13

聖嬰中學
HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class

Index Number

80

PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
MATHEMATICS PAPER ONE

40/16/01 & 4048/01

Date: 16 Aug 2016

Duration: 2 hours

Time : 11 00 – 13 00

Additional Materials: **Nil**
Students answer on Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer ALL questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total number of marks for this paper is **80**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

Answer **all** questions.

1 **(a)** Calculate $\dfrac{\sqrt[3]{27.58}+17.9\times 2.08}{(60.69)^2}$.

Write down the first seven digits on your calculator display.

Answer (a) ..... [1]

(b) Write your answer to part (a) correct to 4 significant figures.

Answer (b) ..... [1]

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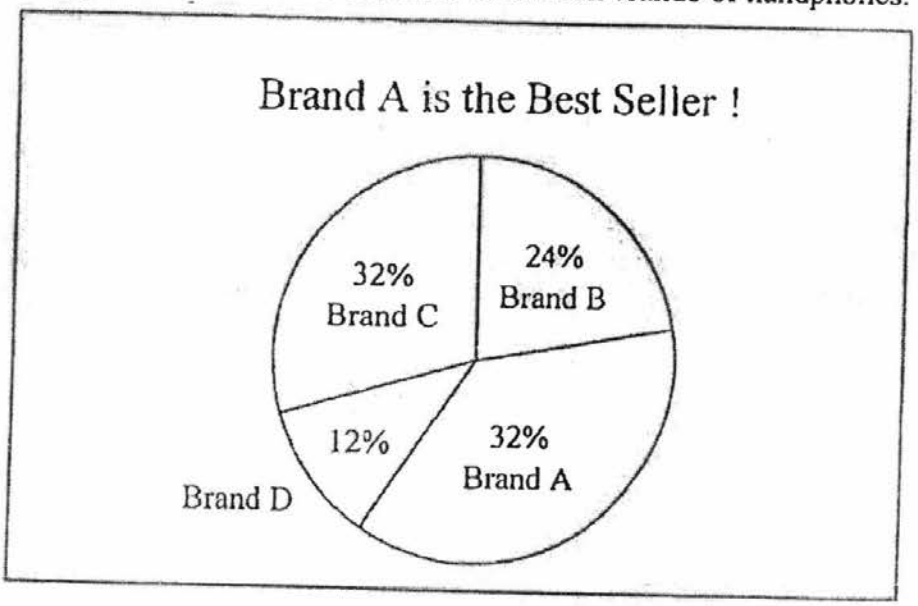
2 The pie chart shows the sales for 4 different brands of handphones.



Explain one way in which the pie chart is misleading.

Answer .....

.....

.....

..... [2]

An antique vase was auctioned off at $1200 at a profit of 140%.
Find the original value of the vase.

Answer $..... [2]

Write the following in order of size, smallest first.

$$3.1\times10^{-1}, \quad \pi, \quad 3.2, \quad -3.14.$$

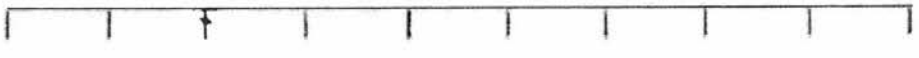
Answer ..... [2]

(a) Solve the inequality $1-\frac{1}{2}x \le 3 < 5-x$.

Answer (a) ..... [2]

(b) Represent your solution on the number line below. [1]

Answer (b)



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6 The volume of air, V cm^3, inside a bicycle pump is inversely proportional to the cube root of the air pressure, P units.

(a) If the air pressure is 343 units when 30 cm^3 of air is pumped, find an equation for V in terms of P.

Answer (a) ..... [2]

(b) Find the volume of air when the air pressure is 999 units.

Answer (b) ..... cm^3 [1]

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7 The diameter of a spherical micro-organism is 648 picometres.
Calculate the total surface area of this micro-organism in mm^2, giving your answer in standard form.

Answer ..... mm^2 [4]

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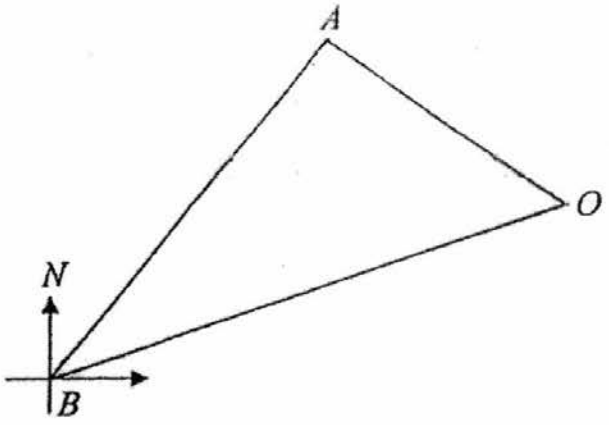
8 ABO is a triangle.
If A is on a bearing of 320° from O and the bearing of B from A is 245°, find the bearing of

(a) O from A,



Answer (a) ..... ° [1]

(b) A from B.

Answer (b) ..... ° [2]

9 (a) Solve the equation $\sqrt{625^x} = \frac{1}{5}$.

Answer (a) $x =$ ..... [2]

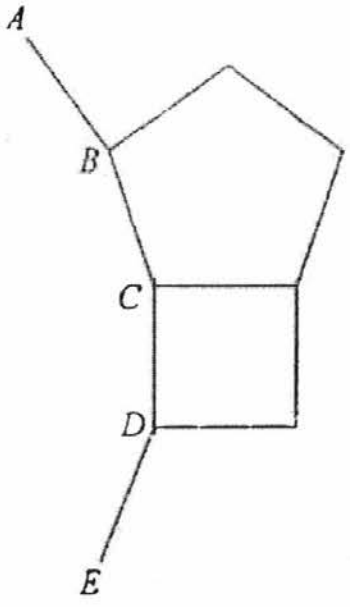
(b) Simplify $\left(49q^4\right)^{-\frac{1}{2}} \times 14q^2$.

Answer (b) ..... [2]

18 The diagram is made up of a square, a regular pentagon and an incomplete regular polygon $ABCDE$ of n sides.

Find

(a) the value of obtuse $\angle BCD$,



Answer (a) $\angle BCD =$ ..... ° [2]

(b) the value of each exterior angle of polygon $ABCDE$,

Answer (b) ..... ° [1]

(c) the value of n.

Answer (c) $n =$ ..... [1]

11 It is given that $\frac{1}{5p} = \frac{1}{q} - \frac{1}{3r}$.

(a) Evaluate q when $p = 0.2$ and $r = \frac{1}{3}$.

Answer (a) $q =$ ..... [1]

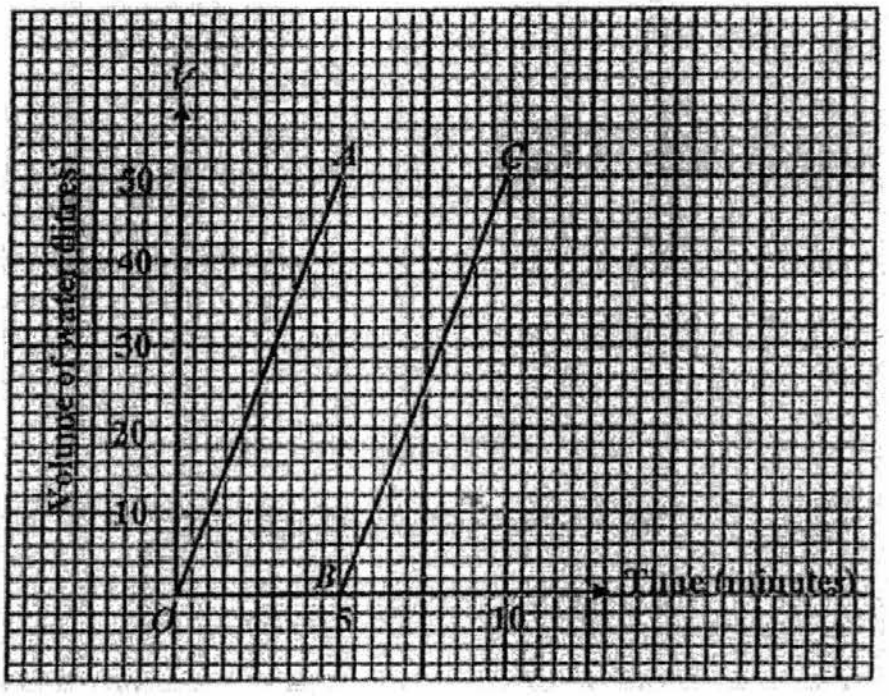
(b) Express r in terms of p and q.

Answer (b) $r =$ ..... [3]

12 In Singapore, giant tipping water buckets are one of the fun features in a water playground.

The bucket is filled with water at a constant rate.
When the bucket is full, the bucket tilts and all the water is poured out instantly.
The bucket then continues to be filled with water.

The graph shows the volume of water (V) in a water bucket for the first 10 minutes.



(a) What is the maximum volume of water that the water bucket can hold?

Answer (a) ..... litres [1]

(b) (i) Find the gradient of the line BC.

Answer (b)(i) ..... [1]

(ii) Briefly explain what has happened at each point A, B and C.

Answer (b)(ii) .....

.....

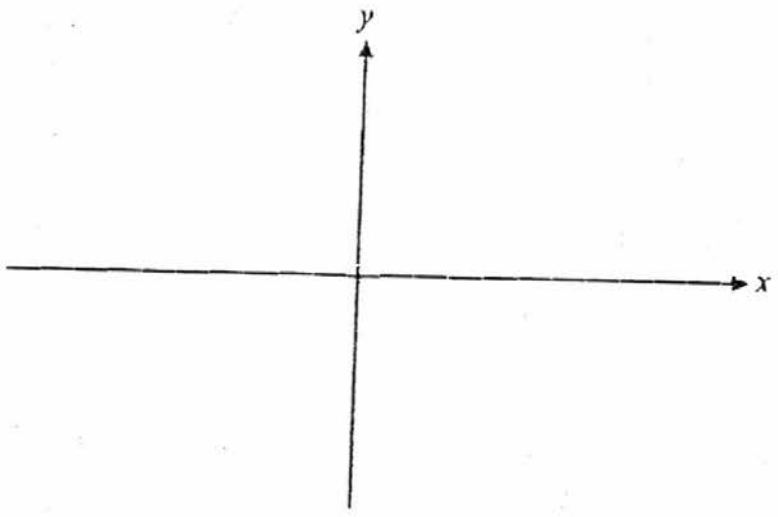
.....

..... [1]

13 (a) (i) On the axes below, sketch the graph of $y = (x + 1)(3 - x)$.

Answer (a)(i)

[2]



(ii) Write down the equation of the line of symmetry of $y = (x + 1)(3 - x)$.

Answer (a)(ii) .....

(b) (i) Express $x^2 - 4x + 7$ in the form $(x - a)^2 + b$.

Answer (b)(i) ..... [1]

(ii) Hence, write down the coordinates of the turning point of the curve $y = x^2 - 4x + 7$.

Answer (b)(ii) (..... , ...) [1]

14 $\varepsilon = \{x : x \text{ is an integer and } 1 \leq x < 10\}$
$A = \{\text{prime numbers}\}$
$B = \{\text{factors of } 6\}$

(a) List the elements in $A \cap B'$.

Answer (a) ..... [1]

(b) List the elements in $(A \cup B)'$.

Answer (b) ..... [1]

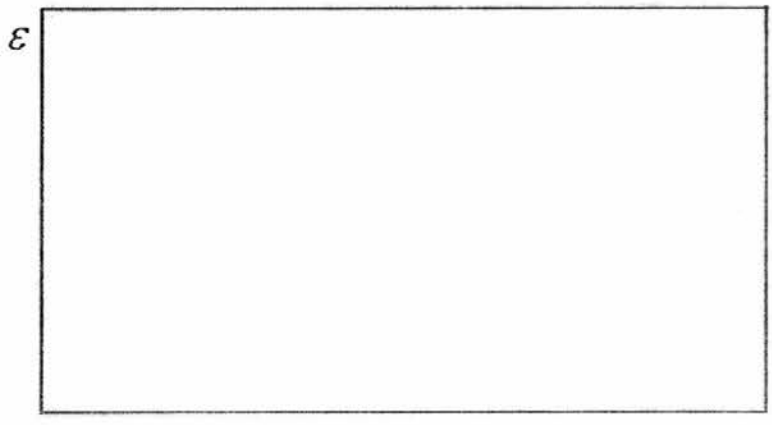
(c) Is $\{2, 3\} \in A$? Justify your answer.

Answer (c) .....

..... [1]

(d) Draw a Venn diagram to represent the sets ε, A and B.

Answer (d) [2]



15 The information below shows the mass of 500 packets of sugar packed by two machines A and B.

Machine A

Mass (x grams)	$980 < x \leq 990$	$990 < x \leq 1000$	$1000 < x \leq 1010$	$1010 < x \leq 1020$
Frequency	62	120	270	48

Machine B

Mean = 1000.2 g
Standard Deviation = 9.8 g

(a) For machine A, calculate the mean and the standard deviation of the mass of the sugar packed.

Answer (a) Mean = ..., Standard deviation = ... [3]

(b) If you are tasked to purchase one of the machines, which one would you choose?
Justify your choice using appropriate statistical measures.

Answer (b) .....

.....

.....

.....

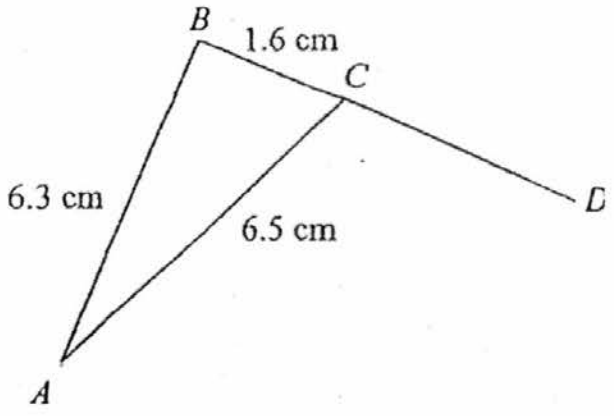
.....

.....

.....

..... [2]

16 In the diagram, $AB = 6.3$ cm, $AC = 6.5$ cm and $BC = 1.6$ cm.
BCD is a straight line.



(a) Prove that triangle ABC is a right-angled triangle.

Answer (a) .....

.....

.....

.....

..... [2]

(b) Express the following as a fraction in its simplest form.

(i) $\tan \angle BCA$,

Answer (b)(i) ..... [1]

(ii) $\cos \angle ACD$.

Answer (b)(ii) ..... [1]

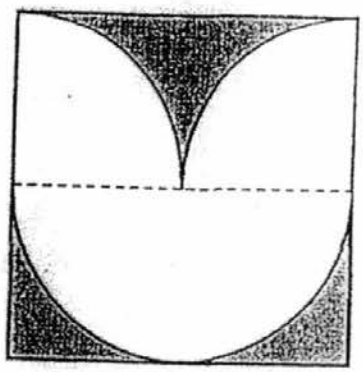
(c) Wayne claims that he can draw a triangle ACD with sides $AC = 6.5$ cm, $CD = 5$ cm and $AD = 12$ cm. Justify whether you agree with his claim.

Answer (c) .....

.....

..... [1]

17 The diagram below shows a semicircle and 2 quadrants inscribed in a square of side $4kr$ cm.



(a) Show that the area of the semicircle is $2\pi k^2 r^2$.

Answer (a) .....

.....

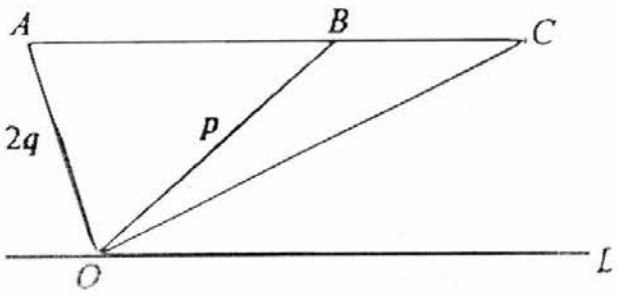
.....

..... [1]

(b) Find, in terms of π and r, the difference in area between the shaded section and the unshaded section when $k = 2$.

Answer (b) ..... cm^2 [4]

18 In the diagram, ABC is a straight line.
B is a point on AC such that $3AB = 2AC$.
$\overrightarrow{OA} = 2q$ and $\overrightarrow{OB} = p$.



(a) Express, in terms of p and/or q,

(i) $\overrightarrow{AB}$,

Answer (a)(i) ..... [1]

(ii) $\overrightarrow{OC}$.

Answer (a)(ii) ..... [2]

(b) It is given that $\overrightarrow{OD} = \frac{3}{2}p - 3q$.

Show that OD is parallel to line AB.

Answer (b) .....

.....

..... [1]

(c) Using vectors, show that $OACD$ is a parallelogram.

Answer (c) .....

.....

.....

.....

.....

..... [2]

19 (a) Given that $\mathbf{N} = \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$, find the scalars a and b such that $\mathbf{N}^2 + a\mathbf{N} - b\mathbf{I} = \mathbf{O}$.

Answer (a) $a =$ ..... , $b =$ ..... [4]

19 (b) A stall sells two different types of food.
They are apple pie and banana cake.
The total number of each item sold on three different days is given in the table below.

	apple pie	banana cake
Monday	22	30
Tuesday	35	27
Wednesday	18	22

The information can be represented by the matrix $\mathbf{S} = \begin{pmatrix} 22 & 30 \\ 35 & 27 \\ 18 & 22 \end{pmatrix}$.

(i) Given that $\mathbf{K} = (1 \quad 1 \quad 1)$, calculate **KS**.

Answer (b)(i) **KS** = [2]

(ii) Explain what each element in **KS** represents.

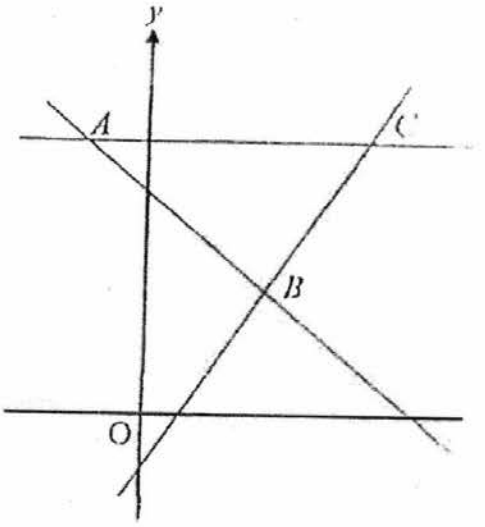
Answer (b)(ii) .....

.....

..... [1]

20 The diagram, which is not drawn to scale, shows the three lines $y = 5 - x$, $y = 2x - 1$ and $y = 6$.

The lines cut each other at points A, B and C.



(a) Find the coordinates of A and B.

Answer (a) A (..... , ...) [1]

B (..... , ...) [1]

(b) Find the area of triangle ABC.

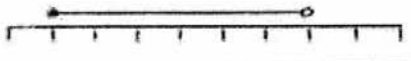
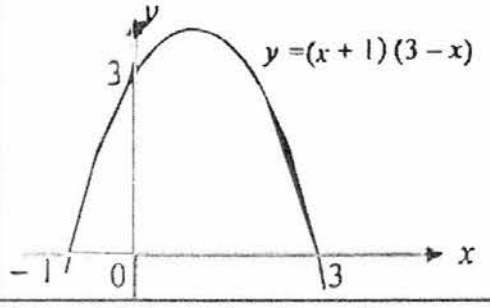
Answer (b) ..... sq units [3]

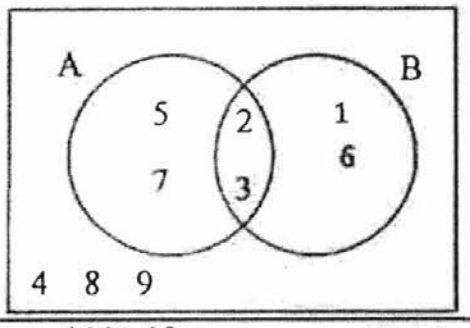
(c) Find the equation of the line which is parallel to AC and passes through $(-3, -1)$.

Answer (c) ..... [3]

~~ End of Paper ~~

ONE PAGE ANSWER

Qn	Answer
1(a)	0.010928
(b)	0.01093
2	See end of next page
3	500
4	$-3.14,\ 3.1\times10^{-1},\ \pi,\ 3.2$
5(a)	$-4 \le x < 2$
(b)	
6(a)	$V = \dfrac{210}{\sqrt[3]{P}}$
(b)	21.0
7	1.32×10^{-12} mm^2
8(a)	140°
(b)	065°
9(a)	$x = -\dfrac{1}{2}$
(b)	2
10(a)	162°
(b)	18°
(c)	$n = 20$
11(a)	$q = \dfrac{1}{2}$
(b)	$r = \dfrac{5pq}{3(5p-q)}$
12(a)	50 litres
(bi)	10
(bii)	At point A, the bucket is full with water. At point B, the bucket is empty. At point C, the bucket is full with water again.
13(ai)	
(aii)	Line of symmetry is $x = 1$
(bi)	$(x-2)^2 + 3$
(bii)	(2, 3) is the turning point

Qn	Answer
14(a)	5, 7
(b)	4, 8, 9
(c)	No. $\{2, 3\} \notin A$ $\{2, 3\}$ is a set. It should be a **subset** of set A.
(d)	
15(a)	Mean = 1001.08 g Standard deviation = 8.24 g
(b)	See end of next page
16(a)	$AC^2 = 6.5^2 = 42.25$ $AB^2 + BC^2 = 6.3^2 + 1.6^2 = 42.25$ Since $AC^2 = AB^2 + BC^2$, thus by converse of Pythagoras' theorem, ΔABC is a right-angled triangle.
(bi)	$\tan\angle BCA = \dfrac{6.3}{1.6} = \dfrac{63}{16}$
(bii)	$\cos\angle ACD = -\dfrac{1.6}{6.5} = -\dfrac{16}{65}$
(c)	From triangle equality theorem, AD = 12 cm AC + CD = 6.5 + 5 = 11.5 < 12 The sum of any 2 sides should be more than the third side. Thus, I disagree with him.
17(a)	Area of semicircle $\dfrac{\pi(2kr)^2}{2} = \dfrac{\pi 4k^2r^2}{2} = 2\pi k^2r^2$
(b)	$32\pi r^2 - 64r^2$
18(ai)	$p - 2q$
(aii)	$\dfrac{3}{2}p - q$

18(b)	$\overrightarrow{OD} = \frac{3}{2}p - 3q = \frac{3}{2}(p - 2q)$ $= \frac{3}{2}\overrightarrow{AB}$ Thus, OD is parallel to line AB.	19(a) (bi)	$a = -5, b = 2$ $(75 \quad 79)_{1\times 2}$
(c)	$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$ $= 3q - \frac{3}{2}p + \frac{3}{2}p - q$ $= 2q$ $= \overrightarrow{OA}$ $\therefore$ Since $\overrightarrow{DC} = \overrightarrow{OA}$, thus $OACD$ is a parallelogram. or $\overrightarrow{AC} = \overrightarrow{OD}$, thus, $OACD$ is a parallelogram.	(bii)	Each element represents the total number of apple pies and banana cakes sold respectively in all the 3 days. OR Each element represents the total number of each type of food sold in all the 3 days.
20(a)	$A(-1, 6), \quad B(2, 3)$		
(b)	6.75 units^2		
(c)	$y = -1$		

Answer to Q2

Misleading feature(s) : Brand A and Brand C have equal percentage but the proportion on the pie chart are not equal / title is biased

Effect of this feature on the graph : It mislead people into believing that Brand A is selling better than Brand C / It does not allow reader to make own judgement.

Answer to Q15b

The standard deviation for the mass of sugar packed by machine A is smaller, thus there is more consistency (or lesser variation) in the mass of each packet.

The mean mass of sugar packed by machine A is higher, indicating that the packets are heavier.

Thus, I will buy machine A.

~~ The End ~~

Answer all questions

1 (a) (i) Factorise completely $3m^2 - 12n^2$. [2]

(ii) Simplify $\dfrac{3m^2 - 12n^2}{6m^2 - 24mn + 24n^2}$. [2]

(b) Simplify

$$\left(\frac{2}{xy^2}\right)^{-3} \div \left(\frac{2x}{3y}\right).$$ [2]

(c) Express as a single fraction in its simplest form

$$\frac{2x}{x-2} - \frac{3}{5}.$$ [2]

(d) Solve the equation $\sqrt{\dfrac{w}{3}} = 3w$. [3]

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2 Eugene ran a marathon of 42 km.

For the first $\frac{2}{3}$ of his run, his average speed was x km/h.

(a) Write down an expression, in terms of x, for the time taken, in hours, for the first $\frac{2}{3}$ of his run. [1]

He then reduced his speed by 2 km/h for the remaining part of the run.

(b) Given that the total time taken by Eugene to complete the run was $4\frac{2}{5}$ hours, form and equation in x and show that it reduces to

$$11x^2 - 127x + 140 = 0.$$ [3]

(c) Solve this equation and hence find his average speed for the last part of the run, giving your answer correct to 1 decimal place. [4]

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3 (a) The value of an antique clock increased by 115% from the year 2014 to the year 2015.
In 2015, the antique clock was valued at $120 000.

Calculate the value of the clock in 2014. [2]

(b) The cash price of a new oven is $1400.
Gordon buys the oven on hire purchase.
He pays a deposit of 40% of the cash price, followed by 10 monthly payments of $90.

Calculate the interest that Gordon paid. [2]

(c) A sales agent is paid a basic salary of $1000 plus 5% commission on all sales if the sales amount exceeds $5000.
The agent's salary for June was $1620.

Calculate his total sales for June. [2]

(d) A bank pays 2% interest compounded annually on all deposits.
At the start of 2010, Adrian deposited $1000 into his account.
At the start of 2012, Adrian deposited a further $800 into his account.
He withdrew all his money at the end of 2015.

Calculate the amount of money Adrian withdrew at the end of 2015. [3]

(e) On a particular day, the exchange rate between the Malaysian Ringgit (MYR) and the Singapore dollar (SGD) was 1 SGD = 2.97 MYR.
Jack purchased some herbs online for 2000 MYR.
The shipping rate for the herbs was 50 MYR.
For shipment into Singapore, a 7% Goods and Services Tax (GST) is payable on the cost of goods and shipment that exceeds 400 SGD

Calculate the amount of money, in Singapore dollars, that Jack had to pay to import the herbs. [3]

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = Ax^2 + Bx + 18.$$

Some corresponding values of x and y are given in the table below.

x	0	1	2	3	4	5	6	7
y	18	26	30	30	26	18	6	−10

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal axis for $0 \leq x \leq 7$.
Using a scale of 2 cm to represent 5 units, draw a vertical axis for $-10 \leq y \leq 35$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

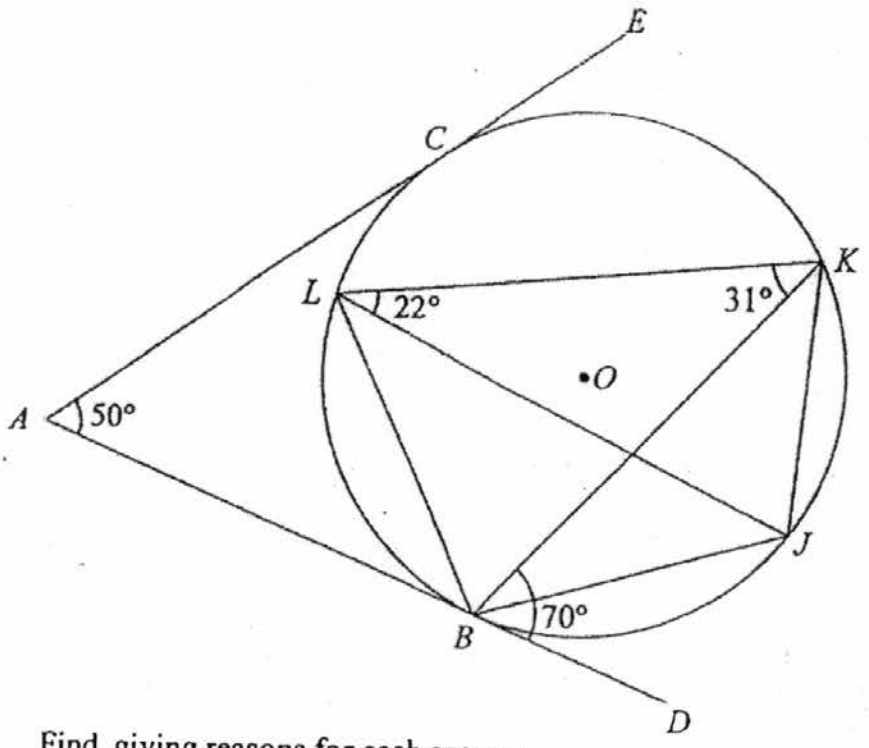
(b) Use your graph to find
(i) the value of y when $x = 4.5$, [1]
(ii) the values of x when $y = 20$. [1]

(c) By drawing a tangent, find the gradient of the curve at $x = 0.5$. [2]

(d) The line $y = k$ is a tangent to the curve.
By drawing this tangent on your graph, find the value of k. [2]

(e) Find the value of A and of B. [3]

5 The diagram shows a circle $BLCKJ$, centre O.
AE and AD are tangents to the circle.
Angle JLB is equal to angle JBD.
Angle $BAC = 50°$, angle $JLK = 22°$, angle $KBD = 70°$ and angle $BKL = 31°$.



(a) Find, giving reasons for each answer,

(i) angle JBK, [1]

(ii) angle KBL, [2]

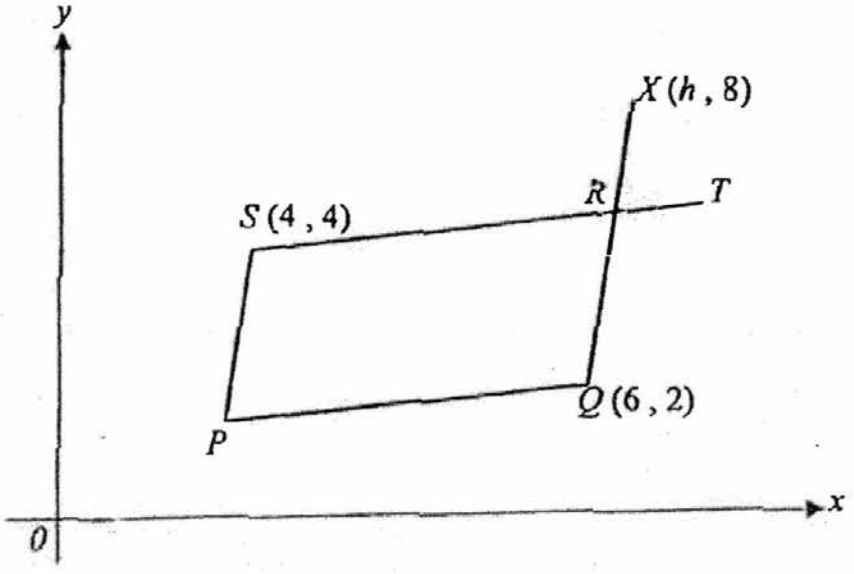
(iii) angle KOL, [1]

(iv) angle COB. [2]

(b) An arc of a circle, with centre A, is drawn through the points C, O and B.
The length of the arc COB is $\frac{35}{18}\pi$ cm.
Calculate the area of the sector $ACOB$, leaving your answer in terms of π. [3]

6 The diagram shows a parallelogram $PQRS$ not drawn to scale.
Q and S are the points (6, 2) and (4, 4) respectively.
T lies on SR produced and X lies on QR produced.

It is given that $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.



(a) Given that $\overrightarrow{ST} = 5\overrightarrow{RT}$, find

(i) the column vector $\overrightarrow{ST}$, [1]

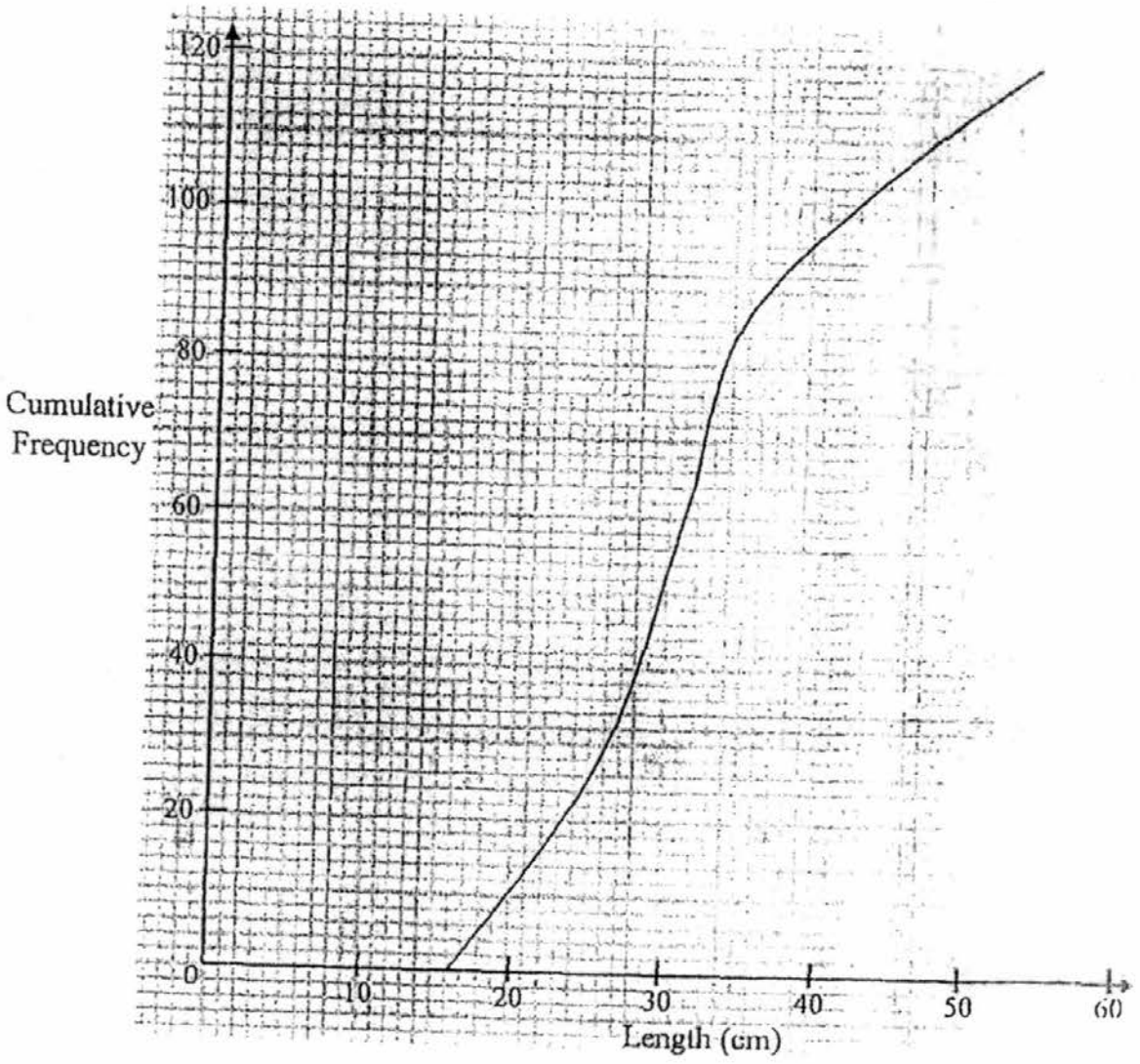
(ii) the coordinates of point T, [1]

(iii) the column vector $\overrightarrow{QS}$. [2]

(b) Find $\dfrac{\text{area of triangle } QTR}{\text{area of triangle } QTS}$. [1]

(c) Given that X is the point $(h, 8)$ and $\overrightarrow{QR} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, find the value of h. [2]

7 (a) The lengths of 120 fishes in a pond were measured.
The cumulative frequency curve below shows the distribution of their lengths.



(i) Use the curve to estimate

(a) the median length, [1]

(b) the interquartile range of the lengths, [2]

(c) the percentage of fishes which are at least 35 cm in length. [2]

(ii) It was discovered that the lengths were measured incorrectly.
The correct lengths should be 3 cm longer than those recorded.

Explain how the median and the interquartile range of the lengths would be affected by this error. [2]

(b) A survey is carried out on a group of 150 people.
72 of the people surveyed are men.
Find

(i) the probability that one person, chosen at random, will be a woman. [1]

(ii) the probability that two people, chosen at random, will be a man and a woman. [1]

---

8 The first six rows in a sequence of numbers, Row 1, Row 2, Row 3, Row 4, Row 5 and Row 6 are given below.

Row 1						2					
Row 2					4	6	8				
Row 3				10	12	14	16	18			
Row 4			20	22	24	26	28	30	32		
Row 5		34	36	38	40	42	44	46	48	50	
Row 6	52	54	56	58	60	62	64	66	68	70	72

The table below shows the row number, the number of terms, the first term and the last term for each row.

Row number, n	Number of terms in the n^{th} row, M_n	1st term in the n^{th} row, F_n	Last term in the n^{th} row, L_n
1	1	2	2
2	3	4	8
3	5	10	18
4	7	20	32
5	9	34	50
6	11	52	72
7	a	74	b

(a) Write down the values of a, and b for the 7th row. [2]

(b) Explain why 221 cannot appear in column L. [1]

(c) Express M_n in terms of n. [1]

(d) (i) Express L_{n-1} in terms of n. [2]

(ii) Hence express F_n in terms of n. [1]

(e) Write down the first and the last term on row 15. [2]

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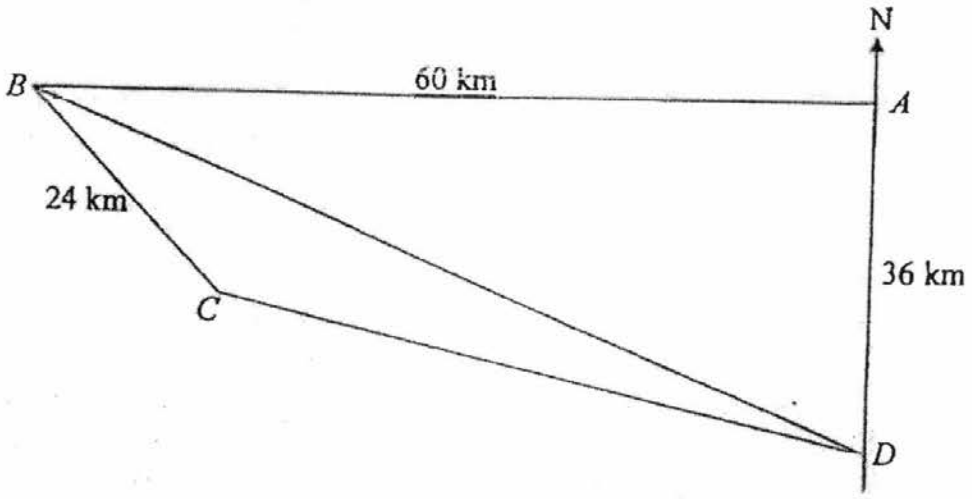
9 $ABCD$ represents a plot of land.
A is due north of D, B is due west of A.
$AB = 60$ km, $BC = 24$ km, and $AD = 36$ km.
The bearing of C from B is $142.5°$.



(a) (i) Show that angle $CBD = 21.5°$. [3]

(ii) Find the bearing of B from D. [1]

(b) Find
(i) BD, [2]

(ii) CD. [3]

(c) A hot air balloon floats 300 m above the ground.
It travels parallel to the ground from A to B.
Find the greatest possible angle of elevation of the balloon from C. [4]

10 Here is some information about a tank used to store gaseous chemicals.

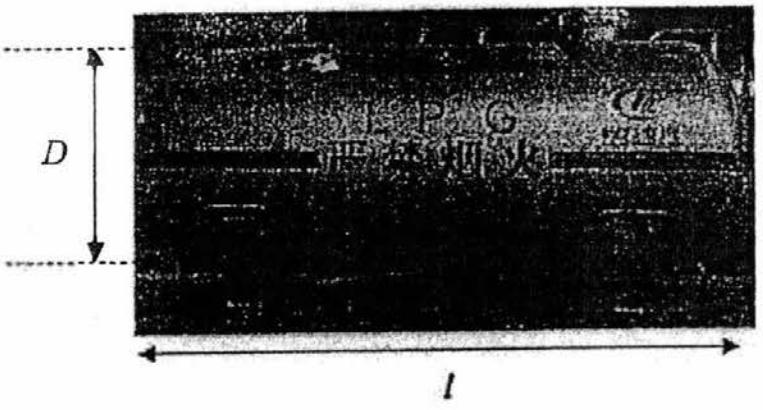
Gas Tank

Length (l): 486 cm

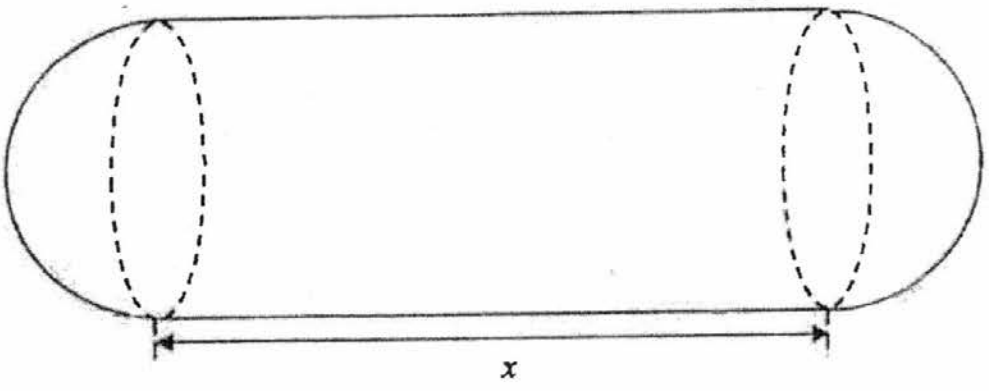
Diameter (D): 105 cm

Maximum Pressure: 25 psi

*psi: pounds per square inch



In this question, the gas tank can be modelled as a cylinder joined to two hemispheres.



(a) Find the length of the cylindrical part, x, in cm. [1]

(b) The gas tank is to be painted.
A tin of 10 litres of paint can cover 9500 cm^2 of the surface of the tank.
Calculate the number of tins of paint that must be bought to paint the entire tank. [4]

(c) According to Boyle's Law, the product of the pressure and volume of a fixed mass of gas is a constant. 10 kg of chemical X has the following properties.

Mass (kg)	Volume (m^3)	Pressure (psi)
10	4.3	19

Determine if the tank can be filled with 10 kg of gaseous chemical X.
State your reasoning and calculations clearly. [5]

---------- *End of Paper* ----------

1(ai)	$3(m+2n)(m-2n)$
1(aii)	$\dfrac{m+2n}{2(m-2n)}$
1(b)	$\dfrac{3x^2y^7}{16}$
1(c)	$\dfrac{7x+6}{5(x-2)}$
1(d)	$w=0 \quad \text{or} \quad w=\dfrac{1}{27}$
2(a)	Time taken $= \dfrac{28}{x}$ hours
2(b)	$\dfrac{28}{x}+\dfrac{14}{x-2}=\dfrac{22}{5}$ $\dfrac{28x-56+14x}{x(x-2)}=\dfrac{22}{5}$ $210x-280=22x^2-44x$ $22x^2-254x+280=0$ $11x^2-127x+140=0$ (shown)
2(c)	Average Speed $= 10.311-2 = 8.3$ km/h
3(a)	\$55813.95
3(b)	Interest = \$1460 − \$1400 = \$60
3(c)	\$12400
3(d)	\$1992.11
3(e)	SGD738.55
4(b)(i)	22.5
4(b)(ii)	0.2, 4.8
4(c)	7.63
4(d)	$k = 30.5$
4(e)	$A=-2, \ B=10$
5(a)(i)	22°
5(a)(ii)	79°
5(a)(iii)	158°
5(a)(iv)	130°
5(b)	$\dfrac{245}{36}\pi \text{ cm}^2$
6(a)(i)	$\begin{pmatrix}5\\1.25\end{pmatrix}$
6(a)(ii)	(9, 5.25)
6(a)(iii)	$\begin{pmatrix}-2\\2\end{pmatrix}$

6(b)	$\frac{1}{5}$
6(c)	$h = 10$
7(a)(i)(a)	31 cm
7(a)(i) (b)	10 cm
7(a)(i) (c)	27.5%
7(a)(ii)	The median would be 3 cm more. There would be no change in the interquartile range.
7(b)(i)	$\frac{13}{25}$
7(b)(ii)	$\frac{1872}{3725}$
8(a)	13, 98
8(b)	221 is odd while the numbers in the column are even.
8(c)	$M_n = 2n-1$
8(d)(i)	$L_{n-1} = 2(n-1)^2$
8(d)(ii)	$2(n-1)^2 + 2$
e)	$L_{15} = 2(15)^2 = 450$ $F_{15} = 2(15-1)^2 + 2 = 394$
9(a)(i)	$\angle ABD = \tan^{-1}\left(\frac{36}{60}\right)$ $= 30.963°$ $\angle CBD = 142.5° - 30.963° - 90°$ $= 21.536°$ $= 21.5°$ (shown)
(a)(ii)	301°
(b)(i)	By pythagoras' theorem, 70.0 km
(b)(ii)	48.4 km
(c)	$\theta = 0.9°$
0(a)	381 cm
0(b)	17 tins
0(c)	Constant $= 4.3 \times 19 = 81.7$ Volume of tank $= \frac{4}{3}\pi(0.525)^3 + \pi(0.525)^2(3.81)$ $= 1.24306875\pi$ Let Pressure be P $P(1.24306875\pi) = 81.7$ $P = 20.9$ Since the calculated pressure is less than 25, the tank is able to withstand 10 kg of gaseous chemical X.