7

Name : _____ Class | Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2016
Secondary 4

Thursday
4 August 2016

MATHEMATICS
Paper 1

4048/01
2 h

INSTRUCTIONS TO CANDIDATES
Write your name, class and index number on the question paper.
Write in dark blue or black ink on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

INFORMATION FOR CANDIDATES
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Marks
80

This question paper consists of 18 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $7\frac{1}{3}-\sqrt[3]{\frac{5.25+13.5^2}{\sin 28°}}$.

Write down the first six digits on your calculator display.

(b) Write your answer to part **(a)** correct to 2 significant figures.

Answer **(a)** ..... [1]

(b) ..... [1]

---

2 **(a)** Arrange the following numbers in ascending order:

$\frac{1}{20}$, $5\frac{1}{4}\%$, 5.22×10^{-3}, $0.\dot{0}\dot{5}$.

Answer **(a)** ..... [1]

(b) State which of the following number(s) is / are irrational:

$0.\dot{3}$, $\frac{\pi}{5}$, $\sqrt{7}\times2\sqrt{7}$, $3\sqrt{3}$.

Answer **(b)** ..... [1]

---

3 The length of each side of a cube is increased by 40%.
Find the percentage increase in the total surface area of the cube.

Answer ..... % [2]

---

4 Given that $(2x - 5)(x + a) = 2x^2 + bx - 5$ for all values of x, find the values of a and b.

Answer $a =$ ..... , $b =$ ..... [2]

---

5 Two numbers p and q, written as the products of their prime factors, are $p = 2^2 \times 3^5 \times 5^6$ and $q = 2^2 \times 3^3$.

(a) Find the HCF of p and q.

(b) Find the smallest positive integer k such that $(p \times q \times k)$ is a perfect cube.

Answer **(a)** ..... [1]

(b) $k =$ ..... [1]

---

6 Local time in Singapore is 7 hours ahead of local time in London. Singapore Airlines SQ007 departed London on Monday at 19 16 London time. The flight arrived at Singapore on Tuesday at 15 51 Singapore time. Calculate how long the flight took, giving your answer in hours and minutes.

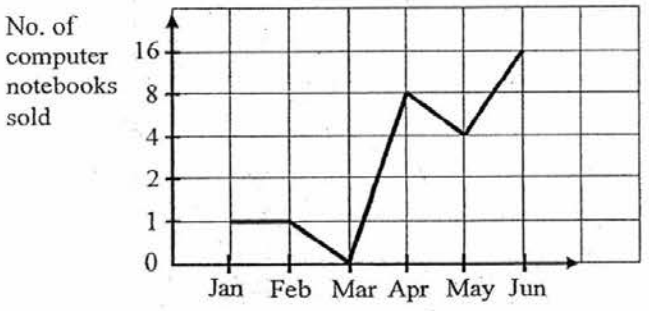
Answer ..... hours ..... minutes [2]

7 The diameter of a spherical micro-organism is 9.04 micrometres. Find the surface area in square millimetres, of the micro-organism, giving your answer in standard form.

Answer ..... mm^2 [2]

---

8 The graph below shows the sales of computer notebooks made by Angie over a period of 6 months in 2016.



Explain why the graph is misleading.

Answer .....

.....

.....

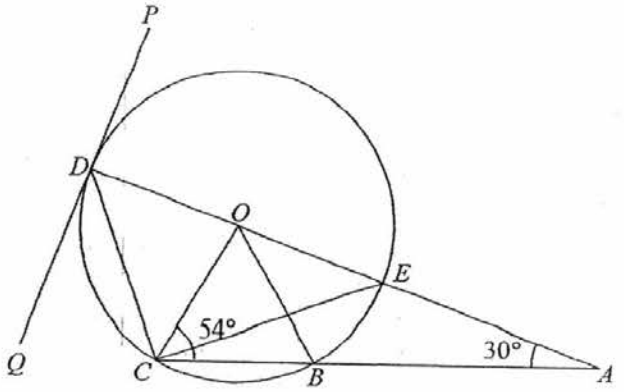
..... [2]

---

9 Two of the interior angles of a hexagon are $2x°$ and $(5x-200)°$. The remaining interior angles are $90°$ each. By forming an equation in x, find the value of x.

Answer $x =$ ..... [2]

10 In the diagram, the points B, C, D and E lie on a circle with centre O. PQ is a tangent to the circle at D. ABC and $AEOD$ are straight lines. $\angle OCB = 54°$ and $\angle OAB = 30°$.



Find, giving reasons for each answer,

(a) $\angle ADC$,

(b) $\angle CDQ$,

(c) $\angle ACE$,

(d) $\angle CBE$.

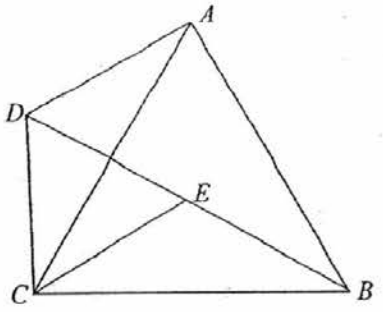
Answer **(a)**..... ° [2]

(b)..... ° [1]

(c)..... ° [2]

(d)..... ° [1]

11 *ABCD* is a quadrilateral. *ABC* and *CDE* are equilateral triangles. Using a pair of congruent triangles, show that $AD = BE$. State your reasons clearly.



Answer In triangles .....

.....

.....

.....

.....

.....

.....

.....

..... [2]

---

12 Janet has \$50000 to invest for 3 years. She invests her money in a unit trust with returns equivalent to 2% per annum interest, compounded every 3 months. Calculate the amount of interest she will get at the end of 3 years.

Answer \$ ..... [2]

13 **(a)** Given that $\left(\frac{1}{4}\right)^p \times 8 = 1$, find the value of p.

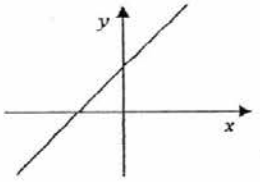
(b) Simplify $\left(\frac{2^{x+1}\sqrt{2}}{2^x}\right)^{-2}$.

Answer **(a)** p = ... [2]

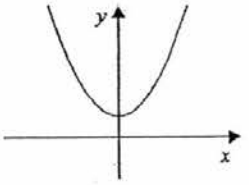
(b) ..... [2]

14 The equations of the three graphs shown below are in the form $y = n + x^{n-1}$.
State the value of n for each of the following graph.

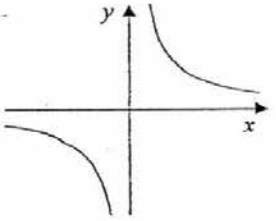
(a)



(b)



(c)



Answer **(a)** $n =$ ..... [1]

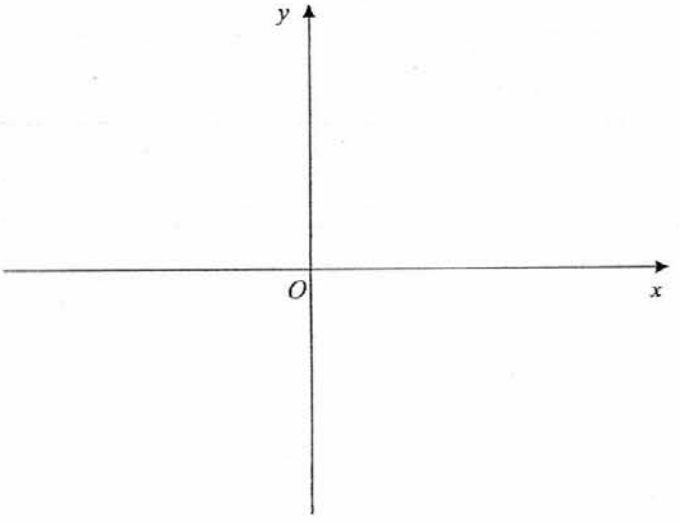
(b) $n =$ ..... [1]

(c) $n =$ ..... [1]

---

15 In the answer space, sketch the graph of $y = 5 - (x+1)^2$, indicate clearly the turning point and the intercepts on the x and y-axes (if any).

Answer [2]



16 **(a)** $\varepsilon = \{ x : x \text{ is an integer and } 1 \leq x < 24 \}$

$A = \{ x : x \text{ is a perfect square} \}$

$B = \{ x : x \text{ is a factor of the number } 24 \}$

$C = \{ x : x + 1 \text{ is divisible by } 6 \}$

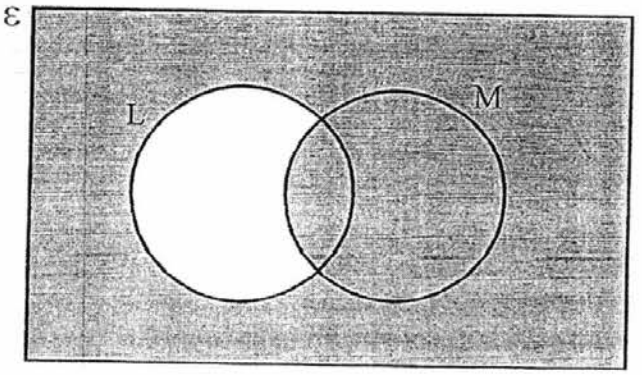
(i) List the elements in $A \cap C$.

(ii) Find $n(B' \cup C)$.

Answer **(a)(i)** ..... [1]

(ii) ..... [1]

(b) State the set notation of the shaded region in following Venn Diagram.



Answer **(b)**..... [1]

17 Given that point $A(4, 2)$ and $\overrightarrow{AC} = \begin{pmatrix} -7 \\ 3 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{CA}\right|$.

Answer **(a)** ..... units [1]

(b) The point P lies on CA such that $\overrightarrow{PA} = k\overrightarrow{CA}$.

(i) Show that $\overrightarrow{OP} = \begin{pmatrix} 4-7k \\ 2+3k \end{pmatrix}$.

Answer **(b)(i)** [1]

(ii) Given that point P lies on the y-axis, find the coordinates of P.

Answer **(b)(ii)** $P($ ..... , ... $)$ [2]

18 Consider the number patterns in the table below. The first three terms of each column have been given.

Row, n	S	T	U
1	4	16	16
2	8	32	30
3	12	48	44
7	p	q	r
n			

(a) Find values of p, q and r.

(b) Write down the equation connecting S and T.

(c) Write down the equation connecting U and n.

(d) Betty said that 256 can be found in column U.
Write whether you agree or disagree with Betty. Give reason(s) for your answer.

Answer **(a)** $p =$ ....., $q =$ ....., $r =$ ..... [1]

(b) ..... [1]

(c) ..... [1]

(d) I .....with Betty. This is because .....

.....

.....

.....

..... [1]

19 The frequency table shows the number of countries that a group of students had visited.

Number of countries	0	1	2	3	4
Number of students	2	8	6	x	4

(a) Given that the mode is 1, state the largest possible value of x.

(b) Given that the median number of countries visited is 2, find the largest possible value of x.

(c) Given that the mean number of countries is more than 2, find the smallest possible value of x.

Answer **(a)** $x =$ ..... [1]

(b) $x =$ ..... [1]

(c) $x =$ ..... [2]

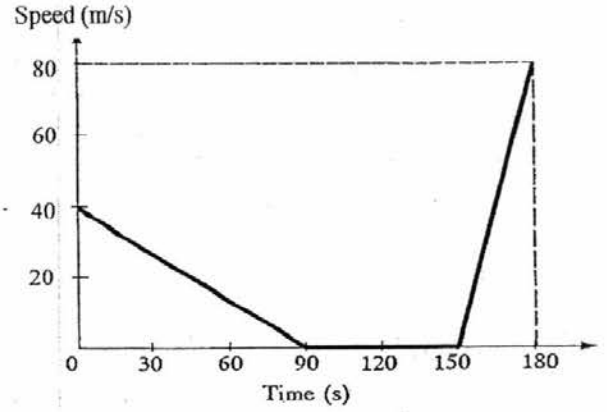
20 **(a)** The air resistance, R, is directly proportional to the square of the speed, V, of an object when it is falling. The air resistance is 24 newtons at a certain speed. Find the air resistance when the speed is increased by 50%.

(b) 48 men can build 2 huts in 60 hours. How many more men are needed if 3 huts are to be built in 72 hours?

Answer **(a)** ..... newtons [2]

(b) ..... men [2]

21 The diagram below shows the speed-time graph of the journey for the first 3 minutes of a train. The train slows down to a stop when entering station *J*. After a brief stop of 60 seconds, it starts to move off with acceleration for 30 seconds before it gets out of station *J*.



(a) Find the deceleration of the train as it enters station *J*.

(b) Calculate

(i) the total distance travelled by the train in the first 3 minutes,

(ii) the average speed of the train, in km/h, in the first 3 minutes.

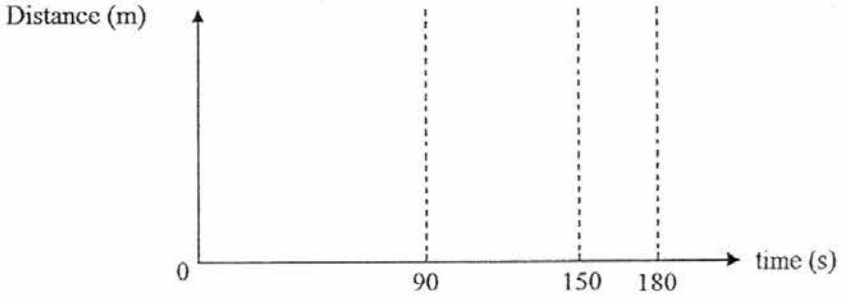
Answer **(a)** ..... m/s^2 [1]

(b)(i) ..... m [1]

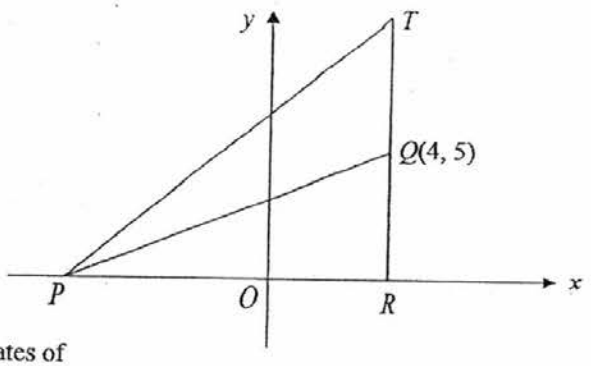
(ii) ..... km/h [2]

(c) On the axes below, sketch the distance-time graph of the train for the first 3 minutes of its journey.

Answer **(c)** [2]



22 P and R are points on the x-axis. TQR is a straight line parallel to the y-axis. Area of $\Delta PQR = 30$ units^2.



(a) Find the coordinates of
- **(i)** point R,
- **(ii)** point P.

(b) Find the length of PQ.

(c) Find $\cos\angle PQT$, giving your answer as a fraction.

(d) Given that $PR = TR$, find the equation of PT.

Answer **(a)(i)** R (..... , ...) [1]

(ii) P (..... , ...) [2]

(b) ..... units [1]

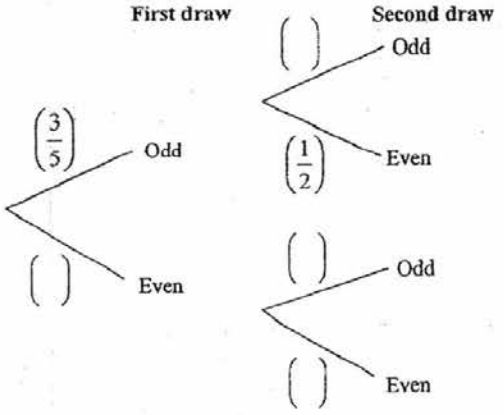
(c) ..... [1]

(d) ..... [1]

23 Five discs numbered 1, 3, 4, 6 and 7 are placed in a bag. A disc is drawn out of the bag at random. Without replacing the first disc into the bag, a second disc is drawn.

(a) Complete the following probability tree diagram.

Answer **(a)** [2]



(b) Find

(i) the probability that one disc is odd and the other is even,

(ii) the probability that both numbers drawn are smaller than 4.

(c) By drawing a possibility diagram in the space below, find the probability that the sum of both numbers is a prime number.

Answer **(b)(i)** ..... [1]

(ii) ..... [1]

(c) ..... [2]

24 The diagram below shows a horizontal field ABC.
A is due north of B and C is due west of B.
Use a scale of 1 cm to 40 m, show all the constructions clearly.

(a) A lamp post, L, is located on a bearing of 290° from A, and 300 m from A.

(i) By construction, mark and label clearly the position of the lamp post L. [1]

(ii) Measure and write down the bearing of the lamp post L from point C.

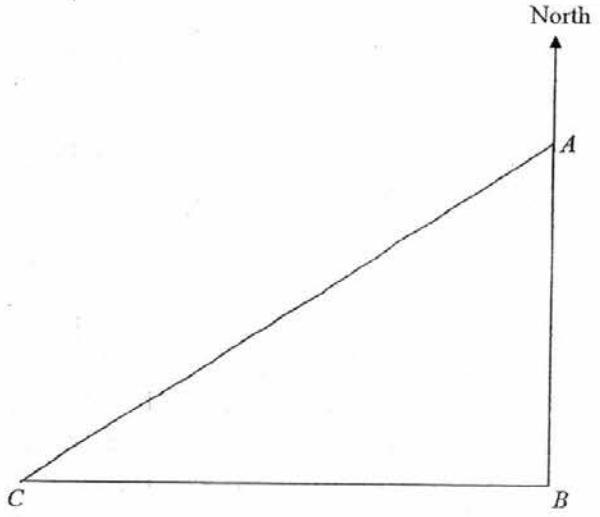
(b) A gate, G, is located along the path of BC, equidistant from B and C.
By construction, mark and label clearly the position of the gate G. [1]

(c) A circular flower bed is built such that it touches each side of the field at one point.

(i) By constructing two angle bisectors, draw the circular flower bed and label its centre O. [2]

(ii) Hence, measure and write down the actual radius of the flower bed.

Answer **(a)(i)**
(b)
(c)(i)



Answer **(a)(ii)** ..... ° [1]

(c)(ii) ..... m [1]

End of Paper 1

Name : _____	Class	Index Number

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4048/01
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Marks
80

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$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $7\frac{1}{3} - \sqrt[3]{\frac{5.25+13.5^2}{\sin 28^\circ}}$.

Write down the first six digits on your calculator display.

(b) Write your answer to part **(a)** correct to 2 significant figures.

B1

Answer **(a)** **–0.03095** ..... [1]

B1

(b) **–0.031** ..... [1]

---

2 **(a)** Arrange the following numbers in ascending order:

$\frac{1}{20}$, $5\frac{1}{4}\%$, 5.22×10^{-3}, $0.\dot{0}\dot{5}$.

0.05 **0.0525** **0.00522** **0.050505...**

B1

Answer **(a)** 5.22×10^{-3}, $\frac{1}{20}$, $0.\dot{0}\dot{5}$, $5\frac{1}{4}\%$ ..... [1]

(b) State which of the following number(s) is / are irrational:

$0.\dot{3}$, $\frac{\pi}{5}$, $\sqrt{7} \times 2\sqrt{7}$, $3\sqrt{3}$.

B1

Answer **(b)** $\frac{\pi}{5}$, $3\sqrt{3}$ ..... [1]

---

3 The length of each side of a cube is increased by 40%.
Find the percentage increase in the total surface area of the cube.

$$\% \text{ increase in surface area} = \frac{6(1.4l)^2 - 6l^2}{6l^2} \times 100\% \quad \textbf{M1}$$
$$= \frac{11.76 - 6}{6} \times 100\%$$
$$= 96\%$$

A1

Answer **96** ..... % [2]

---

4 Given that $(2x - 5)(x + a) = 2x^2 + bx - 5$ for all values of x, find the values of a and b.

$2x^2 + 2ax - 5x - 5a = 2x^2 + bx - 5$

$-5a = -5$ | $2a - 5 = b$

$a = 1$ | $b = 2(1) - 5$

$= -3$

B1 B1

Answer $a =$ 1 , $b =$ −3 [2]

---

5 Two numbers p and q, written as the products of their prime factors, are $p = 2^2 \times 3^5 \times 5^6$ and $q = 2^2 \times 3^3$.

(a) Find the HCF of p and q.

(b) Find the smallest positive integer k such that $(p \times q \times k)$ is a perfect cube.

(a) HCF $= 2^2 \times 3^3 = 108$

(b) $(p \times q \times k) = 2^4 \times 3^5 \times 5^6 \times k$

$k = 2^2 \times 3$

$= 12$

B1

Answer **(a)** 108 [1]

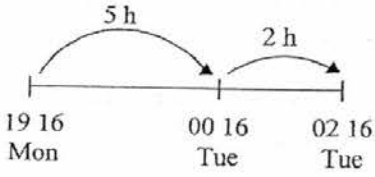
B1

(b) $k =$ 12 [1]

---

6 Local time in Singapore is 7 hours ahead of local time in London. Singapore Airlines SQ007 departed London on Monday at 19 16 London time. The flight arrived at Singapore on Tuesday at 15 51 Singapore time. Calculate how long the flight took, giving your answer in hours and minutes.

Departure time from London (Singapore time)
= 02 16 Tuesday **M1**



Arrival time at Singapore (Singapore time)
= 15 51 Tuesday

Duration of Journey
= 13 h 35 min

h	min
15	51
− 02	16
13	35

A1

Answer 13 hours 35 minutes [2]

7 The diameter of a spherical micro-organism is 9.04 micrometres. Find the surface area in square millimetres, of the micro-organism, giving your answer in standard form.

Radius $= \frac{1}{2} \times 9.04 \times 10^{-6}$ m
$= 4.52 \times 10^{-6} \times 10^{3}$ mm
$= 4.52 \times 10^{-3}$ mm **M1**

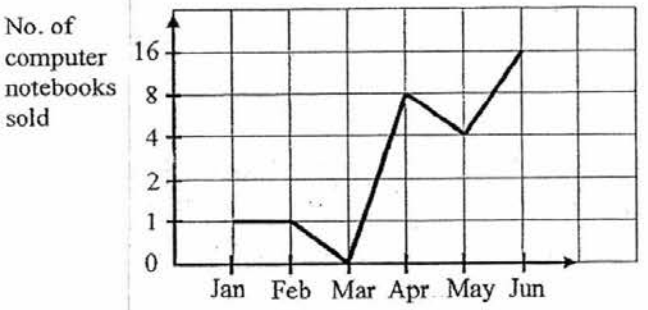
Surface area $= 4\pi (4.52 \times 10^{-3})^2$
$= 2.57 \times 10^{-4}$ mm^2

A1

Answer 2.57×10^{-4} mm^2 [2]

---

8 The graph below shows the sales of computer notebooks made by Angie over a period of 6 months in 2016.



Explain why the graph is misleading.

Answer The scale of the vertical axis is **not consistent.** **B1**

This distorts the graph, making the **sales from May to June** (16 – 4 = 12 units)

seemed to be **less than** the **sales from March to April** (8 – 0 = 8 units).

[2]

---

9 Two of the interior angles of a hexagon are $2x°$ and $(5x - 200)°$. The remaining interior angles are 90° each. By forming an equation in x, find the value of x.

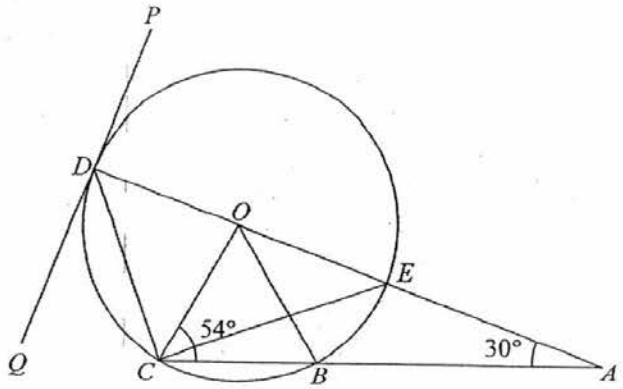
$2x + (5x - 200) + 4(90) = (6 - 2) \times 180$ **M1**
$7x + 160 = 720$
$7x = 560$
$x = 80$

A1

Answer $x =$ 80 [2]

---

10 In the diagram, the points B, C, D and E lie on a circle with centre O. PQ is a tangent to the circle at D. ABC and $AEOD$ are straight lines. $\angle OCB = 54°$ and $\angle OAB = 30°$.



Find, giving reasons for each answer,

(a) $\angle ADC$,

(b) $\angle CDQ$,

(c) $\angle ACE$,

(d) $\angle CBE$.

(a) $\angle COD = 54° + 30°$ (Ext $\angle$ of Δ) } **M1**
$= 84°$

$\angle ADC = \dfrac{180° - 84°}{2}$ (Base $\angle$s of isos. Δ) } **A1**
$= 48°$

(b) $\angle CDQ = 90° - 48°$ (tan $\perp$ rad) } **A1**
$= 42°$

(c) $\angle DCE = 90°$ (Rt. $\angle$ in semi-circle) **M1**
$\angle ADC = 180° - 90° - 48° - 30°$ ($\angle$ sum of Δ) } **A1**
$= 12°$

or $\angle COE = 48° \times 2$ ($\angle$ at centre = 2 $\angle$ at circumfere
$= 96°$
$\angle ACE = \dfrac{180° - 96°}{2}$ (Base $\angle$s of isos. Δ)
$= 42°$
$\angle ADC = 54° - 42°$
$= 12°$

(d) $\angle CBE = 180° - 48°$ ($\angle$s in opp segments are supp) } **A1**
$= 132°$

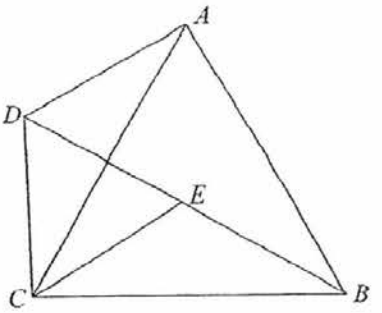
Answer **(a)** 48 ° [2]

(b) 42 ° [1]

(c) 12 ° [2]

(d) 132 ° [1]

11 $ABCD$ is a quadrilateral. ABC and CDE are equilateral triangles. Using a pair of congruent triangles, show that $AD = BE$. State your reasons clearly.



Answer In triangles ACD and BCE,

CD and CE (sides of equil. ΔCDE)

AB and BC (sides of equil. ΔABC)

$\angle ACD = 60° - \angle ACE$ ($\angle$ of equil. ΔCDE) **M1**

$\angle BCE = 60° - \angle ACE$ ($\angle$ of equil. ΔABC) **(all criteria must be correct)**

$\therefore \angle ACD = \angle BCE$

B1

$\therefore \Delta ACD = \Delta BCE$ (SAS) **(criteria must tally with test)**

Hence, $AD = BE$

..... [2]

---

12 Janet has \$50000 to invest for 3 years. She invests her money in a unit trust with returns equivalent to 2% per annum interest, compounded every 3 months. Calculate the amount of interest she will get at the end of 3 years.

$$\text{Amount} = 50000\left(1+\frac{0.02}{4}\right)^{12} \quad \textbf{M1}$$

$$= \$53083.8905$$

$$\text{Interest} = \$53083.8905 - \$50000$$

$$= \$3083.89 \text{ (to 2 dp)}$$

A1

Answer \$ 3083.89 ..... [2]

13 **(a)** Given that $\left(\frac{1}{4}\right)^p \times 8 = 1$, find the value of p.

$$\left(2^{-2}\right)^p \times 2^3 = 2^0$$
$$2^{-2p+3} = 2^0 \qquad \textbf{M1}$$
$$-2p+3=0$$
$$p = 1\frac{1}{2}$$

(b) Simplify $\left(\frac{2^{y+1}\sqrt{2}}{2^y}\right)^{-2}$.

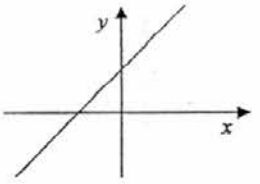
$$\left(\frac{2^{y+1}\sqrt{2}}{2^y}\right)^{-2}$$
$$= \left(2^{y+1+\frac{1}{2}-y}\right)^{-2} \qquad \textbf{M1}$$
$$= \left(2^{\frac{3}{2}}\right)^{-2}$$
$$= 2^{-3}$$
$$= \frac{1}{8}$$

Answer **(a)** $p =$ $1\frac{1}{2}$ **A1** [2]

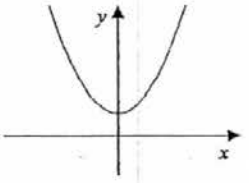
(b) $-\frac{1}{8}$ **A1** [2]

14 The equations of the three graphs shown below are in the form $y = n + x^{n-1}$.
State the value of n for each of the following graph.

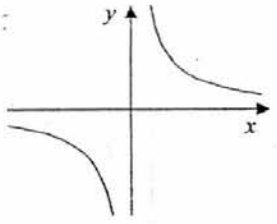
(a)



(b)



(c)



Answer **(a)** $n =$ 2 **B1** [1]

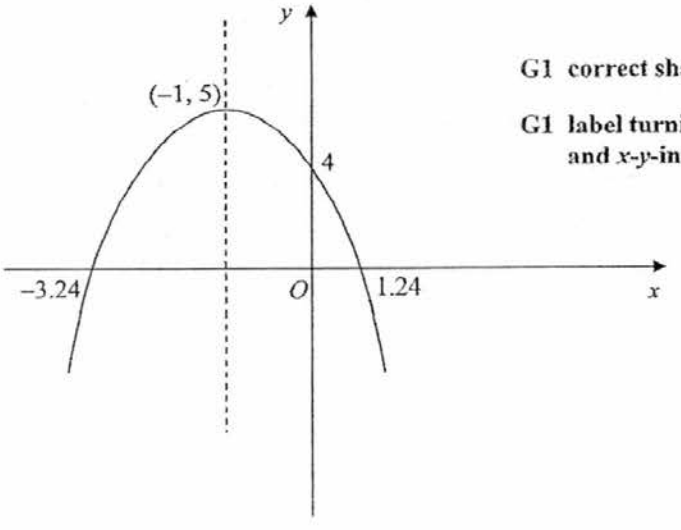
(b) $n =$ 3 **B1** [1]

(c) $n =$ 0 **B1** [1]

---

15 In the answer space, sketch the graph of $y = 5 - (x+1)^2$, indicate clearly the turning point and the intercepts on the x and y-axes (if any).

Answer [2]



G1 correct shape

G1 label turning point and x-y-intercepts

16 **(a)** $\varepsilon = \{ x : x \text{ is an integer and } 1 \leq x < 24 \} = \{ 1, 2, 3, \ldots 23 \}$
$A = \{ x : x \text{ is a perfect square} \} = \{ 1, 4, 9, 16 \}$
$B = \{ x : x \text{ is a factor of the number } 24 \} = \{ 1, 2, 3, 4, 6, 8, 12 \}$
$C = \{ x : x + 1 \text{ is divisible by } 6 \} = \{ 5, 11, 17, 23 \}$

(i) List the elements in $A \cap C$.

(ii) Find $n(B' \cup C)$.

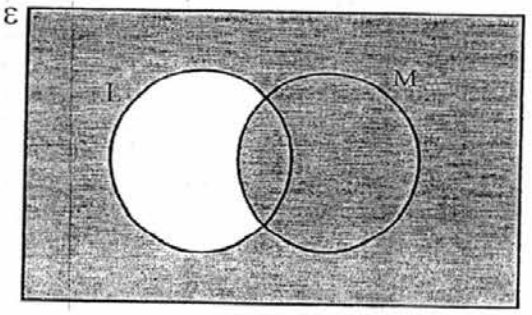
(a) (ii) $B' = \{ 5, 7, 9, 10, 11, 13, 14, 15, 16, \ldots 23 \}$

$$\begin{aligned} n(B' \cup C) &= n(B') \\ &= n(\varepsilon) - n(B) \\ &= 23 - 7 \end{aligned}$$

Answer **(a)(i)** ϕ or $\{\ \}$ **B1** ..... [1]

(ii) 16 **B1** ..... [1]

(b) State the set notation of the shaded region in following Venn Diagram.



Answer **(b)** $L' \cup M$ **B1** ..... [1]

17 Given that point $A(4, 2)$ and $\overrightarrow{AC} = \begin{pmatrix} -7 \\ 3 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{CA}\right|$.

$$\overrightarrow{CA} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$\left|\overrightarrow{CA}\right| = \sqrt{7^2 + (-3)^2}$$

$$= 7.62 \text{ (to 3 sf)}$$

B1

Answer **(a)** ..... 7.62 ..... units [1]

(b) The point P lies on CA such that $\overrightarrow{PA} = k\overrightarrow{CA}$.

(i) Show that $\overrightarrow{OP} = \begin{pmatrix} 4-7k \\ 2+3k \end{pmatrix}$.

Answer **(b)(i)** [1]

$$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} + k\overrightarrow{AC}$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} + k\begin{pmatrix} -7 \\ 3 \end{pmatrix}$$ **A1**

$$= \begin{pmatrix} 4-7k \\ 2+3k \end{pmatrix}$$ (shown)

(ii) Given that point P lies on the y-axis, find the coordinates of P.

$$4 - 7k = 0$$

$$k = \frac{4}{7}$$ **B1**

$$2 + 3\left(\frac{4}{7}\right) = 3\frac{5}{7}$$

A1

Answer **(b)(ii)** $P($ ..... 0 ..... , ... $3\frac{5}{7}$ ..... $)$ [2]

18 Consider the number patterns in the table below. The first three terms of each column have been given.

Row, n	S	T	U
1	4	16	16
2	8	32	30
3	12	48	44
7	p	q	r
n			

(a) Find values of p, q and r.

(b) Write down the equation connecting S and T.

(c) Write down the equation connecting U and n.

(d) Betty said that 256 can be found in column U.
Write whether you agree or disagree with Betty. Give reason(s) for your answer.

(d) $14n + 2 = 256$

$14n = 254$

$n = \dfrac{254}{14}$

$= 18\dfrac{1}{7}$

B1
(All 3 must be correct)

Answer **(a)** $p =$ 28 , $q =$ 112 , $r =$ 100 [1]

(b) $T = 4S$ **B1** [1]

(c) $U = 14n + 2$ **B1** [1]

(d) I **disagree** with Betty. This is because If $N = 256$, $n = 18\dfrac{1}{7}$ which is not a natural number. (is not a positive integer). **B1**

OR

When 2 is deducted from 256, the result 254 is not divisible by 14. (is not a multiple of 14). [1]

19 The frequency table shows the number of countries that a group of students had visited.

Number of countries	0	1	2	3	4
Number of students	2	8	6	x	4

(a) Given that the mode is 1, state the largest possible value of x.

(b) Given that the median number of countries visited is 2, find the largest possible value of x.

(c) Given that the mean number of countries is more than 2, find the smallest possible value of x.

(b) $2 + 8 + (6 - 1) = x + 4$

$15 = x + 4$

$x = 11$

(c) Mean $= \dfrac{0(2) + 1(8) + 2(6) + 3x + 4(4)}{2 + 8 + 6 + x + 4} > 2$

$\dfrac{3x + 36}{x + 20} > 2$ **M1**

$3x + 36 > 2(x + 20)$

$3x + 36 > 2x + 40$

$x > 4$

smallest $x = 5$

Answer **(a)** $x =$ 7 **B1** [1]

(b) $x =$ 11 **B1** [1]

(c) $x =$ 5 **B1** [2]

20 **(a)** The air resistance, R, is directly proportional to the square of the speed, V, of an object when it is falling. The air resistance is 24 newtons at a certain speed. Find the air resistance when the speed is increased by 50%.

(b) 48 men can build 2 huts in 60 hours. How many more men are needed if 3 huts are to be built in 72 hours?

(a) $R = kV^2$, k constant

$24 = kV^2 \Rightarrow k = \frac{24}{V^2}$ **M1**

$R_{new} = k(1.5V)^2$

$= \frac{24}{V^2} \times 2.25V^2$

$= 54$ newtons

(b) No. of men required to build 3 huts in 72 h

$= \frac{3}{2} \times \frac{60}{72} \times 48$

$= 60$

$\therefore$ Extra no. of men needed $= 60 - 48$

$= 12$

OR

48 men ---- 2 huts ---- 60 h

48 men ---- 1 hut ---- 30 h

1 man ---- 1 hut ---- 1440 h **M1**

1 man ---- 3 huts ---- 4320 h

60 men ---- 3 huts ---- 72 h

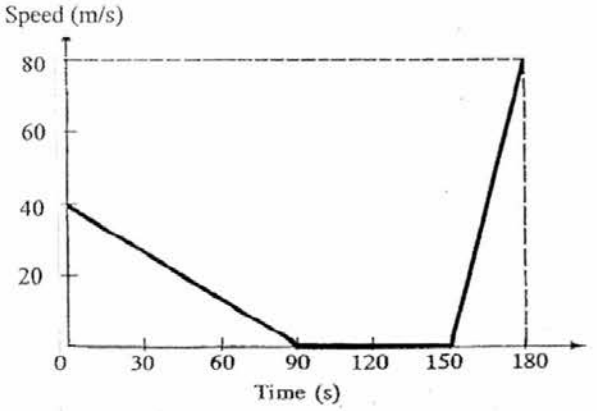
$\therefore$ Extra no. of men needed $= 60 - 48$

$= 12$

Answer **(a)** ..... 54 **A1** ..... newtons [2]

(b) ..... 12 **-A1** ..... men [2]

21 The diagram below shows the speed-time graph of the journey for the first 3 minutes of a train. The train slows down to a stop when entering station *J*. After a brief stop of 60 seconds, it starts to move off with acceleration for 30 seconds before it gets out of station *J*.



(a) Find the deceleration of the train as it enters station *J*.

(b) Calculate

(i) the total distance travelled by the train in the first 3 minutes,

(ii) the average speed of the train, in km/h, in the first 3 minutes.

(a) Acceleration $= \dfrac{40-0}{0-90} = -\dfrac{4}{9}$ m/s^2 $\quad \therefore$ Deceleration $= \dfrac{4}{9}$ m/s^2

(b) (i) Total distance $= \dfrac{1}{2}(90)(40)+\dfrac{1}{2}(30)(80)$

$= 1800 + 1200$

$= 3000$ m

(ii) Average speed $= \dfrac{3000\text{ m}}{3\text{ min}}$ **M1**

$= \dfrac{3\text{ km}}{\left(\frac{3}{60}\text{h}\right)}$

$= 60$ km/h

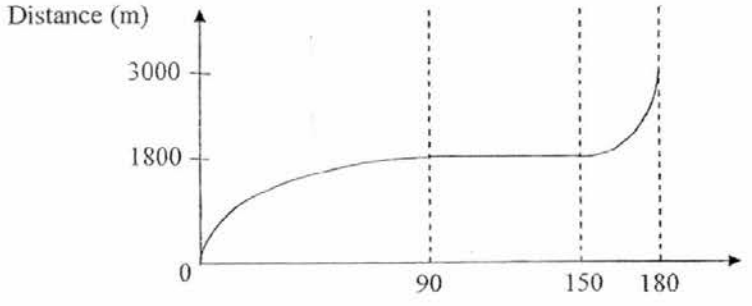
Answer **(a)** ..... $\frac{4}{9}$ **B1** ..... m/s^2 [1]

(b)(i) ..... 3000 **A1** ..... m [1]

(ii) ..... 60 **A1** ..... km/h [2]

(c) On the axes below, sketch the distance-time graph of the train for the first 3 minutes of its journey.

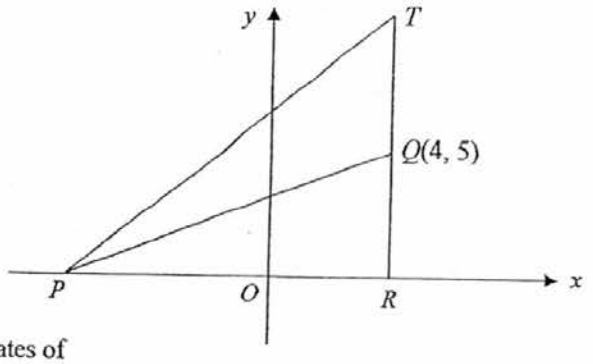
Answer **(c)** [2]



G1 correct shape

G1 label correct distance

22 P and R are points on the x-axis. TQR is a straight line parallel to the y-axis. Area of $\Delta PQR = 30$ units^2.



(a) Find the coordinates of
- **(i)** point R,
- **(ii)** point P.

(b) Find the length of PQ.

(c) Find $\cos\angle PQT$, giving your answer as a fraction.

(d) Given that $PR = TR$, find the equation of PT.

(a)(i) $R\,(4, 0)$

(ii) $\frac{1}{2} \times PR \times 5 = 30$ **M1**

$PR = \frac{2 \times 30}{5} = 12$ units

$\therefore P\,(-8, 0)$

(b) $P\,(-8, 0)$ $\quad Q\,(4, 5)$

$PQ = \sqrt{[4-(-8)]^2 + (5-0)^2}$

$= \sqrt{144 + 25}$

$= 13$ units

(c) $\cos\angle PQT = -\cos\angle PQR$

$= -\frac{5}{13}$

(d) $P\,(-8, 0)$ $\quad T\,(4, 12)$

$m = \frac{12-0}{4-(-8)} = 1$

Equation of PT is

$y - 0 = 1\,[x-(-8)]$

$y = x + 8$

Answer **(a)(i)** R (.....4..... , ...0.....) [1] **B1**

(ii) P (.....−8..... , ...0.....) [2] **A1**

(b) .....13..... units [1] **B1**

(c) .....$-\frac{5}{13}$..... [1]

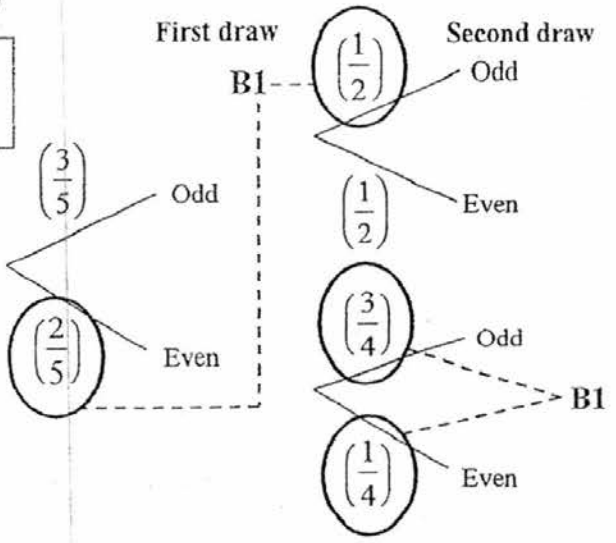
(d) .....$y = x + 8$..... [1] **A1**

23 Five discs numbered 1, 3, 4, 6 and 7 are placed in a bag. A disc is drawn out of the bag at random. Without replacing the first disc into the bag, a second disc is drawn.

(a) Complete the following probability tree diagram.

Answer **(a)** [2]

1, 3, 4, 6, 7
3 odd nos. , 2 even nos.



(b) Find

(i) the probability that one disc is odd and the other is even,

(ii) the probability that both numbers drawn are smaller than 4.

(c) By drawing a possibility diagram in the space below, find the probability that the sum of both numbers is a prime number.

(b) (i) $P(\text{odd, even}) + P(\text{even, odd}) = \frac{3}{5}\times\frac{1}{2} + \frac{2}{5}\times\frac{3}{4}$ or $= 2\times\frac{3}{5}\times\frac{1}{2}$

$= \frac{3}{5}$

(ii) $P(\text{both nos.} < 4) = \frac{2}{5}\times\frac{1}{4}$

$= \frac{1}{10}$

(c)

B1

+	1	3	4	6	7
1		4	5	7	8
3	4		7	9	10
4	5	7		10	11
6	7	9	10		13
7	8	10	11	13	

$P(\text{sum = prime no.}) = \frac{10}{20}$

$= \frac{1}{2}$

Answer **(b)(i)** $\frac{3}{5}$ **B1** ..... [1]

(ii) $\frac{1}{10}$ **B1** ..... [1]

(c) $\frac{1}{2}$ **B1** ..... [2]

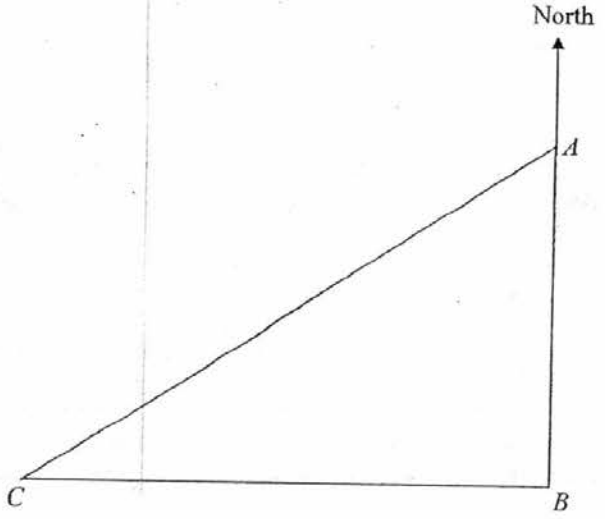
24 The diagram below shows a horizontal field ABC.
A is due north of B and C is due west of B.
Use a scale of 1 cm to 40 m, show all the constructions clearly.

(a) A lamp post, L, is located on a bearing of 290° from A, and 300 m from A.
- **(i)** By construction, mark and label clearly the position of the lamp post L. [1]
- **(ii)** Measure and write down the bearing of the lamp post L from point C.

(b) A gate, G, is located along the path of BC, equidistant from B and C.
By construction, mark and label clearly the position of the gate G. [1]

(c) A circular flower bed is built such that it touches each side of the field at one point.
- **(i)** By constructing two angle bisectors, draw the circular flower bed and label its centre O. [2]
- **(ii)** Hence, measure and write down the actual radius of the flower bed.

Answer **(a)(i)**
(b)
(c)(i)



Answer **(a)(ii)** ..... ° [1]

(c)(ii) ..... m [1]

End of Paper 1

mgs Paper 2

Answer **all** the questions.

1 **(a)** Given that $-8 \le x \le 4$ and $-3 \le y \le 2$, find

(i) the least value of xy, [1]

(ii) the greatest value of $x^2 - y^2$. [1]

(b) Express as a single fraction in its simplest form

(i) $\frac{x-y}{xy} + \frac{y-z}{yz}$, [2]

(ii) $\frac{2x^3}{x+y+z} \times \frac{(x+y)^2 - z^2}{6x}$. [2]

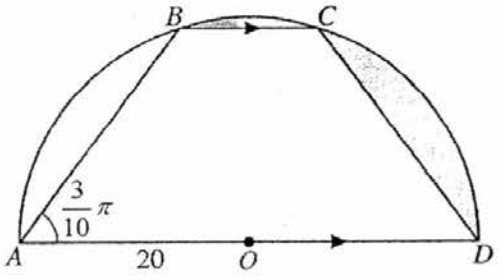
(c) It is given that $2pq = \sqrt{\frac{4q^2 + p^2}{2}}$.

Express q in terms of p. [3]

---

2 In the diagram, $OABCD$ is a semicircle with centre at O.

AD // BC, angle CDA = angle BAD = $\frac{3}{10}\pi$ radians and $OA = 20$ mm.



(a) Show that angle $BOA = \frac{2}{5}\pi$ rad. [1]

(b) Find the length of arc AB, leaving your answer in terms of π. [1]

(c) Find angle BOC. [1]

(d) Calculate the area of the shaded region. [3]

(e) Find angle BOA in degrees. [1]

(f) The unshaded region forms a company logo. An enlarged copy of the logo is made. In the enlargement, $AD = 60$ mm. Find the area of the enlarged logo. [2]

---

3 The cash price of a car is \$74 000. Mr Smith is introduced to two types of payment schemes.

	Scheme *A*	**Scheme *B***
Down payment	40%	60%
Simple interest rate (per annum)	3.28%	R %
Loan period (years)	5	5

(a) Find the total amount that Mr Smith has to pay for the car, if he chose **Scheme *A***. [2]

(b) If Mr Smith chose **Scheme *B***, the monthly instalment he has to pay over 5 years is \$572.76. Calculate the value of R. [3]

(c) One day the exchange rate between US dollar (US\$) and Singapore dollars (S\$) was US\$1 = S\$1.27 .

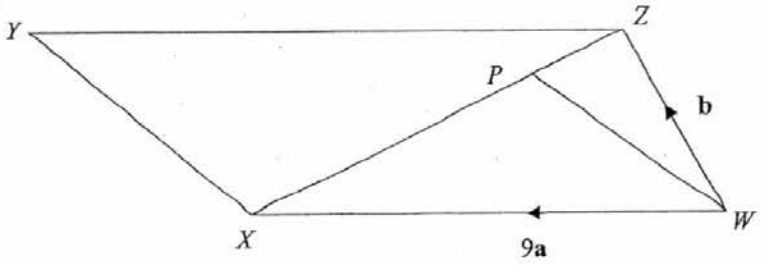
On the same day, the exchange rate between British pound (£) and US dollar was £1 = US\$1.33.

Calculate the cash price of the car in pounds, correct to the nearest pound. [2]

4 In the diagram, $WXYZ$ is a trapezium and WX is parallel to ZY.

The point P on XZ is such that $ZP : PX = 1 : 3$ and $WX : ZY = 3 : 4$.

It is given that $\overrightarrow{WX} = 9\mathbf{a}$ and $\overrightarrow{WZ} = \mathbf{b}$.



(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{ZX}$, [1]

(ii) $\overrightarrow{WP}$, [1]

(iii) $\overrightarrow{YW}$. [1]

(b) Show that the line XY is parallel to the line WP. [2]

(c) Find, as a fraction in its simplest form,

(i) $\dfrac{\text{area of } \Delta WZP}{\text{area of } \Delta WXP}$, [1]

(ii) $\dfrac{\text{area of } \Delta WZP}{\text{area of } \Delta YXZ}$. [2]

5 **Answer the whole of this question on a sheet of graph paper.**

A group of friends founded a new social networking website. The table below shows the number of members at the beginning of each week over a period of 7 weeks.

Week (x)	0	1	2	3	4	5	6	7
Total number of members (y)	5	15	35	p	90	145	230	400

(a) Using a scale of 2 cm to 1 week, draw a horizontal x-axis for $0 \le x \le 7$.
Using a scale of 2 cm to 50 members, draw a vertical y-axis for $0 \le y \le 400$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to estimate

(i) the value of p, [1]

(ii) the week that the total number of members reaches 300. [1]

(c) **(i)** By drawing a tangent, find the gradient of the curve at $x = 4$. [2]

(ii) What does this gradient represent? [2]

(d) The group of friends wish to estimate what the total number of members will be in one year's time. They propose to extend the graph line up to week, $x = 52$.
Explain why is it not possible to estimate the total number of members in this way. [1]

6 The distance between two houses, P and Q, is 200 km. Joe travelled by car from P to Q at an average speed of x km/h.

(a) Write down an expression, in terms of x, for the number of hours he took to travel from P to Q. [1]

(b) He returned from Q to P at an average speed of which was 5 km/h more than the first journey.

Write down an expression, in terms of x, for the number of hours he took to travel from Q to P. [1]

(c) The difference between the two times was 24 minutes.

Write down an equation in x to represent this information, and show that it reduces to

$$x^2 + 5x - 2500 = 0.$$ [3]

(d) Solve the equation $x^2 + 5x - 2500 = 0$, giving each answer correct to three decimal places. [3]

(e) Calculate the time that Joe took to travel from P to Q, giving your answer in hours, minutes and seconds, correct to the nearest second. [2]

7 **(a)** Jim exercises on Monday and Wednesday.

On Monday, he jogs for 10 minutes, cycles for 20 minutes and swims for 30 minutes.

On Wednesday, he jogs for 20 minutes, cycles for 10 minutes and swims for 15 minutes.

This information can be represented by the matrix $\mathbf{Q} = \begin{matrix} \\ \text{Mon} \\ \text{Wed} \end{matrix} \begin{matrix} \begin{matrix} \text{J} & \text{C} & \text{S} \end{matrix} \\ \begin{pmatrix} 10 & 20 & 30 \\ 20 & 10 & 15 \end{pmatrix} \end{matrix}$.

(i) Evaluate the matrix $\mathbf{P} = 60\mathbf{Q}$. [1]

(ii) Jim's exercising speeds are the same for Monday and Wednesday.

His jogging speed is 4 m/s, cycling speed is 5.5 m/s and swimming speed is 1.3 m/s.

Represent his exercising speeds in a 3×1 column matrix $\mathbf{S}$. [1]

(iii) Evaluate the matrix $\mathbf{R} = \mathbf{PS}$. [2]

(iv) State what the elements of $\mathbf{R}$ represent. [1]

(b) The cost of a shirt is $\$C$. If the shirt is sold at \$60, a shop makes a profit of $x\%$ on the cost price.

(i) Write down an equation in C and x to represent this information and show that it simplifies to

$$6000 - 100C = Cx.$$

[1]

If the shirt is sold at \$24, the shop makes a loss of $2x$ % on the cost price.

(ii) Write down an equation in C and x to represent this information. [1]

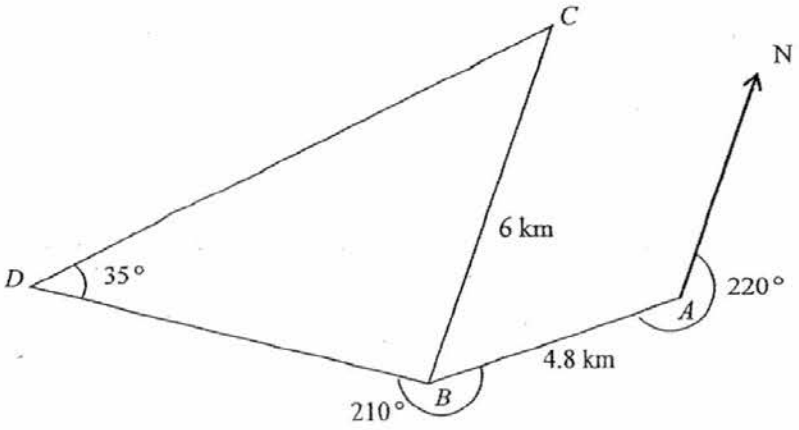
(iii) Solve these two equations to find the value of C and the value of x. [3]

(iv) Calculate the selling price of the shirt if the profit is 45% of the cost price. [2]

8 The diagram shows a triangular park BCD and the route that Ali has cycled.

Ali cycles from his home, A, on a bearing of 220° towards point B of the park. The distance from A to B is 4.8 km. From B, he cycles to C, which is 6 km away, and he continues to D.

C is due north of B. Reflex angle $ABD = 210°$ and angle $BDC = 35°$.



(a) Show that ΔBCD is an isosceles triangle. [1]

(b) Calculate the

(i) distance of AC, [3]

(ii) area of the park BCD, [2]

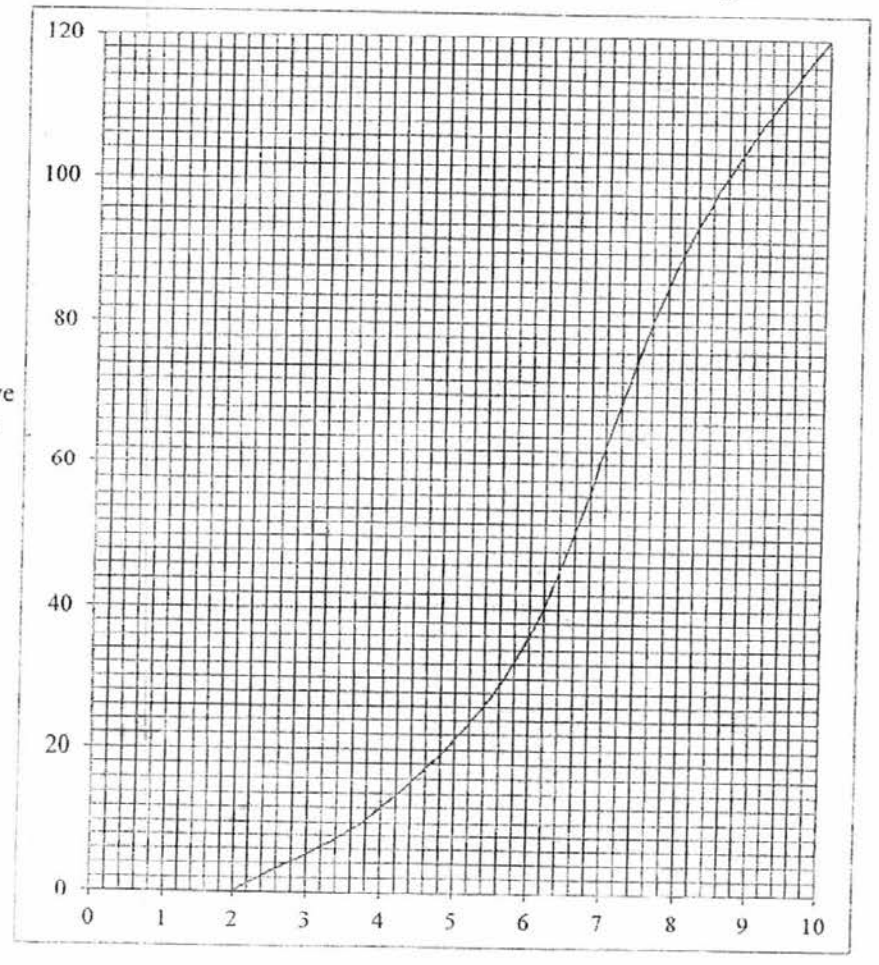
(iii) angle BAC, [2]

(iv) shortest distance from B to CD. [2]

(c) A building stands vertically at B. The angle of depression of C when viewed from the top of the building is 40°. Find the height of the building. [2]

9 120 visitors took a survey on the number of hours they spent at the Gardens by the Bay in February 2016.

The cumulative frequency curve below shows the distribution of the time spent.



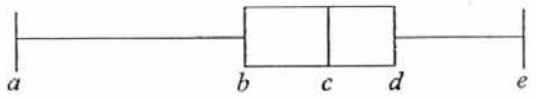
(a) Use the curve to estimate

(i) the median time, [1]

(ii) the interquartile range of the times, [2]

(iii) the percentage of visitors who spent at least 4 hours at the Gardens by the Bay. [2]

(b) It was discovered that the number of hours has been recorded incorrectly. The correct number of hours was all 1 hour less than those recorded.
The box-and-whisker plot shows the correct distribution of hours.



Find the value of

(i) c, [1]

(ii) $e-a$. [1]

(c) The table below shows the results of the survey conducted on another 120 visitors on the number of hours they spent at the Gardens by the Bay in June 2016.

Number of hours spent (x h)	Number of visitors
$2 < x \leq 4$	33
$4 < x \leq 6$	46
$6 < x \leq 8$	30
$8 < x \leq 10$	11

Calculate an estimate of the

(i) mean time that the visitors spent in June, [1]

(ii) standard deviation. [2]

(d) The programme management team at the Gardens by the Bay commented that the visitors generally spent longer hours in February 2016 than in June 2016.
Justify if the comment is valid. [2]

10 A solid cone is cut into 2 parts, X and Y, by a plane parallel to the base.
The length of AB = the length of BC.

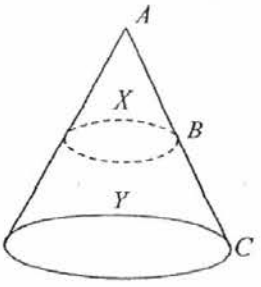


Diagram I

(a) Given that the volume of the solid cone is $\frac{64}{3}\pi$ m^3, find the volume, in terms of π, of the frustum, Y. [3]

(b) In **Diagram II**, a rocket can be modelled from a cylinder of height, h, 94.2 m with a cone, X, on top and a frustum, Y, at the bottom. The cone, X, has a diameter, d_2, of 4 m and the frustum, Y, has a base diameter, d_1, of 8 m. The parts X and Y are taken from **Diagram I** above.

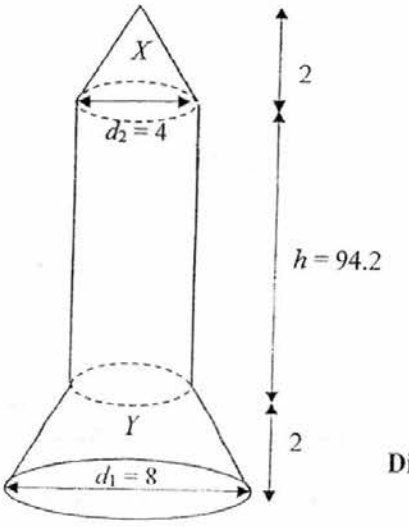


Diagram II

(i) Calculate the total surface area of the rocket. Give your answer correct to the nearest square meter. [3]

(ii) Calculate the volume, in cubic metres, of the rocket. [1]

(iii) The rocket is designed to launch to the moon.

Useful information

- Distance of moon from earth: 384 400 km
- Speed of rocket: 800 km /minute
- $1\ m^3 = 264$ gallon
- The rocket is filled with liquid fuel to a maximum of 95% of its volume.
- Rate of fuel consumption: 20 000 gallons /minute
- Capacity of each external fuel tank: 3.2×10^6 gallons

How many external fuel tanks will the rocket require to sustain its journey to the moon?

Justify your answer with calculations. [4]

Name : _____ Class | Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2016
Secondary 4

Tuesday
16 August 2016

MATHEMATICS
Paper 2

4048/02
2 h 30 mins

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

INFORMATION FOR CANDIDATES

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of 13 printed pages

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1	(a)	Given that $-8 \le x \le 4$ and $-3 \le y \le 2$, find		
		(i)	the least value of xy, **Least value of $xy = (-8)(2) = -16$ ---- B1**	[1]
		(ii)	the greatest value of $x^2 - y^2$. **Greatest value of $x^2 - y^2 = (-8)^2 - 0 = 64$ ---- B1**	[1]
	(b)	Express as a single fraction in its simplest form		
		(i)	$\frac{x-y}{xy} + \frac{y-z}{yz}$, $\frac{xz-yz+xy-xz}{xyz}$ ----- M1 $= \frac{xy-yz}{xyz}$ $= \frac{y(x-z)}{xyz}$ $= \frac{x-z}{xz}$ ----- A1	[2]
		(ii)	$\frac{2x^3}{x+y+z} \times \frac{(x+y)^2 - z^2}{6x}$. $\frac{2x^3}{x+y+z} \times \frac{(x+y-z)(x+y+z)}{6x}$ ----- M1 $= \frac{x^2(x+y-z)}{3}$ ---- A1	[2]
	(c)	It is given that $2pq = \sqrt{\frac{4q^2+p^2}{2}}$. Express q in terms of p.		[3]

$$2pq = \sqrt{\frac{4q^2+p^2}{2}}$$

$$4p^2q^2 = \frac{4q^2+p^2}{2} \quad \text{----- M1}$$

$$8p^2q^2 = 4q^2+p^2$$

$$8p^2q^2 - 4q^2 = p^2$$

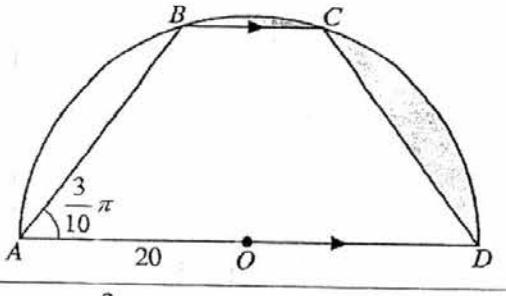
$$q^2(8p^2-4) = p^2 \quad \text{----- M1}$$

$$q^2 = \frac{p^2}{(8p^2-4)}$$

$$q = \pm\sqrt{\frac{p^2}{4(2p^2-1)}} \quad \text{---- A1} \quad \text{or} \quad q = \pm\frac{p}{2\sqrt{2p^2-1}} \quad \text{or} \quad q = \pm\sqrt{\frac{p^2}{8p^2-4}}$$

2 In the diagram, $OABCD$ is a semicircle with centre at O.

AD // BC, angle CDA = angle BAD = $\frac{3}{10}\pi$ radians and OA = 20 mm.



(a) Show that angle $BOA = \frac{2}{5}\pi$ rad. [1]

ΔBOA is an isosceles triangle

$\angle BOA = \pi - 2\left(\frac{3\pi}{10}\right)$ ----- B1

$= \frac{2\pi}{5}$ rad

(b) Find the length of arc AB, leaving your answer in terms of π. [1]

arc length $AB = (20)\left(\frac{2\pi}{5}\right)$

$= 8\pi$ mm ----- B1

(c) Find angle BOC. [1]

		$\angle BOC = \pi - 2\left(\frac{2\pi}{5}\right)$ (adj ∠s on a st line) $= \frac{\pi}{5}$ rad ----- B1 (or 0.628 rad (3 s.f.) or 36°)	
	(d)	Calculate the area of the shaded region. $\angle BOD = \pi - \frac{2\pi}{5}$ $= \frac{3\pi}{5}$ rad area of sector $BOD = \frac{1}{2}(20)^2\left(\frac{3\pi}{5}\right)$ ----- M1 $= 120\pi$ mm^2 area of ΔBOD and $\Delta COD = \frac{1}{2}(20)^2\left(\sin\frac{\pi}{5} + \sin\frac{2\pi}{5}\right)$ ----- M1 shaded area $= 120\pi - 200\left(\sin\frac{\pi}{5} + \sin\frac{2\pi}{5}\right)$ $= 69.2$ mm^2 (3 s.f.) ----- A1 OR shaded area $= \frac{1}{2}(20)^2\left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right) + \frac{1}{2}(20)^2\left(\frac{2\pi}{5} - \sin\frac{2\pi}{5}\right)$ ----- M1+M1 $= 69.2$ mm^2 (3 s.f.) ----- A1	[3]
	(e)	Find angle BOA in degrees. $\angle BOA = \frac{2\pi}{5}$ $= 72°$ ----- B1	[1]
	(f)	The unshaded region forms a company logo. An enlarged copy of the logo is made. In the enlargement, $AD = 60$ mm. Find the area of the enlarged logo.	[2]

unshaded area $= \frac{1}{2}\pi(20)^2 - 69.22276$ ----- M1

$= 559.0957 \text{ mm}^2$

$\left(\frac{AD}{AO}\right)^2 = \frac{\text{area of enlarged logo}}{559.0957}$

$\left(\frac{60}{40}\right)^2 = \frac{\text{area of enlarged logo}}{559.0957}$

$\frac{9}{4} = \frac{\text{area of enlarged logo}}{559.0957}$

area of enlarged logo $= \frac{9}{4} \times 559.0957$

$= 1260 \text{ mm}^2$ (3 s.f.) ----- A1

or by using enlarged radius = 30

3 The cash price of a car is \$74 000. Mr Smith is introduced to two types of payment schemes.

	Scheme *A*	**Scheme *B***
Down payment	40%	60%
Simple interest rate (per annum)	3.28%	R %
Loan period (years)	5	5

(a) Find the total amount that Mr Smith has to pay for the car, if he chose **Scheme *A***. [2]

Amount loaned $= 0.6 \times 74000$

$= \$44400$

Simple interest $= 44400 \times \frac{3.28}{100} \times 5$ ----- M1

$= \$7281.60$

Total amount $= 7281.60 + 74000$

$= \$81281.60$ ----- A1

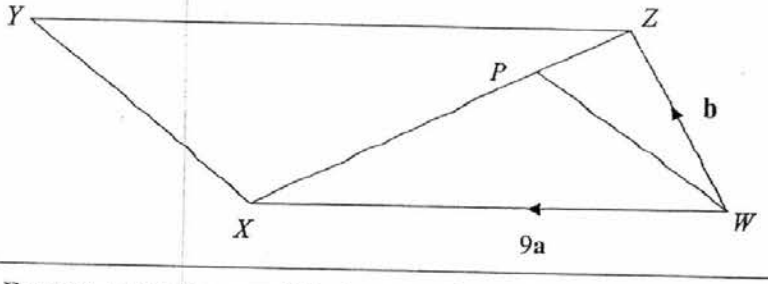
(b) If Mr Smith chose **Scheme *B***, the monthly instalment he has to pay over 5 years is \$572.76. Calculate the value of R. [3]

		Amount loaned $= 0.4 \times 74000$ $= \$29600$ $572.76 \times 12 \times 5 = 29600 + 29600 \times \frac{R}{100} \times 5$ M1- instalments paid (LHS) + M1- simple interest (RHS) $R = 3.22$ ----- A1	
	(c)	One day the exchange rate between US dollar (US$) and Singapore dollars (S$) was US$1 = S$1.27 . On the same day, the exchange rate between British pound (£) and US dollar was £1 = US$1.33. Calculate the cash price of the car in pounds, correct to the nearest pound. Amount in US$ $= 74000 \div 1.27$ ------- M1 here $= US\$58267.71654$ Amount in pounds $= 58267.71654 \div 1.33$ ----- or M1 here $= £43810$ (to nearest pound) ----- A1 *or* $£1 = US\$1.33 \times 1.27$ ----- M1 $= US\$1.6891$ cost of car in pounds $= \frac{74000}{1.6891}$ $= £43810$ (to nearest pound)	[2]

4 In the diagram, $WXYZ$ is a trapezium and WX is parallel to ZY.

The point P on XZ is such that $ZP : PX = 1 : 3$ and $WX : ZY = 3 : 4$.

It is given that $\overrightarrow{WX} = 9\mathbf{a}$ and $\overrightarrow{WZ} = \mathbf{b}$.



(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{ZX}$, [1]

$\overrightarrow{ZX} = -\mathbf{b} + 9\mathbf{a}$ ---- **B1**

(ii) $\overrightarrow{WP}$, [1]

$\overrightarrow{WP} = \mathbf{b} + \overrightarrow{ZP}$

$= \mathbf{b} + \frac{1}{4}(-\mathbf{b} + 9\mathbf{a})$

$= \frac{3}{4}(\mathbf{b} + 3\mathbf{a})$ ---- **B1**

(iii) $\overrightarrow{YW}$. [1]

$\overrightarrow{WY} = \mathbf{b} + \overrightarrow{ZY}$

$= \mathbf{b} + \frac{4}{3}(9\mathbf{a})$

$= \mathbf{b} + 12\mathbf{a}$

$\overrightarrow{YW} = -\mathbf{b} - 12\mathbf{a}$ ---- **B1**

or

$\overrightarrow{YW} = \overrightarrow{YZ} - \mathbf{b}$

$= -\mathbf{b} - 12\mathbf{a}$

(b) Show that the line XY is parallel to the line WP. [2]

$\overrightarrow{XY} = \overrightarrow{XW} + \overrightarrow{WY}$

$= -9\mathbf{a} + 12\mathbf{a} + \mathbf{b}$

$= 3\mathbf{a} + \mathbf{b}$ ----- M1

$\overrightarrow{WP} = \frac{9}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

$= \frac{3}{4}(3\mathbf{a} + \mathbf{b})$

Since $\overrightarrow{WP} = \frac{3}{4}\overrightarrow{XY}$ ----- A1

XY is parallel to WP.

(c) Find, as a fraction in its simplest form,

(i) $\frac{\text{area of } \Delta WZP}{\text{area of } \Delta WXP}$, [1]

$= \frac{1}{3}$

(ii) $\frac{\text{area of } \Delta WZP}{\text{area of } \Delta YXZ}$. [2]

WZP	:	WXZ	:	YXZ	
1	:	4			
		3	:	4	------ M1
3	:	12	:	16	

$\frac{\text{area of } \Delta WZP}{\text{area of } \Delta YXZ} = \frac{3}{16}$ ------ A1

Or

$\frac{\text{area of } \Delta WZP}{\text{area of } \Delta YXZ} = \frac{1}{4} \times \frac{3}{4} = \frac{3}{6}$

5 Answer the whole of this question on a sheet of graph paper.

A group of friends founded a new social networking website. The table below shows the number of members at the beginning of each week over a period of 7 weeks.

Week (x)	0	1	2	3	4	5	6	7
Total number of members (y)	5	15	35	p	90	145	230	400

(a) Using a scale of 2 cm to 1 week, draw a horizontal x-axis for $0 \le x \le 7$. Using a scale of 2 cm to 50 members, draw a vertical y-axis for $0 \le y \le 400$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to estimate

(i) the value of p, [1]

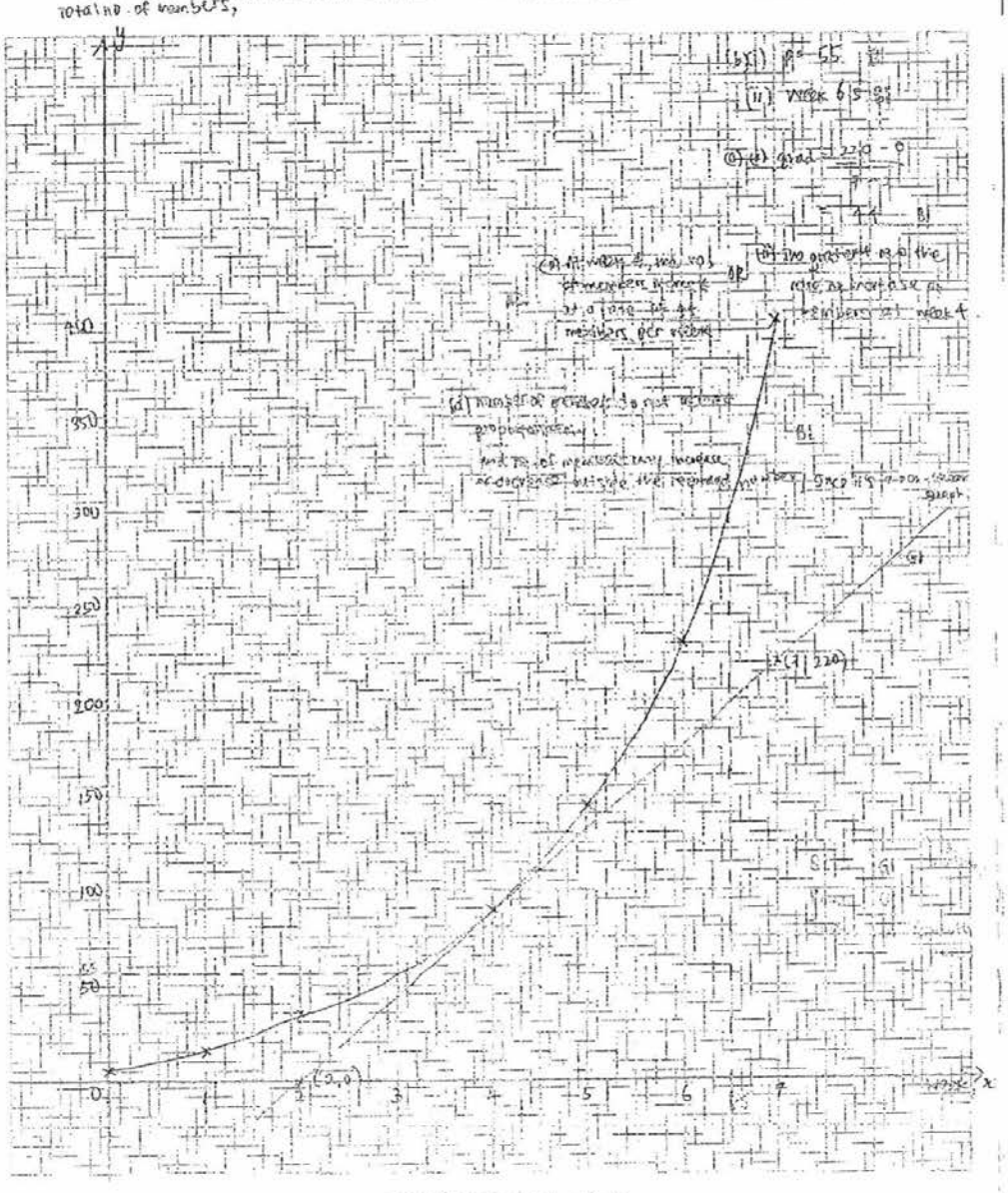
(ii) the week that the total number of members reaches 300. [1]

(c) **(i)** By drawing a tangent, find the gradient of the curve at $x = 4$. [2]

(ii) What does this gradient represent? [2]

(d) The group of friends wish to estimate what the total number of members will be in one year's time. They propose to extend the graph line up to week, $x = 52$. Explain why is it not possible to estimate the total number of members in this way. [1]

Name _____ ( ) Class _____ Date _____



6	The distance between two houses, P and Q, is 200 km. Joe travelled by car from P to Q at an average speed of x km/h.		
	(a)	Write down an expression, in terms of x, for the number of hours he took to travel from P to Q. time $= \frac{200}{x} h$	[1]
	(b)	He returned from Q to P at an average speed of which was 5 km/h more than the first journey. Write down an expression, in terms of x, for the number of hours he took to travel from Q to P. time $= \frac{200}{x+5} h$	[1]
	(c)	The difference between the two times was 24 minutes. Write down an equation in x to represent this information, and show that it reduces to $x^2 + 5x - 2500 = 0$. $\frac{200}{x} - \frac{200}{x+5} = \frac{24}{60}$ ----- M1 $200(x+5) - 200x = \frac{2}{5}(x)(x+5)$ ----- M1 $1000(x+5) - 1000x = 2x^2 + 10x$ $1000x + 5000 - 1000x = 2x^2 + 10x$ $2x^2 + 10x - 5000 = 0$ ----- M1 $x^2 + 5x - 2500 = 0$	[3]
	(d)	Solve the equation $x^2 + 5x - 2500 = 0$, giving each answer correct to three decimal places. $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-2500)}}{2(1)}$ ----- M1 $= 47.562$ or -52.562 ----- A1 +A1	[3]
	(e)	Calculate the time that Joe took to travel from P to Q, giving your answer in hours, minutes and seconds, correct to the nearest second. time $= \frac{200}{47.562} =$ 4h 12min 18sec (nearest sec) ---- M1 + A1	[2]

7	(a)	Jim exercises on Monday and Wednesday. On Monday, he jogs for 10 minutes, cycles for 20 minutes and swims for 30 minutes. On Wednesday, he jogs for 20 minutes, cycles for 10 minutes and swims for 15 minutes. This information can be represented by the matrix $\mathbf{Q} = \begin{matrix} \\ \text{Mon} \\ \text{Wed} \end{matrix}\begin{matrix} \begin{matrix} \text{J} & \text{C} & \text{S} \end{matrix} \\ \begin{pmatrix} 10 & 20 & 30 \\ 20 & 10 & 15 \end{pmatrix} \end{matrix}$.		
		(i)	Evaluate the matrix $\mathbf{P} = 60\mathbf{Q}$. $\mathrm{P} = 60\begin{pmatrix} 10 & 20 & 30 \\ 20 & 10 & 15 \end{pmatrix}$ $= \begin{pmatrix} 600 & 1200 & 1800 \\ 1200 & 600 & 900 \end{pmatrix}$ B1	[1]
		(ii)	Jim's exercising speeds are the same for Monday and Wednesday. His jogging speed is 4 m/s, cycling speed is 5.5 m/s and swimming speed is 1.3 m/s. Represent his exercising speeds in a 3×1 column matrix **S**. $\mathbf{S} = \begin{pmatrix} 4 \\ 5.5 \\ 1.3 \end{pmatrix}$ B1	[1]
		(iii)	Evaluate the matrix $\mathbf{R} = \mathbf{PS}$. $\mathbf{R} = \begin{pmatrix} 600 & 1200 & 1800 \\ 1200 & 600 & 900 \end{pmatrix}\begin{pmatrix} 4 \\ 5.5 \\ 1.3 \end{pmatrix}$ M1 $= \begin{pmatrix} 11340 \\ 9270 \end{pmatrix}$ A1	[2]
		(iv)	State what the elements of **R** represent. The elements of **R** represent the distance, in metres, that Jim has exercised on Monday and Wednesday, respectively. A1	[1]

(b) The cost of a shirt is \$$C$. If the shirt is sold at \$60, a shop makes a profit of x% on the cost price.

(i) Write down an equation in C and x to represent this information and show that it simplifies to

$$6000-100C=Cx.$$

[1]

Percentage profit $= x$ %

$$\frac{60-C}{C}\times 100 = x$$
$$100(60-C) = Cx$$ } M1

$$6000-100C=Cx \quad \text{(shown)}$$

If the shirt is sold at \$24, the shop makes a loss of $2x$ % on the cost price.

(ii) Write down an equation in C and x to represent this information. [1]

$$2x=\frac{C-24}{C}\times 100$$
$$2x=\frac{100C-2400}{C}$$
$$100C-2400=2Cx \quad \text{A1}$$

(iii) Solve these two equations to find the value of C and the value of x. [3]

$6000-100C=Cx$ --- (1)

$100C-2400=2Cx$ --- (2)

(1) × 2 − (2),

$$(12000-200C)-(100C-2400)=0$$ } M1
$$1400=300C$$

$C=48$

$x=25$ A1+A1

(iv) Calculate the selling price of the shirt if the profit is 45% of the cost price. [2]

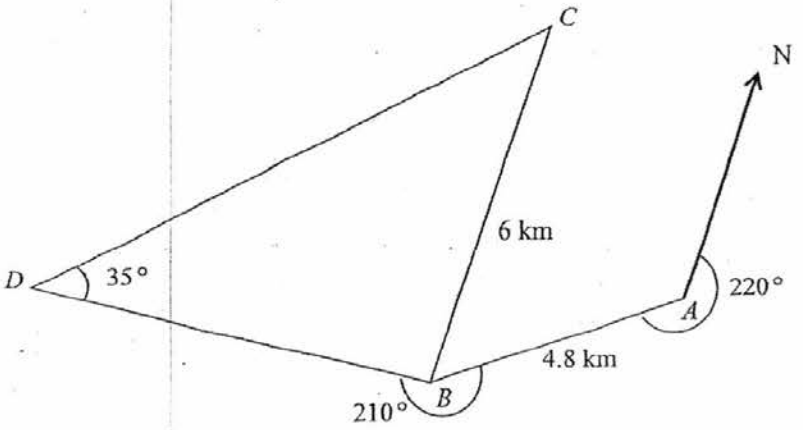
Selling price $= 1.45\times 48$ M1

$=$ \$69.60 A1

8 The diagram shows a triangular park BCD and the route that Ali has cycled.

Ali cycles from his home, A, on a bearing of 220° towards point B of the park. The distance from A to B is 4.8 km. From B, he cycles to C, which is 6 km away, and he continues to D.

C is due north of B. Reflex angle $ABD = 210°$ and angle $BDC = 35°$.



(a) Show that ΔBCD is an isosceles triangle. [1]

$\angle CBA = 180° - (360° - 220°)$ (int ∠s, ∠s at a point)

$= 40°$

$\angle DBC = 360° - 210° - 40°$ (∠s at a point)

$= 110°$

$\angle DCB = 180° - 35° - 110°$ (∠ sum of Δ)

$= 35°$

Since $\angle DCB = \angle CDB = 35°$, ΔBCD is an isosceles triangle. B1

(b) Calculate the

(i) distance of AC, [3]

$AC^2 = 6^2 + 4.8^2 - 2(6)(4.8)\cos 40°$ M2, 1

$AC = \sqrt{14.91584008}$

$= 3.86$ km^2 (to 3 sf) A1

(ii) area of the park BCD. [2]

Area of $\Delta BCD = \frac{1}{2}(6)(6)\sin 110°$ M1

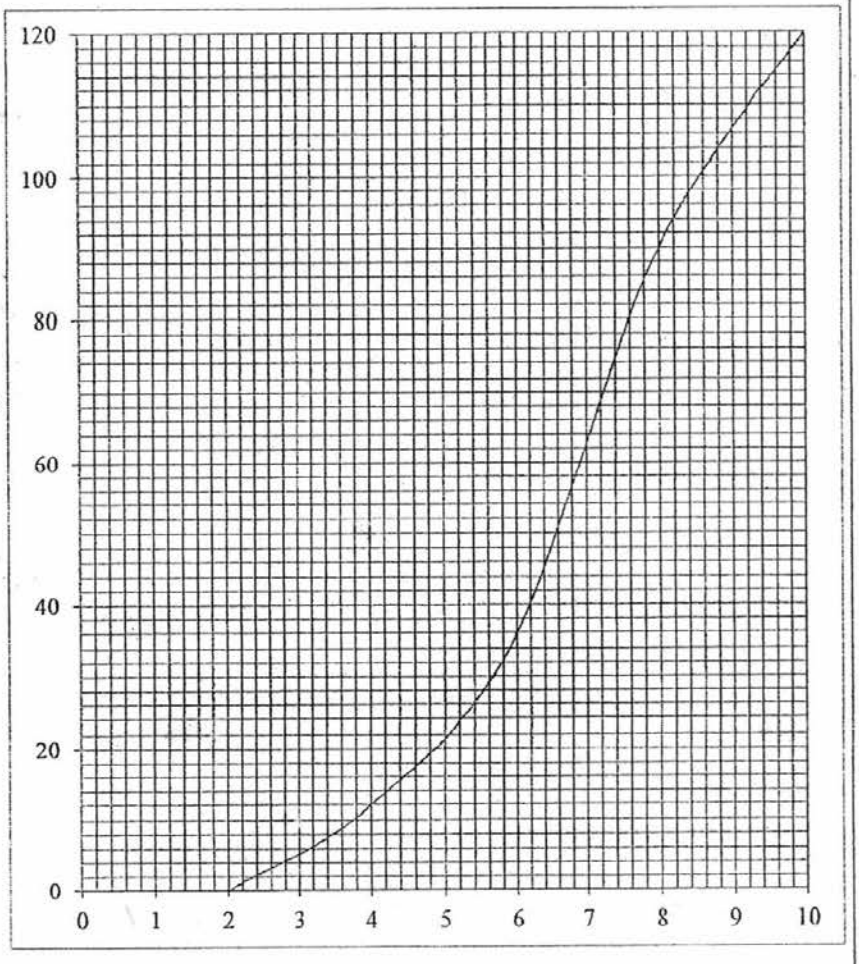
$= 16.9$ km^2 (to 3 sf) A1

(iii) angle BAC, [2]

			$\frac{\sin\angle BAC}{6} = \frac{\sin 40°}{3.862103}$ **M1** $\angle BAC = \sin^{-1}\left(\frac{\sin 40°}{3.862103} \times 6\right)$ $= 87.0°$ (to 1 dp) **A1**	
		(iv)	shortest distance from B to CD. Shortest distance $= 60 \times \sin 35°$ **M1** $= 3.44$ km (to 3 sf) **A1**	[2]
	(c)	A building stands vertically at B. The angle of depression of C when viewed from the top of the building is $40°$. Find the height of the building. Height of the building $= 6 \times \tan 40°$ **M1** $= 5.03$ km (to 3 sf) **A1**		[2]

9 120 visitors took a survey on the number of hours they spent at the Gardens by the Bay in February 2016.

The cumulative frequency curve below shows the distribution of the time spent.



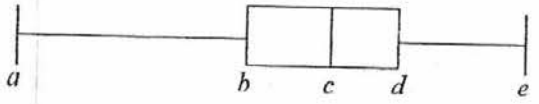
	(a)	Use the curve to estimate		
		(i)	the median time, **median = 6.9 hours B1**	[1]
		(ii)	the interquartile range of the times, **IQR = 8 – 5.7 M1** **= 2.3 hours A1**	[2]
		(iii)	the percentage of visitors who spent at least 4 hours at the Gardens by the Bay.	[2]

$percentage = \frac{120-12}{120} \times 100\%$ M1

$= 90\%$ A1

(b) It was discovered that the number of hours has been recorded incorrectly. The correct number of hours was all 1 hour less than those recorded.

The box-and-whisker plot shows the correct distribution of hours.



Find the value of

(i) c,

$c = 5.9$ hours B1 [1]

(ii) $e - a$.

$e - a = 8$ hours B1 [1]

(c) The table below shows the results of the survey conducted on another 120 visitors on the number of hours they spent at the Gardens by the Bay in June 2016.

Number of hours spent (x h)	Number of visitors
$2 < x \leq 4$	33
$4 < x \leq 6$	46
$6 < x \leq 8$	30
$8 < x \leq 10$	11

Calculate an estimate of the

(i) mean time that the visitors spent in June, [1]

$mean = \frac{3 \times 33 + 5 \times 46 + 7 \times 30 + 9 \times 11}{120}$

$= 5.32$ hours (to 3 sf) B1

(ii) standard deviation. [2]

standard deviation $= 1.86$ hours (to 3 sf) B2 or M1+A1

	(d)	The programme management team at the Gardens by the Bay commented that the visitors generally spent longer hours in February 2016 than in June 2016. Justify if the comment is valid. Median in June is $4 < x \leq 6$. M1 The comment is invalid as median is in February (5.9 hours) is within the median class in June ($4 < x \leq 6$). A1	[2]

10 A solid cone is cut into 2 parts, X and Y, by a plane parallel to the base.
The length of AB = the length of BC.

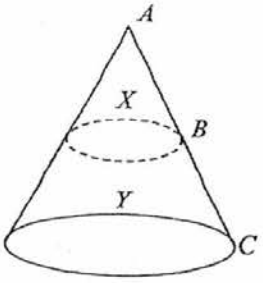


Diagram I

(a) Given that the volume of the solid cone is $\frac{64}{3}\pi$ m^3, find the volume, in terms of π, of the frustum, Y. [3]

$$\left(\frac{\text{length of } AB}{\text{length of BC}}\right)^2 = \frac{\text{vol of } X}{\text{vol of } X+Y}$$

$$\left(\frac{1}{2}\right)^2 = \frac{\text{vol of } X}{\frac{64}{3}} \quad \text{M1}$$

$$\text{Vol of } X = \frac{8\pi}{3}$$

$$\text{Vol of } Y = \frac{64\pi}{3} - \frac{8\pi}{3} \quad \text{M1}$$

$$= \frac{56}{3}\pi \text{ m}^3 \quad \text{A1}$$

(b) In **Diagram II**, a rocket can be modelled from a cylinder of height, h, 94.2 m with a cone, X, on top and a frustum, Y, at the bottom. The cone, X, has a diameter, d_2, of 4 m and the frustum, Y, has a base diameter, d_1, of 8 m. The parts X and Y are taken from **Diagram I** above.

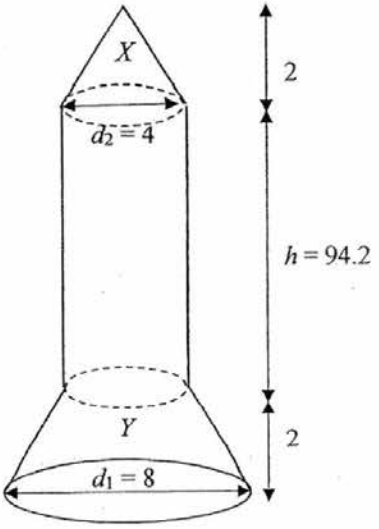


Diagram II

(i) Calculate the total surface area of the rocket. Give your answer correct to the nearest square meter. [3]

total surface area $= \pi(4)(\sqrt{4^2+4^2})+2\pi(2)(94.2)+\pi(4)^2$ **M2**

$=1305.1037...$

$=1305 \text{ m}^2$ (to nearest square metre) **A1**

(ii) Calculate the volume, in cubic metres, of the rocket. [1]

$$\text{vol} = \frac{1}{3}\pi(4)^2(4)+\pi(2)^2(94.2)$$

$=1250.7727...$

$=1250 \text{ m}^3$ (to 3 sf) **A1**

(iii) The rocket is designed to launch to the moon.

Useful information

- Distance of moon from earth: 384 400 km
- Speed of rocket: 800 km /minute
- $1\ m^3 = 264$ gallon
- The rocket is filled with liquid fuel to a maximum of 95% of its volume.
- Rate of fuel consumption: 20 000 gallons /minute
- Capacity of each external fuel tank: 3.2×10^6 gallons

[4]

How many external fuel tanks will the rocket require to sustain its journey to the moon?

Justify your answer with calculations.

Amount of fuel in rocket
$= 0.95 \times 1250.7727$
$= 1188.234\ m^3$

Gallons of fuel
$= 1188.234 \times 264$
$= 313693.807$ gallons

M1

Time taken to travel to moon
$= \dfrac{384400\ km}{800\ km/min}$
$= 480.5\ min$

Amount of fuel needed
$= 20000 \times 480.5$ A1
$= 9610000$ gallons

number of tanks
$= \dfrac{9610000 - 313693.807}{3.2 \times 10^6}$ M1
$\approx 2.905...$
$= 3$ A1 (must arrive 2.905...)

Therefore, number of external tanks required is 3.