· 9

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

PRELIMINARY EXAMINATION ONE

SECONDARY FOUR

MATHEMATICS

Paper 1

13 May 2016

2 hours

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use		
Presentation		80
Unit		
Rounding off		

This document consists of **20** printed pages inclusive of cover page.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = p\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **ALL** the questions in the spaces provided.

1 Samuel makes a loss of 20 % on his cost price by selling his electric guitar at $90. Find the price he has to sell his electric guitar in order to make a profit of 160 % on the cost price.

Answer $_____ [2]

---

2 Express as a product of its prime factors $6048 = 2^5 \times 3^x \times 7^y$.

(a) Find the values of x and y.

(b) Find the smallest positive integer k such that $\frac{6048}{k}$ is a cubic number.

Answer (a) $x =$ _____, $y =$ _____ [2]

(b) $k =$ _____ [1]

---

3 Given ε = { students in a secondary school }

P = { students who watch news on Channel 5 }

Q = { students who watch news on Channel 8 }

(a) Express in set notation, as simply as possible, the statement :

' Some students watch News on neither Channel 5 nor Channel 8'

(b) Given that $n(\varepsilon) = 100$, $n(P) = 43$ and $n(Q) = 38$. Find the maximum number of students who watch News on neither Channel 5 nor Channel 8.

Answer (a) _____ [1]

(b) _____ [1]

---

4 Factorise completely $5pr + 3qs - 3qr - 5ps$.

Answer _____ [2]

5 A map is drawn to a scale of 1 : 40 000. The actual area of a nature reserve is 68 km^2. Calculate the area of the nature reserve on the map, giving your answer in square centimetres.

Answer _____ cm^2 [2]

---

6 Factorise $22x^2 - 71x - 21$ and hence deduce the prime factors of 212 879.

Answer _____ [1]

212 879 = _____ × _____ [2]

7 The atomic radius of hydrogen is 0.025 nanometres and that of nitrogen atom is 65 picometres. Find the difference in metres, between the atomic radii of hydrogen and nitrogen atoms, expressing your answer in standard form.

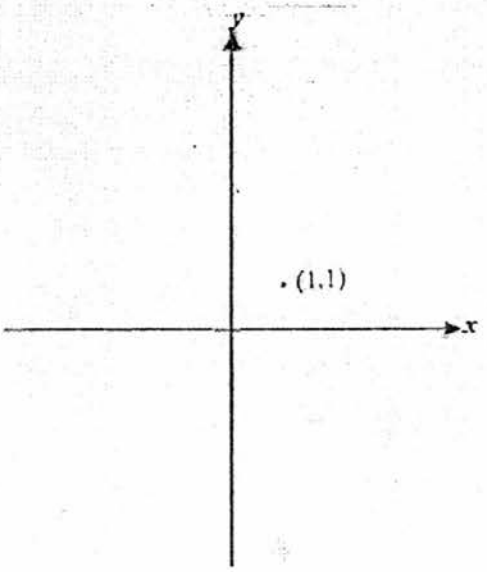
Answer _____ m [2]

8 Two geometrically similar vases have volumes 375 cm^3 and 1029 cm^3.
Given that the cost of painting the larger vase is $58.80, find the cost of painting the smaller vase.

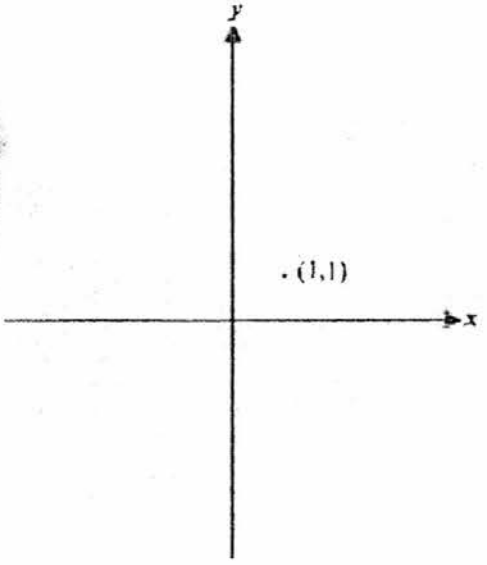
Answer $_____ [2]

9 Sketch the following graphs on the axes given below, indicating clearly any x and y or y intercepts whenever possible. The point (1,1) has been marked on the graphs.

(a) $y = -\frac{1}{2}x^3$ [1]



(b) $y = 1 + 2^{-x}$ [2]



10 Two of the exterior angles of a n-sided polygon are 42° and 78°.
Given that the other $(n-2)$ exterior angles are 20° each, calculate the value of n.

Answer $n =$ _____ [2]

11 Simplify the expression $\sqrt{\frac{(64a^{-1}b^3c)^{-2}}{-a^{-1}b^0c^7}}$, leaving your answer in positive index form.

Answer _____ [3]

12 Solve the equations

(a) $49^x \div 7 - \frac{1}{343^x} = 0$

(b) $x(3x - 1) = 5$

Answer (a) _____ [2]

(b) _____ [3]

13 The gravitational force between two particles is inversely proportional to the square of the distance between them.

(a) Calculate the percentage change in the gravitational force if the distance between the two particles is doubled.

Answer (a) _____ % [2]

(b) Given further that when the distance between the two particles is 5 mm, the gravitational force between two particles is 0.09 N. Find the distance between the two particles when the gravitational force between them is 0.04 N.

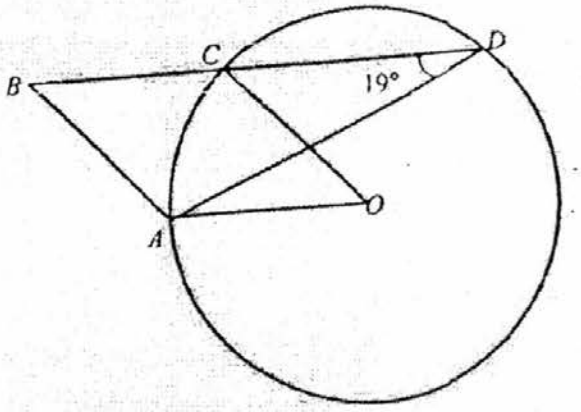
Answer (b) distance = _____ mm [2]

14 $OABC$ is a rhombus where O is the centre of the circle and D is a point on the circumference such that BCD is a straight line.

If angle $ADB = 19°$, calculate, stating the reasons clearly,

(a) angle ABC.

(b) angle CAD.



Answer (a) _____ °[2]

(b) _____ °[2]

15 Alan and Ben sat for a test with 40-questions. Table 1 below shows the results of the test.

	Correct	No attempt	Incorrect
Alan	29	6	5
Ben	24	x	y

Table 2 shows the marks awarded for each question.

	Correct	No attempt	Incorrect
Marks	3	0	−2

(a) Write down two matrices A and B such that the product AB will give the total marks that Alan and Ben each score.

(b) **Hence**, using matrix multiplication, calculate the values of x and y, given that the total score by Ben is 68 marks.

Answer *(a)* $A =$ _____ [1]

$B =$ _____

(b) $x =$ _____ $y =$ _____ [3]

16 (a) Solve $-1 < \frac{2x}{3} - \frac{4-x}{2} \leq 5$.

(b) Given $0 < x < 7$ and $-1 \leq y \leq 3$.

For all integer values of x and y, find

(i) the minimum value of $y^2 - x$.

(ii) the maximum value of $\frac{y}{x}$.

Answer (a) _____ [2]

(b) (i) _____ [1]

(ii) _____ [1]

17 (a) Express $5 + 4x - x^2$ in the form $p - (x + q)^2$.

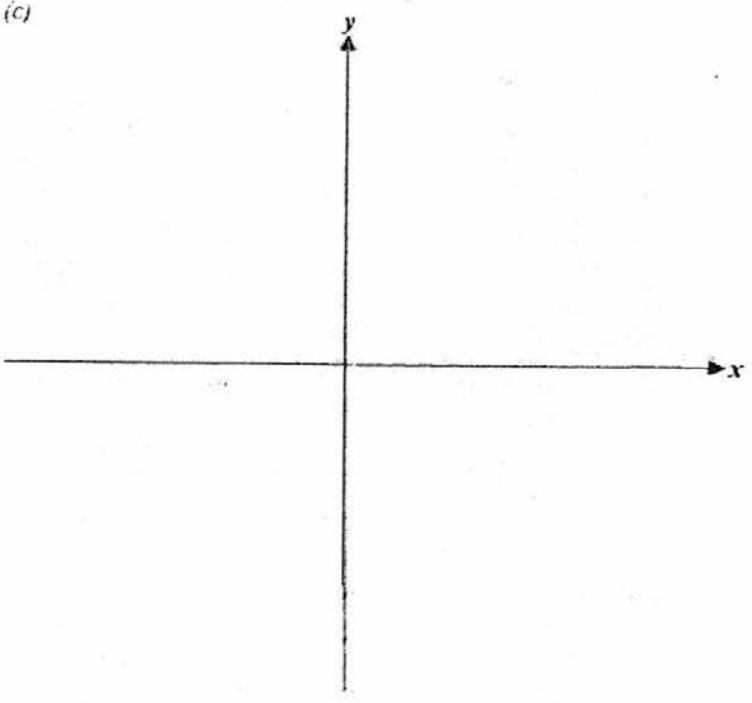
(b) State the equation of the line of symmetry.

(c) Sketch the graph of $y = 5 + 4x - x^2$.

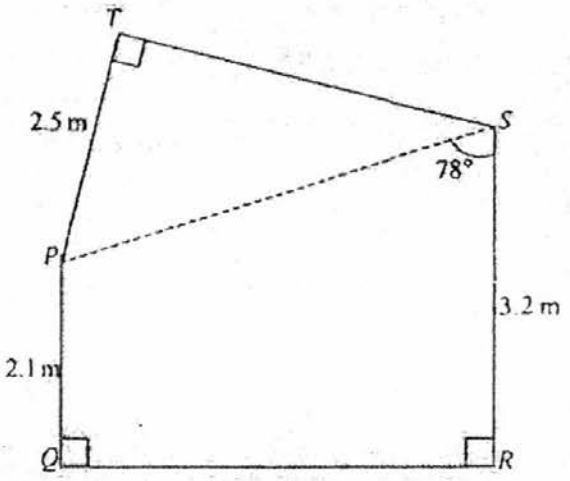
Answer (a) _____ [2]

(b) _____ [1]

Answer for (c) [2]



18 The diagram shows points on horizontal ground where P and S are due north of Q and R respectively. Given that $PQ = 2.1$ m, $PT = 2.5$ m, $RS = 3.2$ m, angle $PSR = 78°$ and $\angle PTS = \angle PQR = \angle SRQ = 90°$



(a) Calculate

(i) QR,

(ii) angle PST.

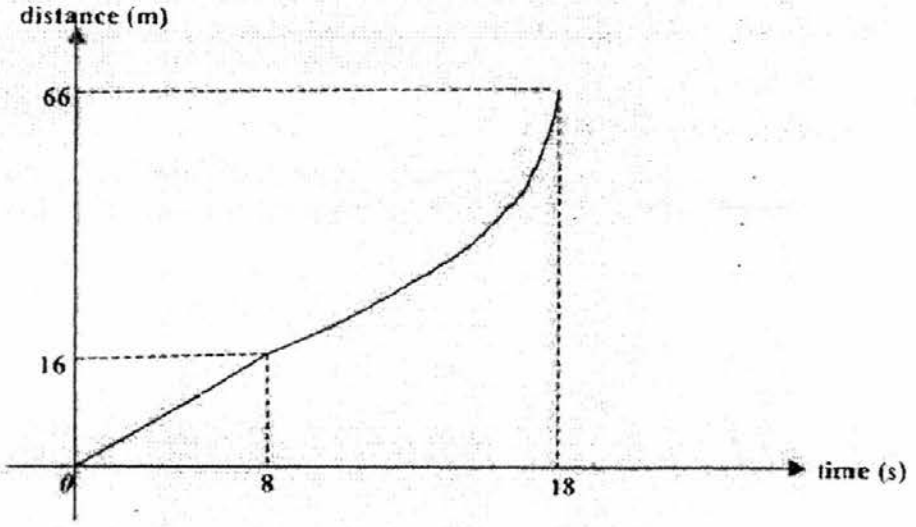
(b) Find the bearing of S from T.

Answer (a) (i) _____ m [2]

(ii) _____ ° [3]

(b) _____ ° [1]

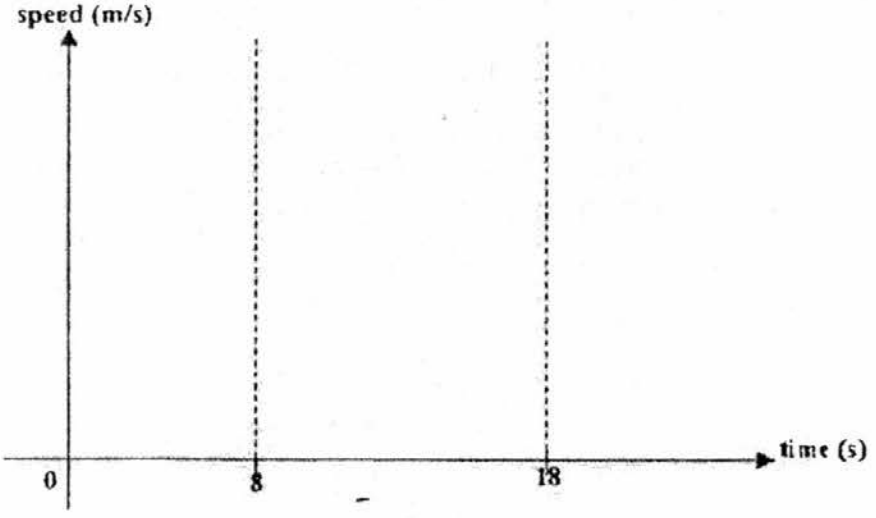
19 The diagram is the distance-time graph for the first 18 seconds of a car's journey.



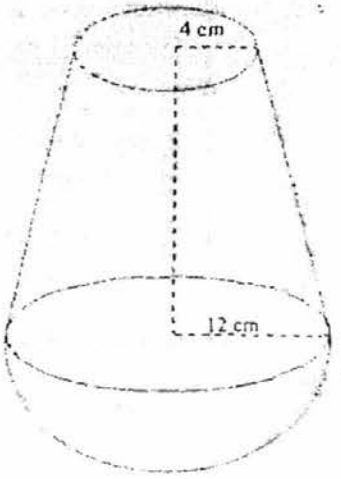
(a) Find the average speed of the car for the whole journey.

Answer (a) _____ m/s [1]

(b) The car moved with a constant acceleration for the last 10 seconds.
On the axes below, sketch the speed-time graph for the first 18 seconds of the car's journey. [4]

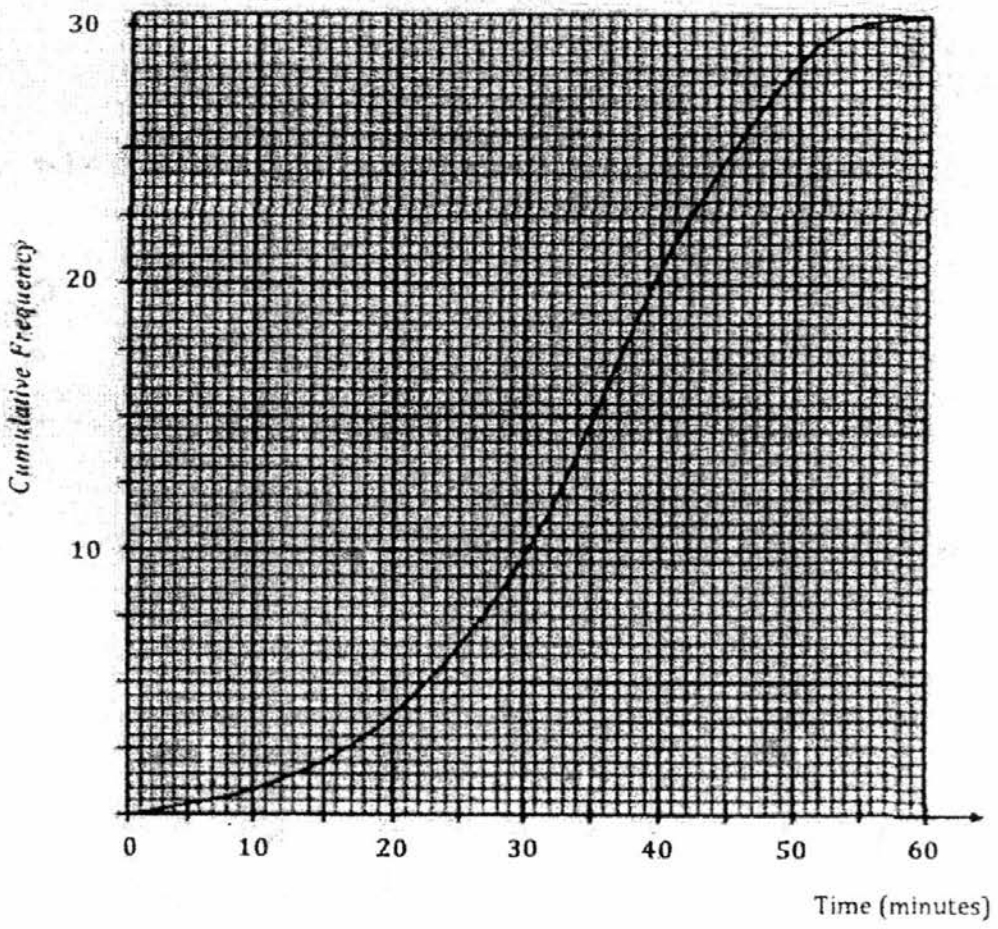


20 The diagram below shows a solid made up of two sections. The lower section is a solid hemisphere of radius 12 cm. The upper section is the frustum of a cone. Given that the original height of the cone is 27 cm and the radius of the upper surface of the frustum is 4 cm. Calculate the total volume of the solid, leaving your answer in terms of π.



Answer _____ cm^3 [4]

21 The cumulative frequency graph below shows the time taken to travel from home to school, in minutes, by the students of class A in a particular school.



(a) Use your graph to estimate

(i) the median time,

(ii) the inter-quartile range,

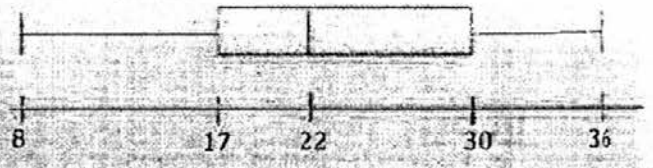
(iii) the percentage of students who took more than 36 minutes to travel to school.

Answer (a) (i) _____ [1]

(ii) _____ [2]

(iii) _____ [1]

(b) The results of the time taken to travel from home to school, in minutes, by the students from class *B* were illustrated in a box and whisker diagram as shown below.



Compare and comment on the time taken to travel from home to school by the students from class *A* and class *B* in two different ways.

Answer : _____ [2]

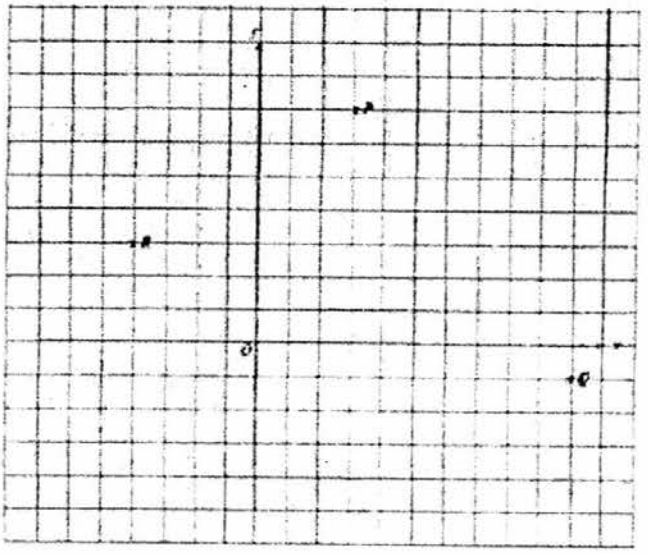
22 A translation maps the point $P(3,7)$ onto $Q(10,-1)$. The same translation maps the point $R(-4,3)$ onto the point S. Find

(a) the coordinates of the point S,

(b) the area of the quadrilateral $PQSR$,

(c) the equation of the line that passes through the mid-point of RQ and is parallel to PQ,

(d) the acute angle that the line PQ makes with the x-axis.



Answer (a) S = (,) [1]

(b) Area = _____ unit^2 [1]

(c) _____ [3]

(d) _____ [2]

END OF PAPER

新加坡海星中学

MARIS STELLA HIGH SCHOOL

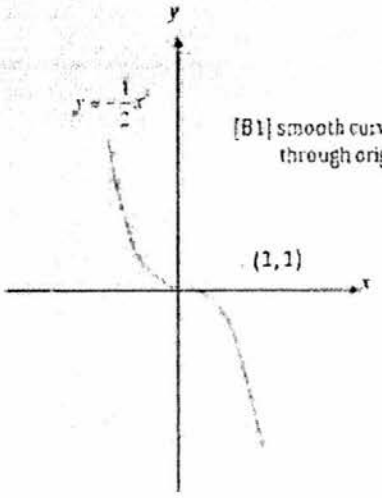
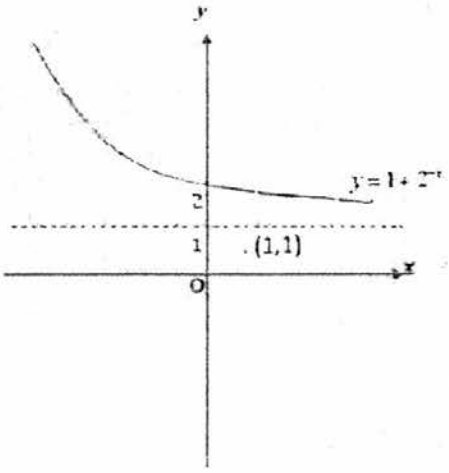
PRELIMINARY EXAMINATION ONE

SECONDARY FOUR

2016 Mathematics Paper One solution

1	Selling price = $\$\left(\frac{90}{86} \times 260\right)$ [M1] = $292.50 [A1] (No A1 mark if $292.5 is given.)	2
2	(a) $6048 = 2^x \times 3^y \times 7^1$ $x = 3$ [B1] $y = 1$ [B1] (b) $\frac{6048}{k} = \frac{6048}{2^2 \times 7} = 2^x \times 3^1$ $k = 2^2 \times 7$ $k = 28$ [A1]	3
3	(a) $(P \cup Q)' \neq \varphi$ [B1] (b) max. no. of students who neither watch news on Channel nor Channel 8 = 100 – 43 = 57 [B1]	2
4	$5pr + 3qs - 3qr - 5ps$ $= 5pr - 3qr + 3qs - 5ps$ $= r(5p - 3q) + s(3q - 5p)$ [M1] $= (5p - 3q)(r - s)$ [A1]	2

5	1 : 40000 $1\text{ cm} : \frac{40000}{100000}\text{ km}$ $1\text{ cm} : \frac{2}{5}\text{ km}$ $1\text{ cm}^2 : \frac{4}{25}\text{ km}^2$ Area of the nature reserve on the map $= 68 \times \frac{25}{4}$ [M1] $= 425\text{ cm}^2$ [A1]	2
6	$22x^2 - 71x - 21$ $= (2x - 7)(11x + 3)$ [B1] $212\,879 = 22(100)^2 - 71(100) - 21$ [M1] $212\,879 = [2(100) - 7][11(100) + 3]$ $212\,879 = 193 \times 1103$ [A1]	3
7	Difference in the atomic radii $= 65 \times 10^{-12} - 0.025 \times 10^{-9}$ [M1] $= 65 \times 10^{-12} - 25 \times 10^{-12}$ $= 40 \times 10^{-12}$ $= 4.0 \times 10^{-11}\text{ m}$ [A1]	2

8	Let the cost of painting the smaller vase be $\$x$ $\left(\frac{l_1}{l_2}\right)^3 = \frac{375}{1029}$ $\left(\frac{l_1}{l_2}\right)^3 = \frac{125}{343}$ $\left(\frac{l_1}{l_2}\right) = \frac{5}{7}$ $\frac{x}{58.80} = \left(\frac{5}{7}\right)^2$ [M1] $x = 58.80 \times \frac{25}{49}$ $x = 530$ [A1]	2
9	(a)  	1
9	(b) [B1] for smooth curve passing through (0,2) [B1] for asymptote y=1  	2

10	$42° + 78° + 20°(n-2) = 360°$ [M1] $20°(n-2) = 240°$ $(n-2) = 12$ $n = 14$ [A1]	2
11	$\dfrac{\sqrt[3]{(64a^{-1}b^{3}c)^{-2}}}{-a^{-1}b^{0}c^{2}}$ $= \dfrac{(4^{3})^{-\frac{2}{3}}a^{\frac{2}{3}}b^{-2}c^{-\frac{2}{3}}}{-a^{-\frac{1}{3}}b^{0}c^{\frac{7}{3}}}$ $= \dfrac{a}{-16b^{2}c^{3}}$ [A3, 2, 1]	3
12	(a) $49^{x} + 7 - \dfrac{1}{343^{x}} = 0$ $49^{x} + 7 = \dfrac{1}{343^{x}}$ $7^{2x-1} = 7^{-3x}$ By comparing the power of 7, $2x - 1 = -3x$ [M1] $5x = 1$ $x = \dfrac{1}{5}$ [A1] (b) $x(3x-1) = 5$ $3x^{2} - x - 5 = 0$ [M1] $x = \dfrac{-(-1) \pm \sqrt{(-1)^{2} - 4(3)(-5)}}{2(3)}$ [M1] $x = \dfrac{1 \pm \sqrt{61}}{6}$ $x = -1.14$ or 1.47 (3sig. fig.) [B1]	5

13

(a) Let F be the force between the particles and d be the distance between them

$F = \frac{k}{d^2}$, k is a constant

When d is doubled, new force, F'

$F' = \frac{k}{(2d)^2}$, k is a constant

$F' = \frac{k}{4d^2}$ [M1]

$F' = \frac{1}{4}F$

$\%$ change in the gravitational force $= \left(\frac{\frac{1}{4}F - F}{F}\right) \times 100\%$

$= -75\%$ [A1]

$F = \frac{k}{d^2}$, k is a constant

When $d = 5mm$, $F = 0.09N$

$0.09 = \frac{k}{(5)^2}$

$k = \frac{9}{4}$

$F = \frac{9}{4d^2}$ [M1]

When $F = 0.04N$,

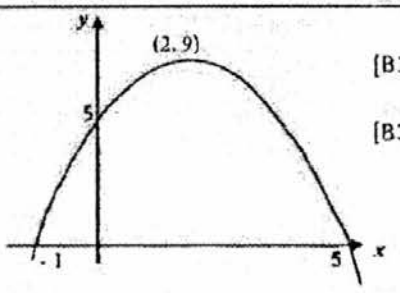
$0.04 = \frac{9}{4d^2}$

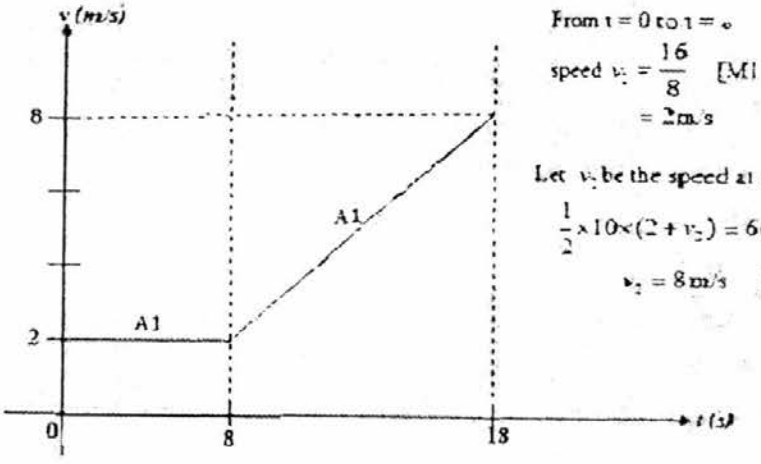
$d^2 = 56.25$ [A1]

$d = 7.5mm$

4

14	$A\hat{O}C = 2 \times 19°$ ($\angle$ at centre = 2 times $\angle$ at circumference) [M1] $= 38°$ $A\hat{B}C = A\hat{O}C$ (opposite angle of rhombus) $= 38°$ [A1] $O\hat{A}D = A\hat{D}C = 19°$ (alt $\angle$s, CD *is parallel to* AO) [M 0.5] $O\hat{A}C = O\hat{C}A$ (base $\angle$s of isosceles triangle) [M 0.5] $= \frac{180° - 38°}{2}$ $= 71°$ $O\hat{A}D = 19°$ (alt $\angle$. $BD // AO$) $C\hat{A}D = 71° - 19°$ $= 52°$ [A1]	4
15	(a) $A = \begin{pmatrix} 29 & 6 & 5 \\ 24 & x & y \end{pmatrix}$ [B1] $B = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$ (b) $AB = \begin{pmatrix} 29 & 6 & 5 \\ 24 & x & y \end{pmatrix}\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} 77 \\ 72 - 2y \end{pmatrix}$ [M1] $72 - 2y = 68$ $y = 2$ [A1] $24 + x + y = 40$ $24 + x + 2 = 40$ [A1] $x = 14$	4

16	(a) $-1 < \frac{2x}{3} - \frac{4-x}{2} \le 5$ $-1 < \frac{2x}{3} - \frac{4-x}{2}$ and $\frac{2x}{3} - \frac{4-x}{2} \le 5$ $-6 < 4x - 12 + 3x$ and $4x - 12 + 3x \le 30$ [M1] $6 < 7x$ $7x \le 42$ $x > \frac{6}{7}$ $x \le 6$ $\therefore \frac{6}{7} < x \le 6$ [A1]	2
16	(b) (i) minimum value of $y^2 - x$ $= 0 - 6$ $= -6$ [A1] (ii) maximum value of $\frac{y}{x}$ $= \frac{3}{1}$ $= 3$ [A1]	2
17	(a) $5 + 4x - x^2$ $= -(x^2 - 4x - 5)$ $= -(x^2 - 4x + 2^2 - 2^2 - 5)$ [M1] $= -[(x-2)^2 - 9]$ $= 9 - (x-2)^2$ [A1]	2
17	(b) Equation of the line of symmetry is $x = 2$ [B1]	1
17	(c)  [B1] ∩ shape with appropriate scale [B1] turning point (2, 9) y-intercept : (0, 5), x-intercepts : (-1, 0), (5,0) $y = 5 + 4x - x^2$	2

18	(a) (a) $\tan 78° = \frac{QR}{3.2-2.1}$ [M1] $QR = 1.1\tan 78°$ $QR = 5.1751$ $QR = 5.18$ m [A1]	2
	(b) $PS^2 = 5.1751^2 + 1.1^2$ [M1] $PS = 5.2907$ m $\sin P\hat{S}T = \frac{2.5}{5.2907}$ [M1] $P\hat{S}T = 28.198°$ $P\hat{S}T = 28.2°$ [A1]	3
	(c) Bearing of T from S = 78° + 28.2° = 106.2° [A1]	1
19	(a) average speed $= \frac{66}{18}$ $= 3\frac{2}{3}$ m/s [A1] (U if unit is omitted)	1
	(b)  From t = 0 to t = 8 speed $v_1 = \frac{16}{8}$ [M1] = 2m/s Let v_2 be the speed at t = 8 $\frac{1}{2}\times 10\times(2+v_2) = 66-16$ $v_2 = 8$ m/s [M1]	4

20	Let the height of smaller cone be x cm $\frac{x}{27} = \frac{4}{12}$ $x = 9$ cm [M1] Volume of the solid $= \frac{2}{3}\pi(12)^3 + \frac{1}{3}\pi(12)^2(27) - \frac{1}{3}\pi(4)^2(9)$ [M1 - volume of sphere, M1 - volume of cone] $= \frac{\pi}{3}(3456 + 3888 - 144)$ $= \frac{\pi}{3} \times 7200$ $= 2400\pi$ cm^3 [A1]	4
21	(a) (i) median = 35 min [B1] (ii) inter-quartile range = 42-27 [M1] = 15 min [A1] (iii) % of students who took more than 36 min $= \left(\frac{30-16}{30}\right) \times 100\%$ $= 46\frac{2}{3}\%$ or 46.7% [A1]	4
	(b) For class B : median = 22 min Inter-quartile range = 30-17 = 13 min The **median** travel time taken by the students from Class A is higher than that of Class B. The students from Class A take a **longer** travel time from home to school. [B1] The **inter-quartile range** of the travel time taken by the students from classes A is higher than that of Class B . The travel time taken from home to school by the student from Class A is slightly **more widely spread** than that of Class B. [B1]	2

22	(a) $S = (7+(-4), -8+3)$ $S = (3, -5)$ [B1]	1
	(b) the area of the polygon $PQSR$, $= 2\times\frac{1}{2}\times[7-(-5)]\times(10-3)]$ $= 2\times\frac{1}{2}\times12\times7$ $= 84$ units^2 [A1] (U if unit is omitted)	1
	(c) mid point of RQ $=\left(\frac{10-4}{2}, \frac{-1+3}{2}\right)$ $= (3, 1)$ [M1] gradient of PQ $=\frac{7+1}{3-10}$ $=-\frac{8}{7}$ [M1] Sub (3, 1) into $y=-\frac{8}{7}x+c$ $1=-\frac{8}{7}(3)+c$ $c=\frac{31}{7}$ Equation of PQ : $y=-\frac{8}{7}x+\frac{31}{7}$ [A1]	3
	(d) $\tan\theta=\frac{8}{7}$ [M1] $\theta=\tan^{-1}\left(\frac{8}{7}\right)$ Angle PQ makes with the x-axis = 48.8° [A1]	2
	The End	

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

PRELIMINARY EXAMINATION ONE

SECONDARY FOUR

MATHEMATICS
PAPER 2

4048/02

6 May 2016

Additional Materials:
Writing paper (6 sheets)
Graph paper (1 sheet)

2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of 11 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = p\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1. **(a)** It is given that $y = 1-(2x)^2$.

   (i) Find the value of y when $x = -1$. [1]

   (ii) Find the value(s) of x when $y = -3$. [2]

   (iii) Express x in terms of y. [2]

   (b) **(i)** Factorise completely $50a^2 - 2$. [2]

   (ii) Simplify $\dfrac{50a^2-2}{5a^2+14a-3}$. [2]

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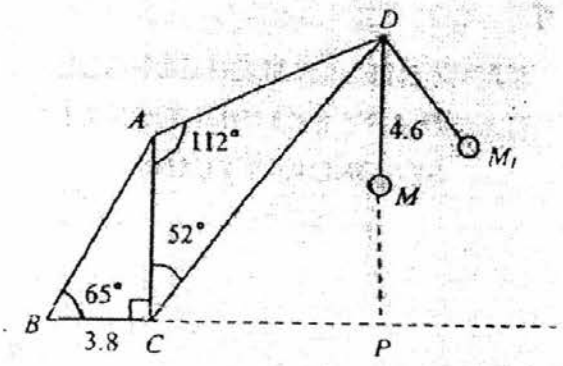
2. **(a)** The cash price of a new car is $95 000. Chris buys the car on hire purchase, he would need to pay a deposit of 50% followed by 42 equal monthly payments at a simple interest rate of 3 % per annum. Calculate the total amount that Chris pays for the car. [3]

   (b) Janice was having holiday in USA. She decided to change her remaining US$480 dollars back to Singapore dollars. The exchange rate in USA was S$1 = US$0.75 whereas the exchange rate in Singapore was US$100 = S$138.80. Should Janice change her remaining US dollars in USA or Singapore? Justify your answer. [3]

   (c) In Indonesia, the forest area was 1.185 million km^2 in 1990. Due to deforestation, the forest area was 0.931 million km^2 in 2012, calculate the percentage decrease in the forest area from 1990 to 2012. [2]

3. In the diagram, $ABCD$ is a steel structure. A chain carrying a load M is attached to the steel structure at D. $\angle ABC = 65°$, $\angle ACB = 90°$, $\angle ACD = 52°$, $\angle CAD = 112°$, $BC = 3.8$ m and $DM = 4.6$ m. P is a point such that DP is perpendicular to BC produced.



[Not drawn to scale]

(a) Calculate AC. [1]

(b) Show that $CD = 27.4$ m, correct to three significant figures. [2]

(c) Calculate the height of the load M above the horizontal level BC extended. [2]

(d) Find the angle of elevation of load M from point C. [3]

(e) If the load M, being blown by a strong wind, is swung to the position M_1 such that distance between M and M_1 is 3.5 m. Calculate $\angle MDM_1$. [2]

4. Study the number pattern below.

$$\frac{1}{1\times 2}=\frac{1}{1}-\frac{1}{2}$$
$$\frac{1}{2\times 3}=\frac{1}{2}-\frac{1}{3}$$
$$\frac{1}{3\times 4}=\frac{1}{3}-\frac{1}{4}$$
$$\vdots \qquad \vdots$$
$$\frac{1}{20\times 21}=\frac{1}{a}-\frac{1}{b}$$
$$\vdots \qquad \vdots$$
$$\frac{1}{c\times d}=\frac{1}{n}-\frac{1}{n+1}$$

(a) **(i)** Write down the value of a and of b. [1]

(ii) Write down the number pattern at $n = 99$. [1]

(iii) Find the value of $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{99\times 100}$. [2]

(iv) Hence, or otherwise, express $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}$ as a single fraction, in terms of n. [2]

(b) Find the value of $\frac{1}{1\times 2}-\frac{1}{2\times 3}-\frac{1}{3\times 4}-\ldots-\frac{1}{99\times 100}$. [2]

5. **Answer the whole of this question on a sheet of graph paper.**

During an experiment, the temperatures of an unknown liquid were recorded over an seven hour period. The table below shows the temperature, y °C, at various times.

Time (x hours)	0	1	2	3	4	5	6	7
Temperature (y °C)	2	−1	−2	−1	2	2.9	3.2	3.3

(a) Using a scale of 2 cm to represent 1 hour, draw a horizontal x-axis for $0 \le x \le 7$.
Using a scale of 2 cm to represent 1 °C, draw a vertical y-axis for $-2 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find an estimate for

(i) the temperature at the 4.3th hour. [1]

(ii) the duration, in hours, for which the temperature was below 0 °C. [2]

(c) (i) By drawing a tangent, find the gradient of the curve at the point where $x = 3$. [2]

(ii) State **briefly** what this gradient represents. [1]

(d) The curve from $x = 0$ to $x = 2$ has the equation $y = x^2 + Bx + C$.
Find the value of C and the value of B. [2]

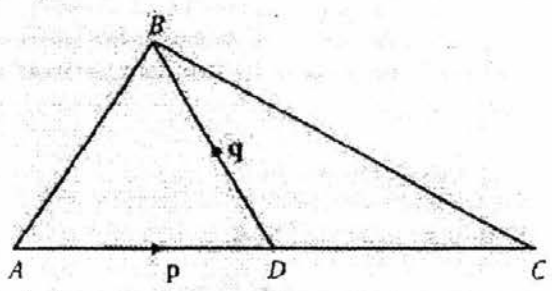
6. **(a)** The position vector of $R = \begin{pmatrix} 7 \\ -2 \\ 5 \end{pmatrix}$, the point S is (3, 1) and equation of line RS is $y = mx + k$, where k is a constant.

(i) Find the value of m. [1]

(ii) Find $\left|\overrightarrow{RS}\right|$. [2]

(iii) Given $\overrightarrow{TU} = \begin{pmatrix} 35 \\ n \end{pmatrix}$ is parallel to the line RS, find the value of n. [2]

(b)



In the above diagram, $\overrightarrow{AD} = \mathbf{p}$, $\overrightarrow{DB} = \mathbf{q}$ and $AC = 2AD$.

(i) Express, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(a) $\overrightarrow{AB}$, [1]

(b) $\overrightarrow{CB}$. [1]

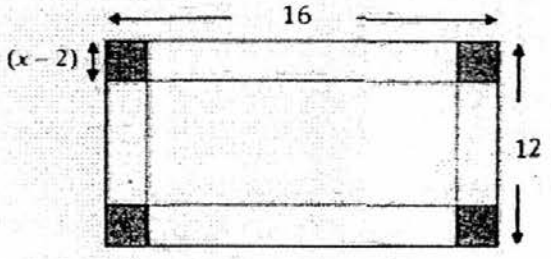
(ii) E is a point such that $\overrightarrow{AE} = \mathbf{q}$.

(a) Express $\overrightarrow{DE}$ in terms of of $\mathbf{p}$ and $\mathbf{q}$. [1]

(b) Show that $AEBC$ is a trapezium. [2]

(c) Calculate $\dfrac{Area\ of\ \Delta BDC}{Area\ of\ Trapezium\ AEBC}$. [1]

7. A open rectangular box is made up by cutting out squares of side $(x-2)$ cm from each corner of a piece of rectangular cardboard measuring 16 cm by 12 cm.



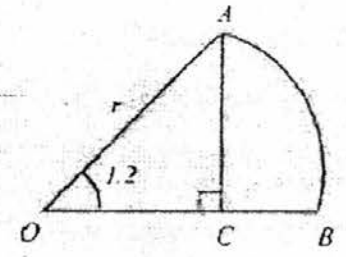
(a) Write down an expression, in terms of x, for the length of the base of the completed rectangular box. [1]

(b) If the area of the base of the completed box is 96 cm^2, write down an equation in x and show that it reduces to $x^2 - 18x + 56 = 0$. [3]

(c) Solve the equation $x^2 - 18x + 56 = 0$. [3]

(d) Find the volume of the box. [2]

8. (a) The following diagram shows a sector of a circle with centre O and radius r cm. $\angle AOB = 1.2$ radians and C is a point on OB such that AC is perpendicular to OB.

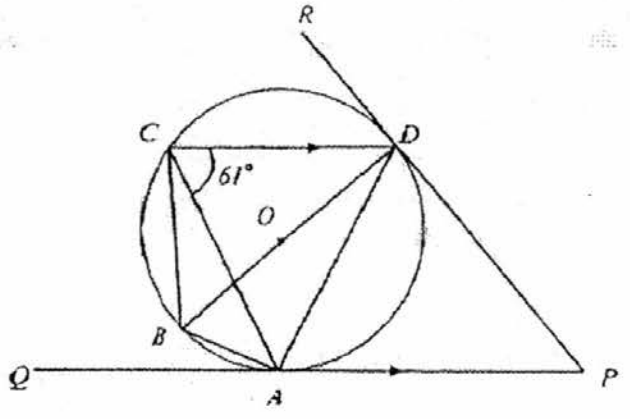


(i) Show that $OC = r \cos 1.2$. [1]

(ii) Find area of sector OAB in terms of r. [1]

(iii) Given that the area of the shaded region is 30 cm^2, find the value of r. [5]

(b)



In the above figure, O is the centre of the circle with tangents PAQ and PDR at A and D respectively. It is given that $\angle ACD = 61°$, CD is parallel to QAP, and BOD is a diameter. Find

(i) $\angle ADB$, [2]

(ii) $\angle APD$, [2]

(iii) $\angle BDC$. [2]

9. **(a)** The heights, in centimeters, of the 12 girls in the class are represented in the stem and leaf diagram below:

13	1 2
14	2 5 5 5 6
15	0 0 7
16	3 3

Key 14|2 means 142 cm

(i) Find the percentage of girls who are at most 150 cm. [1]

(ii) Find the modal height of the girls. [1]

(iii) Find the median height of the girls. [1]

(iv) Find the mean and standard deviation of the heights of the girls. [3]

(b) Box *A* contains four red marbles and three yellow marbles.

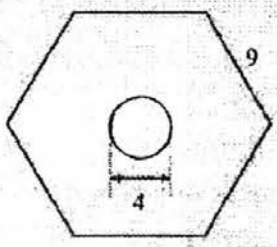
(i) Two marbles are drawn from the box, one after the other, without replacement. Find the probability that exactly one marble is yellow. [2]

Box *B* contains five red marbles and two yellow marbles. A fair six-sided die is tossed. If the score on the die is 3 or 5, a marble is drawn from Box *A*. Otherwise, a marble is drawn from Box *B*.

(ii) Given that the marble was drawn from Box *B*, write down the probability that it is red. [1]

(iii) Find the probability that a yellow marble is drawn from Box *B*. [2]

10. A pendant in the form of a regular hexagonal prism of **thickness** 3.5 mm with a circular hole drilled symmetrically along the axis of the prism. The cross section of the pendant is shown in the diagram. Each side of the hexagon is of length 9 mm and the diameter of the circular hole is 4 mm.



(a) Find the cross-section area, in mm^2, of the pendant. [3]

(b) Find the volume, in mm^3, of the pendant. [2]

Useful information:

Density of pure gold = 19.3 grams per cubic centimetre (g/cm^3)
1 ounce (oz) = 28.35 gram (g)

(c) A seller claims that the pendant is made of pure gold and has a mass of 0.3 oz.
Would you buy this pendant from the seller? Justify your answer. [5]

~ End of Paper ~

Marking Scheme Emaths P2 Prelim 1 2016 MSHS

1) (i)

$y = 1 - (2x)^2$

$y = 1 - (-2)^2$

$y = -3$ [A1]

1)(ii)

$-3 = 1 - (2x)^2$

$4x^2 = 4$ [M1]

$x^2 = 1$

$x = \pm 1$ [A1]

1)(iii)

$y = 1 - 4x^2$

$4x^2 = 1 - y$

$x^2 = \frac{1-y}{4}$ [M1]

$x = \pm\sqrt{\frac{1-y}{4}}$

$x = \pm\frac{\sqrt{1-y}}{2}$ [A1]

1)(bi)

$50a^2 - 2$

$= 2(25a^2 - 1)$ [M1]

$= 2(5a-1)(5a+1)$ [A1]

1)(bii)

$\frac{2(5a-1)(5a+1)}{5a^2+14a-3}$

$= \frac{2(5a-1)(5a+1)}{(5a-1)(a+3)}$ [M1]

$= \frac{2(5a+1)}{a+3}$ [A1]

2)(a) Balance for installment

$= \frac{50}{100} \times 95000$ [M1]

$= \$47500$

Simple interest

$= \frac{47500 \times 3 \times 3.5}{100}$ [M1]

$= \$4987.50$

Total amount Chris has to pay

$= 95000 + 4987.50$

$= \$99987.50$ [A1]

2)(b) In USA, she will get $S\$(480 \div 0.75) = S\$640$ [M1]

In Singapore, she will get $S\$(0.48 \times 138.80) = S\$666.24$ [M1]

Janice should exchange the US$480 in Singapore since she will be getting more S$ in Singapore. [A1]

2)(c) Percentage decrease in the forest area in Indonesia from 1990 to 2012

$= \frac{1185 - 931}{1185} \times 100$ [M1]

$= 21.4\% (3s.f.)$ [A1]

3)(a)

$$\tan 65° = \frac{AC}{3.8}$$

$$AC = 8.1491 (5s.f.)$$

$$= 8.15m (3s.f.) \qquad [A1]$$

3)(b)

$$\angle ADC = 16°$$

$$\frac{CD}{\sin 112°} = \frac{AC}{\sin 16°} \qquad [M1]$$

$$CD = \frac{8.1491}{\sin 16°} \times \sin 112°$$

$$CD = 27.412 (5s.f.)$$

$$CD = 27.4\ m\ (3s.f.) \quad (shown) \qquad [A1]$$

3)(c)

$$\sin 38° = \frac{DP}{27.412} \qquad [M1]$$

$$DP = 16.877\ m\ (5s.f.)$$

Height of load M above the horizontal level BC extended

$$= 16.877 - 4.6$$

$$= 12.277$$

$$= 12.3\ m\ (3s.f.) \qquad [A1]$$

3)(d)

$$\cos 38° = \frac{CP}{27.412} \qquad [M1]$$

$$CP = 21.601\ m\ (5s.f.)$$

$$\tan \angle MCP = \frac{12.277}{21.601} \qquad [M1]$$

$$\angle MCP = 29.612° \ (3d.p.)$$

Thus, angle of elevation of load M from point C is 29.6° (1d.p.) [A1]

3)(e)

$$3.5^2 = 4.6^2 + 4.6^2 - 2(4.6)(4.6)\cos \angle MDM_1 \qquad [M1]$$

$$\cos \angle MDM_1 = \frac{3.5^2 - 4.6^2 - 4.6^2}{-2(4.6)(4.6)}$$

$$\angle MDM_1 = 44.721° (3d.p.)$$

$$= 44.7° (1d.p.) \qquad [A1]$$

4)(ai) $a = 20, b = 21$ [A1]

4)(aii) $\frac{1}{99\times100} = \frac{1}{99} - \frac{1}{100}$ [A1]

4)(aiii)

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{99\times100}$$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{99}-\frac{1}{100}\right) \quad [M1]$$

$$=1-\frac{1}{100}$$

$$=\frac{99}{100} \quad [A1]$$

4)(aiv)

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n(n+1)}$$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right) \quad [M1]$$

$$=1-\frac{1}{n+1}$$

$$=\frac{n}{n+1} \quad [A1]$$

OR by observation from part (aiii), answer = $\frac{n}{n+1}$. [B2]

4)(b)

$$\frac{1}{1\times2}-\frac{1}{2\times3}-\frac{1}{3\times4}-\ldots-\frac{1}{99\times100}$$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{3}-\frac{1}{4}\right)-\ldots-\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=\frac{1}{1}-\frac{1}{2}-\frac{1}{2}+\frac{1}{100} \quad [M1]$$

$$=\frac{1}{100} \quad [A1]$$

6)(ai)

R is $(-2, 5)$

$\text{Grad. of } RS = \dfrac{5-1}{-2-3}$

$\therefore m = -\dfrac{4}{5}$ [A1]

6)(aii)

$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$

$= \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ [M1]

$|\overrightarrow{RS}| = \sqrt{(-4)^2 + 5^2}$

$= \sqrt{41}$

$= 6.40$ units (3 s.f.) [A1]

6)(aiii)

$\begin{pmatrix} 35 \\ n \end{pmatrix} // \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

$\therefore \begin{pmatrix} 35 \\ n \end{pmatrix} = k \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, k is a constant [M1]

$\therefore k = 7$

$\therefore n = -28$ [A1]

6)(bi)(a)

$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$

$= \mathbf{p} + \mathbf{q}$ [A1]

6)(bi)(b)

$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$

$= -2\mathbf{p} + (\mathbf{p} + \mathbf{q})$

$= -\mathbf{p} + \mathbf{q}$ [A1]

6)(bii)(a)

$\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE}$

$= -\mathbf{p} + \mathbf{q}$ [A1]

6)(bii)(b)

$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$

$= -\mathbf{q} + (\mathbf{p} + \mathbf{q})$

$= \mathbf{p}$ [M1]

Since $\overrightarrow{EB} = \dfrac{1}{2}\overrightarrow{AC}$, $EB // AC$

Thus, $AEBC$ is a trapezium. (shown) [A1]

6)(bii)(c) $\dfrac{\text{Area of } \Delta BDC}{\text{Area of trapezium } AEBC} = \dfrac{1}{3}$ [B1]

7)(a) Expression for length of the base of the completed box = $-2x-20$ [A1]

7)(b) Expression for the width of the completed box = $-2x+16$

$(-2x+20)(-2x+16)=96$ [M1]

$4x^2-32x-40x+320=96$ [M1]

$4x^2-72x+224=0$

$x^2-18x+56=0$ *(shown)* [A1]

7)(c)

$$x=\frac{-(-18)\pm\sqrt{(-18)^2-4(1)(56)}}{2(1)}$$ [M1]

$$x=\frac{18\pm\sqrt{100}}{2}$$

$x=4$ *or* 14 [A2]

7)(d) Reject $x=14$ (since $[-2(14)+20]$ is negative)

Volume of box

$=96\times(4-2)$ [M1]

$=192\ cm^3$ [A1]

8)(ai) $\cos 1.2 = \frac{OC}{r}$

$OC = r\cos 1.2$ *(shown)* [A1]

8)(aii)

Area of sector OAB

$= \frac{1}{2}r^2(1.2)$

$= \frac{3}{5}r^2$ [A1]

8)(aiii)

$\sin 1.2 = \frac{AC}{r}$

$AC = r\sin 1.2$ [M1]

Area of shaded region

$= \frac{3}{5}r^2 - \frac{1}{2}(r\cos 1.2)(r\sin 1.2)$ [M1]

$\therefore \frac{3}{5}r^2 - \frac{1}{2}r^2\cos 1.2\sin 1.2 = 30$ [M1]

$r^2(\frac{3}{5} - \frac{1}{2}\cos 1.2\sin 1.2) = 30$

$r = \pm\sqrt{\frac{30}{\frac{3}{5} - \frac{1}{2}\cos 1.2\sin 1.2}}$ [M1]

$r = 8.3417$ *(5s.f.)* *reject −ve r*

$r = 8.34$ *(3s.f.)* [A1]

8) (bi) $\angle BCD = 90°$ (rt. $\angle$ in a semicircle) [A1]

$\angle BCA = 29°$

$\angle ADB = 29°$ ($\angle$ s in the same segments) [A1]

(bii) $\angle BDP = 90°$ (tan $\perp$ rad) [M1]

$\angle ADP = 90° - 29°$

$= 61°$

$\angle APD = 180° - 61° - 61°$

$= 58°$ [A1]

(Triangle ADP is isos. since tangents from ext. pts.)

(biii) $\angle RDC = \angle APD = 58°$ (Corr. $\angle$s, CD//AP) [M1]

$\angle BDR = 90°$ (tan $\perp$ rad)

$\angle BDC = 90° - 58°$

$= 32°$ [A1]

9)(ai) Percentage of students who are at most 150 cm $= \frac{9}{12} \times 100$

$= 75\%$ [A1]

(aii) Mode height = 145 cm [A1]

(aiii) Median height = 145.5 cm [A1]

(aiv) Mean height

$= \frac{1769}{12}$

$= 147\frac{5}{12}\ cm$

(accept also 147 cm to 3 s.f.) [A1]

Standard deviation of heights

$= \sqrt{\frac{261927}{12} - \left(\frac{1769}{12}\right)^2}$

$= 9.78\ cm\ (3s.f.)$ [A1]

9)(bi) P(exactly one marble is yellow)

$= \frac{3}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{3}{6}$ [M1]

$= \frac{4}{7}$ [A1]

(bii) P(Given marble was drawn from Box B, the marble is red)

$= \frac{5}{7}$ [A1]

(biii) P(yellow marble is drawn from Box B)

$= \frac{4}{6} \times \frac{2}{7}$ [M1]

$= \frac{4}{21}$ [A1]

10)(a) Area of the regular hexagon

$= \frac{1}{2}(9)(9)\sin 60° \times 6$ [M1]

$= 210.44\ mm^2$ (5s.f.)

Cross-section area of the pendant

$= 210.44 - \pi(2)^2$ [M1]

$= 197.87$ (5s.f.)

$= 198\ mm^2$ [A1]

10)(b) Volume of the pendant

$= 197.87 \times 3.5$ [M1]

$= 692.55$ (5s.f.)

$= 693\ mm^3$ (3s.f.) [A1]

10)(c) Volume of pendant in cm^3

$= \frac{692.56}{1000} cm^3$

$= 0.69256\ cm^3$ [M1]

Density of the pendant

$= \frac{0.3 \times 28.35}{0.69256}$ [M1]

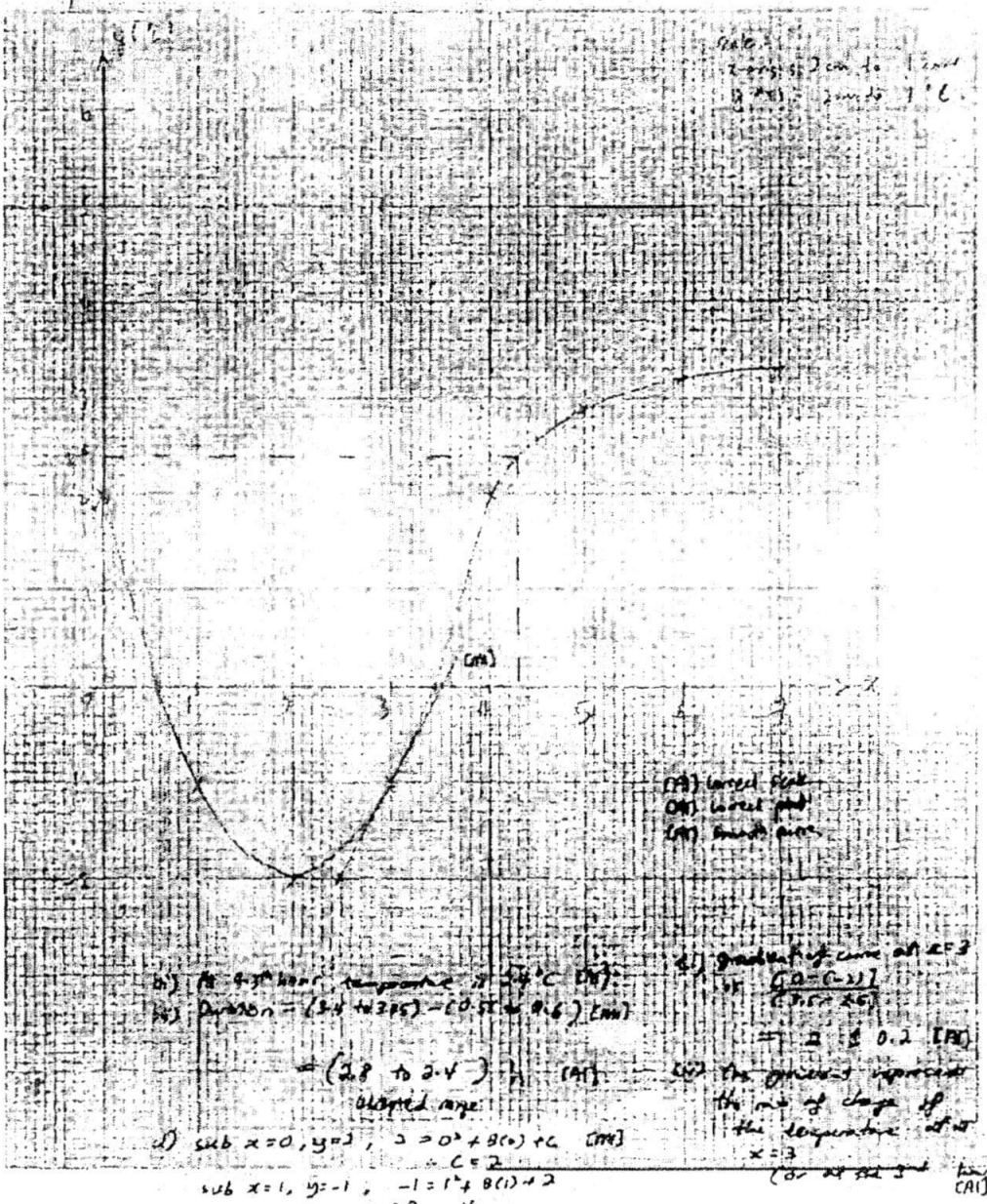
$= 12.3\ g/cm^3$ [A1]

Since the density of the pendant is below 19.3 g/cm^3, [M1]

Thus, this pendant is not made of pure gold,

I will not buy this pendant from the seller. [A1]

5)



b) At 4:30 pm temperature is 2.4 °C [A1]

b(ii) Duration = (3.4 + 3.25) − (0.55 + 0.6) [M1]

= (2.8 to 2.9) hrs [A1]

accepted range

c) sub $x=0, y=2$, $2 = 0^2 + B(0) + C$ [M1]

$\therefore C = 2$

sub $x=1, y=-1$, $-1 = 1^2 + B(1) + 2$

$\therefore B = -4$

$\therefore y = x^2 - 4x + 2$ [A1]

d(i) gradient of curve at $x=3$

$\approx \frac{(2.0 - (-1))}{(3.5 - 2.5)}$

$= 3 \pm 0.2$ [A1]

d(ii) The gradient represents the rate of change of the temperature at $x=3$ (or 4:30 pm) hours [A1]