scale, shows the speed-time graph of a car and a bus during a period of 48 seconds. The car and the bus start from the same point, at the same time and travel in the same direction.



(a) Calculate the value(s) of t when the car and bus have the same speed. [3]

(b) Find the value of t when the car overtakes the bus. [3]

(c) Use the grid below to sketch the distance-time graph of the car for the same journey. [3]



Answer Scheme

Answer **all** the questions.

1 Write the following numbers in order of size, starting with the **smallest**.

$-\frac{4}{7}, -\frac{4}{5}, -0.8^2, -0.\dot{8}$

$-0.\dot{8}, -\frac{4}{5}, -0.8^2, -\frac{4}{7}$

2 During a children's day celebration, a charity organization distributed 825 files, 495 pens and 660 pencils equally among the children in a children's home. Each child received the same number of files, pens and pencils.

(a) Find the largest possible number of children.

3	825	495	660
5	275	165	220
11	55	33	44
	5	3	4

OR

$825 = 3 \times 5^2 \times 11$

$495 = 3^2 \times 5 \times 11$

$660 = 2^2 \times 3 \times 5 \times 11$

HCF = $3 \times 5 \times 11 = 165$

(b) Hence, find thenumber of files, pens and pencils each child received.

5 files, 3 pens, 4 pencils

3 It is given that $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

(a) Find f when $u = 1.2$ and $v = 0.4$.

$$\frac{1}{f} = \frac{1}{1.2} + \frac{1}{0.4}$$
$$= \frac{5}{6} + \frac{5}{2}$$
$$= \frac{20}{6}$$
$$f = 0.3$$

(b) Express u in terms of f and v.

$$\frac{1}{u} = \frac{v-f}{fv}$$
$$u(v-f) = fv$$
$$u = \frac{fv}{v-f}$$

4 A restaurant charges \$27.80 per person for a buffet lunch. On a particular day, 114 people dined in the restaurant.
By approximating both the charge and the number of diners to 2 significant figures, **estimate** the total amount received by the restaurant on that particular day.

Show your working and give your answer to a reasonable degree of accuracy.

110×28
$= 3080$
$= 3100$ (2 sf)

---

5 A piece of metal is heated to 375°C and then left to cool for 15 minutes.
The temperature of the metal decreases at a rate of 18°C/min for the first 5 minutes and then decreases at a rate of 7°C/min for the next 10 minutes.

Find the time taken for the metal to cool to a temperature of 250°C.

Decrease in temp in 1st 5 min = 18×5
$= 90$°C

Further decr. in temp req after 1st 5 min
$= 375 - 125 - 90$
$= 35$°C

$$\text{Time taken} = \frac{35}{7} + 5$$
$$= 10 \text{ min}$$

---

6 (a) Solve the inequality $1 - x \leq 4 + x < 13 - 2x$.

$1 - x \leq 4 + x$ and $4 + x < 13 - 2x$
$-3 \leq 2x$ $\quad$ $3x < 9$

$x \geq -1\frac{1}{2}$ $\quad$ $x < 3$

$\therefore -1\frac{1}{2} \leq x < 3$

(b) Write down all the integers which satisfy $1 - x \leq 4 + x < 13 - 2x$.

$-1, 0, 1, 2$

---

7 The current, I amperes, passing through a circuit is inversely proportional to its resistance, R ohms. When the resistance of the circuit is 3 ohms, the current passing through it is 2 amperes.

(a) Find an equation connecting I and R.

$$I = \frac{k}{R}$$
$$2 = \frac{k}{3}$$
$$k = 6$$

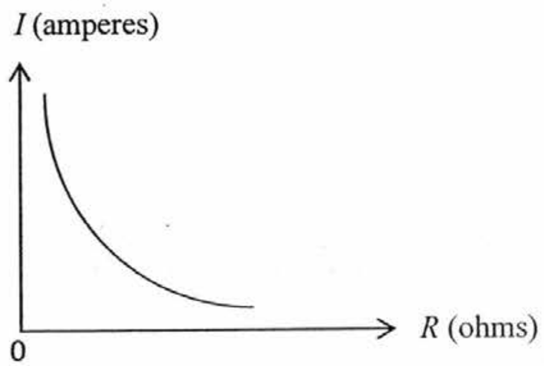
$$I = \frac{6}{R}$$

(b) Calculate the resistance of the circuit when 1.5 amperes of current passes through it.

$$1.5 = \frac{6}{R}$$
$$R = 4 \text{ ohms}$$

(c) Sketch the graph of I against R.

Answer (c)



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8 Two containers are geometrically similar.
The surface area of the larger container is 63 cm^2 and the surface area of the smaller container is 28 cm^2.
The height of the smaller container is 5 cm.
Calculate the height of the larger container.

$$\left(\frac{h_l}{h_s}\right)^2 = \frac{63}{28}$$
$$\frac{h_l}{h_s} = \sqrt{\frac{63}{28}}$$
$$\frac{h_l}{5} = \frac{3}{2}$$
$$h_l = 7.5\ cm$$

9 Between 2014 and 2015, the number of pupils who applied for a particular school as their first choice school increased by 25%.
In 2015, the number of applicants for that school was 425.

Calculate the number of applicants in 2014.

$$\frac{100}{125}\times 425$$
$$= 340$$

---

10 The probability that it will rain on any particular day is 0.3.

Calculate the probability that on two consecutive days, it will rain on only one of the days.

$$0.3\times 0.7\times 2$$
$$= 0.42$$

---

11 The table below shows the number of internet-connected devices in some households.

Number of devices	1	2	3	4	5	6
Number of households	2	4	x	7	5	3

(a) If the modal number of devices is 4, state the maximum possible value of x.

6

(b) If the mean number of devices is 3.6, calculate the value of x.

$$\frac{1(2)+2(4)+3x+4(7)+5(5)+6(3)}{2+4+x+7+5+3}=3.6$$
$$\frac{81+3x}{21+x}=3.6$$
$$81+3x=3.6(21+x)$$
$$81+3x=75.6+3.6x$$
$$5.4=0.6x$$
$$x=9$$

(c) If the median number of devices is 4, write down all the possible values of x.

0, 1,2,3……8 or $0\le x\le 8$, x is an integer

12 Peter drove from Town X to Town Z, passing by Town Y along the way.
He took 40 minutes to drive from Town X to Town Y at an average speed of 72 km/h.
He rested in Town Y for 10 minutes before continuing his journey to Town Z.
The distance between Town Y and Town Z is 52 km.
His average speed for the whole journey is 60 km/h.

Calculate

(a) the distance between Town X and Town Y,

Distance between X and Y $= 72 \times \dfrac{2}{3} = 48$ km

(b) the average speed for the journey between Town Y and Town Z.

$$\text{Total time for the whole journey} = \frac{48+52}{60}$$
$$= 1\frac{2}{3} \text{ h}$$

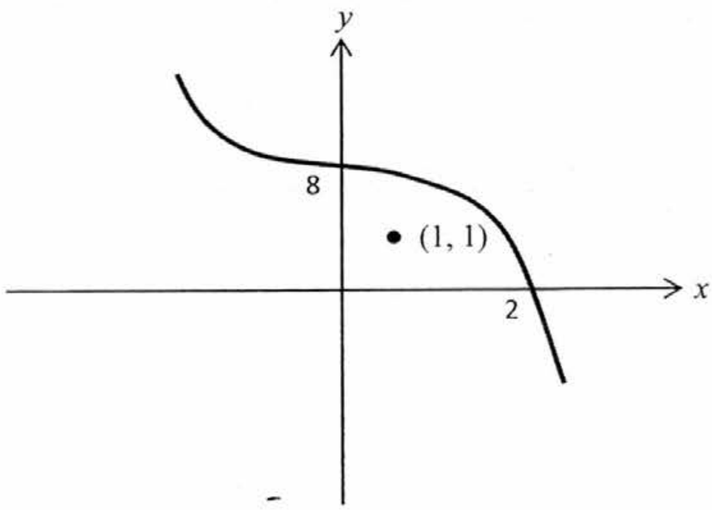
$$\text{Time taken for Y to Z} = 1\frac{2}{3} - \frac{2}{3} - \frac{1}{6}$$
$$= \frac{5}{6} \text{ h}$$

$$\text{Average speed} = 52 \div \frac{5}{6}$$
$$= 62.4 \text{ km/h}$$

13 The point (1, 1) is marked on the diagram.

Sketch the graph of $y = 8 - x^3$ in the answer space below.

Answer



14 David wants to invest \$500 for 3 years.
Company A offers 8% simple interest per year.
Company B offers 6% interest per year compounded quarterly.

In which company should David invest his money? Justify your answer.

Company A $Interest = \frac{500 \times 8 \times 3}{100}$ $= 120$		Company B $Amount = 500\left(1 + \frac{6}{4 \times 100}\right)^{12}$ $= 597.81$	
Total = 500 + 120 =620		$Interest = 597.81 - 500$ $= 97.81$	
Company A			

15 $\xi = \{x: x \text{is an integer}, 1 \leq x \leq 100\}$

$A = \{x: x \text{ is divisible by } 11\}$

$B = \{x: x \text{ is divisible by } 22\}$

$C = \{x: x \text{ is divisible by } 33\}$

(a) List the elements of $A \cap (B \cup C)'$.

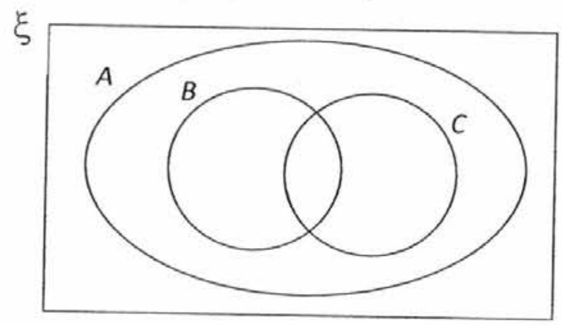
$A = \{x: x \text{ is divisible by } 11\} = \{11,22,33,44,55,66,77,88,99\}$

$B = \{x: x \text{ is divisible by } 22\} = \{22,44,66,88\}$

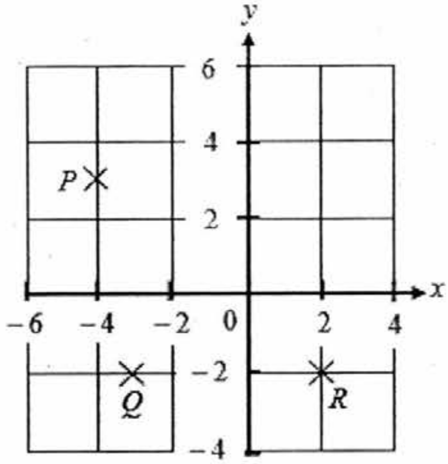
$C = \{x: x \text{ is divisible by } 33\} = \{33,66,99\}$

$A \cap (B \cup C)' = \{11,55,77\}$

(b) Draw, in the answer space, a clearly labelled Venn diagram to illustrate the three sets A, B and C.



16 On the axes shown, P is $(-4, 3)$, Q is $(-3, -2)$ and R is $(2, -2)$.



Find

(a) the gradient of PQ,

$$Gradient\ PQ = \frac{-2-3}{-3-(-4)}$$
$$= -5$$

(b) $\tan P\hat{R}Q$,

$$\tan \measuredangle PRQ = \frac{5}{6}$$

(c) the equation of the line PR,

$$Gradient\ PR = -\frac{5}{6}$$
$$-2 = \left(-\frac{5}{6}\right)(2) + c$$
$$c = -\frac{1}{3}$$
$$Equation: y = -\frac{5}{6}x - \frac{1}{3} \text{ OR}$$
$$6y = -5x - 2$$

(d) the area of triangle PQR,

$$Area\ PQR = \frac{1}{2} \times 5 \times 5$$
$$= 12.5\ square\ units$$

(e) the coordinates of two possible points S, such that the four points P, Q, R and S are the four vertices of a parallelogram.

$S(1,3)$ *or* $S(3,-7)$ *or* $S(-9,3)$

17

$T_1 T_2$ $T_3\ T_4$

The figures T_1, T_2, T_3 ….. are made up of squares.
N is the number of rows of squares in each shape.
S is the number of squares in each shape.
D is the number of dots in each shape.
The values of N, S and D in T_1, T_2, T_3 and T_4 are recorded in the table below.

Figure	T_1	T_2	T_3	T_4
N	1	2	3	4
S	1	4	p	16
D	4	10	q	28
$D-N^2$	3	6	r	s

(a) Find the values of p, q, r and s.

p = 9, q = 18, r = 9, s = 12

(b) Express S in terms of N.

$S = N^2$

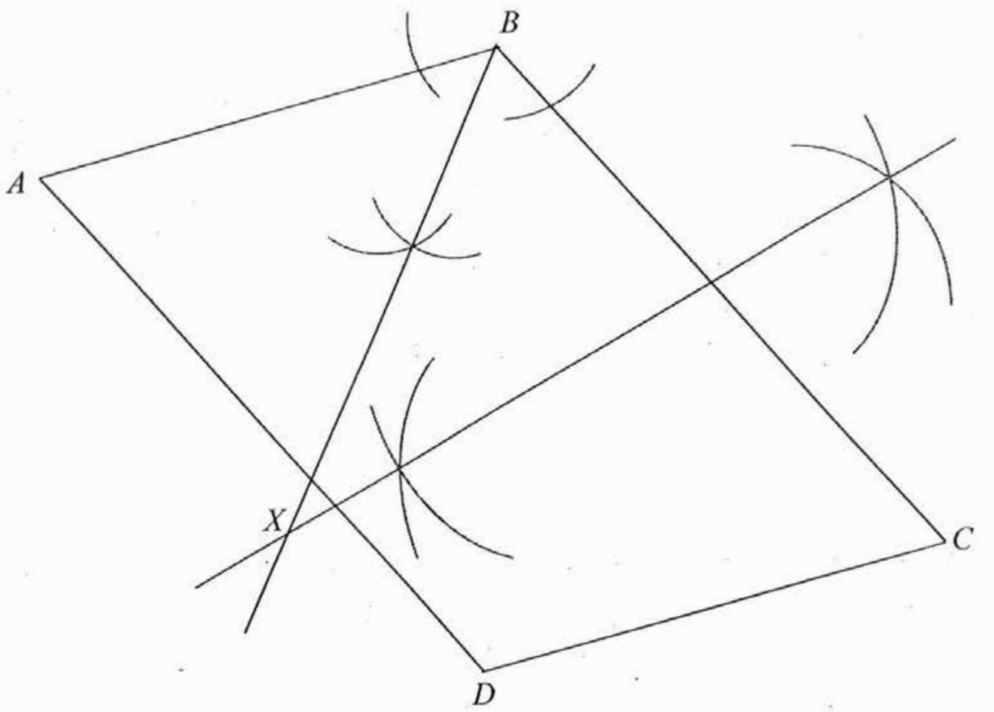
(c) Express D in terms of N.

$D = 3N + N^2$

(d) Explain why the number of dots cannot be 42.

$If\ D = 42$
$3N + N^2 = 42$
$N^2 + 3N - 42 = 0$
$N = \frac{-3\pm\sqrt{9+168}}{2}$ which is no a whole number
Hence D cannot be 42

18 Three points A, B and C are shown below.



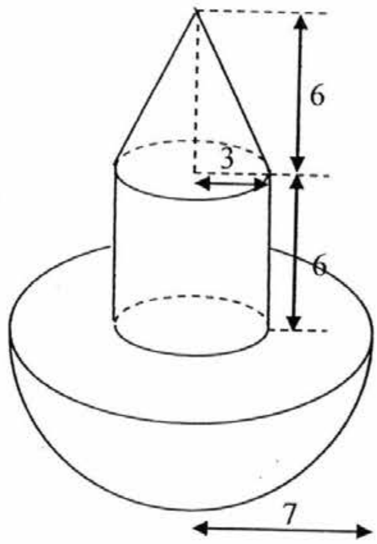
(a) Construct the perpendicular bisector of BC. [1]

(b) Construct the bisector of angle ABC. [1]

(c) Mark clearly a possible point, X, which is equidistant from the lines AB and BC, and equidistant from the points B than$C$. [1]

(d) The point D is such that $ABCD$ is a parallelogram. Find the position of D.[1]

19 A gold solid is formed by joining the plane faces of a cone, a cylinder and a hemisphere.
The cone and cylinder have a base radius of 3 cm and height 6 cm.
The hemisphere has a radius of 7 cm.



Calculate

(a) the length of the slant height of the cone,

$$\begin{aligned} Slant\ height &= \sqrt{3^2 + 6^2} \\ &= 6.708 \\ &= 6.71\ cm \end{aligned}$$

(b) the surface area of the gold solid,

$$\begin{aligned} Total\ SA &= \pi(3)(6.708) + 2\pi(3)(6) \\ &\quad + 2\pi(7)^2 + [\pi(7^2 - 3^2)] \\ &= 609.8 \\ &\approx 610\ cm^2 \end{aligned}$$

(c) the surface area of the gold solid.

$$\begin{aligned} Volume &= \left[\frac{1}{3} \times \pi(3)^2 \times 6\right] + [\pi(3)^2 \times 6] \\ &\quad + \left[\frac{2}{3} \times \pi(7)^3\right] \\ &= 944.5 \\ &\approx 945\ cm^3 \end{aligned}$$

The density of gold is 19.32 g/cm^3.
A gold bar has length 25 cm, width 7 cm and height 3.5 cm.
Five gold bars were melted down and all the gold was used to make a large number of these gold solids.

(d) Calculate the mass of gold that remains after the gold solids are made, giving your answer correct to two significant figures.

$Volume\ five\ gold\ bars$
$= 5 \times (25 \times 7 \times 3.5)$
$= 3062.5\ cm^3$
$No\ of\ figures = 3062.5 \div 944.5$
$= 3.24$
$\approx 3\ figures$
$Volume\ of\ gold\ that\ remains$
$= [3062.5 - (3 \times 944.5)]\ or$
$(3.242 - 3)(944.5)$
$= 229.0\ cm^3\ or\ 228.7cm^3$
$Mass\ of\ gold\ that\ remains$
$= 229 \times 19.32\ or\ 228.7 \times 19.32$
$= 4424.28\ or\ 4420.0$
$\approx 4400\ g$

20 O is the origin. A is the point $(3, p)$. B is the point $(-4, 5)$. $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$.

(a) If $\overrightarrow{BC}$ is parallel to $\overrightarrow{OA}$, find the value of p.

$\overrightarrow{BC} = m\,\overrightarrow{OA}$

$\begin{pmatrix} 6 \\ 5 \end{pmatrix} = \text{m} \begin{pmatrix} 3 \\ p \end{pmatrix}$

$6 = 3\text{ m}$

$\text{m} = 2$

$5 = 2p$

$p = 2.5$

(b) Find the ratio $OA : BC$.

$\overrightarrow{BC} = 2\ \overrightarrow{OA} \Rightarrow OA : BC = 1 : 2$

(c) Find the position vector of M such that $OAMB$ is a parallelogram.

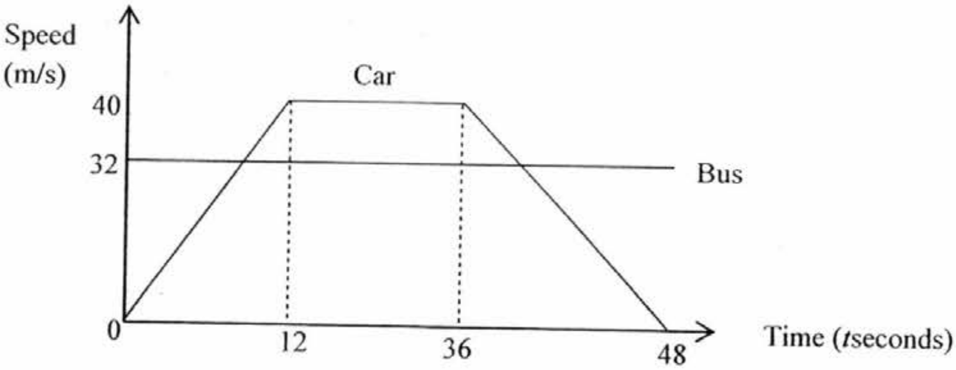
$\overrightarrow{BM} = \overrightarrow{OA}$ or $\overrightarrow{OB} = \overrightarrow{AM}$

$\overrightarrow{OM} - \begin{pmatrix} -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$ $\overrightarrow{OM} - \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$\overrightarrow{OM} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ $\overrightarrow{OM} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 7.5 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ 7.5 \end{pmatrix}$

21 The diagram, not drawn to scale, shows the speed-time graph of a car and a bus during a period of 48 seconds. The car and the bus start from the same point, at the same time and travel in the same direction.



(a) Calculate the value(s) of t when the car and bus have the same speed.

$$\frac{40}{12}=\frac{32}{t}$$

$$t=32\times\frac{12}{40}$$

$$=9\tfrac{3}{5}\quad or\quad 9.6$$

$$t_2=48-9\tfrac{3}{5}$$

$$=38\tfrac{2}{5}\quad or\quad 38.4$$

(b) Find the value of t when the car overtakes the bus.

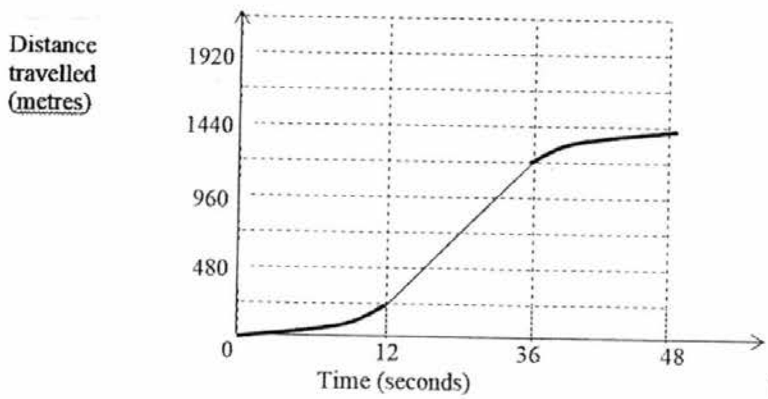
$$\frac{1}{2}(12)(40)+40(t-12)=32t$$

$$240+40t-480=32t$$

$$8t=240$$

$$t=30$$

(c) Use the grid below to sketch the distance-time graph of the car for the same journey.



[3]

1 **(a)** Express as a single fraction in its simplest form $1-\frac{2x}{2x-7}+\frac{7}{(2x-7)^2}$. [3]

(b) Simplify $5a^{-3}b^5 \div \frac{10}{9}a^3b^{-2}$. [2]

(c) **(i)** Factorise fully $11p^2-44pq+4q-p$. [2]

(ii) Factorise fully $30m^2+14mn-4n^2$ [2]

(d) Solve the equation $\frac{1}{x}-\frac{x-5}{2x-3}=1$. [3]

2 Twenty five boys took a quiz.
The marks are shown in the stem-and-leaf diagram.

1	4 7
2	3 5 7 7 9
3	0 1 2 3 3 5 7 7 8 9 9 9
4	3 4 6 6 7
5	0

Key
1 | 4 means 14 marks

(a) Find

(i) the median mark, [1]

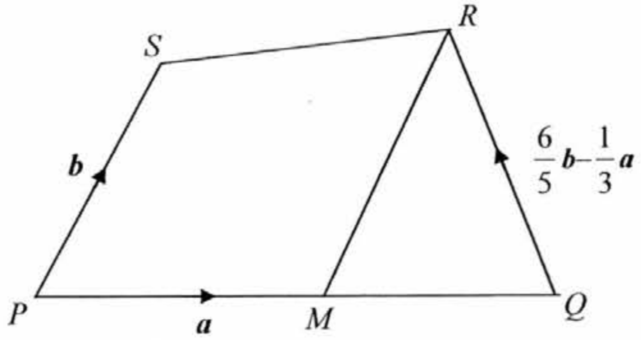
(ii)the interquartile range. [3]

Twenty five girls took the same quiz.
The median mark and interquartile range of the girls' marks are 35 and 6 respectively.

(b) Compare and comment on the performance of the boys and girls in this quiz.[2]

3 $PQRS$ is a quadrilateral. M is the mid-point of PQ.

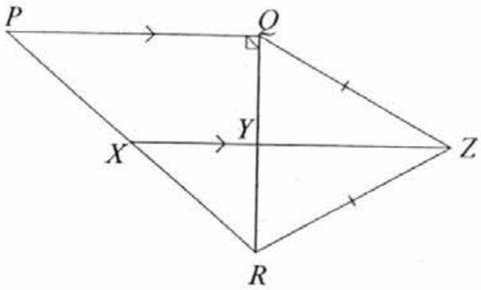
$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PS} = \boldsymbol{b}$ and $\overrightarrow{QR} = \frac{6}{5}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$.



(a) Find $\overrightarrow{SR}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. [1]

(b) Use vectors to show that PS and MR are **not** parallel. [2]

4 In the diagram, PXR, QYR, and XYZ are straight lines.
PQ is parallel to XZ, $QZ = RZ$, $\frac{YZ}{XZ} = \frac{3}{5}$ and $P\hat{Q}R = 90^\circ$.



(a) Show that triangles QYZ and RYZ are congruent. [3]

(b) Show that triangles PQR and XYR are similar. [2]

(c) Find

(i) $\frac{\text{area of } \Delta XYR}{\text{area of } \Delta RYZ}$, [1]

(ii) $\frac{\text{area of } \Delta XYR}{\text{area of } \Delta PQR}$ [1]

5 Jeannie bought some health drink for \$6400. She paid \$$x$ for each litre of the drink.

(a) Find, in terms of x, an expression for the number of litres she bought.[1]

(b) She gave away 8 litres of the drink to her friends. She sold the remainder of the drink for \$50 per litre more than she paid for it. Write down an expression, in terms of x, for the sum of money she received. [1]

(c) She made a profit of \$2960.

(i) Write down an equation in x to represent this information, and show that it reducesto $x^2 + 420x - 40\,000 = 0$. [2]

(ii) Solve the equation $x^2 + 420x - 40\,000 = 0$. [3]

(d) Find the number of litres of drink Jeanniesold. [1]

6 Two satay stalls sell 3 types of satay.
The number of sticks of each type of satay sold per day is given by the matrix **S**.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Chicken} & \text{Mutton} & \text{Beef} \end{array} \\ \mathbf{S} = & \begin{pmatrix} 400 & 300 & 200 \\ 200 & 500 & 300 \end{pmatrix} \begin{array}{l} \text{Stall A} \\ \text{Stall B} \end{array} \end{array}$$

(a) The price of each stick of chicken, mutton and beef satay is \$0.35, \$0.45 and \$0.40 respectively.Represent these pricesin a 3×1 column matrix **P**. [1]

(b) Evaluate the matrix $\mathbf{T} = \mathbf{SP}$. [1]

(c) State what the elements of **T** represent. [1]

(d) In June 2016, Stall A operated 20 days and Stall B operated 25 days.
Use matrix multiplication to find the total amount of money collected by the two stalls in June 2016. [2]

(e) In July, the number of sticks of each type of satay sold per day is increased by 10%. The information is given by the matrix **Q**.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Chicken} & \text{Mutton} & \text{Beef} \end{array} \\ \mathbf{Q} = & \begin{pmatrix} 440 & 330 & 220 \\ 220 & 550 & 330 \end{pmatrix} \begin{array}{l} \text{Stall A} \\ \text{Stall B} \end{array} \end{array}$$

Write down the matrix **R** such that $\mathbf{Q} = \mathbf{SR}$. [1]

7 A box contains 5 Chocolate doughnuts, 3 Glazed doughnuts and 1 Strawberry doughnut.

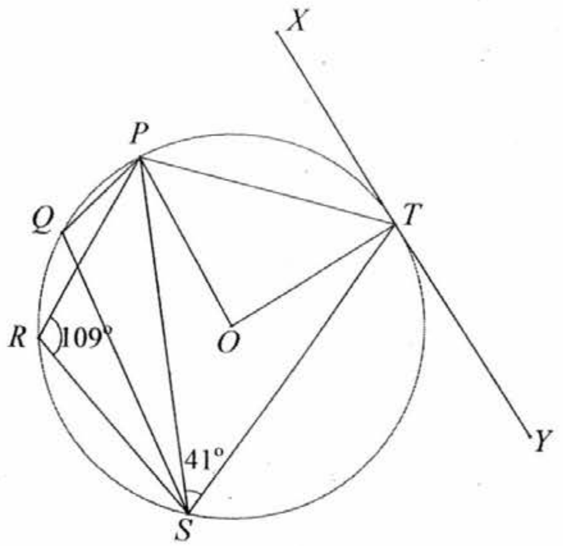
(a) Two doughnuts were taken out of the box at random, without replacement. Copy and complete the tree diagram to show this information.[3]

(b) Find, as a fraction in its simplest form, the probability that

(i) the two doughnuts are the same flavour, [2]

(ii)at least one of the doughnuts is Chocolate. [3]

8 In the diagram, the points P, Q, R, S and T lie on a circle, centre O. XTY is a tangent to the circle. Angle $PRS = 109°$ and angle $PST = 41°$.



(a) Find, giving reasons for each answer,

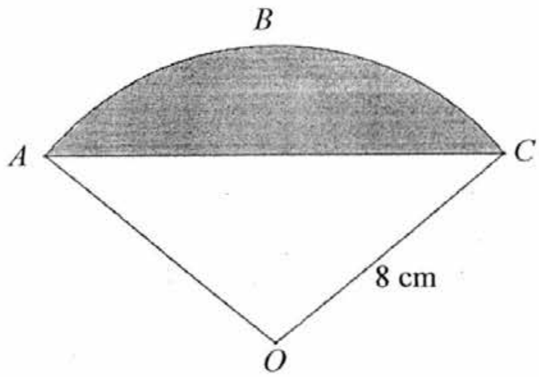
(i) $P\hat{Q}S$, [1]

(ii) $P\hat{T}S$, [1]

(iii) $Y\hat{T}S$. [2]

(iv) $O\hat{T}P$, [2]

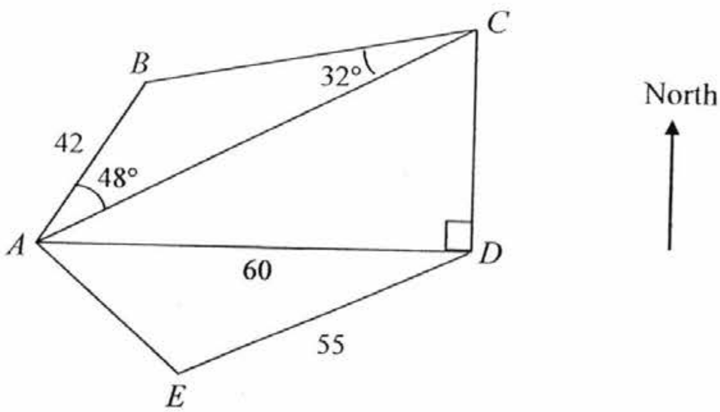
(b) $OABC$ is a sector of a circle, centre O and radius 8 cm. The perimeter of the sector is 30 cm.



(i) Show that angle AOC= 1.75 radians. [1]

(ii) Calculate the area of the shaded region. [3]

9



The diagram shows a field, $ABCDE$, which is crossed by two paths, AC and AD. AD is perpendicular to CD. $AB = 42$ m, $AD = 60$ m, $DE = 55$ m, angle $BAC = 48°$ and angle $ACB = 32°$.

(a) Show that $AC = 78.05$ m, correct to four significant figures. [2]

(b) Calculate CD. [2]

(c) A bird is at P, which is 8 m vertically above E.
Calculate the angle of depression of D from P. [2]

(d) Given that the area of triangle ADE is 1300 m^2, calculate angle ADE. [2]

(e) D is due east of A.
Calculate the bearing of E from A. [3]

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{5x^2}{4} + \frac{60}{x} - 40$.

Some corresponding values of x and y are given in the following table.

x	1	1.5	2	3	3.5	4	4.5	5	6
y	p	2.81	-5	-8.75	-7.54	-5	-1.35	3.25	15

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $1 \le x \le 6$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-10 \le y \le 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [2]

(c) Using your graph, find the range of values of x for which
$\frac{5x^2}{4} + \frac{60}{x} - 40 < 0$. [3]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$.[2]

(e) Draw the tangent to the curve which the gradient is -10.
Write down the equation of this tangent. [2]

(f) The line l intersects the curve $y = \frac{5x^2}{4} + \frac{60}{x} - 40$ at $x = 2$ and $x = 6$.

(i) Find the equation of l. [2]

(ii) By using your answer from **(f)(i)**, find the value of A and of B. [3]

11 **Diagram I** shows a pencil before it is sharpened. It is made up of a piece of cylindrical carbon encased in wood. The length of the pencil is 19 cm.
Diagram II shows the cross-sectional area of the pencil. *ABCDEF* is a regular hexagon with side 0.45 cm. The diameter of the carbon is 0.2 cm.

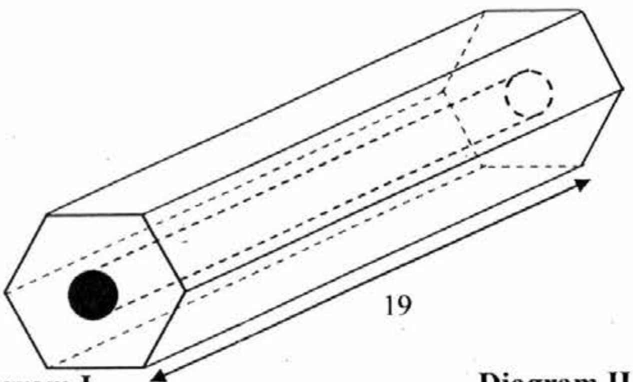


Diagram I

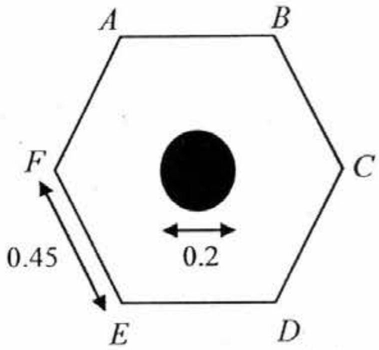


Diagram II

(a) Find

(i) the interior angle of the regular hexagon *ABCDEF*, [2]

(ii) *CF*, [1]

(b) Show that $AE = 0.7794$ cm. [2]

(c) Calculate the area of the regular hexagon *ABCDEF*. [2]

(d) Calculate the volume of the carbon as a percentage of the volume of the pencil. [2]

Diagram III shows ten of these pencils which just fit into a rectangular box which is open on one side.
Diagram IV shows ten of these pencils which just fit into a box whose cross-sectional area is an equilateral triangle which is open on one side.

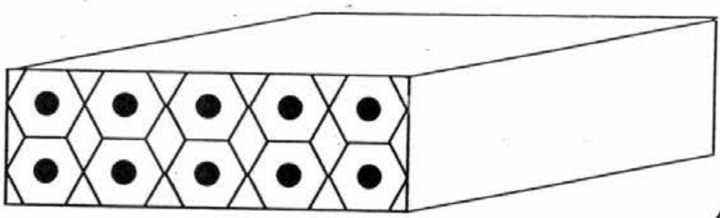


Diagram III

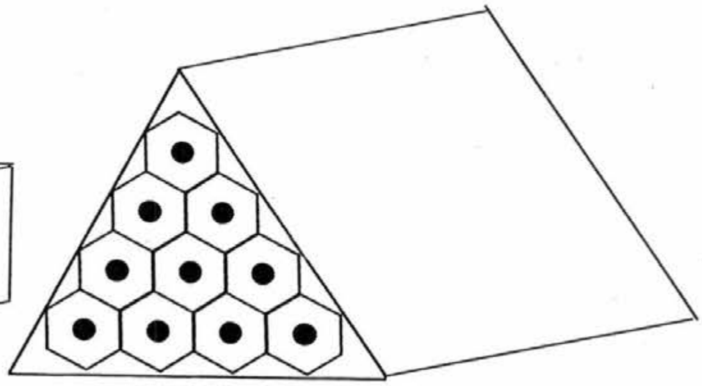


Diagram IV

(e) The boxes are made of cardboard which cost \$10 per m^2.
Determine which box will be cheaper to produce for 1000 boxes.
Justify your decision with calculations. [5]

1 **(a)** Express as a single fraction in its simplest form $1-\frac{2x}{2x-7}+\frac{7}{(2x-7)^2}$. [3]

$$1-\frac{2x}{2x-7}+\frac{7}{(2x-7)^2}$$
$$=\frac{(2x-7)^2-2x(2x-7)+7}{(2x-7)^2}$$
$$=\frac{4x^2-28x+49-4x^2+14x+7}{(2x-7)^2}$$
$$=\frac{56-14x}{(2x-7)^2} \text{ or } \frac{14(4-x)}{(2x-7)^2}$$

(b) Simplify $5a^{-3}b^5 \div \frac{10}{9}a^3b^{-2}$. [2]

$$\frac{5b^5}{a^3} \div \frac{10a^3}{9b^2}$$
$$\frac{5b^5}{a^3} \times \frac{9b^2}{10a^3}$$
$$\frac{9b^7}{2a^6} \ or \ \frac{9}{2}a^{-6}b^7$$

(c) **(i)** Factorise fully $11p^2-44pq+4q-p$. [2]

$$11p^2-44pq+4q-p$$
$$=(11p^2-44pq)-(p-4q)$$
$$=11p(p-4q)-(p-4q)$$
$$=(11p-1)(p-4q)$$

(ii) Factorise fully $30m^2+14mn-4n^2$ [2]

$$30m^2+14mn-4n^2$$
$$=2(15m^2+7mn-2n^2)$$
$$=2(3m+2n)(5m-n)$$

(d) Solve the equation $\frac{1}{x}-\frac{x-5}{2x-3}=1$. [3]

$$\frac{1}{x}-\frac{x-5}{2x-3}=1$$
$$\frac{(2x-3)-x(x-5)}{x(2x-3)}=1$$
$$2x-3-x^2+5x=2x^2-3x$$
$$0=3x^2-10x+3$$
$$(3x-1)(x-3)=0$$
$$x=\frac{1}{3} \quad or \quad 3$$

2 Twenty five boys took a quiz.
The marks are shown in the stem-and-leaf diagram.

1	4 7
2	3 5 7 7 9
3	0 1 2 3 3 5 7 7 8 9 9 9
4	3 4 6 6 7
5	0

Key
1 | 4 means 14 marks

(a) Find

(i) the median mark, [1]
Median = 35

(ii)the interquartile range. [3]

$Q_1 = \dfrac{27+29}{2} = 28 \; Q_3 = \dfrac{39+43}{2} = 41$

Interquartile range = 41 − 28 = 13

Twenty five girls took the same quiz.
The median mark and interquartile range of the girls' marks are 35 and 6 respectively.

(b) Compare and comment on the performance of the boys and girls in this quiz.[2]

$Q_{2B} = Q_{2G} \Rightarrow$ The performance of the boys and girls is the same (is similar).

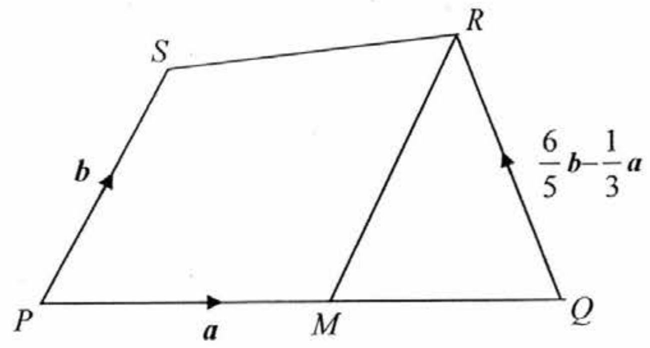
$IQR_G < IQR_B \Rightarrow$ The quiz marks of the girls has a narrower spread.

OR

The results of the girls is more consistent.
Note : Must see$IQR_G < IQR_B$ or in words

3 $PQRS$ is a quadrilateral. M is the mid-point of PQ.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PS} = \boldsymbol{b}$ and $\overrightarrow{QR} = \frac{6}{5}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$.



(a) Find $\overrightarrow{SR}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. [1]

$\overrightarrow{SR} = \overrightarrow{SP} + \overrightarrow{PQ} + \overrightarrow{QR}$

$= -\boldsymbol{b} + \boldsymbol{a} + \frac{6}{5}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$

$= \frac{2}{3}\boldsymbol{a} + \frac{1}{5}\boldsymbol{b}$

(b) Use vectors to show that PS and MR are **not** parallel. [2]

$\overrightarrow{MR} = \overrightarrow{MQ} + \overrightarrow{QR}$

$= \frac{1}{2}\boldsymbol{a} + \frac{6}{5}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$

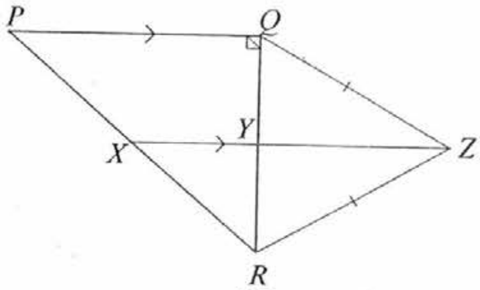
$= \frac{1}{6}\boldsymbol{a} + \frac{6}{5}\boldsymbol{b}$

$\overrightarrow{MR} \neq m\boldsymbol{b}$, where m is a scalar.

OR

$\therefore \overrightarrow{MR}$ is not a multiple of $\overrightarrow{PS}$, hence PS and MR are not parallel

4 In the diagram, PXR, QYR, and XYZ are straight lines.
PQ is parallel to XZ, $QZ = RZ$, $\frac{YZ}{XZ} = \frac{3}{5}$ and $P\hat{Q}R = 90^\circ$.



(a) Show that triangles QYZ and RYZ are congruent. [3]

$QZ = RZ$ (given)
$YZ = YZ$ (common side)
$\angle QYZ = 90^o$ (alt $\angle s$, $PQ \parallel XY$)
$\angle RYZ = 180^\circ - 90^\circ$ (adj $\angle s$ on straight line)
$= 90^\circ$
$\therefore \angle QYZ = \angle RYZ$

$\therefore \Delta QYZ \equiv \Delta RYZ$ (RHS)

Alternate Solution

$QZ = RZ$ (given)
$\angle ZQY = \angle ZRY$ (base angles, isos Δ)
$\angle QYZ = 90^o$ (alt $\angle s$, $PQ \parallel XY$)
$\angle RYZ = 180^\circ - 90^\circ$ (adj $\angle s$ on straight line)
$= 90^\circ$
$\therefore \angle QYZ = \angle RYZ$

$\therefore \Delta QYZ \equiv \Delta RYZ$ (AAS)

(b) Show that triangles PQR and XYR are similar. [2]

$$\left.\begin{array}{l} \angle PQR = \angle XYR \text{ (corr. } \angle s\text{, } PQ \parallel XY) \\ \angle QPR = \angle YXR \text{ (corr. } \angle s\text{, } PQ \parallel XY) \\ \angle PRQ = \angle XRY \text{ (common angle)} \end{array}\right\} \text{Any 2 reasons}$$

$\therefore \Delta PQR$ is similar to ΔXYR.

(c) Find

(i) $\frac{\text{area of } \Delta XYR}{\text{area of } \Delta RYZ}$, [1]

$$\frac{\text{area of } \Delta XYR}{\text{area of } \Delta RYZ} = \frac{2}{3}$$

(ii) $\frac{\text{area of } \Delta XYR}{\text{area of } \Delta PQR}$ [1]

$$\frac{\text{area of } \Delta XYR}{\text{area of } \Delta PQR} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

5 Jeannie bought some health drink for \$6400. She paid \$$x$ for each litre of the drink.

(a) Find, in terms of x, an expression for the number of litres she bought.[1]

No of Litres bought $= \dfrac{6400}{x}$

(b) She gave away 8 litres of the drink to her friends. She sold the remainder of the drink for \$50 per litre more than she paid for it. Write down an expression, in terms of x, for the sum of money she received. [1]

No of litres left $= \dfrac{6400}{x} - 8$

Sum of money received

$= (x + 50)\left(\dfrac{6400}{x} - 8\right)$

$= 6400 - 8x + \dfrac{320\,000}{x} - 400$

$= S\left(\dfrac{320\,000}{x} - 8x + 6000\right)$

(c) She made a profit of \$2960.

(i) Write down an equation in x to represent this information, and show that it reducesto $x^2 + 420x - 40\,000 = 0$. [2]

$\left(\dfrac{320\,000}{x} - 8x + 6000\right) - 6400 = 2960$

$8x^2 + 3360x - 320\,000 = 0$

$x^2 + 420x - 40\,000 = 0$

(ii) Solve the equation $x^2 + 420x - 40\,000 = 0$. [3]

$(x + 500)(x - 80) = 0$

$x = -500$ or $x = 80$

(d) Find the number of litres of drink Jeanniesold. [1]

Number of litres sold $= \dfrac{6400}{80} - 8$

$= 72$

6 Two satay stalls sell 3 types of satay.
The number of sticks of each type of satay sold per day is given by the matrix **S**.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Chicken} & \text{Mutton} & \text{Beef}\end{array} \\ \mathbf{S} = & \begin{pmatrix} 400 & 300 & 200 \\ 200 & 500 & 300 \end{pmatrix} \begin{array}{l}\text{Stall A} \\ \text{Stall B}\end{array} \end{array}$$

(a) The price of each stick of chicken, mutton and beef satay is \$0.35, \$0.45 and \$0.40 respectively.Represent these pricesin a 3×1 column matrix **P**. [1]

$$\mathbf{P} = \begin{pmatrix} 0.35 \\ 0.45 \\ 0.40 \end{pmatrix}$$

(b) Evaluate the matrix $\mathbf{T} = \mathbf{SP}$. [1]

$$\mathbf{T} = \begin{pmatrix} 400 & 300 & 200 \\ 200 & 500 & 300 \end{pmatrix} \begin{pmatrix} 0.35 \\ 0.45 \\ 0.40 \end{pmatrix}$$

$$= \begin{pmatrix} 355 \\ 415 \end{pmatrix}$$

(c) State what the elements of **T** represent. [1]
The total amount of money collected by each stall (per day from the selling the satay)

(d) In June 2016, Stall A operated 20 days and Stall B operated 25 days.
Use matrix multiplication to find the total amount of money collected by the two stalls in June 2016. [2]

$$\begin{pmatrix} 20 & 25 \end{pmatrix} \begin{pmatrix} 355 \\ 415 \end{pmatrix}$$

$$= \begin{pmatrix} 17\,475 \end{pmatrix}$$

Total amount of money collected from the two stalls in June 2016 was \$17 475.

(e) In July, the number of sticks of each type of satay sold per day is increased by 10%. The information is given by the matrix **Q**.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Chicken} & \text{Mutton} & \text{Beef}\end{array} \\ \mathbf{Q} = & \begin{pmatrix} 440 & 330 & 220 \\ 220 & 550 & 330 \end{pmatrix} \begin{array}{l}\text{Stall A} \\ \text{Stall B}\end{array} \end{array}$$

Write down the matrix **R** such that $\mathbf{Q} = \mathbf{SR}$. [1]

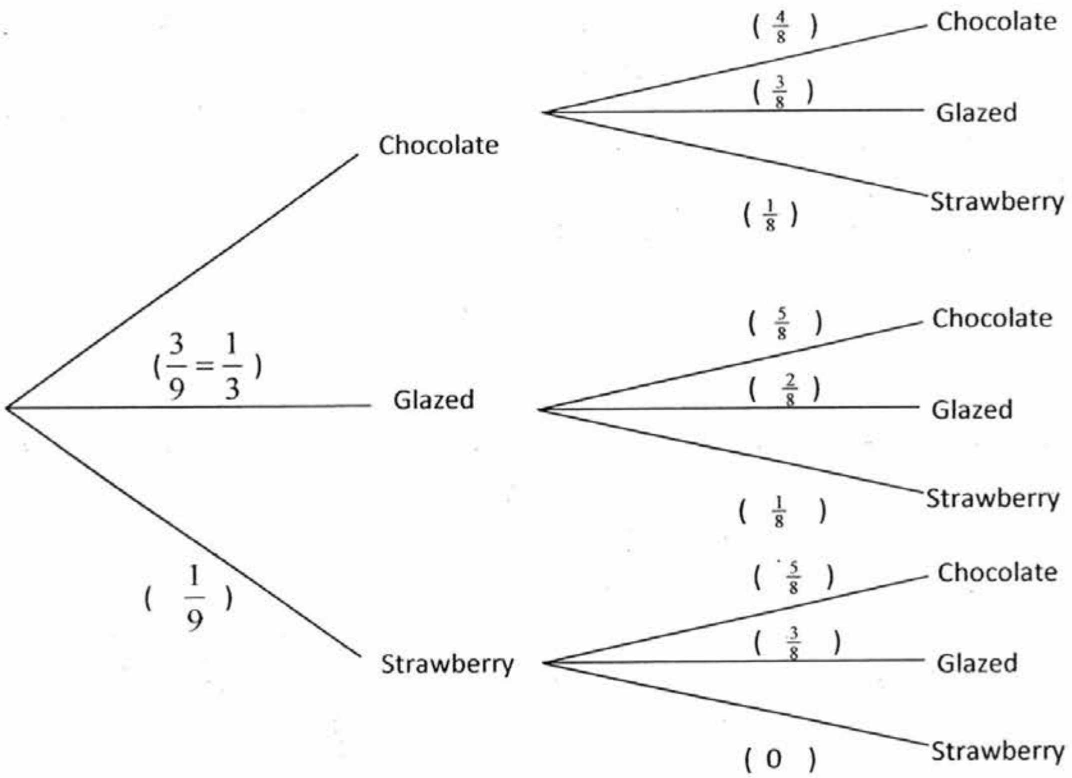
$\mathbf{Q} = \mathbf{SR}$

$$\begin{pmatrix} 440 & 330 & 220 \\ 220 & 550 & 330 \end{pmatrix} = \begin{pmatrix} 400 & 300 & 200 \\ 200 & 500 & 300 \end{pmatrix} \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$$

7 A box contains 5 Chocolate doughnuts, 3 Glazed doughnuts and 1 Strawberry doughnut.

(a) Two doughnuts were taken out of the box at random, without replacement. Copy and complete the tree diagram to show this information.[3]



(b) Find, as a fraction in its simplest form, the probability that

(i) the two doughnuts are the same flavour, [2]

$$\left(\frac{5}{9}\times\frac{4}{8}\right)+\left(\frac{3}{9}\times\frac{2}{8}\right)$$

$$=\frac{20}{72}+\frac{6}{72}$$

$$=\frac{13}{36}$$

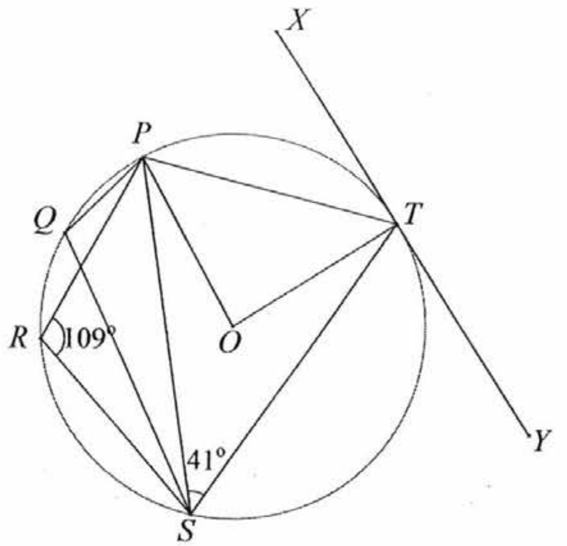
(ii) at least one of the doughnuts is Chocolate. [3]

$$\left(\frac{5}{9}\times\frac{4}{8}\right)+\left[2\left(\frac{5}{9}\times\frac{3}{8}\right)+2\left(\frac{5}{9}\times\frac{1}{8}\right)\right] \quad 1-\left[\left(\frac{3}{9}\times\frac{2}{8}\right)+\left(\frac{1}{9}\times 0\right)+2\left(\frac{3}{9}\times\frac{1}{8}\right)\right]$$

$$=\frac{20}{72}+\frac{40}{72} \qquad\qquad =1-\frac{12}{72}$$

$$=\frac{5}{6} \qquad\qquad =\frac{5}{6}$$

8 In the diagram, the points P, Q, R, S and T lie on a circle, centre O. XTY is a tangent to the circle. Angle $PRS = 109°$ and angle $PST = 41°$.



(a) Find, giving reasons for each answer,

(i) $P\hat{Q}S$, [1]

$P\hat{Q}S = 109°$ (angles in same segment)

(ii) $P\hat{T}S$, [1]

$P\hat{T}S = 180° - 109°$ (angles in opp segment)
$= 71°$

(iii) $Y\hat{T}S$. [2]

$O\hat{T}S = 71^o - 49^o$
$= 22°$

$Y\hat{T}S = 90^o - 22^o$ (rad $\perp$ tan)
$= 68°$

Alternate Solution

$S\hat{P}T = 180^o - 71^o - 41^o$ (angle sum of Δ)

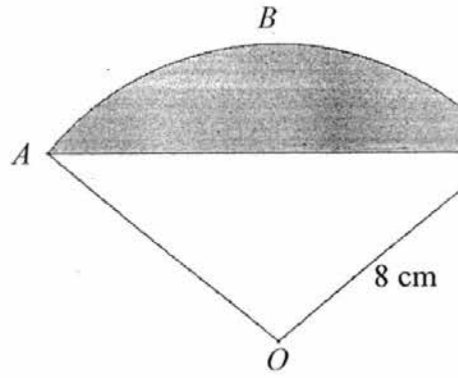
$Y\hat{T}S = 68^o$ (angles in alt segment)

(iv) $O\hat{T}P$, [2]

$P\hat{O}T = 41^o \times 2\,(\measuredangle\ at\ center = 2\measuredangle\ at\ cir)$
$= 82°$

$O\hat{T}P = \dfrac{180^o - 82^o}{2}$ (base angles, isos Δ)
$= 49°$

(b) *OABC* is a sector of a circle, centre *O* and radius 8 cm. The perimeter of the sector is 30 cm.



(i) Show that angle *AOC*= 1.75 radians. [1]

$2(8)+8A\hat{O}C=30$

$A\hat{O}C$ = 1.75 rad

(ii) Calculate the area of the shaded region. [3]

area of ΔOAC = $\frac{1}{2}\times 8^2\times \sin 1.75$

= 31.487 cm^2

Area of sector = $\frac{1}{2}\times 8^2\times 1.75$

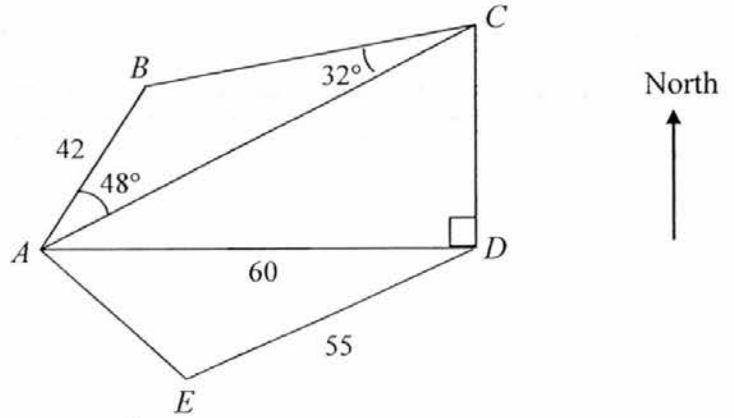
= 56 cm^2

Shaded area = 56 – 31.487

= 24.513

= 24.5 cm^2 (3 sf)

9



The diagram shows a field, *ABCDE*, which is crossed by two paths, *AC* and *AD*. *AD* is perpendicular to *CD*.*AB* = 42 m, *AD* = 60 m, *DE* = 55 m, angle *BAC* = 48° and angle *ACB* = 32°.

(a) Show that $AC = 78.05$ m, correct to four significant figures. [2]

$$\measuredangle ABC = 180 - (48 + 32)$$
$$= 100° \ (\measuredangle\ sum\ of\ \Delta)$$
$$\frac{AC}{\sin 100°} = \frac{42}{\sin 32°}$$
$$AC = \frac{42}{\sin 32°} \times \sin 100°$$
$$= 78.053$$
$$\approx 78.05\ cm$$

(b) Calculate CD. [2]

$$CD = \sqrt{78.053^2 - 60^2}$$
$$= 49.92$$
$$\approx 49.9\ cm$$

(c) A bird is at P, which is 8 m vertically above E.
Calculate the angle of depression of D from P. [2]
Let angle of depression be θ

$$\tan\theta = \frac{8}{55}$$
$$\theta = 8.27$$
$$\approx 8.3°$$

(d) Given that the area of triangle ADE is 1300 m^2, calculate angle ADE. [2]

$$\frac{1}{2} \times 60 \times 55 \times \sin \measuredangle ADE = 1300$$
$$\sin \measuredangle ADE = \frac{2 \times 1300}{60 \times 55}$$
$$\measuredangle ADE = 51.98\ or\ 128.02$$
$$\approx 52.0°\ or\ 128.0°\ (NA)$$

(e) D is due east of A.
Calculate the bearing of E from A. [3]

$$AE^2 = 60^2 + 55^2 - 2(60)(55)\cos 51.98°$$
$$AE = \sqrt{60^2 + 55^2 - 2(60)(55)\cos 51.98°}$$
$$= 50.59$$

$$\frac{\sin \measuredangle EAD}{55} = \frac{\sin 51.98}{50.59}$$
$$\sin \measuredangle EAD = \frac{\sin 51.98}{50.59} \times 55$$
$$\measuredangle EAD = 58.92\ or\ 121.08(NA)$$

$$Bearing\ of E\ from\ A = 90 + 58.92$$
$$= 148.92$$
$$\approx 148.9°$$

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{5x^2}{4} + \frac{60}{x} - 40$.

Some corresponding values of x and y are given in the following table.

x	1	1.5	2	3	3.5	4	4.5	5	6
y	p	2.81	-5	-8.75	-7.54	-5	-1.35	3.25	15

(a) Find the value of p. [1]

$$y = \frac{5x^2}{4} + \frac{60}{x} - 40$$

$$p = \frac{5(1)}{4} + \frac{60}{1} - 40$$

$$= 21.25$$

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $1 \le x \le 6$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-10 \le y \le 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [2]

Correct Scale + Label axes

7 Correct Points + Curve

(c) Using your graph, find the range of values of x for which

$\frac{5x^2}{4} + \frac{60}{x} - 40 < 0$. [3]

$1.65 < x < 4.65$

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$.[2]

Draw tangent

m = 6.25

(e) Draw the tangent to the curve which the gradient is -10.
Write down the equation of this tangent. [2]

Draw any tangent line with gradient -10

Draw tangent at/near x = 2 and y = -10x+15

(f) The line l intersects the curve $y=\frac{5x^2}{4}+\frac{60}{x}-40$ at $x=2$ and $x=6$.

(i) Find the equation of l. [2]

$(2,-5),(6,15)$

$m=\frac{15+5}{6-2}$

$=5$

$y+5=5(x-2)$

$y=5x-15$

It is given that $x=2$ and $x=6$ are solutions of the equation $5x^3+Ax^2+Bx+240=0$.

(ii) By using your answer from **(f)(i)**, find the value of A and of B. [3]

$\frac{5x^2}{4}+\frac{60}{x}-40=5x-15$

$\frac{5x^2}{4}-5x-25+\frac{60}{x}=0$

$5x^3-20x^2-100x+240=0$

$A=-20,$

$B=-100$

11 **Diagram I** shows a pencil before it is sharpened. It is made up of a piece of cylindrical carbon encased in wood. The length of the pencil is 19 cm.
Diagram II shows the cross-sectional area of the pencil. *ABCDEF* is a regular hexagon with side 0.45 cm. The diameter of the carbon is 0.2 cm.

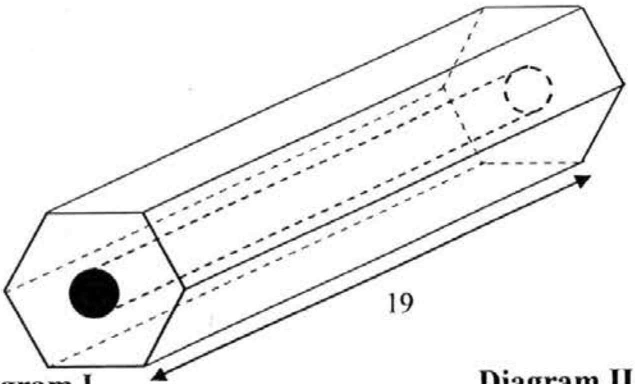


Diagram I

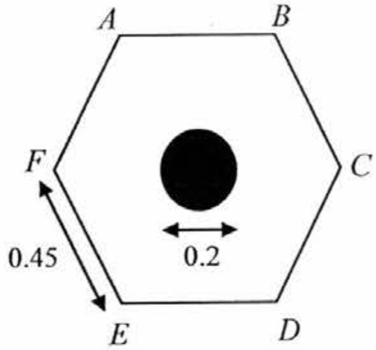


Diagram II

(a) Find

(i) the interior angle of the regular hexagon *ABCDEF*, [2]

$Interior \measuredangle = (180 \times 4) \div 6$ or $Interior \measuredangle = 180 - (\frac{360}{6})$
$= 120° = 120°$

(ii) *CF*, [1]

$\measuredangle ABF = \measuredangle AFB$
$= 30° \ (base \measuredangle \ in \ iso \ \Delta)$
$\measuredangle FBC = 90°$
$\measuredangle AFC = 60° \ (int \measuredangle, AB \parallel CF)$
$\measuredangle BFC = 60 - 30$
$= 30°$

$$\sin 30° = \frac{0.45}{CF}$$

$$CF = \frac{0.45}{\sin 30°} \quad CF = 0.45 \times 2$$
$$= 0.9\ cm = 0.9\ cm$$

(b) Show that $AE = 0.7794$ cm. [2]

$AE^2 = 0.45^2 + 0.45^2$
$- 2(0.45)(0.45)cos120°$
$AE = \sqrt{0.6075}$
$AE = 0.77942$
$\approx 0.7794\ cm$

(c) Calculate the area of the regular hexagon $ABCDEF$. [2]

$$\sin 60^\circ = \frac{h}{0.45}$$

$h = 0.45 \sin 60^\circ$

$$Area\ ABCDEF Area\ ABCDEF Area\ ABCDEF$$
$$= 2[\frac{1}{2} \times (0.45 + 0.9) = 2[\frac{1}{2}(0.45)^2 \sin 120^\circ] = 6[\frac{1}{2}(0.45)^2 \sin 60^\circ]$$
$$\times 0.45 \sin 60^\circ] + (0.7794 \times 0.45)$$
$$= 0.5261 = 0.5261 = 0.5261$$
$$\approx 0.526\ cm^2 \approx 0.526\ cm^2 \approx 0.526\ cm^2$$

(d) Calculate the volume of the carbon as a percentage of the volume of the pencil. [2]

$$Percentage = \frac{\pi(0.1)^2 \times 19}{0.5261 \times 19} \times 100\%$$
$$= 5.971$$
$$\approx 5.97\%$$

Diagram III shows ten of these pencils which just fit into a rectangular box which is open on one side.
Diagram IV shows ten of these pencils which just fit into a box whose cross-sectional area is an equilateral triangle which is open on one side.

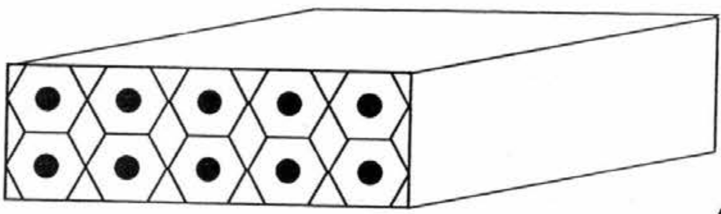


Diagram III

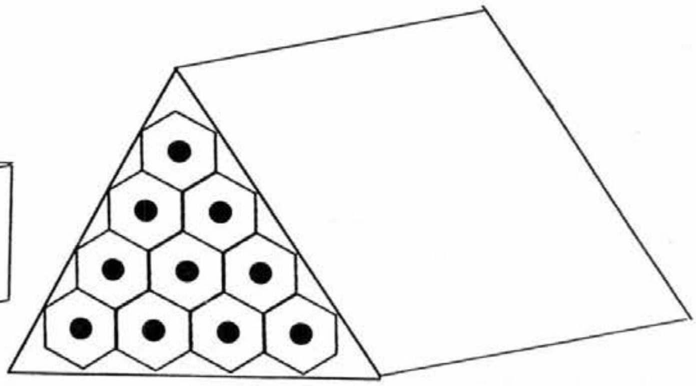


Diagram IV

(e) The boxes are made of cardboard which cost \$10 per m^2.
Determine which box will be cheaper to produce for 1000 boxes.
Justify your decision with calculations. [5]

Diagram III

$$Length\ of\ III = 5 \times 0.9$$
$$= 4.5\ cm$$
$$Width\ of\ III = 2 \times 0.7794$$
$$= 1.5588\ cm$$
$$SA = 2(4.5 \times 19) + 2(1.5588 \times 19)$$
$$+(4.5 \times 1.5588)$$
$$= 237.249\ cm^2$$
$$= 2.37249 \times 10^{-2} m^2$$
$$Cost = 2.37249 \times 10^{-2} \times 10 \times 1000$$
$$= 237.249$$
$$\approx \$237.25$$

Diagram IV

$$Length\ of\ \Delta = 5 \times 0.7794$$
$$= 3.897\ cm$$
$$SA = 3(3.897 \times 19)$$
$$+\frac{1}{2}(3.897)(3.897)\sin 60°$$
$$= 228.704\ cm^2$$
$$= 2.28704 \times 10^{-2} m^2$$
$$Cost = 2.28704 \times 10^{-2} \times 10 \times 1000$$
$$= 228.704$$
$$\approx \$228.70$$

Design IV will be cheaper to produce
for 1000 boxes