14

Calculator Model: _____	4E/5N/4NA 'O'

Name: _____ ( ) Class: _____

KRANJI SECONDARY SCHOOL
Preliminary Examination I
Secondary 4 Express / 5 Normal Academic/ 4NA 'O'

ELEMENTARY MATHEMATICS **4048/1**
4016/1

PAPER 1

Friday **6 May 2016** **2 hours**

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

1. Write your name, class and register number on this cover page.
2. Answer all questions.
3. Write your answers in the spaces provided on the question paper.
4. If working is needed for any question, show it in the space below that question.
5. **Omission of essential working will result in loss of marks.**
6. If the degree of accuracy is not specified in the question, and that if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
7. For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

Calculators should be used where appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Set by : **Ms Sim Chin Chin**

This question paper consists of 18 printed pages, including the cover page.

[Turn over

Mathematical formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

For Examiner's Use

Answer **ALL** the questions.

1 Evaluate

(a) $3+\sqrt[3]{5.222}$,

(b) $\dfrac{\sqrt{\pi \div 2}+0.967}{676\times10^{-2}}$,

Give your answers correct to 2 decimal places.

Answer *(a)* ..... [1]

(b) ..... [1]

2 **(a)** Ray bought a house at $720 000. Five years later, he sold the house at $640000. Calculate the loss as a percentage of his selling price.

(b) Rose's weight is 51 kg after losing 15% of her original weight. What was her original weight?

Answer *(a)* ..... [1]

(b) ..... [1]

For Examiner's Use

For Examiner's Use

3 (a) Write down the following numbers in ascending order.

$$-1.53,\ \sqrt{2.34},\ \frac{3}{2},\ -\frac{8}{5},\ 0.56,\ 0.5\dot{6}$$

(b) Find the integer(s) which satisfy both $3 + 2x \leq 1$ and $5x > x - 12$

Answer *(a)* ..... [1]

(b) ..... [3]

4 (a) Simplify $\left(2x^2\right)^2 \times \left(-\frac{4}{3}x\right)^n \div \left(\frac{xy^2}{-4}\right)^2$.

(b) The reciprocal of 5^{-6} is 5^{1-p}. Write down the value of p.

Answer *(a)* ..... [2]

(b) $p =$ ..... [1]

For Examiner's Use

For Examiner's Use

5 y is directly proportional to x^3. It is known that $y = 8$ for a particular value of x. Find the value of y when this value of x is doubled.

Answer $y =$ ..... [3]

---

6 Written as a product of its prime factors, $300 = 2^2 \times 3 \times 5^2$.

(a) Write 72 as a product of its prime factors.

(b) **Using the above results,** find

(i) the highest common factor of 72 and 300,

(ii) the smallest positive integer k such that $72k$ is a perfect cube,

(iii) the smallest integer that is a multiple of both 72 and 300.

Answer *(a)* $72 =$ ..... [1]

(b) *(i)* ..... [1]

(ii) $k =$ ..... [1]

(iii) ..... [1]

For Examiner's Use

For Examiner's Use

7 (i) Factorise $9x^2 - 1$.

(ii) **Use the result in part (i) to express 899 as a product of its prime factors.**

Answer (i) ..... [1]

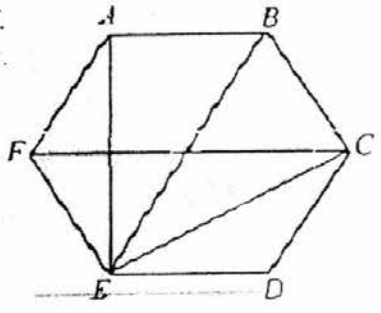
(ii) ..... [2]

---

8 The diagram shows a regular hexagon $ABCDEF$.

(a) Find $\angle BCE$, showing your working clearly.

(b) Show that ΔFEC is congruent to ΔBCE.



Answer (a) ..... [2]

(b) ..... [2]

For Examiner's Use

For Examiner's Use

9 The lifetime of a sample of 120 brand A batteries is given in the table below.

Lifetime (t hours)	$0 < t \leq 30$	$30 < t \leq 60$	$60 < t \leq 90$	$90 < t \leq 120$	$120 < t \leq 150$
Freq	16	20	33	41	10

Calculate

(a) the mean,

(b) the standard deviation.

The mean lifetime of a sample of 120 brand B batteries is 78.0 hours and the standard deviation is 40.2 hours.

(c) Compare, briefly, the results for the two brands of batteries.

Answer *(a)* ..... [1]

(b) ..... [2]

(c) .....

..... [2]

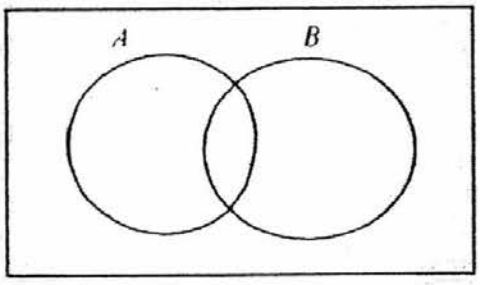
For Examiner's Use

For Examiner's Use

10 **(a)** On the Venn diagram shown in answer space, shade the region, $A \cup B'$. [1]

Answer (a)

ξ



(b) $\xi = \{ x: x \text{ is an integer and } 3 < x \leq 30 \}$
$P = \{x: x \text{ is a prime number}\}$
$R = \{x: x \text{ is an integer ending in 3}\}$

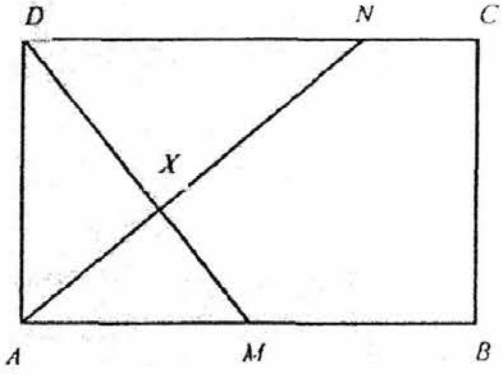
Find $n(P \cap R)$

Answer (b) ..... [1]

For Examiner's Use

For Examiner's Use

11 $ABCD$ is a rectangle. M is the mid-point of AB and N is a point on CD. MD and AN meet at X.



(a) Prove that ΔAXM and ΔNXD are similar.

Answer *(a)* .....

.....

.....

.....

..... [2]

(b) Given that $DN = 3NC$, find the value of

(i) $\dfrac{area\ of\ \Delta NXD}{area\ of\ \Delta AXM}$,

(ii) $\dfrac{area\ of\ \Delta AXD}{area\ of\ ABCD}$.

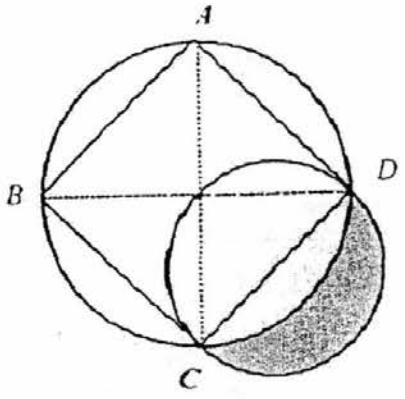
Answer *(b)* *(i)* ..... [1]

(ii) ..... [2]

For Examiner's Use

For Examiner's Use

12 In the diagram, $ABCD$ is a square whose diagonals are 3 cm each. BD and CD are the diameters of the bigger and smaller circle respectively. Find the area and perimeter of the shaded region.



Answer *Area* = ... cm^2 [3]

Perimeter = ... cm [2]

---

13 The approximate time for light to travel 1 metre is 3.30 nanoseconds.

(a) Express the time taken, in seconds, for light to travel 10 metres. Give your answer in standard form.

(b) Find the distance travelled, correct to the nearest kilometre, in 1 second.

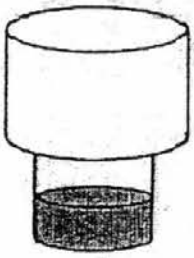
Answer *(a)* ..... sec [2]

(b) ..... km [2]

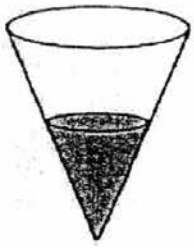
For Examiner's Use

For Examiner's Use

14 Water is poured into each of the containers below at a constant rate of 250 ml per second. The graphs show the depth d of the water varies with time t. Match the container to its corresponding graph.

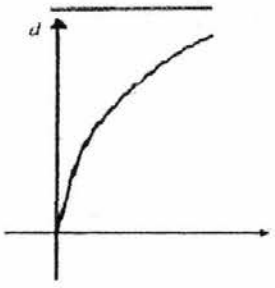


Container A

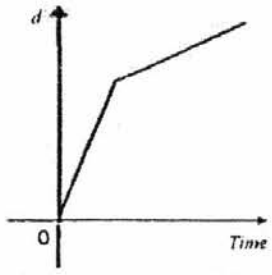


Container B

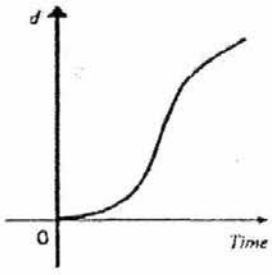
Container C



Graph 1



Graph 2



Graph 3

Answer Container A Graph .....

Container B Graph .....

Container C Graph ..... [3]

For Examiner's Use

For Examiner's Use

15 Mrs Lim wants to buy some antiseptic germicide. Below are the rates of the germicide sold at 2 different shops. Which shop should Mrs Lim buy the germicide from? Explain your choice.

Shop *A*
$13.50 for each 1-litre bottle.
Buy 2 get 1 free now!

Shop *B*
Usual Price : $18.50 for a 2-litre bottle.
5% discount NOW!

Answer .....

.....

.....

.....

..... [3]

For Examiner's Use

For Examiner's Use

16 **(a)** Express $x^2 - 4x + 3$ in the form $(x - p)^2 + q$.

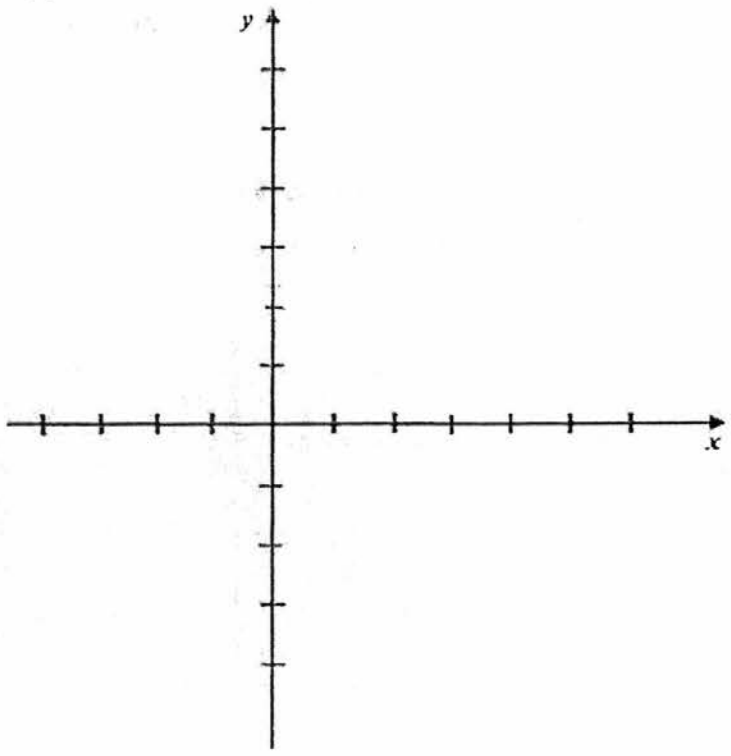
(b) The equation of a quadratic curve is $y = x^2 - 4x + 3$.
State the the coordinates of its turning point.

(c) Sketch the graph of $y = x^2 - 4x + 3$ on the axes in the answer space, indicating clearly the intercepts and turning point.

Answer *(a)* ..... [2]

(b) (..... , ...) [1]

(c)



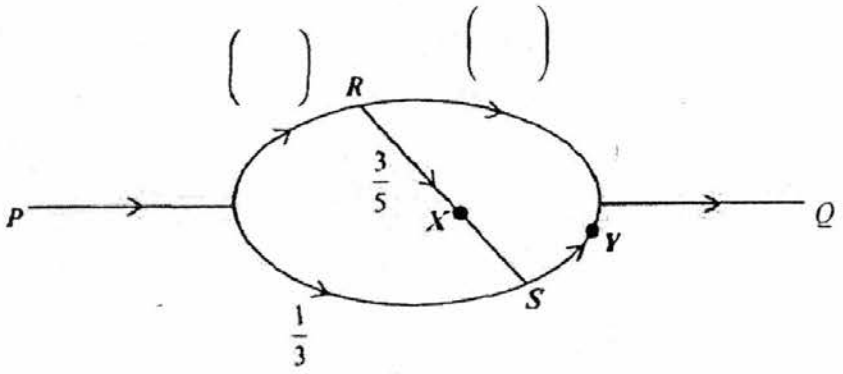
[2]

For Examiner's Use

For Examiner's Use

17 The diagram shows a one-way road system between two junctions P and Q. The probability that a car travels along PS is $\frac{1}{3}$. At junction R, the probability that the car travels along RS is $\frac{3}{5}$.

(a) Complete the diagram below.



Answer (a) [1]

(b) Find the probability that the car travelling from P to Q

(i) will pass point X.

(ii) will pass point Y.

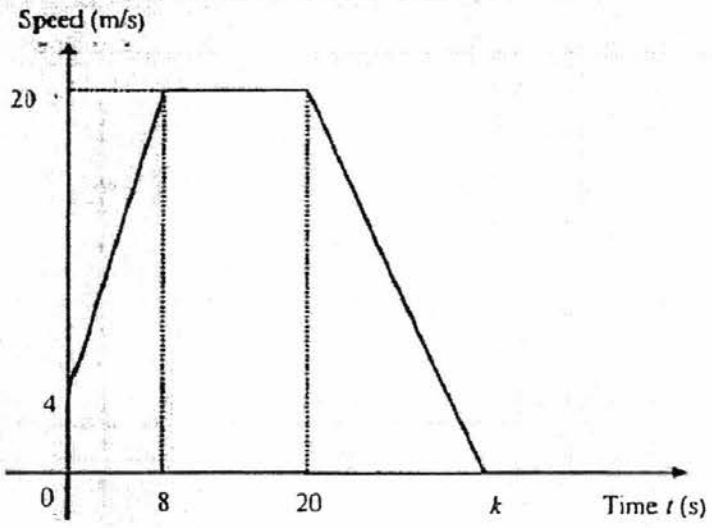
(b) (i) ..... [1]

(ii) ..... [2]

For Examiner's Use

For Examiner's Use

18 The diagram is the speed-time graph of a car's journey.



Calculate

(a) the speed of the car when $t = 3$,

(b) the total distance travelled in the first 20 seconds of the journey,

(c) the value of k, if the retardation is twice that of the acceleration.

Answer *(a)* ..... m/s [2]

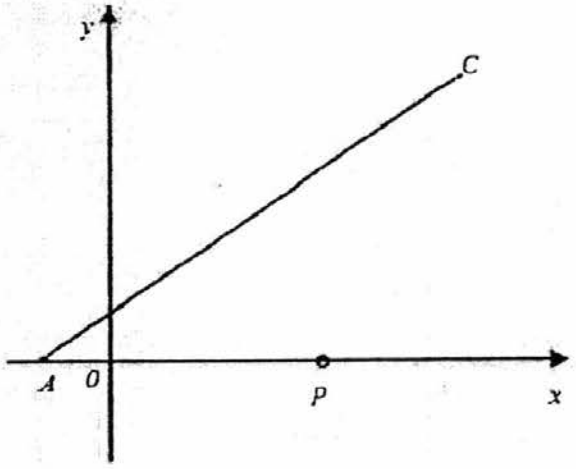
(b) ..... m [1]

(c) t = ... sec [2]

For Examiner's Use

For Examiner's Use

19 A straight line AC, whose equation $2y - 5x = 4$, is drawn below.
P is the point (4, 0).



Find

(a) the gradient of AC

(b) the coordinates of A,

(c) the equation of the line through P which is parallel to AC.

Answer (a) ..... [1]

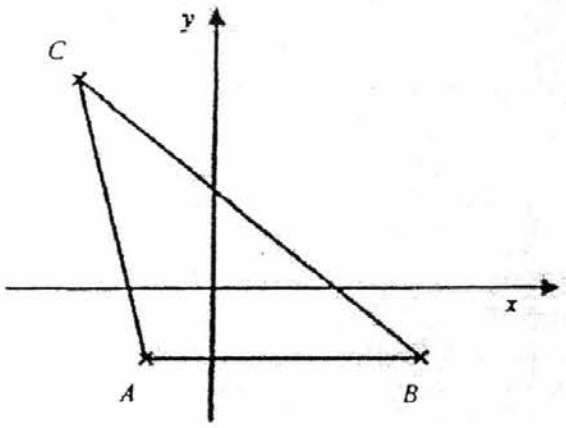
(b) (..... , ...) [1]

(c) ..... [2]

For Examiner's Use

For Examiner's Use

20 The points $A(-1,-1)$, $B(3,-1)$ and $C(-2,3)$, are shown in the diagram below.



Find

(a) the value of $\sin \angle BAC$,

(b) the area of ΔABC.

Answer *(a)* ..... [2]

(b) ..... units^2 [1]

---

21 A length of 2 cm on the map represents an actual distance of 5 km. Calculate

(i) the scale of the map, in the form of $1:n$.

(ii) the area on the map, in square centimetres, which represents an actual area of 15 km^2.

Answer *(i)* 1 : ..... [1]

(ii) ..... cm^2 [2]

For Examiner's Use

For Examiner's Use

22 The lines PQ and QR are shown in the space below.

(a) **Using compass and ruler only**, construct a parallelogram $PQRS$. [1]

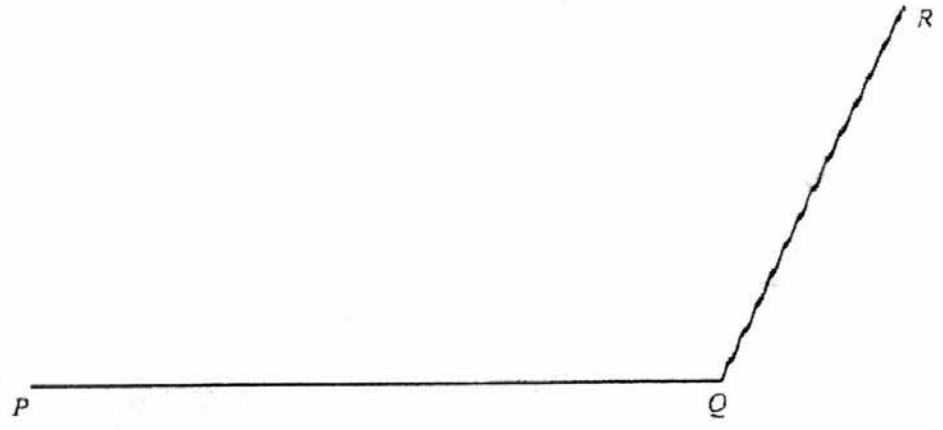
(b) Measure $\angle SPQ$.

(c) On the parallelogram, construct

(i) the perpendicular bisector of PQ, [1]

(ii) the angle bisector of $\angle RSP$. [1]

Answer



Answer (b) $\angle SPQ =$ ..... ° [1]

~ *End of Paper* ~

For Examiner's Use

For Examiner's Use

Answer ALL the questions.

1 Evaluate

(a) $3+\sqrt[3]{5.222}$, [1]

(b) $\dfrac{\sqrt{\pi \div 2}+0.967}{676\times 10^{-2}}$, [1]

Give your answers correct to 2 decimal places.

Ans:

(a) 4.73 ------- B1

(b) 0.33 ------- B1

---

2 (a) Ray bought a house at \$720 000. Five years later, he sold the house at \$640000. Calculate the loss as a percentage of his selling price. [1]

(b) Rose's weight is 51 kg after losing 15% of her original weight. What was her original weight? [1]

Ans:

(a) $\dfrac{80000}{640000}\times 100\% = 12.5\%$ ——B1

(b) $\dfrac{100}{85}\times 51 = 60$ ——B1

For Examiner's Use

For Examiner's Use

3 (a) Write down the following numbers in ascending order. [1]

$$-1.53,\ \sqrt{2.34},\ \frac{3}{2},\ -\frac{8}{5},\ 0.56,\ 0.\dot{5}\dot{6}$$

(b) Find the integer(s) which satisfy both $3+2x \le 1$ and $5x > x-12$.

Ans: [3]

(a) $-\frac{8}{5},\ -1.53,\ 0.56,\ 0.\dot{5}\dot{6},\ \frac{3}{2},\ \sqrt{2.34}$ ----------B1

(b) $3+2x \le 1$ and $5x > x-12$

$2x \le -2$ $\quad 4x > -12$

$x \le -1$ ------B1 $\quad x > -3$ ------ B1

$-3 < x \le -1$

Hence, integer $x = -2, -1$ ------- B1 (both correct)

---

4 (a) Simplify $(2x^2)^2 \times \left(-\frac{4}{3}x\right)^n \div \left(\frac{xy^2}{-4}\right)^2$. [2]

(b) The reciprocal of 5^{-6} is 5^{2p}. Write down the value of p. [1]

Ans:

(a) $(2x^2)^2 \times \left(-\frac{4}{3}x\right)^n \div \left(\frac{xy^2}{-4}\right)^2$

$= 4x^4 \div \left(\frac{x^2y^4}{16}\right)$ ----------M1 (expand bracket correctly & power 0)

$= 4x^4 \times \frac{16}{x^2y^4}$

$= \frac{64x^2}{y^4}$ ----------B1

(b) $5^{2p} = 5^6$

$p = 3$ ----------B1

For Examiner's Use

For Examiner's Use

5 y is directly proportional to x^3. It is known that $y = 8$ for a particular value of x. Find the value of y when this value of x is doubled. [3]

Ans:

$y = kx^3$ ------B1

$8 = kx^3$

$k = \frac{8}{x^3}$ --------M1

$y = \frac{8}{x^3}(2x)^3 = 64$ -----A1

---

6 Written as a product of its prime factors, $300 = 2^2 \times 3 \times 5^2$.

(a) Write 72 as a product of its prime factors. [1]

(b) Using the above results, find

(i) the highest common factor of 72 and 300, [1]

(ii) the smallest positive integer k such that $72k$ is a perfect cube, [1]

(iii) the smallest integer that is a multiple of both 72 and 300. [1]

Ans:

(a) $72 = 2^3 \times 3^2$ -----B1

(b) (i) $HCF = 2^2 \times 3 = 12$ ------B1

(ii) $k = 3$ ------B1

(iii) $LCM = 2^3 \times 3^2 \times 5^2 = 1800$ ------B1

No marks awarded for (b) if answers are not obtained from prime factorisation method

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7 (i) Factorise $9x^2 - 1$. [1]

(ii) Use the result in part (i) to express 899 as a product of its prime factors. [2]

Ans:

(i) $9x^2 - 1 = (3x + 1)(3x - 1)$ ----------B1

(ii) $899 = 9(10)^2 - 1 = (3(10) + 1)(3(10) - 1)$ ---------- B1

$899 = (31)(29)$ ---------- B1

No marks awarded if not using (i)

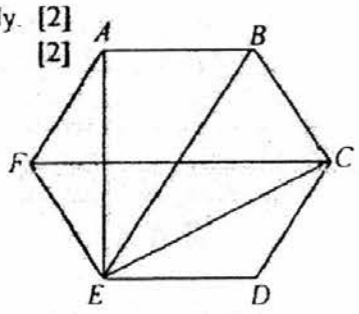
For Examiner's Use

For Examiner's Use

8 The diagram shows a regular hexagon $ABCDEF$.

(a) Find $\angle BCE$, showing your working clearly. [2]

(b) Show that ΔFEC is congruent to ΔBCE. [2]



Ans:

(a) $\angle CDE = \dfrac{4 \times 180}{6} = 120°$ — M1

$\angle DCE = (180 - 120) \div 2 = 30°$

$\angle BCE = 120 - 30 = 90°$ -----A1

(b) $FE = BC$ (regular hexagon)

EC is the common side

$\angle FEC = \angle BCE = 90°$

$\Delta FEC \equiv \Delta BCE$ (SAS) o.e. proof

-1 m for each incorrect statement

All 3 correct facts and Reasons ---- B2

For Examiner's Use

9 The lifetime of a sample of 120 brand *A* batteries is given in the table below.

Lifetime (t hours)	$0 < t \leq 30$	$30 < t \leq 60$	$60 < t \leq 90$	$90 < t \leq 120$	$120 < t \leq 150$
Freq	16	20	33	41	10

Calculate

(a) the mean, [1]

(b) the standard deviation. [2]

The mean lifetime of a sample of 120 brand *B* batteries is 78.0 hours and the standard deviation is 40.2 hours.

(c) Compare, briefly, the results for the two brands of batteries. [2]

Ans:

(a) mean = $9270 \div 120 = 77.25$ -----A1

(b) s.d. = $\sqrt{\frac{864000}{120} - (77.25)^2} = 35.1$ (3sf) -----M1, A1

(c) Brand *B* batteries have higher mean lifetimethan Brand *A* batteries. Brand *A* batteries has lower standard deviation than Brand *B* batteries. -----B2 (compare mean & Standard dev)

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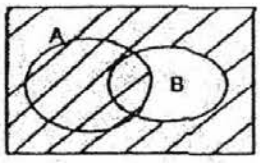
10 **(a)** On the Venn diagram shown in answer space, shade the region, $A \cup B$. [1]

Answer (a)



ξ ———B1

(b) $\xi = \{ x: x \text{ is an integer and } 3 < x \leq 30 \}$

$P = \{x: x \text{ is a prime number}\}$

$R = \{x: x \text{ is an integer ending in 3}\}$

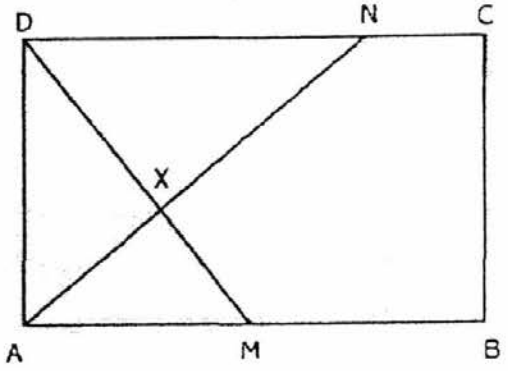
Find $n(P \cap R)$

Ans: $n(P \cap R) = 2$ ----------A1

For Examiner's Use

For Examiner's Use

11 $ABCD$ is a rectangle. M is the mid-point of AB and N is a point on CD. MD and AN meet at X.



(a) Prove that ΔAXM and ΔNXD are similar. [2]

In ΔAXM and ΔNXD,
$\angle AXM = \angle NXD$ (vert. opp $\angle s$)
$\angle XAM = \angle XND$ (alt. $\angle s$, $DN // AM$)
$\angle XMA = \angle XDN$ (alt. $\angle s$, $DN // AM$)
Hence ΔAXM and ΔNXD are similar

2 m for any 2 conditions with correct reasons stated and concluding statement.
-1 m for every incorrect reason or statement

(b) Given that $DN = 3NC$, find the value of

(i) $\dfrac{area\ of\ \Delta NXD}{area\ of\ \Delta AXM}$, [1]

(ii) $\dfrac{area\ of\ \Delta AXD}{area\ of\ ABCD}$. [2]

(i) $\dfrac{area\ of\ \Delta NXD}{area\ of\ \Delta AXM} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$ -----A1

(ii) $\dfrac{area\ of\ \Delta AXD}{area\ of\ ABCD}$

$= \dfrac{area\ of\ \Delta AXD}{4 \times area\ of\ \Delta AMD}$ ----------M1

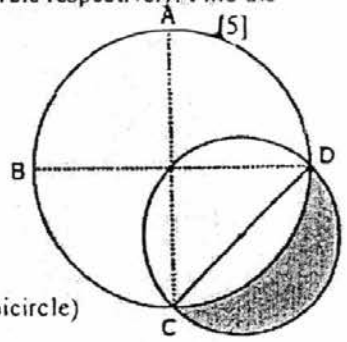
$= \dfrac{1}{4} \times \dfrac{XD}{MD}$

$= \dfrac{1}{4} \times \dfrac{3}{5} = \dfrac{3}{20}$ ---------- A1

For Examiner's Use

For Examiner's Use

12 In the diagram, $ABCD$ is a square whose diagonals are 3 cm each. BD and CD are the diameters of the bigger and smaller circle respectively. Find the area and perimeter of the shaded region. [5]



Ans:

$BD = 3cm$

$CD = \sqrt{1.5^2 + 1.5^2} = 2.12132cm$ ---M1

Area of shaded region

$= \frac{1}{2} \times \pi \times \left(\frac{\sqrt{4.5}}{2}\right)^2 -$ ----M1 (find A of semicircle)

$\left[\frac{1}{4} \times \pi \times 1.5^2 - \frac{1}{2} \times 1.5 \times 1.5\right]$

$= \frac{9}{16}\pi - \left(\frac{9}{16}\pi - \frac{9}{8}\right)$

$= 1\frac{1}{8}$ or 1.125 cm^2 ---- A1

Perimeter of shaded region

$= \frac{1}{4} \times 2 \times \pi \times 1.5 + \frac{1}{2} \times 2 \times \pi \times \frac{\sqrt{4.5}}{2}$ ------M1

$= 5.69$ cm -----A1

13 The approximate time for light to travel 1 metre is 3.30 nanoseconds.

(a) Express the time taken, in seconds, for light to travel 10 metres. Give your answer in standard form. [2]

(b) Find the distance travelled, correct to the nearest kilometre, in 1 second. [2]

Ans:

(a) Time taken $= 10 \times 3.30 \times 10^{-9}$ ------M1

$= 3.30 \times 10^{-8}$ seconds ----------A1

(b) 3.30×10^{-9} seconds ---------- 1 m

1 second ---------- $\frac{1}{3.30 \times 10^{-9}}$ m

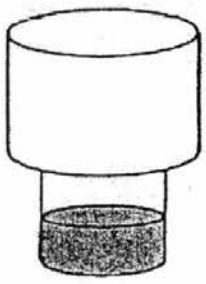
$= \frac{1}{3.30 \times 10^{-9}} \div 1000$ km -----M1

$= 303030$ km -----A1

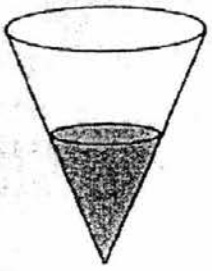
For Examiner's Use

For Examiner's Use

14 Water is poured into each of the containers below at a constant rate of 250 ml per second. The graphs show the depth d of the water varies with time t. Match the container to its corresponding graph.

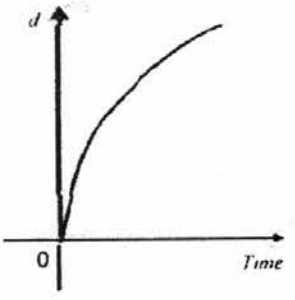


Container A

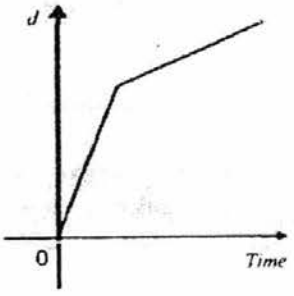


Container B

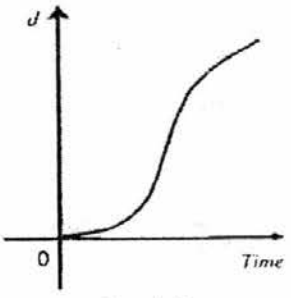
Container C



Graph 1



Graph 2



Graph 3

Answer

Container A — Graph 2 -----B1

Container B — Graph 1 ------B1

Container C — Graph 3 -----B1 [3]

For Examiner's Use

For Examiner's Use

15 Mrs Lim wants to buy some antiseptic germicide. Below are the rates of the germicide sold at 2 different shops. Which shop should Mrs Lim buy the germicide from? Explain your choice. [3]

Shop A
$13.50 for each 1-litre bottle.
Buy 2 get 1 free now!

Shop B
Usual Price : $18.50 for a 2-litre bottle.
5% discount NOW!

Ans:

For Shop A :
3 litres ------ $27.00
1 litre ------ $9.00 **(M1)**

For Shop B :
2 litres ----- 95% × $18.50 = $17.58
1 litre ----- $8.79 **(M1)**

Hence, Mrs Lim should buy from **Shop B** as it is cheaper. **(B1)**

If students compared 3 litres from Shop A and 2 litres from Shop B, Marks are not awarded since the comparison is unfair.

For Examiner's Use

For Examiner's Use

16 (a) Express x^2-4x+3 in the form $(x-p)^2+q$. [2]

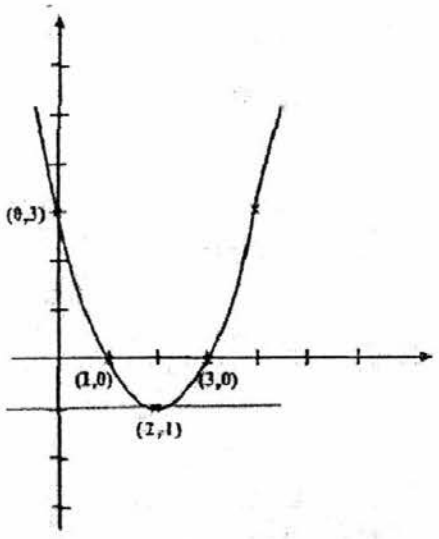
(b) The function of a quadratic curve is $y=x^2-4x+3$.
State the the coordinates of its turning point. [1]

(c) Sketch the graph of $y=x^2-4x+3$ for on the axes in the answer space, indicating clearly the intercepts and turning point. [2]

Ans:

(a) x^2-4x+3
$=x^2-2x(2)+2^2-2^2+3$ ----M1
$=(x-2)^2-1$ -----A1

(b)
$(2,-1)$ -----B1



1m for correct shape & smooth curve
1m for correct x intercepts & y-intercepts & turning point

For Examiner's Use

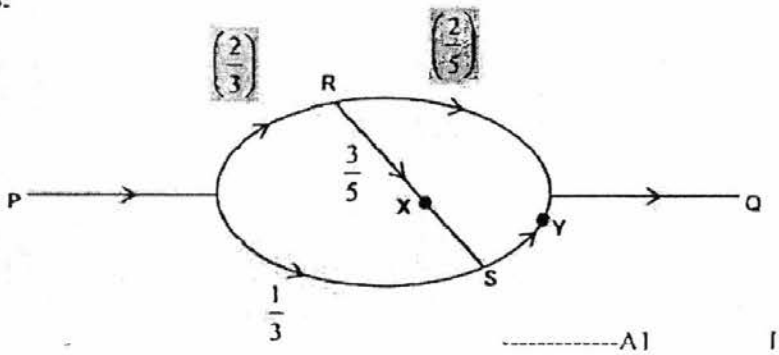
For Examiner's Use

17 The diagram shows a one-way road system between two junctions P and Q.

The probability that a car travels along PS is $\frac{1}{3}$. At junction R, the probability that the car travels along RS is $\frac{3}{5}$.

(a) Complete the diagram below.

Ans:



----------A1 [1]

(b) Find the probability that the car travelling from P to Q

(i) will pass point X. [1]

(ii) will pass point Y. [2]

Ans:

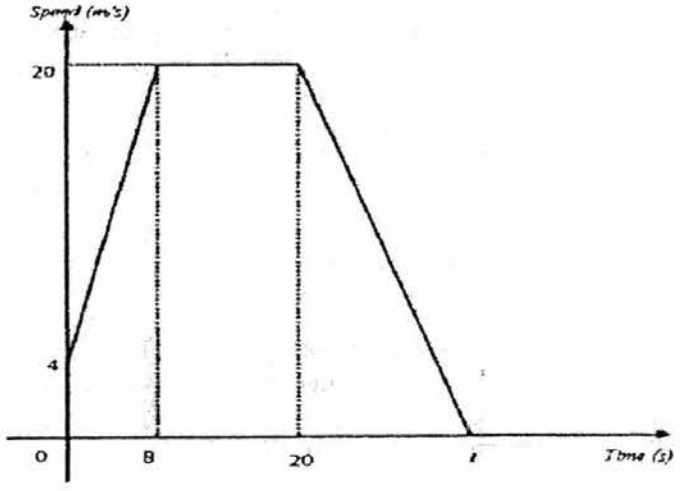
(i) $P(\text{pass X}) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$ ——A1

(ii) $P(\text{pass Y}) = \frac{2}{3} \times \frac{3}{5} + \frac{1}{3} = \frac{11}{15}$ ——M1, A1

For Examiner's Use

For Examiner's Use

18 The diagram is the speed-time graph of a car's journey.



Calculate

(a) the speed of the car when $t = 3$, [2]

(b) the total distance travelled in the first 20 seconds of the journey, [1]

(c) the value of t, if the retardation is twice that of the acceleration. [2]

(a) $\frac{20-4}{8-0} = \frac{20-v}{8-3}$ ————M1

$2 = \frac{20-v}{5}$

$v = 10$ ----------A1

(b) $\frac{1}{2} \times (4+20) \times 8 + 12 \times 20 = 336\,\text{m}$ ----A1

(c) $\frac{20-0}{t-20} = 4$ ————M1

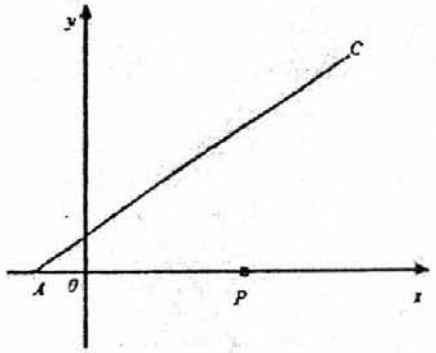
$t - 20 = 5$

$t = 25$ ———— A1

For Examiner's Use

For Examiner's Use

19 A straight line AC, whose equation $2y-5x=4$, is drawn below. P is the point $(4,0)$.



Find

(a) the gradient of AC, [1]

(b) the coordinates of A, [1]

(c) the equation of the line through P which is parallel to AC. [2]

Ans:

(a) $2y-5x=4$

$2y=5x+4$

$y=2.5x+2$

Gradient $=2.5$ ----------B1

(b) $0=2.5x+2$

$x=-\frac{4}{5}$ *or* -0.8

$A=\left(-\frac{4}{5},0\right)$ ---------- B1

(c) $y=2.5x+c$

$0=2.5(4)+c$ ------M1 (Subst pt)

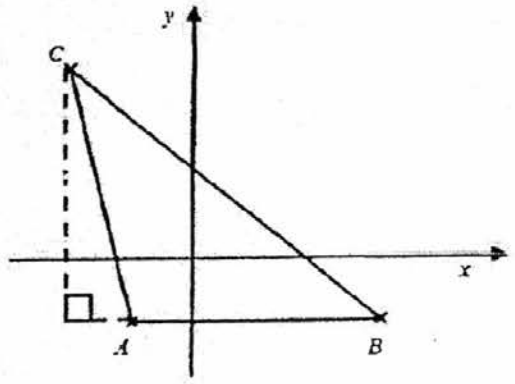
$c=-10$

$y=2.5x-10$ -----B1

For Examiner's Use

For Examiner's Use

20 The points $A(-1,-1)$, $B(3,-1)$ and $C(-2,3)$ are shown in the diagram below.



Find

(a) the value of $\sin \angle BAC$, [2]

(b) the area of ΔABC. [1]

Ans:

(a) $AC = \sqrt{(-1+2)^2 + (-1-3)^2} = \sqrt{17}$ ------ M1

$\sin \angle BAC = \dfrac{4}{\sqrt{17}} = 0.970$ -------A1

Mtd 1

(b) Area of ΔABC

$= \dfrac{1}{2} \times 4 \times 4 = 8$ units^2 ---B1

or

Mtd 2

Area of ΔABC

$= \dfrac{1}{2} \times AB \times AC \times \sin \angle BAC$

$= \dfrac{1}{2} \times 4 \times \sqrt{17} \times \dfrac{4}{\sqrt{17}} = 8$ units^2

----B1

---

21 A length of 2 cm on the map represents an actual distance of 5 km. Calculate

(i) the scale of the map, in the form of $1:n$. [1]

(ii) the area on the map, in square centimetres, which represents an actual area of 15 km^2. [2]

Ans:

(i) 2 cm ------ 5 km

1 cm ------ 2.5 km = 2500 m = 250 000 cm

Hence, scale = 1: 250000 ----------B1

(ii) 5 km ------ 2 cm

1 km ------ $\dfrac{2}{5}$ cm

1 km^2 ------ $\dfrac{4}{25}$ cm^2 --------M1 (squaring)

15 km^2 ------ $\dfrac{4}{25} \times 15$ cm^2 $= 2\dfrac{2}{5}$ *or* 2.4 cm^2 ----A1

For Examiner's Use

For Examiner's Use

22 The lines PQ and QR are shown in the space below.

(a) Using **compass only**, construct a parallelogram $PQRS$. [1]

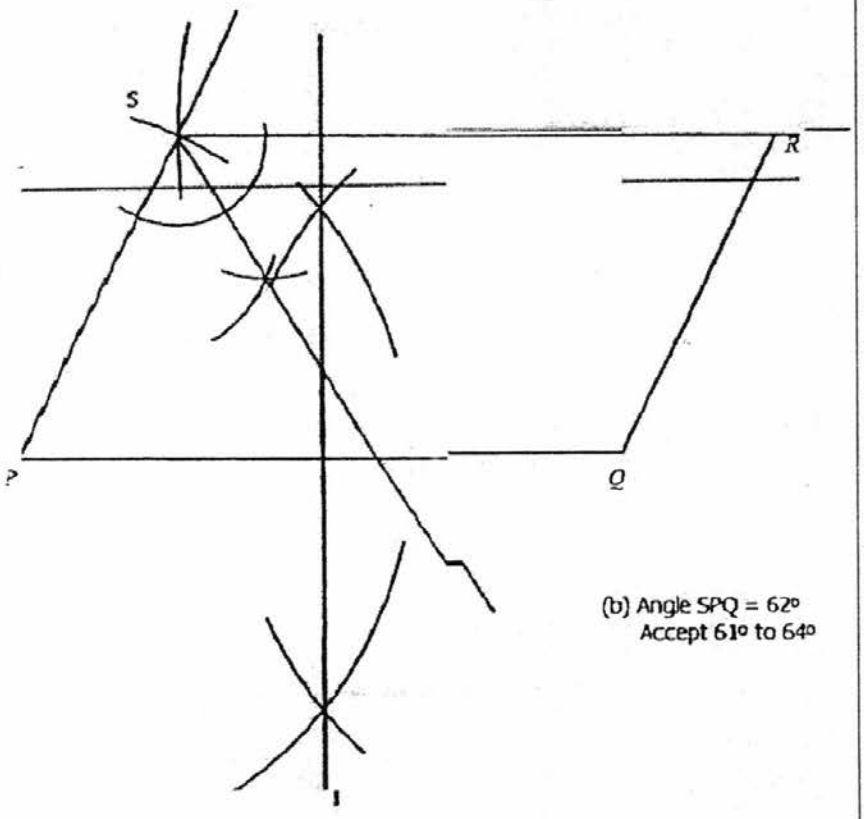
(b) Measure $\angle SPQ$. [1]

(c) On the parallelogram, construct

(i) the perpendicular bisector of PQ, [1]

(ii) the angle bisector of $\angle RSP$. [1]

Answer



(b) Angle SPQ = 62°
Accept 61° to 64°

No marks awarded for (a), (c)(i) and (ii) if construction arcs are not clearly shown.

* For perpendicular bisector (c)(i), arcs MUST be shown above and below the line PQ

1 (a) Factorise $x^3 - xy^2 + x^2y - y^3$ completely. [3]

(b) Make a the subject of the formula $a^2 = k + ra^2$. [2]

(c) Express $\frac{x+2}{x-2} + \frac{1-x}{4-2x}$ as a single fraction in its simplest form. [3]

(d) (i) Express $x^2 - 6x - 2$ in the form $(x+a)^2 + b$. [1]

(ii) Hence, solve the equation $x^2 - 6x - 2 = 0$, giving your answers correct to two decimal places. [3]

2 (a) A triathlon is a competition involving the completion of three continuous and sequential endurance disciplines – swimming, cycling and running.

The Olympic course requires participants to swim 1.5 km, cycle 40 km and run 10 km. The time taken to transit between the stages is also factored into the overall timing.

Adam took part in the Singapore Triathlon 2015. The following table shows his results.

Stages	Swim	Transit 1	Cycle	Transit 2	Run	Overall
Time	x mins	1.5 mins	$4x$ mins	1.5 mins	y mins	2 h 43 mins

(i) Form an equation in x and y based on the information given in the table above, and show that it reduces to $y = 160 - 5x$. [1]

(ii) Given that his average cycling speed, in km/min, is three times that of his average running speed, form another equation in x and y. [1]

(iii) Solve these two equations to find the value of x and of y. [3]

(b) At a graduation party, it was decided that every student will buy a gift for each one of their classmates and teachers. If the total number of gifts at the party was 1050 and that there are 6 teachers, find the number of students in the class. [3]

3 (a) The Singapore Government announced changes to the individual income tax rates in 2015, to be effected from 2017 onwards. The following table shows the revised individual income tax rate structure based on the yearly chargeable income.

Yearly chargeable income	Rate (%)	Gross Tax Payable ($)
On the first 20,000	0	0
On the next 10,000	2	200
On the first 30,000	-	200
On the next 10,000	3.50	350
On the first 40,000	-	550
On the next 40,000	7	2,800
On the first 80,000	-	3,350
On the next 40,000	11.5	4,600
On the first 120,000	-	7,950
On the next 40,000	15	6,000
On the first 160,000	-	13,950
On the next 40,000	18	7,200
On the first 200,000	-	21,150
On the next 40,000	19	7,600
On the first 240,000	-	28,750
On the next 40,000	19.5	7,800
On the first 280,000	-	36,550
On the next 40,000	20	8,000
On the first 320,000	-	44,550
In excess of 320,000	22	

(i) Gerrard's yearly chargeable income in 2017 is projected to be $42 000. Calculate his gross tax payable. [1]

(ii) John calculated from his chargeable income that he will have to pay a gross tax of $9750. Calculate his monthly chargeable income. [2]

(b) Jordan wants to invest $100 000 in a bank for 5 years. The following shows the rates advertised by two banks.

Bank A: Simple interest of 2.5% per annum.
Bank B: Compound interest of 2.3% per annum, compounded half-yearly.

Justify, with calculations, which bank will provide a higher amount of returns for his investment. [4]

(c) Daniel owns a smartphone with 16 gigabytes storage capacity. 45% of the phone's capacity was used up by his existing applications and pictures. Given that the average file size of a picture taken on his phone is 1.8 megabytes, calculate the number of pictures he can take with the remaining capacity. [4]

4 (a) The first four terms in a sequence are 78, 73, 68 and 63.

(i) Find an expression, in terms of n, for the nth term, T_n, of this sequence. [2]

(ii) Evaluate T_{21}. [1]

(b) Study the pattern below and answer the questions that follow based on the pattern.

Line	Equation
Line 1	$\frac{1}{2} = \frac{1}{1} - \frac{1}{2}$
Line 2	$\frac{1}{6} = \frac{1}{2} - \frac{1}{3}$
Line 3	$\frac{1}{12} = \frac{1}{3} - \frac{1}{4}$
Line 4	$\frac{1}{20} = \frac{1}{4} - \frac{1}{5}$

(i) Write down the equation corresponding to line k in terms of k. [1]

(ii) Justify, with appropriate mathematical working, why $\frac{1}{340}$ will not appear on the left hand side of the equation in any of the lines. [2]

(iii) Find the sum of the following series without the use of a calculator,

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}.$$

Hence, find the sum of the following series,

$$1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \frac{1}{36} + \frac{1}{45}.$$ [4]

5 **Answer the whole of this question on a piece of graph paper.**

The flight path of a football can be modelled by the equation $y = -12x^2 + 45x$, where y metres is the height of the football x seconds after it was kicked.

Some corresponding values of x and y are given in the table below.

x	0	0.5	1	1.5	2	2.5	3	3.5
y	0	19.5	33	40.5	42	p	27	10.5

(a) Calculate the value of p. [1]

(b) Using a scale of 4 cm to represent 1 second, draw a horizontal axis for $0 \le x \le 4$.
Using a scale of 1 cm to represent 2 metres, draw a vertical axis for $0 \le y \le 44$.

On your axes, plot the points given in the table and draw the graph of $y = -12x^2 + 45x$ for $y \ge 0$. [3]

(c) Using your graph, find

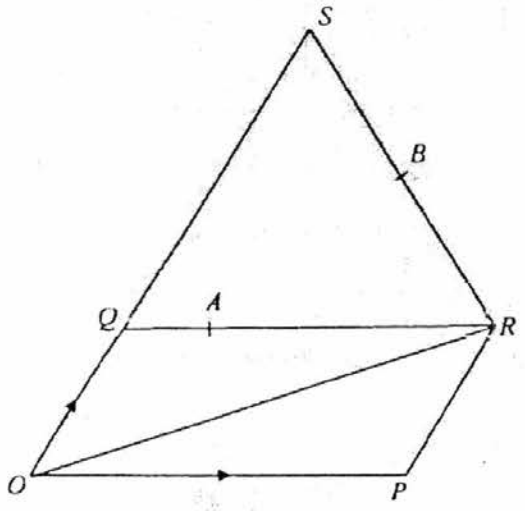
(i) the maximum height of the football reached. [1]

(ii) the time duration the ball was in the air. [1]

(d) (i) By drawing a tangent, find the gradient of the curve at (3, 27). [2]

(ii) What does the sign of the gradient found in **(d)(i)** mean? [1]

6 In the diagram below, $OPRQ$ is a parallelogram. S is a point on OQ produced such that $OS = 3OQ$. A is a point on QR such that $QA = \frac{1}{3}AR$. B is the mid-point of RS.



It is given that $\overrightarrow{OP} = 2\mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

(a) Express the following, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

(i) $\overrightarrow{OR}$ [1]

(ii) $\overrightarrow{RS}$ [1]

(iii) $\overrightarrow{QB}$ [1]

(iv) $\overrightarrow{QA}$ [2]

(b) Do the points O, A and B lie on a straight line?
Justify your answer using vectors. [2]

(c) Find the value of

(i) $\dfrac{\text{Area of } \Delta OAS}{\text{Area of } \Delta OBS}$, [1]

(ii) $\dfrac{\text{Area of } \Delta BAR}{\text{Area of } \Delta BOX}$, where X is the point on SR and OP produced. [1]

7 The ticket sales for four movies – Blue Man, Sunken Ship, Star Force and Revengers, on a particular day at three different cinemas, is shown in Table I.

Cinema	Blue Man	Sunken Ship	Star Force	Revengers
Tachay	1200	1000	1500	1300
Wahs	1300	1200	1100	1200
Silver Village	1000	1100	1600	1400

Table I

The information in Table I can be represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 1200 & 1000 & 1500 & 1300 \\ 1300 & 1200 & 1100 & 1200 \\ 1000 & 1100 & 1600 & 1400 \end{pmatrix}.$$

The price of a movie ticket in the three cinemas are given in Table II.

Cinema	Tachay	Wahs	Silver Village
Price of a Movie Ticket	\$10	\$11	\$9.50

Table II

(a) (i) Represent the information in Table II by a 1×3 matrix B. [1]

(ii) Evaluate the matrix $\mathbf{T} = \mathbf{BA}$. [1]

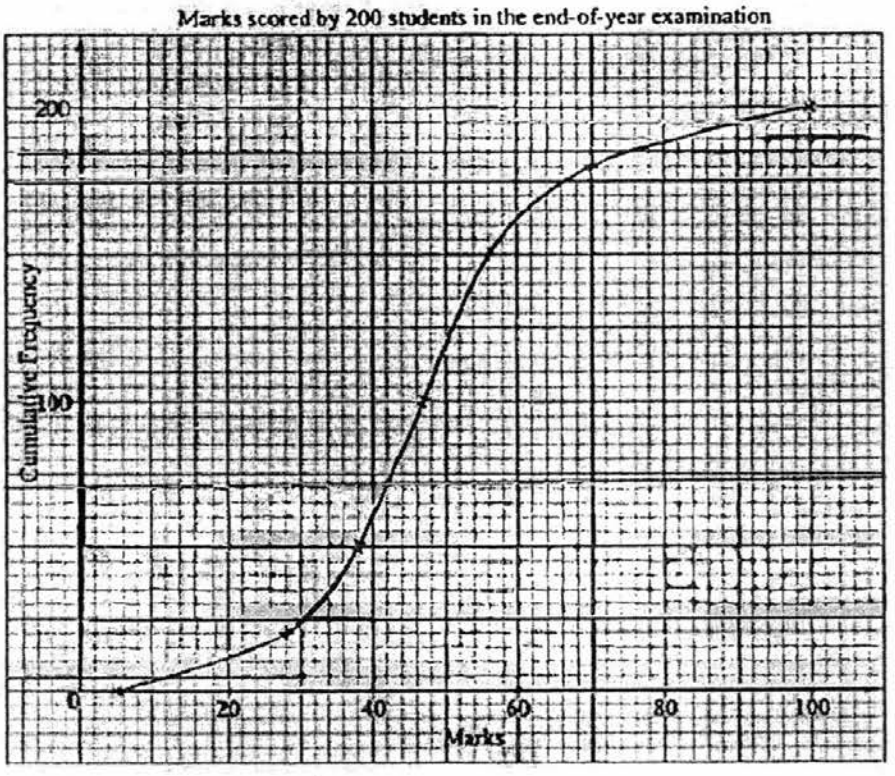
(iii) State what the elements of T represent. [1]

(b) (i) Write down a matrix C such that the elements in AC represent the total number of tickets sold at Tachay, Wahs and Silver Village on that day respectively.

Evaluate the matrix $\mathbf{K} = \mathbf{AC}$. [2]

(ii) Hence, using matrix multiplication, find the total amount of money each cinema earned from the ticket sales on that day. [2]

8 The cumulative frequency graph below represents the end-of-year examination marks of a group of 200 students for Mathematics.



Use the graph to estimate

(i) the median and the interquartile range, [3]

(ii) the number of students who scored 30 marks or less, [1]

(iii) the number of students who scored more than 75 marks, [1]

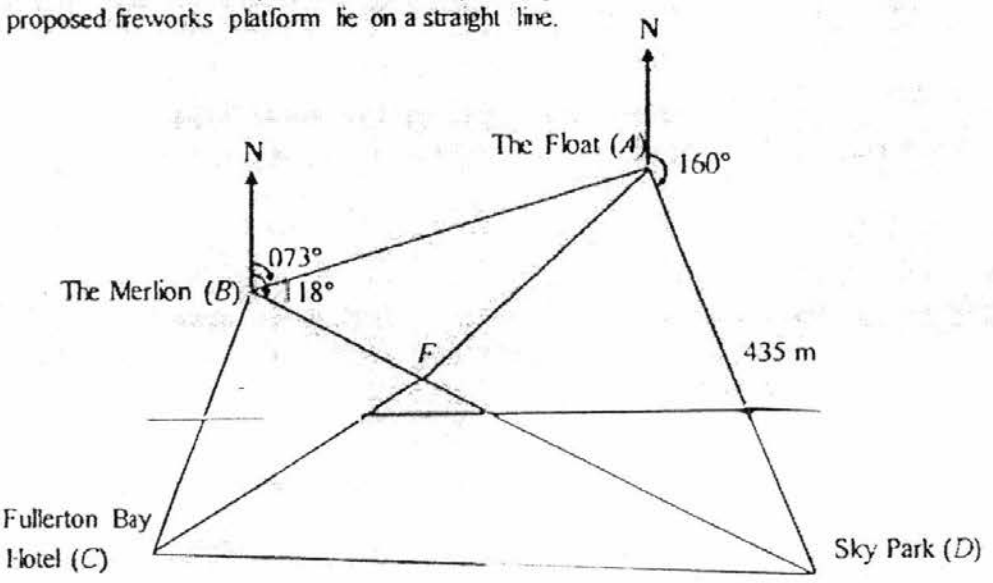
(iv) the passing mark if 65% of the students passed the examination. [1]

(v) Two students are selected randomly. Calculate the probability that one student scored 30 marks or less and the other scored more than 75 marks. [2]

(vi) Draw a box and whisker diagram to represent the data above. [2]

9 The diagram below shows a simplified map of the Marina Bay area.

In the preparations for the upcoming National Day Parade, it is proposed that a platform to store and shoot fireworks off be constructed at point F. The Float, The Merlion, Fullerton Bay Hotel and Sky Park are popular places to view the fireworks, and are labelled as A, B, C and D respectively. The Merlion, Sky Park and the proposed fireworks platform lie on a straight line.



It is given that the bearing of The Float and Sky Park from The Merlion is 073° and 118° respectively. The Sky Park is 435 metres away on a bearing of 160° from The Float.

(i) Show that the distance between the Merlion and the Sky Park is 614.34 m. [2]

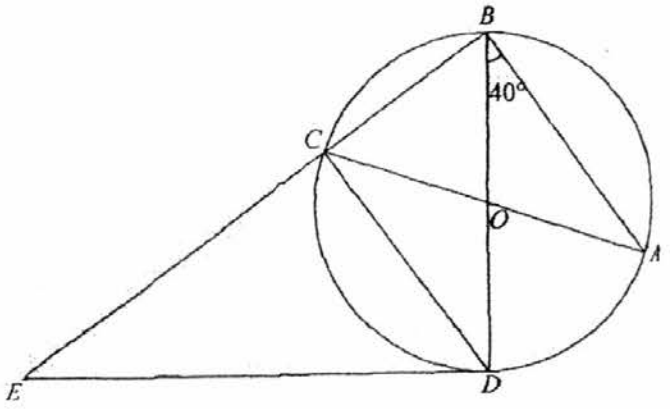
(ii) Given further that the Fullerton Bay Hotel is 650 m away on a bearing of 275° from the Sky Park, and that the area of triangle FCD is 54600 m^2, find the distance of F from the Sky Park. [3]

(iii) Find the distance of F from The Float. [3]

(iv) As fireworks are explosives, it is required by law that the fireworks platform have a safety radius of 200 m. Determine if F is a suitable place to place the platform. [2]

(v) The Sky Park is on top of the Marina Bay Sands Hotel. Given that the angle of elevation of the Sky Park from The Float is 24.7°, find the angle of depression of the base of The Merlion from the Sky Park. [3]

10 (a)



In the diagram above, the points A, B, C and D lie on a circle of centre O. The line DE touches the circle at point D. BCE, BOD and AOC are straight lines.

Given that $\angle ABD = 40°$, find, stating your reasons clearly,

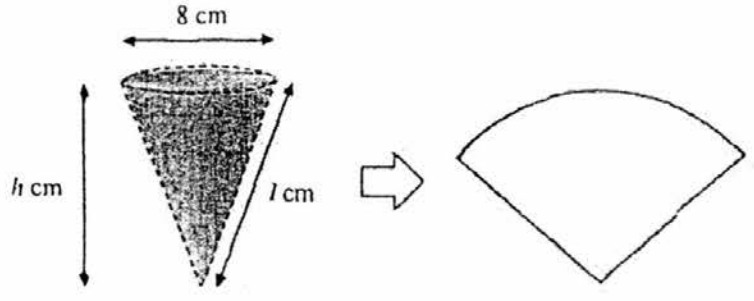
(i) $\angle ACD$, [1]

(ii) $\angle AOD$, [1]

(iii) $\angle CBD$, [1]

(iv) $\angle CED$. [2]

(b) Conical paper cups are commonly used at water dispenser stations. One such cup can be modelled by the diagram below. When the cup is cut along one of its slant height, l cm, a sector is obtained.



Find the angle subtended by the arc at the centre in the sector if the volume of water the cup can hold is 120 cm^3. Leave your answer in radians. [6]

End of Paper

MARKING SCHEME

1(a)	$x^3 - xy^2 + x^2y - y^3$ $= x(x^2 - y^2) + y(x^2 - y^2)$ $= (x^2 - y^2)(x + y)$ $= (x - y)(x + y)(x + y)$ OR $(x - y)(x + y)^2$	 B1 B1 B1
1(b)	$a^2 = k + ra^2$ $a^2 - ra^2 = k$ $a^2(1 - r) = k$ $a^2 = \frac{k}{1 - r}$ $a = \pm\sqrt{\frac{k}{1 - r}}$	 M1: Factorizing a^2 A1

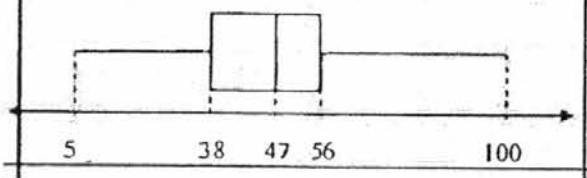
1(c)	Method 1 $\frac{x+2}{x-2}+\frac{1-x}{4-2x}$ $=\frac{x+2}{x-2}+\frac{1-x}{2(2-x)}$ $=\frac{x+2}{x-2}-\frac{1-x}{2(x-2)}$ $=\frac{2(x+2)-(1-x)}{2(x-2)}$ $=\frac{3x+3}{2(x-2)}$ *or* $\frac{3(x+1)}{2(x-2)}$ Method 2 $\frac{x+2}{x-2}+\frac{1-x}{4-2x}$ $=\frac{(x+2)(4-2x)+(1-x)(x-2)}{(x-2)(4-2x)}$ $=\frac{-2x^2+8+(-x^2+3x-2)}{2(x-2)(2-x)}$ $=\frac{-3x^2+3x+6}{-2(x-2)^2}$ $=\frac{-3(x^2-x-2)}{-2(x-2)^2}$ $=\frac{3(x-2)(x+1)}{2(x-2)^2}$ $=\frac{3(x+1)}{2(x-2)}$	M1: Simplifying denominator M1: Combine into a single fraction A1 M1: Combine into a single fraction M1: Factorization of numerator and denominator A1
1(d)(i)	x^2-6x-2 $=(x-3)^2-11$	B1
1(d)(ii)	$x^2-6x-2=0$ $(x-3)^2-11=0$ $(x-3)^2=11$ $x-3=\pm\sqrt{11}$ $x=3\pm\sqrt{11}$ $x=6.32$ or $x=-0.32$	M1 A2

2(a)(i)	$x+1.5+4x+15+y=163$ $y=160-5x$	B1
2(a)(ii)	$\frac{40}{4x}=3\times\frac{10}{y}$	B1
2(a)(iii)	$x+1.5+4x+15+y=163$ $y=160-5x$ **(1)** $\frac{40}{4x}=3\times\frac{10}{y}$ $\frac{10}{x}=\frac{30}{y}$ $y=3x$ **(2)** Sub (1) into (2) $3x=160-5x$ $8x=160$ $x=20$	M1: Substitution or elimination A1
	$y=60$	A1
2(b)	Let the number of students be x. Number of gifts each student will buy: $x-1+6$ i.e. $x+5$ $x(x+5)=1050$ $x^2+5x-1050=0$ $(x-30)(x+35)=0$ $x=30$ or $x=-35$ (rejected) The number of students is 30.	B1 M1 A1
3(a)(i)	Gross tax payable $=550+\frac{7}{100}\times(42000-40000)$ $=\$690$	B1
3(a)(ii)	Let his yearly chargeable income be $\$120000+x$. $\frac{15}{100}\times x=9750-7950$ $0.15x=1800$ $x=12000$ His monthly chargeable income $=\frac{120000+12000}{12}$ $=\$11000$	B1 B1

3(b)	Bank A $I = \frac{PRT}{100}$ $= \frac{100000 \times 2.5 \times 5}{100}$ $= 12500$ Total amount: \$112500 Bank B $T = P\left(1 + \frac{R}{100}\right)^n$ $= 100000\left(1 + \frac{\frac{2.3}{2}}{100}\right)^{10}$ $= \$112113.75$ Since Bank A's total amount at the end of 5 years is higher than that of Bank B's, Bank A has a higher amount of returns for his investment.	M1 M1: $R = \frac{2.3}{2}$ M1: $n = 10$ A1: Compare total or interest and conclude that Bank A has higher returns.
3(c)	$16\ \text{Gb} = 16 \times 10^9\ \text{b}$ $1.8\ \text{Mb} = 1.8 \times 10^6\ \text{b}$ Remaining capacity $= \frac{55}{100} \times 16 \times 10^9 = 8.8 \times 10^9\ \text{b}$ Number of pictures $= \frac{8.8 \times 10^9}{1.8 \times 10^6}$ $= 4888$ (round down)	M1: Making same base counting unit M1 DM1 A1
4(a)(i)	$T_n = a + (n-1)d$ $= 78 + (n-1)(-5)$ $= 83 - 5n$	B2
4(a)(ii)	$T_{21} = 83 - 5 \times 21 = -22$	B1

4(b)(i)	$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$	B1
4(b)(ii)	$k(k+1) = 340$ $k^2 + k - 340 = 0$ $k = 17.9$ or $k = -18.9$ No integer solutions for k, so $\frac{1}{340}$ will not appear on the left hand side of the equation in any of the lines.	B1 B1
4(b)(iii)	$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$ $= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{9} - \frac{1}{10}$ $= 1 - \frac{1}{10} = \frac{9}{10}$ $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \frac{1}{36} + \frac{1}{45}$ $= 2\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}\right)$ $= 2\left(\frac{9}{10}\right)$ $= 1\frac{4}{5}$	M1: Substituting and cancelling A1 M1 A1
6(a)(i)	$\overrightarrow{PR} = \overrightarrow{OQ} = \mathbf{q}$ $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = 2\mathbf{p} + \mathbf{q}$	 B1
6(a)(ii)	$\overrightarrow{OS} = 3\overrightarrow{OQ} = 3\mathbf{q}$ $\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS} = -\overrightarrow{OR} + \overrightarrow{OS} = -2\mathbf{p} + 2\mathbf{q}$	 B1
6(a)(iii)	$\overrightarrow{RB} = \frac{1}{2}\overrightarrow{RS} = -\mathbf{p} + \mathbf{q}$ $\overrightarrow{OB} = \overrightarrow{OR} + \overrightarrow{RB} = \mathbf{p} + 2\mathbf{q}$	 B1
6(a)(iv)	$\overrightarrow{QA} = \frac{1}{4}\overrightarrow{QR}$ $\overrightarrow{QA} = \frac{1}{2}\mathbf{p}$ $\overrightarrow{OA} = \frac{1}{2}\mathbf{p} + \mathbf{q}$	M1 A1

6(b)	$\overrightarrow{OB} = 2\overrightarrow{OA}$ Hence, $\overrightarrow{OB}$ and $\overrightarrow{OA}$ are parallel vectors. Since $\overrightarrow{OB}$ and $\overrightarrow{OA}$ share a common point O, the three points O, A, B lie on a straight line	B1 B1
6(c)(i)	$\frac{\text{Area of } \Delta OAS}{\text{Area of } \Delta OBS} = \frac{OA}{OB} = \frac{1}{2}$	B1
6(c)(ii)	$\frac{\text{Area of } \Delta BAR}{\text{Area of } \Delta BOX} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	B1
7(a)(i)	$(10 \quad 11 \quad 9.5)$	B1
7(a)(ii)	$\mathbf{T} = \mathbf{AB}$ $= (10 \quad 11 \quad 9.5)\begin{pmatrix} 1200 & 1000 & 1500 & 1300 \\ 1300 & 1200 & 1100 & 1200 \\ 1000 & 1100 & 1600 & 1400 \end{pmatrix}$ $= (35800 \quad 33650 \quad 42300 \quad 39500)$	B1
7(a)(iii)	The elements in T represent the total amount of money collected from the ticket sales at all three cinemas for Blue Man, Sunken Ship, Star Force and Revengers respectively.	B1
7(b)(i)	$\mathbf{C} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$	B1
	$\mathbf{K} = \mathbf{AC}$ $= \begin{pmatrix} 1200 & 1000 & 1500 & 1300 \\ 1300 & 1200 & 1100 & 1200 \\ 1000 & 1100 & 1600 & 1400 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 5000 \\ 4800 \\ 5100 \end{pmatrix}$	B1
7(b)(ii)	$\begin{pmatrix} 10 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 9.5 \end{pmatrix}\begin{pmatrix} 5000 \\ 4800 \\ 5100 \end{pmatrix} = \begin{pmatrix} 50000 \\ 52800 \\ 48450 \end{pmatrix}$ Tachay earned \$50000, Wahs earned \$52800 and Silver Village earned \$48450.	M1 A1

8(i)	Median = 47 marks Lower quartile = 38 marks Upper quartile = 56 marks IQR = 56 – 38 = 18 marks	B1 B2
8(ii)	25 students	B1
8(iii)	15 students	B1
8(iv)	42 marks	B1
8(v)	$\frac{25}{200}\times\frac{15}{199}\times 2=\frac{15}{796}$	B2 If ans wrong, award B1 for either 199 or 2.
8(vi)	Smallest = 5 marks Largest = 100 marks  	B1 (for smallest and largest) B1 (plot with correct 5 numbers)
9(i)	$\angle ABD = 118° - 73° = 45°$ $\angle BAN = 180° - 73° = 107°$ (int. $\angle$s, // lines) $\angle BAD = 360° - 160° - 107° = 93°$ ($\angle$s at a point) By Sine Rule, $\frac{BD}{\sin\angle BAD}=\frac{AD}{\sin\angle ABD}$ $BD=\frac{435}{\sin 45°}\times\sin 93°$ $BD = 614.34$ m	B1 B1
9(ii)	$\angle BDN = 180° - 118° = 62°$ (int. $\angle$s, // lines) $\angle FDC = 360° - 275° - 62° = 23°$ ($\angle$s at a point) $\frac{1}{2}\times 650\times FD\times\sin 23° = 54600$ $FD=\frac{54600\times 2}{650\times\sin 23°}$ $FD = 429.96 = 430$ m	M1 M1 A1
9(iii)	$\angle ADF = 180° - 45° - 93° = 42°$ ($\angle$s sum of Δ) By Cosine Rule, $AF^2 = AD^2 + FD^2 - 2(AD)(FD)\cos\angle ADF$ $AF=\sqrt{435^2+429.96^2-2(435)(429.96)\cos 42°}$ $AF = 310.01 = 310$ m	M1 M1 A1

9(iv)	$FB = BD - FD = 614.34 - 429.96 = 184.38$ m $FB = 184.38$ m < 200 m It is not suitable as The Merlion falls within the safety radius of the platform.	B1 B1
9(v)	Let the height of the Sky Park be h m. $\tan 24.7° = \frac{h}{435}$ $h = 435 \tan 24.7° = 200.08$ Let the angle of depression be θ. $\tan\theta = \frac{200.08}{614.34}$ $\theta = 18.0°$	M1 M1 A1
10(a)(i)	$\angle ACD = \angle ABD = 40°$ ($\angle$s in same segment)	B1
10(a)(ii)	$\angle AOD = 2 \times \angle ABD = 80°$ ($\angle$ at centre $= 2 \times \angle$ at circumference)	B1
10(a)(iii)	$\angle CBA = 90°$ ($\angle$ in semicircle) $\angle CBD = 90° - 40° = 50°$	B1
10(a)(iv)	$\angle BDE = 90°$ (tan perp rad) $\angle CED = 180° - 90° - 50° = 40°$	M1 A1
10(b)	Volume of cup $= \frac{1}{3}\pi(4)^2 h = 120$ $h = \frac{360}{16\pi} = \frac{45}{2\pi} = 7.1620$ By Pyth Thm, $l^2 = h^2 + \left(\frac{4}{2}\right)^2$ $l = \sqrt{7.1620^2 + \left(\frac{4}{2}\right)^2} = 7.4360$ Circumference of cup $= \pi l = 4\pi$ Let the angle be θ rads. $7.4360\theta = 4\pi$ $\theta = 1.59$ rads	M1 M1 M1 M1: slant height is radius of sector M1: circumference is arc length of sector A1

END