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SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 1 **4048/01**

15 AUGUST 2016 (Monday) **2 hours**

CANDIDATE NAME

CLASS

INDEX NUMBER

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a pencil for any diagrams or graphs.

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INFORMATION FOR CANDIDATES

Answer all the questions.

Write your answers in the space provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use		
Q1	2	
Q2	3	
Q3	3	
Q4	3	
Q5	4	
Q6	4	
Q7	4	
Q8	4	
Q9	4	
Q10	4	
Q11	4	
Q12	4	
Q13	5	
Q14	5	
Q15	5	
Q16	6	
Q17	7	
Q18	9	
Total	/ 80	

This document consists of 26 printed pages including the Cover Sheet.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

For Examiner's Use

1 For a regular decagon, state

(a) its number of lines of symmetry,

Answer (a) ..... lines of symmetry [1]

(b) its order of rotational symmetry.

Answer (b) Order of rotational symmetry = ... [1]

2 In Semester 1, there are three Mathematics tests. The full score of each test is 100 marks. To get Grade A2 for Mathematics in Semester 1, a student must have a mean score of at least 80 marks but less than 95 marks based on the three tests.

Ahmad scored 60 marks, 83 marks and x marks in the three tests. Given that Ahmad gets Grade A2 in Semester 1, find the inequality satisfied by x.

Answer ..... [3]

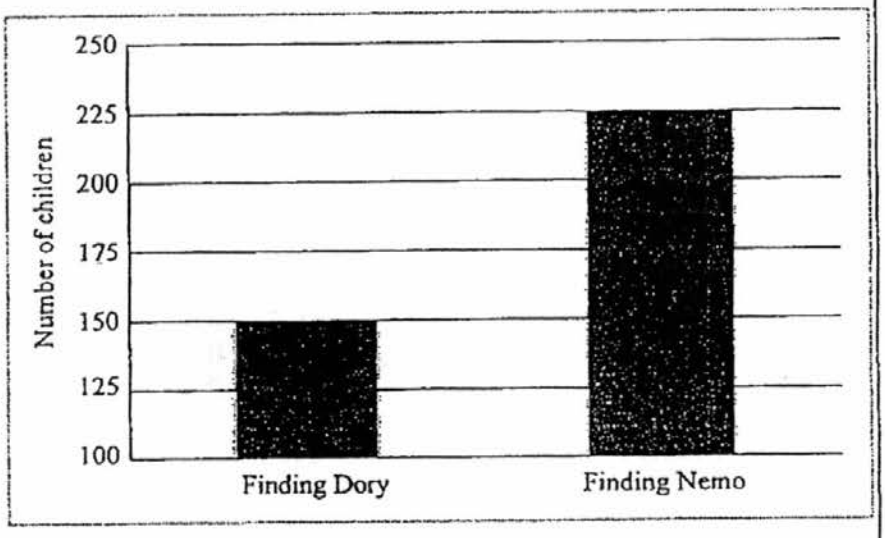
For Examiner's Use

For Examiner's Use

3

(a) A group of children watched the movies Finding Nemo and Finding Dory at a movie marathon. At the end of the movie marathon, a survey was conducted to find out which of the two movies the children preferred. The survey results are shown in the bar graph below.

Do you prefer Finding Nemo or Finding Dory?



Conclusion: More than twice as many children preferred Finding Nemo to Finding Dory.

Do you agree with the above conclusion? Explain your answer using the given data.

Answer (a)

I agree / disagree (circle your answer) with the conclusion because

.....

.....

.....

..... [2]

For Examiner's Use

For Examiner's Use

3

(b) A survey was conducted to find the number of students who like each of the following bubble tea cafes: KOI, Share Tea, Sweet Talk and Gong Cha. The survey results are represented in the pie chart below. The percentages of the number of students who like the respective bubble tea cafes are indicated on the pie chart.

List one thing wrong with this pie chart.

Answer (b)

.....

.....

.....

..... [1]

For Examiner's Use

For Examiner's Use

4 It is given that p is directly proportional to q^n.

(a) State the value of n when p m^3 is the volume of a cone with base radius q and a constant height.

Answer (a) $n =$ ..... [1]

(b) If a particular value of p is increased by 300%, how many times does the value of q increase? Leave your answer in terms of n.

Answer (b) ..... times [2]

For Examiner's Use

For Examiner's Use

5 ξ = {quadrilaterals with at least one pair of parallel sides}

A = {quadrilaterals with diagonals that bisect each other}

B = {quadrilaterals with perpendicular diagonals}

(a) Draw a Venn diagram to illustrate the above information. (No need to state the elements in the sets.)

Answer (a)

[2]

(b) Name a quadrilateral in the set $A \cap B$.

Answer (b) ..... [1]

(c) Name a quadrilateral in the set $(A \cup B)'$.

Answer (c) ..... [1]

For Examiner's Use

For Examiner's Use

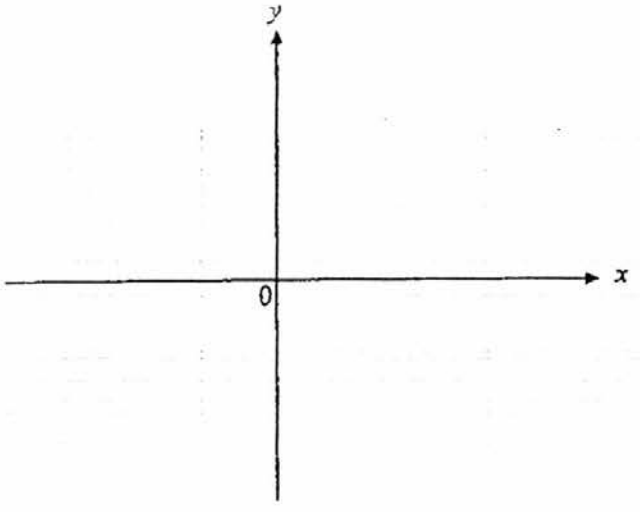
6

(a) Factorise fully $6x^2+2x-4$.

Answer (a) ..... [2]

(b) Sketch the graph of $y=6x^2+2x-4$, indicating the axial intercepts clearly.

Answer (b)



[2]

For Examiner's Use

For Examiner's Use

7 The price of a ticket in each category at the River Safari is given below:

Adult : $30	Child : $20	Senior Citizen : $14

(a) The number of tickets sold on one weekend is given as follow.

	Adult	Child	Senior Citizen
Saturday	57	123	80
Sunday	103	79	45

The information on the number of tickets sold can be represented by the matrix

$$\mathbf{T} = \begin{pmatrix} 57 & 123 & 80 \\ 103 & 79 & 45 \end{pmatrix}.$$

(i) By representing the price of a ticket in each category into a column matrix **P**, find the matrix $\mathbf{S} = \mathbf{TP}$.

Answer (a)(i) $\mathbf{S} =$ [2]

(ii) Explain what the elements of **S** represent.

Answer (a)(ii)

.....

.....

..... [1]

For Examiner's Use

For Examiner's Use

7

(b) The current price of a ticket in each category can be represented by the matrix

$$A = \begin{pmatrix} 30 & 20 & 14 \end{pmatrix}.$$

The River Safari has the following plan to adjust the prices of the tickets:

Adult: increase by 10%

Child: increase by 5%

Senior citizen: decrease by 20%

Write down a matrix **B** such that the product **AB** will be the newly adjusted price of the ticket for each category.

Answer (b) **B** = [1]

For Examiner's Use

For Examiner's Use

8 The menu of MathRox Cafe is shown below:

MathRox Cafe MENU	
Main Course	
Baked Salmon	$ 13.50
Chicken Sandwich	$ 9.50
Seafood Aglio Olio	$ 13.50
Drinks	
Canned Soft Drinks	$ 2.50
Coffee/ Tea	$ 2.00
Desserts	
Sliced Cake	$ 8.00
Single Scoop Ice Cream	$ 4.50

(a) The cafe manager plans to introduce a lunch set option in the menu, offering customers with a discount of 10%. The lunch set consists of the following items:

Seafood Aglio Olio/ Baked Salmon

Coffee/ Tea

Single scoop Ice Cream

What is the price of the lunch set?

Answer (a) $ ..... [2]

For Examiner's Use

For Examiner's Use

8

(b) The prices of all items are subjected to 10% service charge followed by 7% Goods and Services Tax (GST).

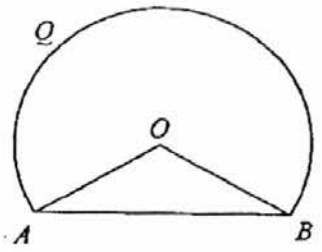
Xiao Ming ordered a chicken sandwich, a slice of cake and a can of soft drink. Calculate the amount of GST he paid.

Answer (b) $ ..... [2]

For Examiner's Use

For Examiner's Use

9 The diagram below shows a dart target board where AQB is a major segment with centre O and radius 6 cm. The length of the major arc AQB is 7π cm.



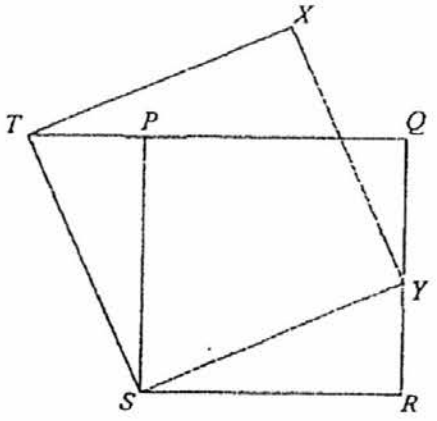
Assuming a dart falls within the dart target board, find the probability that the dart lands within triangle AOB. Leave your answer in terms of π.

Answer ..... [4]

For Examiner's Use

For Examiner's Use

10 In the diagram, $PQRS$ is a rectangle and $STXY$ is a square. It is given that QPT and RYQ are straight lines.



(a) Show that angle RSY = angle PST.

Answer (a)

.....

.....

.....

..... [1]

(b) Hence, prove that triangle RSY is congruent to triangle PST.

Answer (b)

.....

.....

.....

..... [3]

For Examiner's Use

For Examiner's Use

11 An atom of helium has a mass of 6.65×10^{-24} g and radius of 28 picometres.

(a) A balloon contains 2.8×10^{27} atoms of helium. Find the exact mass of helium in the balloon, giving your answer in kilograms.

Answer (a) ..... kg [2]

(b) 7 billion helium atoms were placed side by side without spaces between the atoms. Calculate the exact total length made by these 7 billion helium atoms, giving your answer in micrometres expressed in standard form.

Answer (b) ..... micrometres [2]

For Examiner's Use

For Examiner's Use

12 The currency exchange table at a bank shows the rates in local currency. The currency exchange board is read from the bank's point of view.

The following is the currency exchange table shown at a foreign bank in Singapore at the beginning of 2013.

Currency	Bank Buy	Bank Sell
euros (€)	1.4459	1.4884

At the beginning of 2013, Mr Bala invested Singapore Dollars ($) 480 000 in this foreign bank account.

(i) Calculate the amount invested by Mr Bala in euros (€).

Answer (i) € ..... [1]

(ii) The foreign bank account offers an interest rate of 2.5% per annum compounded half yearly. Calculate the amount in euros (€) that Mr Bala can withdraw at the end of 2016.

Answer (ii) € ..... [2]

(iii) At the end of 2016, Mr Bala decided to close the account and withdraw the money. At the end of 2016, the currency exchange table shown at the foreign bank in Singapore is as follow.

Currency	Bank Buy	Bank Sell
euros (€)	1.5011	1.5383

Calculate the amount he withdrew in Singapore Dollars ($), correct to the nearest dollar.

Answer (iii) $ ..... [1]

For Examiner's Use

For Examiner's Use

13 When written as the product of their prime factors,

$$x = 2^5 \times 3^3 \times 5,$$
$$y = 2 \times 3 \times 5^4,$$
$$z = 3 \times 5 \times 7.$$

(a) Find the value of

(i) the lowest common multiple of y and z,

Answer (a)(i) ..... [1]

(ii) the smallest possible value of m such that the product xm is a perfect cube.

Answer (a)(ii) $m =$ ..... [2]

(b) Sally wants to pack x packets of coffee, y packets of biscuits and z boxes of axe oil into gift packs for elderly. It is given that each gift pack has the same number of axe oil, biscuits and coffee and there are no leftover items.

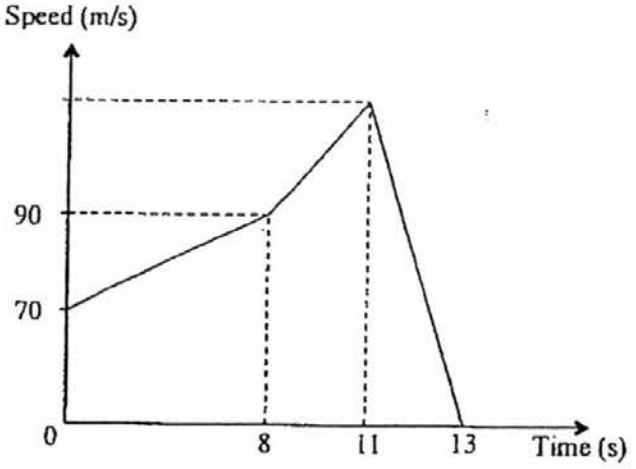
How many packets of biscuits does each gift pack has?

Answer (b) ..... packets [2]

For Examiner's Use

For Examiner's Use

14 The following diagram is the speed-time graph of a particle over a period of 13 seconds.



(i) Calculate the acceleration of the particle in the first 8 seconds.

Answer (i) ..... m/s^2 [1]

(ii) Calculate the speed of the particle at $t = 6$s.

Answer (ii) ..... m/s [2]

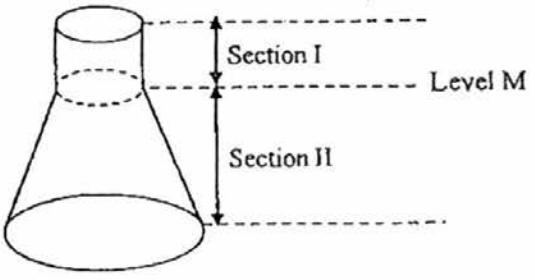
(iii) Given that the particle travelled 315 m from $t = 8$s to $t = 11$s, find the speed of the particle at $t = 11$s.

Answer (iii) ..... m/s [2]

For Examiner's Use

For Examiner's Use

15 The diagram below shows Flask A. Flask A is made up of Section I and Section II. The ratio of the volume of Section I to the volume of Section II is 2 : 7.



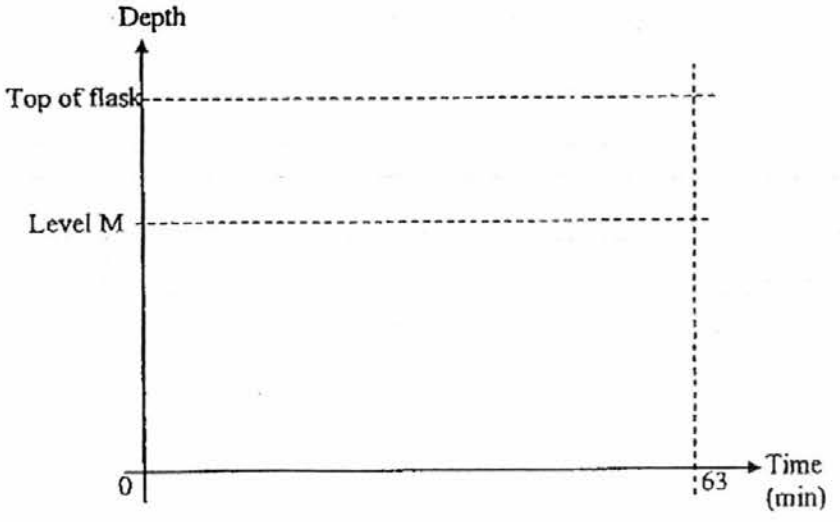
(a) Flask A is initially completely filled with water. There is a small hole at the bottom of the flask and water is leaking through the hole at a constant rate.

(i) Given that it takes 63 minutes for the Flask A to be empty, calculate the time taken for the depth of the water to reach Level M?

Answer (a)(i) .... minutes [1]

(ii) On the axes in the answer space, sketch the graph showing how the depth of the water in the Flask A varies over the 63 minutes.

Answer (a)(ii)



[2]

For Examiner's Use

For Examiner's Use

15

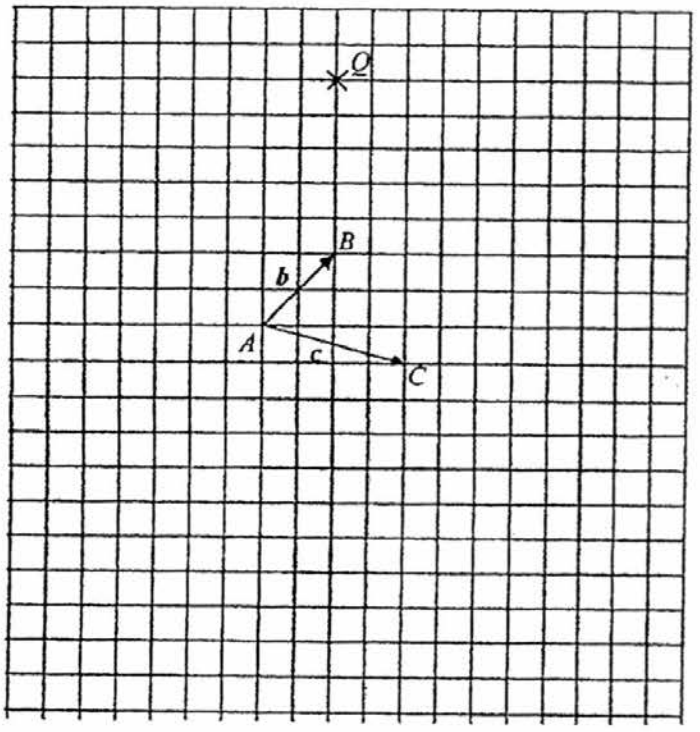
(b) Flask A has a base area of 32 cm^2. Flask A is geometrically similar to Flask B which has a base area of 50 cm^2. It is given that the capacity of Flask A is 960 cm^3. Find the exact capacity of Flask B, giving your answer in litres.

Answer (b) ..... litres [2]

For Examiner's Use

For Examiner's Use

16 In the diagram below, one square represents one square unit. Four points A, B, C and Q are shown on the diagram. The position vectors of B and C with respect to A are $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively.



(i) Calculate $\left|\overrightarrow{AC}\right|$.

Answer (i) $\left|\overrightarrow{AC}\right|$ = ... [1]

(ii) Given that $\overrightarrow{QP} = h\boldsymbol{b}$ and $\overrightarrow{AP} = k\boldsymbol{c}$, mark and label point P on the above diagram. [1]

For Examiner's Use

For Examiner's Use

16

(iii) Express $\overline{CQ}$ as a column vector.

Answer (iii) $\overline{CQ}$ = [1]

(iv) Given that the coordinates of Q is $(7, -3)$, find the coordinates of C.

Answer (iv) C(..... , ...) [2]

(v) Given that $\overline{QM}$ is parallel to $\overline{BA}$, find the equation of line QM.

Answer (v) ..... [1]

For Examiner's Use

For Examiner's Use

17 The scale of a map is 1 cm : 10 km.

(a) The actual area of a park is 3903 km^2. Find the exact area, in square centimetres, of the park on the map.

Answer (a) ..... cm^2 [2]

For Examiner's Use

For Examiner's Use

17

(b) In a park XYZ, $XY = 100$ km and $YZ = 45$ km. X is due west of Y and the bearing of Z from Y is 300°.

(i) The point Y is marked in the space below. Construct the park XYZ using the scale 1 cm : 10 km. [2]

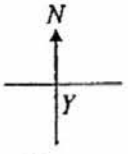
(ii) Construct in the diagram drawn in **(b)(i)**,

(a) the perpendicular bisector of XY, [1]

(b) the bisector of angle ZXY. [1]

(iii) A bench B is located in the park, equidistant from X and Y and nearer to XZ than XY. Mark a possible position of B in the diagram drawn in **(b)(i)**. [1]

Answer (b)



For Examiner's Use

For Examiner's Use

18 A regular pentagon $ABCDE$ and a regular hexagon $AEFGHI$ are joined together along AE, the common side, to form the Figure 1.

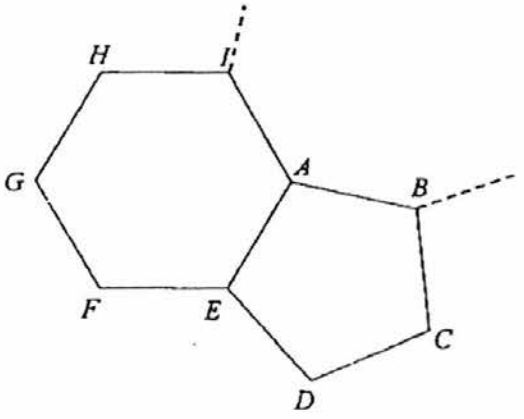


Figure 1

(a) Explain with mathematical calculations why the line segments AB and AI cannot be part of a bigger regular polygon.

Answer (a)

[4]

For Examiner's Use

For Examiner's Use

18

(b) Given that $AE = 8$ cm, calculate the area of the Figure 1.

Answer (b) ..... cm^2 [5]

END OF PAPER

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$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use	Answer all the questions.	For Examiner's Use

1 For a regular decagon, state

(a) its number of lines of symmetry,

Solution:

10 lines of symmetry

Answer (a) ..... lines of symmetry [1]

(b) its order of rotational symmetry.

Solution:

Order of rotational symmetry = **10**

Answer (b) Order of rotational symmetry = ... [1]

2 In Semester 1, there are three Mathematics tests. The full score of each test is 100 marks. To get Grade A2 for Mathematics in Semester 1, a student must have a mean score of at least 80 marks but less than 95 marks based on the three tests.

Ahmad scored 60 marks, 83 marks and x marks in the three tests. Given that Ahmad gets Grade A2 in Semester 1, find the inequality satisfied by x.

Solution:

$$80 \le \frac{60+83+x}{3} < 95$$

$$240 \le 143 + x < 285$$

$$97 \le x < 142$$

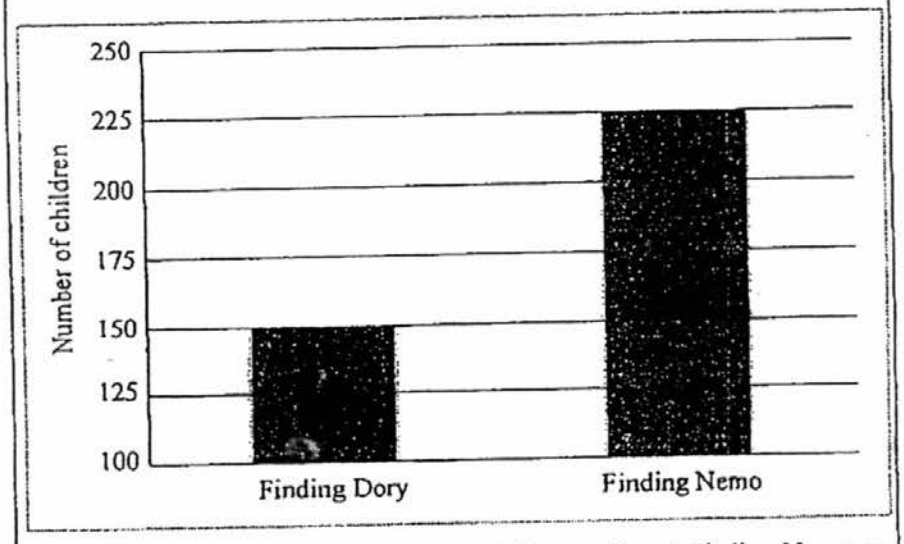
Since full score is 100 marks, range of Ahmad's marks: $97 \le x \le 100$

Answer ..... [3]

3

(a) A group of children watched the movies Finding Nemo and Finding Dory at a movie marathon. At the end of the movie marathon, a survey was conducted to find out which of the two movies the children preferred. The survey results are shown in the bar graph below.

Do you prefer Finding Nemo or Finding Dory?



Conclusion: More than twice as many children preferred Finding Nemo to Finding Dory.

Do you agree with the above conclusion? Explain your answer using the given data.

Solution:

I **disagree** with the conclusion because 225 children preferred Finding Nemo which **is less than twice of 150** children who preferred Finding Dory.

Remarks:

- Stating "Less than twice as many children preferred Finding Nemo to Finding Dory" is insufficient to explain as the statement only states the correct conclusion.
- Stating "vertical axis does not start from 0" or "vertical axis start from 100" is insufficient.

Answer (a)

I agree / disagree (circle your answer) with the conclusion because

.....

.....

....

.... [2]

(b) A survey was conducted to find the number of students who like each of the following bubble tea cafes: KOI, Share Tea, Sweet Talk and Gong Cha. The survey results are represented in the pie chart below. The percentages of the number of students who like the respective bubble tea cafes are indicated on the pie chart.

List one thing wrong with this pie chart.

Solution:	Remarks:
The total percentage is more than 100% **OR** The percentages do not add up to 100% **OR** The total percentage is 103%	Mathematical calculations such as "KOI should be 100-23-10-25=42% not 45%" are not accepted. Explanation such as "the survey does not include students who do not like bubble tea/ like other bubble tea cafes" are not accepted because the question specify something wrong with the pie chart, not the survey. Note: The percentages make sense but the pie chart is not an appropriate diagram to represent this set of data because a student can select more than 1 cafe.

Answer (b)

....

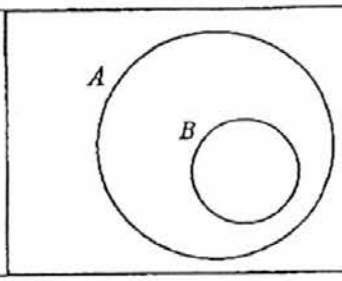
....

....

.... [1]

	4	It is given that p is directly proportional to q^n.	
	(a)	State the value of n when p m^3 is the volume of a cone with base radius q and a constant height.	
		Solution: $n = 2$	
		Answer (a) $n =$ ..... [1]	
	(b)	If a particular value of p is increased by 300%, how many times does the value of q increase? Leave your answer in terms of n.	
		Solution: $p = kq^n \Rightarrow k = \frac{p}{q^n}$ $4p = k(\text{new } q)^n$ $(\text{new } q)^n = 4p \div \frac{p}{q^n}$ $(\text{new } q)^n = 4p \times \frac{q^n}{p}$ $(\text{new } q)^n = 4q^n$ $\text{new } q = \sqrt[n]{4}q$ Hence q increase $\sqrt[n]{4} - 1$ OR $4^{\frac{1}{n}} - 1$ times **OR** new p = 4p new $q^n = 4q^n$ Hence q increase $\sqrt[n]{4} - 1$ OR $4^{\frac{1}{n}} - 1$ times	Remarks: Due to different interpretations of phrasing of question, accept $\sqrt[n]{4}$ OR $4^{\frac{1}{n}}$ times
		Answer (b) ..... times [2]	

	5	ξ = {quadrilaterals with at least one pair of parallel sides} A = {quadrilaterals with diagonals that bisect each other} B = {quadrilaterals with perpendicular diagonals}	
	(a)	Draw a Venn diagram to illustrate the above information. (No need to state the elements in the sets.)	
		Solution:	



Answer (a)

[2]

(b) Name a quadrilateral in the set $A \cap B$.

Solution: rhombus / square

Answer (b) ..... [1]

(c) Name a quadrilateral in the set $(A \cup B)'$.

Solution: trapezium

Answer (c) ..... [1]

6

(a) Factorise fully $6x^2 + 2x - 4$.

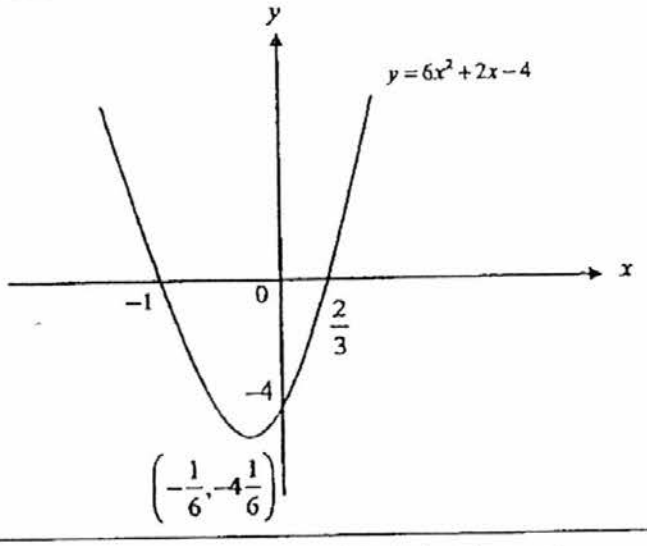
Solution:

$$6x^2 + 2x - 4$$
$$= 2(3x^2 + x - 2)$$
$$= 2(3x - 2)(x + 1)$$

Answer (a) ..... [2]

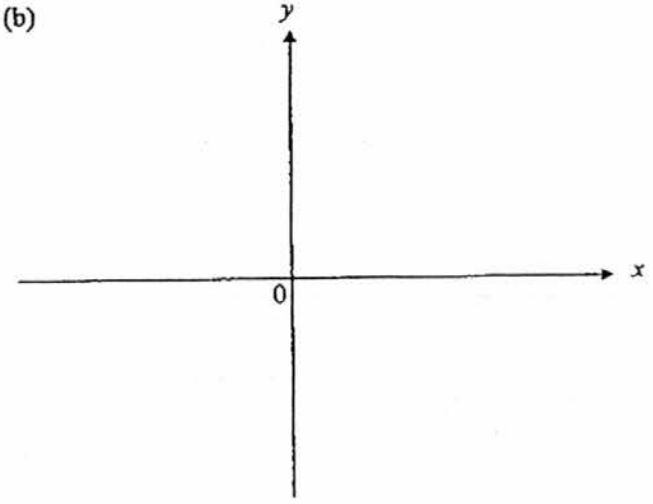
(b) Sketch the graph of $y = 6x^2 + 2x - 4$, indicating the axial intercepts clearly.

Solution:



Remark: Deduct "P" if graph is not labelled with equation

Answer (b)



[2]

7 The price of a ticket in each category at the River Safari is given below:

Adult : $30	Child : $20	Senior Citizen : $14

(a) The number of tickets sold on one weekend is given as follow.

	Adult	Child	Senior Citizen
Saturday	57	123	80
Sunday	103	79	45

The information on the number of tickets sold can be represented by the matrix

$$\mathbf{T} = \begin{pmatrix} 57 & 123 & 80 \\ 103 & 79 & 45 \end{pmatrix}.$$

(i) By representing the price of a ticket in each category into a column matrix **P**, find the matrix $\mathbf{S} = \mathbf{TP}$.

Solution:

$$\mathbf{S} = \begin{pmatrix} 57 & 123 & 80 \\ 103 & 79 & 45 \end{pmatrix}\begin{pmatrix} 30 \\ 20 \\ 14 \end{pmatrix}$$

$$= \begin{pmatrix} 5290 \\ 5300 \end{pmatrix}$$

Answer (a)(i) $\mathbf{S} =$ [2]

(ii) Explain what the elements of S represent.

Solution:
The elements of S represent the amount of money collected from the sales of tickets on Saturday and Sunday **respectively**.

OR

The elements of S represent the revenue on Saturday and Sunday **respectively**.

Answer (a)(ii)

.....

.....

		..... [1]	
	(b)	The current price of a ticket in each category can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 30 & 20 & 14 \end{pmatrix}$. The River Safari has the following plan to adjust the prices of the tickets: Adult: increase by 10% Child: increase by 5% Senior citizen: decrease by 20% Write down a matrix **B** such that the product **AB** will be the newly adjusted price of the ticket for each category.	
		Solution: $\mathbf{B} = \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.05 & 0 \\ 0 & 0 & 0.8 \end{pmatrix}$	
		Answer (b) **B** = [1]	

8 The menu of MathRox Cafe is shown below:

MathRox Cafe

MENU

Main Course	
Baked Salmon	$ 13.50
Chicken Sandwich	$ 9.50
Seafood Aglio Olio	$ 13.50
Drinks	
Canned Soft Drinks	$ 2.50
Coffee/ Tea	$ 2.00
Desserts	
Sliced Cake	$ 3.00
Single Scoop Ice Cream	$ 4.50

(a) The cafe manager plans to introduce a lunch set option in the menu, offering customers with a discount of 10%. The lunch set consists of the following items:

Seafood Aglio Olio/ Baked Salmon

Coffee/ Tea

Single scoop Ice Cream

What is the price of the lunch set?

Solution:

$$\frac{90}{100}\times(13.50+4.50+2)$$
$$=\$18$$

Answer (a) $ ..... [2]

(b) The prices of all items are subjected to 10% service charge followed by 7% Goods and Services Tax (GST).

Xiao Ming ordered a chicken sandwich, a slice of cake and a can of soft drink. Calculate the amount of GST he paid.

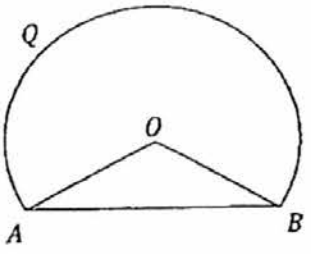
Solution:

$\frac{7}{100}\times\left(\frac{110}{100}\times(9.50+8+2.50)\right)$

$=\$1.54$

Answer (b) $ ..... [2]

9 The diagram below shows a dart target board where AQB is a major segment with centre O and radius 6 cm. The length of the major arc AQB is 7π cm.



Assuming a dart falls within the dart target board, find the probability that the dart lands within triangle AOB. Leave your answer in terms of π.

Solution:

Reflex angle AQB $= \frac{7\pi}{6}$

Area of major sector AQB

$=\frac{1}{2}\times 6^2\times\frac{7\pi}{6}=21\pi$

Obtuse angle AQB $= 2\pi-\frac{7\pi}{6}=\frac{5\pi}{6}$

Area of triangle AOB

$=\frac{1}{2}\times 6^2\times\sin\frac{5\pi}{6}=9$

P (dart lands within triangle AOB)

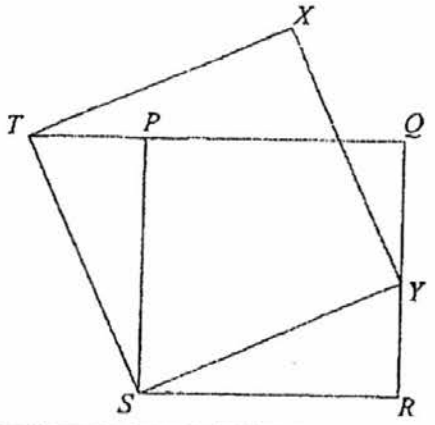
$=\frac{9}{9+21\pi}$

$=\frac{9}{3(3+7\pi)}$

$=\frac{3}{3+7\pi}$

Answer ..... [4]

10 In the diagram, $PQRS$ is a rectangle and $STXY$ is a square. It is given that QPT and RYQ are straight lines.



(a) Show that angle RSY = angle PST.

Solution:

$\angle RSY$

$= 90° - \angle PSY$ (in rectangle PQRS)

$= \angle PST$ (in square STXY) (Shown)

Answer (a) .....

.....

.....

..... [1]

(b) Hence, prove that triangle RSY is congruent to triangle PST.

Solution:

$\angle RSY = \angle PST$ (Prove in (a))

$\angle YRS = \angle TPS = 90°$ (Right angles of rectangle $PQRS$ and square $STXY$)

$SY = ST$ (sides of square $STXY$)

ΔRSY congruent to ΔPST (AAS)

Answer (b)

.....

.....

.....

..... [3]

11 An atom of helium has a mass of 6.65×10^{-24} g and radius of 28 picometres.

(a) A balloon contains 2.8×10^{27} atoms of helium. Find the exact mass of helium in the balloon, giving your answer in kilograms.

Solution:

$2.8\times10^{27}\times6.65\times10^{-24}$

$=18620$g

$=18.62$kg

Answer (a) ..... kg [2]

(b) 7 billion helium atoms were placed side by side without spaces between the atoms. Calculate the exact total length made by these 7 billion helium atoms, giving your answer in micrometres expressed in standard form.

Solution:

$7\times10^{9}\times28\times10^{-12}\times2$

$=0.392$metre

$=392000\times10^{-6}$ metre

$=3.92\times10^{5}$ micrometres

Answer (b) ..... micrometres [2]

	12	The currency exchange table at a bank shows the rates in local currency. The currency exchange board is read from the bank's point of view. The following is the currency exchange table shown at a foreign bank in Singapore at the beginning of 2013. **Currency** | **Bank Buy** | **Bank Sell** euros (€) | 1.4459 | 1.4884 At the beginning of 2013, Mr Bala invested Singapore Dollars ($) 480 000 in this foreign bank account.
	(i)	Calculate the amount invested by Mr Bala in euros (€).
		Solution: $\frac{480000}{1.4884} = €\ 322493.95$ (nearest cent)
		Answer (i) € ..... [1]
	(ii)	The foreign bank account offers an interest rate of 2.5% per annum compounded half yearly. Calculate the amount in euros (€) that Mr Bala can withdraw at the end of 2016.

Solution:

$$\left(\frac{480000}{1.4884}\right)\left(1+\frac{2.5/2}{100}\right)^{4\times2}$$

= € 356190.09 (nearest cent)

Answer (ii) € ..... [2]

(iii) At the end of 2016, Mr Bala decided to close the account and withdraw the money. At the end of 2016, the currency exchange table shown at the foreign bank in Singapore is as follow.

Currency	Bank Buy	Bank Sell
euros (€)	1.5011	1.5383

Calculate the amount he withdrew in Singapore Dollars ($), correct to the nearest dollar.

Solution:

356190.0891×1.5011

= SGD 534677 (nearest dollar)

Answer (iii) SGD ..... [1]

13 When written as the product of their prime factors,

$$x = 2^5 \times 3^3 \times 5,$$
$$y = 2 \times 3 \times 5^4,$$
$$z = 3 \times 5 \times 7.$$

(a) Find the value of

(i) the lowest common multiple of y and z,

Solution:

LCM = $2 \times 3 \times 5^4 \times 7 = 26250$

Answer (a)(i) ..... [1]

(ii) the smallest possible value of m such that the product xm is a perfect

cube.

Solution:

$$m = \frac{1}{2^5 \times 3^3 \times 5}$$

$$m = \frac{1}{4320}$$

Answer (a)(ii) $m =$ ..... [2]

(b) Sally wants to pack x packets of coffee, y packets of biscuits and z boxes of axe oil into gift packs for elderly. It is given that each gift pack has the same number of axe oil, biscuits and coffee and there are no leftover items.

How many packets of biscuits does each gift pack has?

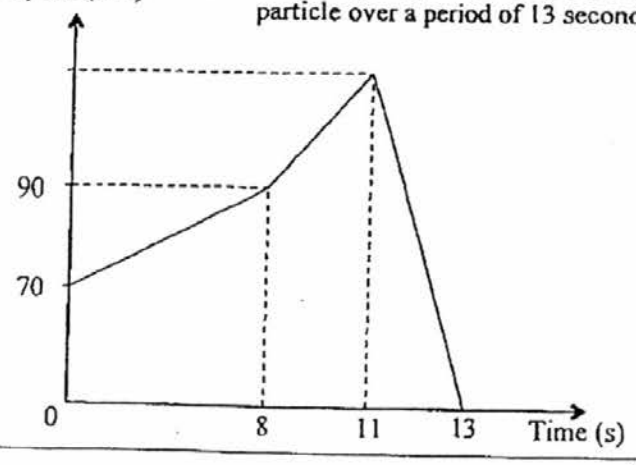
Solution:

$\text{HCF} = 3 \times 5 = 15$

Number of packets of biscuits in each pack $= \frac{2 \times 3 \times 5^4}{15} = 250$ packets

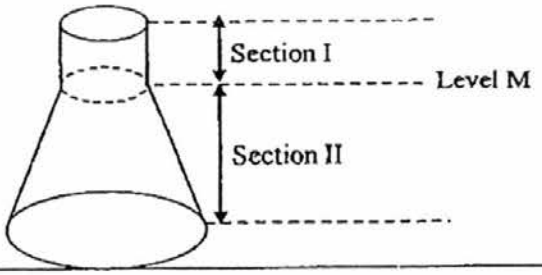
Answer (b) ..... packets [2]

14 The following diagram is the speed-time graph of a particle over a period of 13 seconds.



	(i)	Calculate the acceleration of the particle in the first 8 seconds.	
		Solution: Acceleration $= \frac{90-70}{8} = 2.5 \text{m/s}^2$	
		Answer (i) ..... m/s^2 [1]	
	(ii)	Calculate the speed of the particle at $t = 6$s.	
		Solution: Let speed at t=6s be v m/s $\frac{v-70}{6} = 2.5$ $v = 85\text{m/s}$	
		Answer (ii) ..... m/s [2]	
	(iii)	Given that the particle travelled 315 m from $t = 8$s to $t = 11$s, find the speed of the particle at $t = 11$s.	
		Solution: Let v m/s be speed of the particle at $t = 11$s. $\frac{1}{2} \times (90+v) \times (11-8) = 315$ $v = 120\text{m/s}$	
		Answer (iii) ..... m/s [2]	

15 The diagram below shows Flask A. Flask A is made up of Section I and Section II. The ratio of the volume of Section I to the volume of Section II is 2 : 7.



(a) Flask A is initially completely filled with water. There is a small hole at the bottom of the flask and water is leaking through the hole at a constant rate.

(i) Given that it takes 63 minutes for the Flask A to be empty, calculate the time taken for the depth of the water to reach Level M?

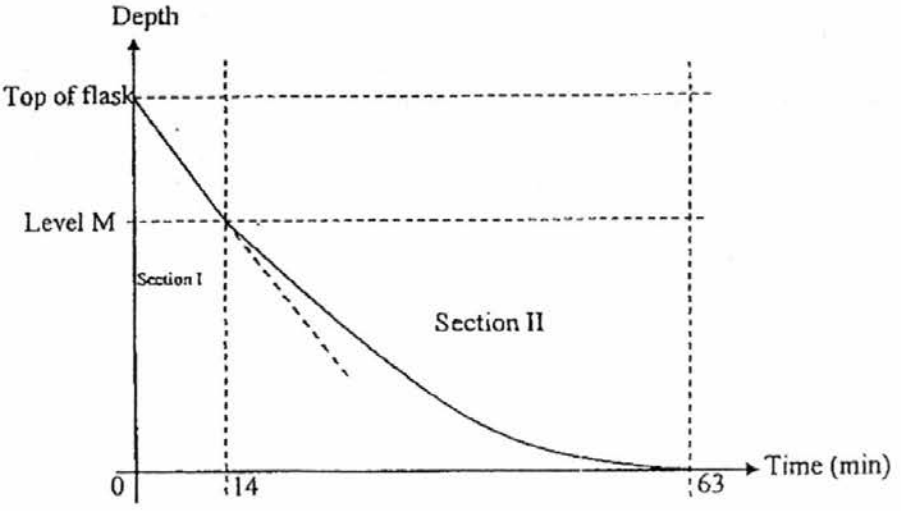
Solution:

$$\frac{63}{9} \times 2 = 14 \text{ min}$$

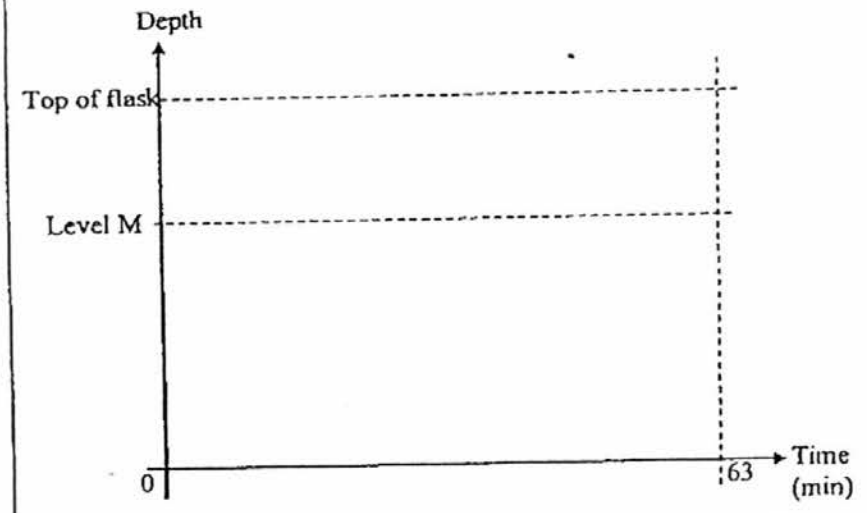
Answer (a)(i) ..... minutes [1]

(ii) On the axes in the answer space, sketch the graph showing how the depth of the water in the Flask A varies over the 63 minutes.

Solution:



Answer (a)(ii)



[2]

(b) Flask A has a base area of 32 cm^2. Flask A is geometrically similar to Flask B which has a base area of 50 cm^2. It is given that the capacity of Flask A is 960 cm^3. Find the exact capacity of Flask B, giving your answer in litres.

Solution:

$$\frac{\text{Height A}}{\text{Height B}} = \sqrt{\frac{32}{50}} = \frac{4}{5}$$

$$\frac{\text{Volume A}}{\text{Volume B}} = \left(\frac{4}{5}\right)^3$$

$$\frac{960}{\text{Volume B}} = \frac{64}{125}$$

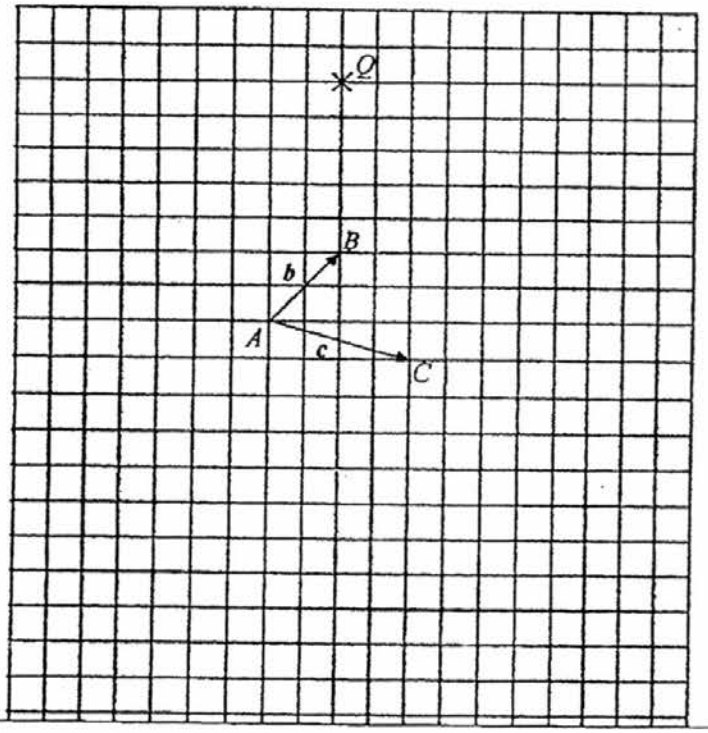
$$\text{Volume of B} = \frac{125}{64} \times 960$$

$$= 1875 \text{ cm}^3$$

$$= 1.875 \text{litres}$$

Answer (b) ..... litres [2]

16 In the diagram below, one square represents one square unit. Four points A, B, C and Q are shown on the diagram. The position vectors of B and C with respect to A are $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively.

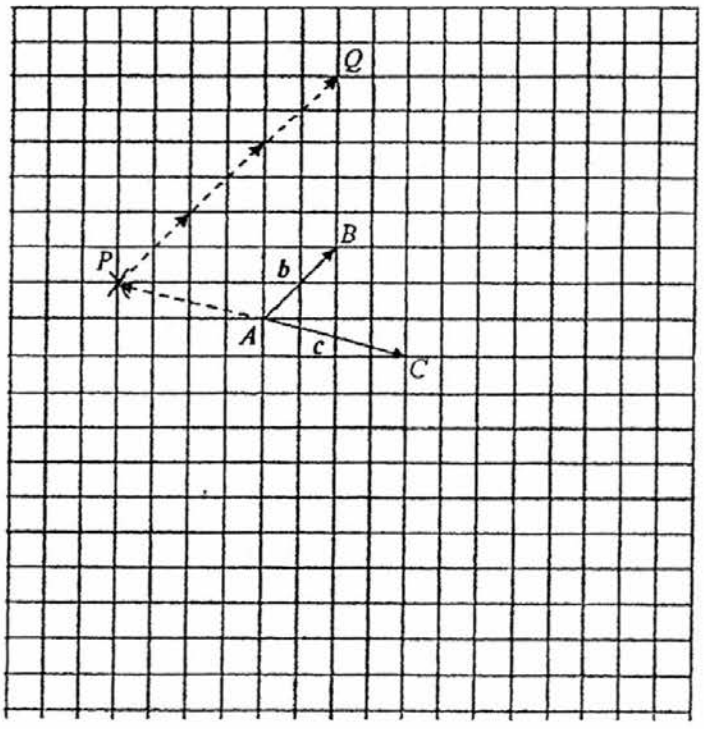


(i) Calculate $|\overrightarrow{AC}|$.

Solution:

$$|\overrightarrow{AC}| = \sqrt{4^2 + (-1)^2}$$
$$= \sqrt{17}$$
$$= 4.12 \text{ units (3sf)}$$

Answer (i) $|\overrightarrow{AC}| =$ ..... [1]

	(ii)	Given that $\overrightarrow{QP} = hb$ and $\overrightarrow{AP} = kc$, mark and label point P on the above diagram.	[1]	
		Solution: **[B1]**: correct position of P 		
	(iii)	Express $\overrightarrow{CQ}$ as a column vector.		
		Solution: $\overrightarrow{CQ} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$		

		–					
			Answer	(iii)	$\overline{CQ}$ =	[1]	
	(iv)	Given that the coordinates of Q is (7, –3), find the coordinates of C.					
		Solution: $\overline{CQ}=\overline{OQ}-\overline{OC}$ $\begin{pmatrix}-2\\8\end{pmatrix}=\begin{pmatrix}7\\-3\end{pmatrix}-\overline{OC}$ $\overline{OC}=\begin{pmatrix}9\\-11\end{pmatrix}$ $C(9,-11)$ **OR** $\overline{OC}=\overline{OQ}+\overline{QC}$ $\overline{OC}=\begin{pmatrix}7\\-3\end{pmatrix}-\begin{pmatrix}-2\\8\end{pmatrix}$ $\overline{OC}=\begin{pmatrix}9\\-11\end{pmatrix}$ $C(9,-11)$					
			Answer	(iv)	C (..... , ...)	[2]	
	(v)	Given that $\overline{QM}$ is parallel to $\overline{BA}$, find the equation of line QM.					
		Solution: Gradient of $QM = 1$ $-3=1(7)+c$ $c=-10$ $y=x-10$					

Answer (v) ..... [1]

17 The scale of a map is 1 cm : 10 km.

(a) The actual area of a park is 3903 km^2. Find the exact area, in square centimetres, of the park on the map.

Solution:
$1\ cm^2 : 10^2\ km^2$

Area of park on map
$= \frac{3903}{100} = 39.03 cm^2$

Answer (a) ..... cm^2 [2]

(b) In a park XYZ, $XY = 100$ km and $YZ = 45$ km. X is due west of Y and the bearing of Z from Y is 300°.

(i) The point Y is marked in the space below. Construct the park XYZ using the scale 1 cm : 10 km. [2]

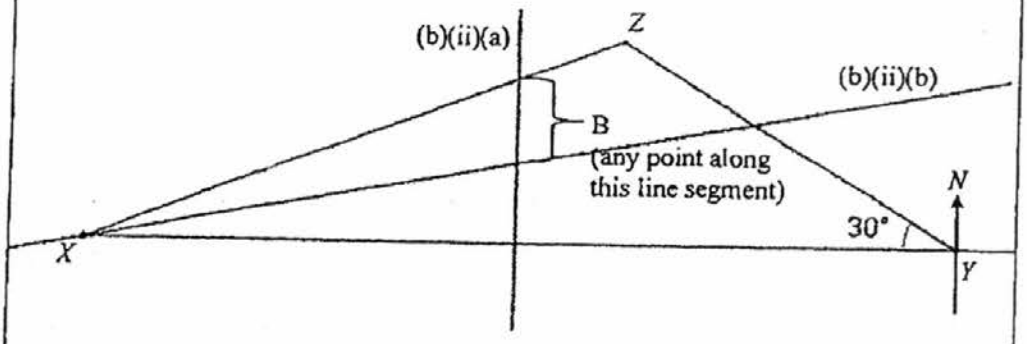
(ii) Construct in the diagram drawn in **(b)(i)**,

(a) the perpendicular bisector of XY, [1]

(b) the bisector of angle ZXY. [1]

(iii) A bench B is located in the park, equidistant from X and Y and nearer to XZ than XY. Mark a possible position of B in the diagram drawn in **(b)(i)**. [1]

Solution:



(b)(i)

Correct shape of triangle with acute angle XYZ

Accurate lengths in cm and accurate angle XYZ

Remark: If any of the following is not labelled on diagram, minus 1 mark.

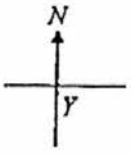
- lengths of XY and YZ in km **OR** cm
- angle XYZ=30° **OR** bearing of Z from Y = 300°

(b)(ii)(a)

correct perpendicular bisector of XY with construction lines

(b)(ii)(b)

correct bisector of angle ZXY with construction lines

		(b)(iii) correct position of B marked and labeled	
		Answer (b)  	

18 A regular pentagon $ABCDE$ and a regular hexagon $AEFGHI$ are joined together along AE, the common side, to form the Figure 1.

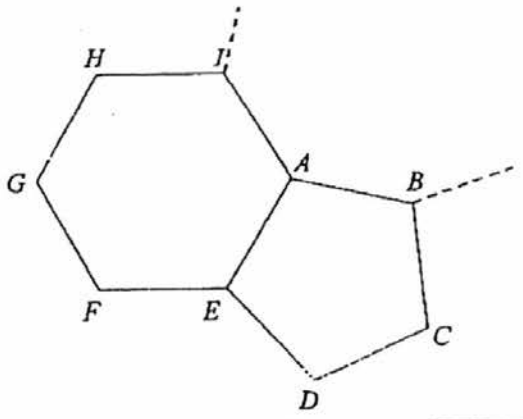


Figure 1

(a) Explain with mathematical calculations why the line segments AB and AI cannot be part of a bigger regular polygon.

Solution:

Interior angle of pentagon

$$= \frac{(5-2)\times 180^\circ}{5} = 108^\circ$$

Interior angle of hexagon

$$= \frac{(6-2)\times 180^\circ}{6} = 120^\circ$$

Angle BAI $= 360^\circ - 108^\circ - 120^\circ = 132^\circ$

Exterior angle of bigger polygon

$= 180^\circ - 132^\circ = 48^\circ$

Number of sides n:

$$n = \frac{360^\circ}{48^\circ} = 7.5$$

OR

$$\frac{(n-2)\times 180^\circ}{n} = 132^\circ$$

$48n = 360$

$n = 7.5$

Since 7.5 is not an integer, hence AB and AI cannot be part of a bigger regular polygon.

Answer (a)

[4]

(b) Given that $AE = 8$ cm, calculate the area of the Figure 1.

Solution:

Hexagon:
Let centre of hexagon be O_x and height of triangle O_xEA be h_x.

$$\frac{360°}{6} = 60° \qquad \frac{60°}{2} = 30°$$

$$\sin 30° = \frac{4}{O_xA} \text{ OR } \tan 30° = \frac{4}{h_x}$$

$$O_xA = \frac{4}{\sin 30°} \text{ OR } h_x = \frac{4}{\tan 30°}$$

Area of triangle O_xEA

$$= \frac{1}{2} \times \left(\frac{4}{\sin 30°} \right)^2 \times \sin 60°$$

OR

$$= \frac{1}{2} \times 8 \times \frac{4}{\tan 30°}$$

Area of hexagon

$$= 6\left(\frac{1}{2} \times \left(\frac{4}{\sin 30°} \right)^2 \times \sin 60° \right)$$

OR

$$= 6\left(\frac{1}{2} \times 8 \times \frac{4}{\tan 30°} \right)$$

$$= 166.2768775$$

Pentagon:
Let centre of pentagon be O_p and height of triangle O_pEA be h_p.

$$\frac{360°}{5} = 72° \qquad \frac{72°}{2} = 36°$$

$$\sin 36° = \frac{4}{O_pA} \text{ OR } \tan 36° = \frac{4}{h_p}$$

$$O_pA = \frac{4}{\sin 36°} \text{ OR } h_p = \frac{4}{\tan 36°}$$

Area of triangle O_pEA

$$= \frac{1}{2} \times \left(\frac{4}{\sin 36°} \right)^2 \times \sin 72°$$

OR

$$=\frac{1}{2}\times 8\times\frac{4}{\tan 36°}$$

Area of hexagon

$$=5\left(\frac{1}{2}\times\left(\frac{4}{\sin 36°}\right)^2\times\sin 72°\right)$$

OR

$$=5\left(\frac{1}{2}\times 8\times\frac{4}{\tan 36°}\right)$$

$=110.1105536$

Area of Figure 1

$$=6\left(\frac{1}{2}\times\left(\frac{4}{\sin 30°}\right)^2\times\sin 60°\right)$$

$$+5\left(\frac{1}{2}\times\left(\frac{4}{\sin 36°}\right)^2\times\sin 72°\right)$$

OR

$$=6\left(\frac{1}{2}\times 8\times\frac{4}{\tan 30°}\right)$$

$$+5\left(\frac{1}{2}\times 8\times\frac{4}{\tan 36°}\right)$$

$=166.2768775+110.1105536$

$=276.3874312$

$=276\ cm^2$ (3sf)

			Answer	(b)	..... cm^2	[5]	

END OF PAPER

4048/2

MATHEMATICS
PAPER 2
SECONDARY 4
2016 Preliminary Examination

18 Aug 2016 (Thursday) **2 hours 30 minutes**

CANDIDATE NAME	**Solution**
CLASS	
INDEX NUMBER	

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

For Examiner's Use		
Q1	12	
Q2	6	
Q3	7	
Q4	9	
Q5	8	
Q6	10	
Q7	11	
Q8	12	
Q9	14	
Q10	11	
Total	/100	

This document consists of 11 printed pages including the Cover Sheet.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

[Turn over

1. (a) Express as a fraction in its simplest form

$$\frac{1}{x+2}+\frac{3}{x-2}-\frac{4x}{4-x^2}$$ [2]

(b) Find the smallest prime number x such that $\frac{1}{3}x-\frac{2x-19}{4}\leq 2$. [2]

(c) Solve the equation $2^{3y-2}=\left(\frac{1}{4}\right)^{y-1}$. [2]

(d) Make p the subject of the formula $s=8\sqrt{2p^2-r}$. [2]

(e) Given that $-5\leq a\leq 2$ and $-1\leq b\leq 8$, find

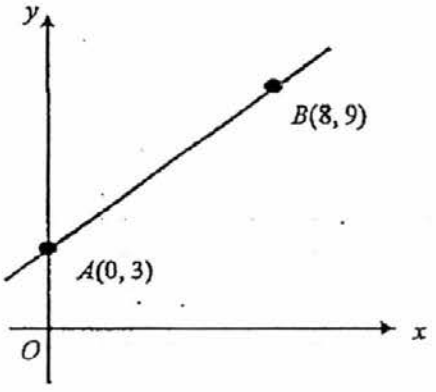
(i) the greatest possible value of a^2-b^2, [1]

(ii) the least possible value of ab. [1]

(f) Factorize the following expression completely

$$x^2y-4y+12-3x^2$$ [2]

2.



The figure above shows a straight line passing through the points $A(0, 3)$ and $B(8, 9)$.

(a) Find the gradient of the line AB. [1]

(b) Find the equation of the line AB. [1]

(c) Calculate the length of the line segment AB. [1]

(d) Another line, parallel to the y-axis and passing through the point (5, 1) meets AB at point C. Calculate the coordinates of the point C. [2]

(e) Find the value of $\cos\angle BAO$. [1]

[Turn over

3. In Singapore, employees are required to contribute part of their monthly salary to Central Provident Fund (CPF) as part of retirement planning.

In 2014, Ahmad's gross monthly salary is $9000. Ahmad's pay slip for January is shown below:

Gross Monthly Salary	$ 9000
CPF contribution (Employee)	$ 1350
Net Monthly Salary	$ 7650

(i) What is the percentage of the gross monthly salary contributed to CPF by Ahmad ? [1]

(ii) The employer is required to pay the employer's share of CPF contribution every month. In 2014, the employer CPF contribution rate is 13% of the employee's gross monthly salary.

How much did Ahmad's employer contribute to CPF in January 2014? [1]

(iii) Annual income is the total income in a year based on the net monthly income. Calculate Ahmad's annual income in 2014. [1]

(iv) Table 1 shows the tax rates on chargeable income in 2014 and Table 2 shows the tax reliefs in 2014:

Table 1			
	Chargeable Income	Income Tax Rate	Gross Tax Payable
	$	(%)	$
On the first	20,000	0	0
On the next	10,000	2	200
On the first	30,000		200
On the next	10,000	3.5	350
On the first	40,000		550
On the next	40,000	7	2,800
On the first	80,000		3,350
On the next	40,000	11.5	4,600
On the first	120,000		7,950
Above	120,000	15	

Table 2	
Type of Relief	Quantum
Parent Relief	$5000 per retired parent
Spouse Relief	$2000
Child Relief	$3000 per child

Ahmad is married with 3 children and he looks after both his retired parents.

Calculate the amount of tax he has to pay for 2014. [3]

[Turn over

(v) To encourage residents of the country to have more children, the parenthood rebate was introduced in 2014. To be eligible for the parenthood rebate, the child must be born on the year of claim. The amount of parenthood rebate is shown in Table 3:

Table 3	
Child Order	**Rebate**
1st	\$50
2nd	\$100
3rd and subsequent child	\$200

Ahmad's youngest child was born in 2014. Calculate the net income tax Ahmad paid in 2014 after the parenthood rebate. [1]

4. Gabriel drove a distance of 195 km from Singapore to Segamat. For the first 150 km, his average speed was x km/h and for the last 45 km, his average speed was 10 km/h more than that of the earlier part.

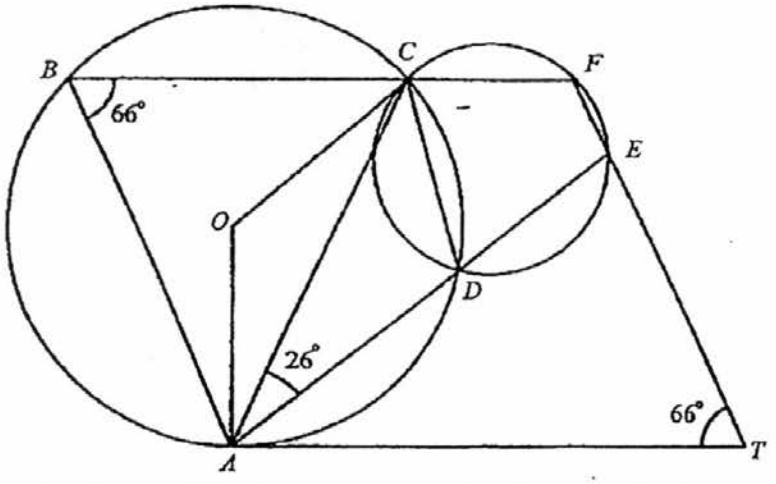
(a) Write down an expression for the time, in term of x, for the number of hours he took for the whole journey. [1]

(b) If the total time taken for the journey was 3 hours 15 minutes, form an equation in x and show that it reduces to $13x^2 - 650x - 6000 = 0$. [3]

(c) Solve the equation $13x^2 - 650x - 6000 = 0$, giving your answer to 2 decimal places. Hence, determine the average speed for the first 150 km of the journey. [3]

(d) How much more time would Gabriel take if he maintained at an average speed of x km/h throughout the journey? Leave your answer to the nearest minute. [2]

5.



In the diagram, TA is a tangent to the circle with centre O, and which passes through the points A, B, C and D.
Another smaller circle, which passes through the points C, D, E and F, is drawn such that BCF, ADE and TEF are straight lines.
Angle ABF = Angle ATE = 66° and Angle CAD = 26°.
Find, giving reasons for each answer,

(a) angle AOC, [1]

(b) angle OCA, [1]

(c) angle ACD, [2]

(d) angle CFE, [1]

(e) angle OCB. [3]

6. **Answer the whole of this question on a sheet of graph paper.**
The table below gives some values of x and the corresponding values of y, where

$$y = 25 + 4x - 3x^2.$$

x	-3	-2	-1	0	1	2	3	4	5
y	-14	5	m	25	26	n	10	-7	-30

(a) (i) Find the value of m and the value of n. [1]

(ii) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-4 \le x \le 5$.
Using a scale of 2 cm to 10 units, draw a vertical y-axis for $-30 \le y \le 30$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

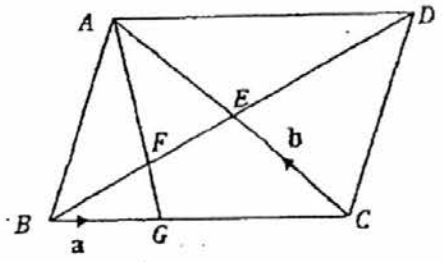
(b) By drawing a tangent, find the gradient of the curve at the point where $x = 3$. [2]

(c) By drawing a suitable straight line, solve the equation $20 + 2x - 3x^2 = 0$. [3]

(d) State the range of values of x for which $25 + 4x - 3x^2 \ge 0$. [1]

[Turn over

7.



In the diagram $ABCD$ is a parallelogram. The diagonal AC and BD intersect at E. F is a point on BD such that $BC = 3BG$. F is the midpoint of $\overline{BE}$. Given that $\overline{BG} = \boldsymbol{a}$ and $CA = \boldsymbol{b}$

(a) Express, as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$.

(i) $\overrightarrow{BA}$, [1]

(ii) $\overrightarrow{BF}$, [2]

(iii) $\overrightarrow{AF}$. [1]

(b) Explain why A, F and G lie on a straight line. [2]

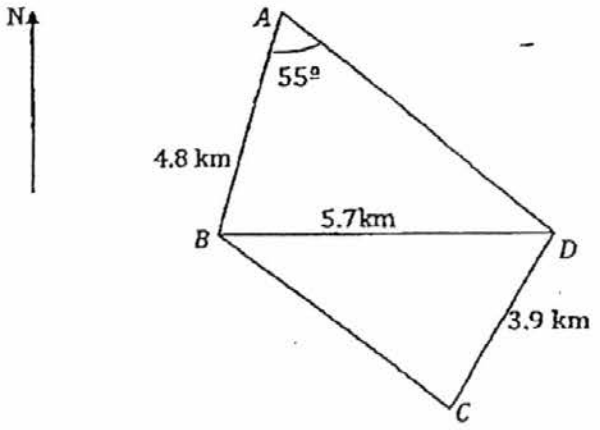
(c) Find

(i) $\dfrac{\text{area of triangle } ABF}{\text{area of triangle } ABD}$, [2]

(ii) $\dfrac{\text{area of triangle } BFG}{\text{area of parallelogram } ABCD}$. [3]

[Turn over

8.



A, B, C and D are four points on an airfield. B is due west of D and the bearing of C from D is 215°. It is given that $B\hat{A}D = 55°$, $AB = 4.8$ km, $DC = 3.9$ km and $BD = 5.7$ km.

(a) Calculate

(i) the bearing of D from C, [1]

(ii) the distance of BC, [2]

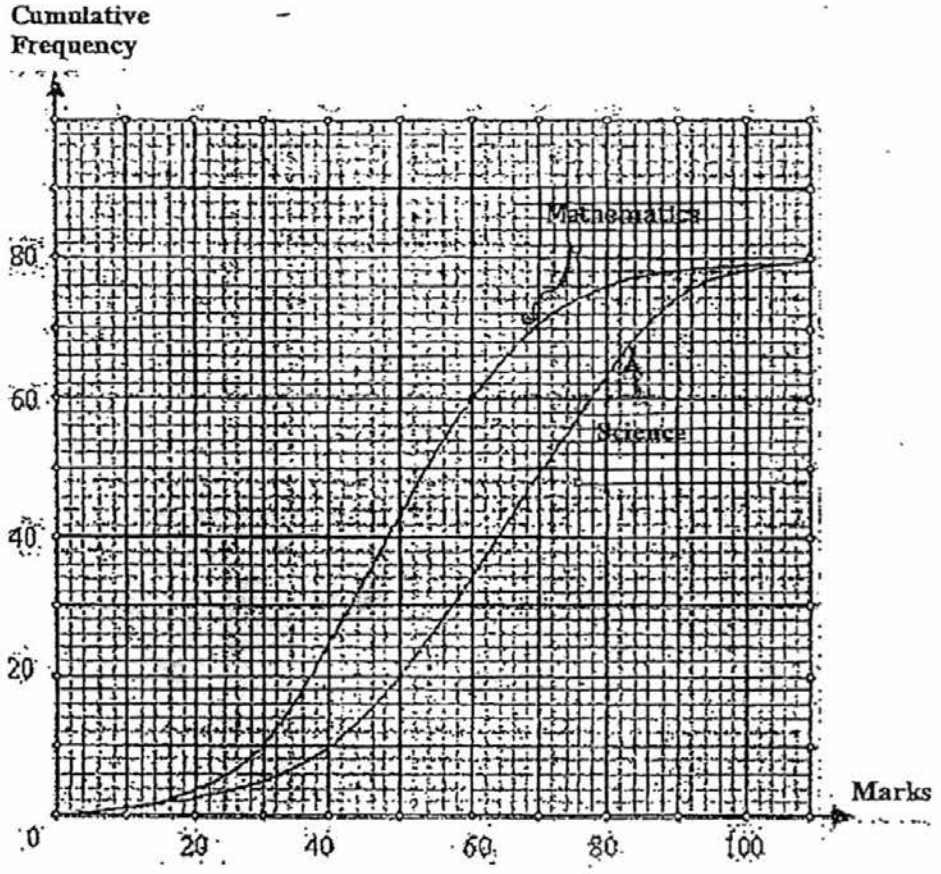
(iii) angle ABD, [2]

(iv) the shortest distance of D to BC. [3]

(b) A control tower is at X such that X is on BD produced and $DX = 1.2$ km. The angle of depression of point B from the top of the tower from is 0.2°. Calculate the height of the tower, correct to the nearest metre. [2]

(c) A landrover travels from point B to point D at a constant speed of 50 km/h. It rests at point D for 35 minutes before it travels to point C at a constant of 40 km/h. Calculate the total time taken by the landrover to travel from point B to point D, and point D to point C, correct to the nearest tenth of a minute. [2]

9. A group of 80 students took a Mathematics examination and a Science examination. The cumulative frequency curves below show the distribution of their marks.



(a) (i) Find the median mark for the Mathematics examination. [1]

(ii) Find the interquartile range for the Science examination. [2]

(iii) Grade A for the Mathematics examination is allocated to the top 15% of the students. Find the mark to achieve Grade A at the Mathematics examination. [1]

(b) Which was the more difficult paper? Justify your answer. [1]

(c) Represent the distribution of marks of the Science examination using a box and whisker plot. [2]

[Turn over

(d) The masses of 15 durians from Shop A are recorded in the table below.

Mass (x kg)	Frequency
$1 \leq x < 2$	1
$2 \leq x < 3$	p
$3 \leq x < 4$	6
$4 \leq x < 5$	q

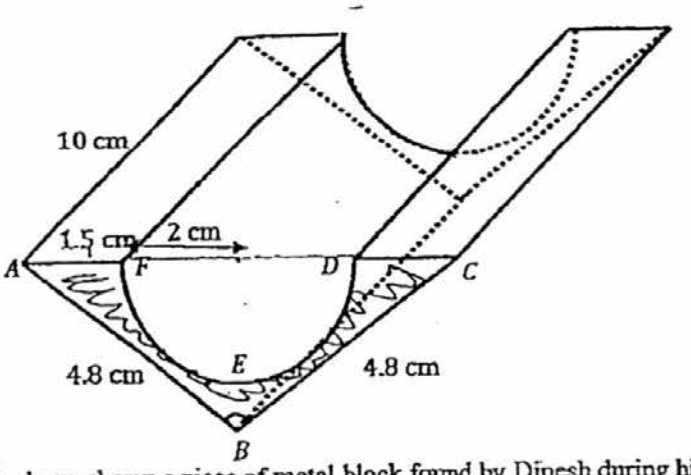
(i) Find the values of p and q such that the estimated mean mass of the durians is 3.5 kg . [3]

(ii) Find an estimate for the standard deviation of the mass of the durians in the shop. [2]

(iii) Given that the mean and standard deviation of the mass of the durians in Shop B are 4.4 kg and 0.4 kg respectively, comment on the distribution of the mass of durians in both shops. [2]

[Turn over

10.



The diagram above shows a piece of metal block found by Dinesh during his expedition to Indonesia. The cross section of the metal block is a triangle, ABC, from which a semicircle, DEF, of radius 2 cm has been removed. The length of the block is 10 cm. $AB = BC = 4.8$ cm and $CD = FA = 1.5$ cm. Calculate

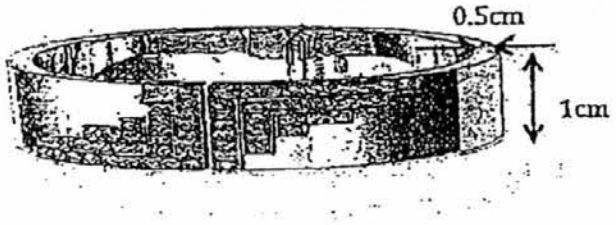
(a) angle ABC, [2]

(b) the shaded area of the cross-section $ABCDEF$ of the metal block, [3]

(c) the volume of the metal block. [1]

(d) Dinesh found out that the metal block is gold. How much can he get by selling this metal block given the density of gold is 19.3 g/cm^3 and the price of gold is \$56/g? [2]

(e) Dinesh decided to go to the goldsmith to melt this metal block of gold to convert it into a bangle for his wife. He intends to make a bangle of thickness of 0.5 cm and width of 1 cm as shown in the diagram below. Will it be possible if his wife's wrist is 15 cm in circumference? Justify your answer. [3]



END OF PAPER

Solution

Q1a) $\frac{1}{x+2} + \frac{3}{x-2} - \frac{4x}{4-x^2}$

$= \frac{1}{x+2} + \frac{3}{x-2} - \frac{4x}{(2+x)(2-x)}$

$= \frac{1}{x+2} + \frac{3}{x-2} + \frac{4x}{(2+x)(x-2)}$

$= \frac{1(x-2) + 3(x+2) + 4x}{(x+2)(x-2)}$

$= \frac{8x+4}{(x+2)(x-2)}$ or $\frac{4(2x+1)}{(x+2)(x-2)}$ or $\frac{8x+4}{x^2-4}$ or $\frac{-4(2x+1)}{(2-x)(2+x)}$

1b) $\frac{1}{3}x - \frac{2x-19}{4} \leq 2$

$\frac{4x - 3(2x-19)}{12} \leq 2$

$4x - 6x + 57 \leq 24$

$-2x \leq -33$

$x \geq \frac{33}{2}$

$x \geq 16\frac{1}{2}$

Smallest prime number is 17

1d)

$$S = 8\sqrt{2p^2 - r}$$

$$\frac{S}{8} = \sqrt{2p^2 - r}$$

$$\frac{S^2}{64} = 2p^2 - r$$

$$2p^2 = \frac{S^2}{64} + r$$

$$p^2 = \frac{1}{2}\left(\frac{S^2}{64} + r\right)$$

$$p = \pm\sqrt{\frac{1}{2}\left(\frac{S^2}{64} + r\right)} \quad \text{or} \quad \pm\sqrt{\frac{1}{2}\left(\frac{S^2+64r}{64}\right)} \quad \text{or} \quad \pm\sqrt{\frac{S^2+64r}{128}}$$

1e) i) $\max(a^2 - b^2) = (-5)^2 - 0^2$

$= 25$

ii) least $ab = (-5)(8) = -40$

1f) $x^2y - 4y + 12 - 3x^2 = x^2(y-3) - 4(y-3)$

$= y(x^2-4) - 3(x^2-4)$

$= (y-3)(x^2-4)$

$= (y-3)(x+2)(x-2)$

1c) $2^{3y-2} = \left(\frac{1}{4}\right)^{y-1}$

$2^{3y-2} = \left(\frac{1}{2^2}\right)^{y-1}$

$2^{3y-2} = 2^{-2(y-1)}$

$3y-2 = -2y+2$

$5y = 4$

$y = \frac{4}{5}$

2. a) $\text{grad } AB = \frac{9-3}{8-0}$

$= \frac{6}{8} = \frac{3}{4}$

b) $y = \frac{3}{4}x + 3$

c) $AB = \sqrt{(0-8)^2 + (3-9)^2}$

$= \sqrt{64 + 36}$

$= 10$ units

d) $y = x - 5$

$y = \frac{3}{4}x + 3$

$y = \frac{3}{4}(x) + 3$

$y = \frac{27}{4}$

$\therefore C(5, \frac{27}{4})$

$(5, 6.75)$

e) $\cos \angle BAO = -\frac{6}{10}$

$= -\frac{3}{5}$

$= -0.6$

3 i) $\frac{1350}{9000} \times 100 = 15\%$

ii) $\frac{13}{100} \times 9000 = \$1170$

iii) $7650 \times 12 = \$91800$

iv) Chargable income $= 91800 - 2000 - 3 \times 3000 - 2 \times 5000$

$= \$70800$

Amount of tax $= 550 + \frac{7}{100} \times (70800 - 40000)$

$= \$2706$

v) $2706 - 200 = \$2506$

4) a) $\left(\frac{150}{x} + \frac{45}{x+10}\right) h$

OR $\left(\frac{195x+1500}{x(x+10)}\right) h$

b) $\frac{150}{x} + \frac{45}{x+10} = 3\frac{1}{4}$

$$\frac{150(x+10)+45x}{x(x+10)} = \frac{13}{4}$$

$$4[150x + 1500 + 45x] = 13(x^2+10x)$$

$$600x + 6000 + 180x = 13x^2 + 130x$$

$$13x^2 - 650x - 6000 = 0$$

c) $$x = \frac{-(-650) \pm \sqrt{650^2 - 4(13)(-6000)}}{2(13)}$$

$x = 57.96268$, -7.96268 (5dp)

$x = 57.96$, -7.96 (2dp)

(reject $x = -7.96$ $\because x > 0$)

$\therefore$ Average speed for 1st 150 km = 58.0 km/h (3sf)

d) Average time $= \frac{195}{x}$

$$= \frac{195}{57.96268}$$

$$= 3.3644 \text{ h}$$

$$\approx 3 \text{ hr } 22 \text{ min (nearest min)}$$

3 hr 22 min − 3 hr 15 min = 7 more mins (nearest min)

5) a) $\angle AOC = 66^\circ \times 2$ ($\angle$ at Centre $= 2\angle$ at Circumference)

$= 132^\circ$

b) $\angle OCA = \angle OAC = \frac{180^\circ - 132^\circ}{2}$ (Base $\angle$ of isosceles Δ)

$= 24^\circ$.

c) $\angle CDA = 180^\circ - 66^\circ$ ($\angle$s in opp segment)

$= 114^\circ$

$\angle ACD = 180^\circ - 114^\circ - 26^\circ = 40^\circ$ ($\angle$ sum of Δ)

d) $\angle CDE = 180^\circ - 114^\circ = 66^\circ$ (sum of adj $\angle$s on a st line)

$\angle CFE = 180^\circ - 66^\circ = 114^\circ$ ($\angle$s in opp. segments)

e) $\angle OAT = 90^\circ$ (tan $\perp$ rad)

$\angle EAT = 90^\circ - 24^\circ - 26^\circ$

$= 40^\circ$

$\angle AET = 180^\circ - 40^\circ - 66^\circ$ ($\angle$ sum of Δ)

$= 74^\circ$

$\angle DEF = 180^\circ - 74^\circ$ (sum of adj $\angle$s on a st. line)

$= 106^\circ$

$\angle DCF = 180^\circ - 106^\circ$ ($\angle$s in opp. segments)

$= 74^\circ$

$\angle OCB = 180^\circ - 74^\circ - 24^\circ - 40^\circ$ (sum of adj $\angle$s on a st line)

$= 42^\circ$

6. (a)(i) $m = 25 + 4(-1) - 3(-1)^2$

$m = 18$

$n = 25 + 4(2) - 3(2)^2$

$n = 21$

(b) Gradient $= \dfrac{38 - (-18)}{1 - 5}$

$= -14$ **(Accept -15.5 to -12.5)**

(c) $20 + 2x - 3x^2 = 0$

$25 + 2x - 3x^2 = 5$

$25 + 4x - 3x^2 = 5 + 2x$

$y = 5 + 2x$

x	-4	0	5
y	-3	5	15

From graph, $x = -2.3$ **(-2.4 to -2.2)** or $x = 2.9$ **(2.8 to 3.0)**

(d) $25 + 4x - 3x^2 \geq 0$

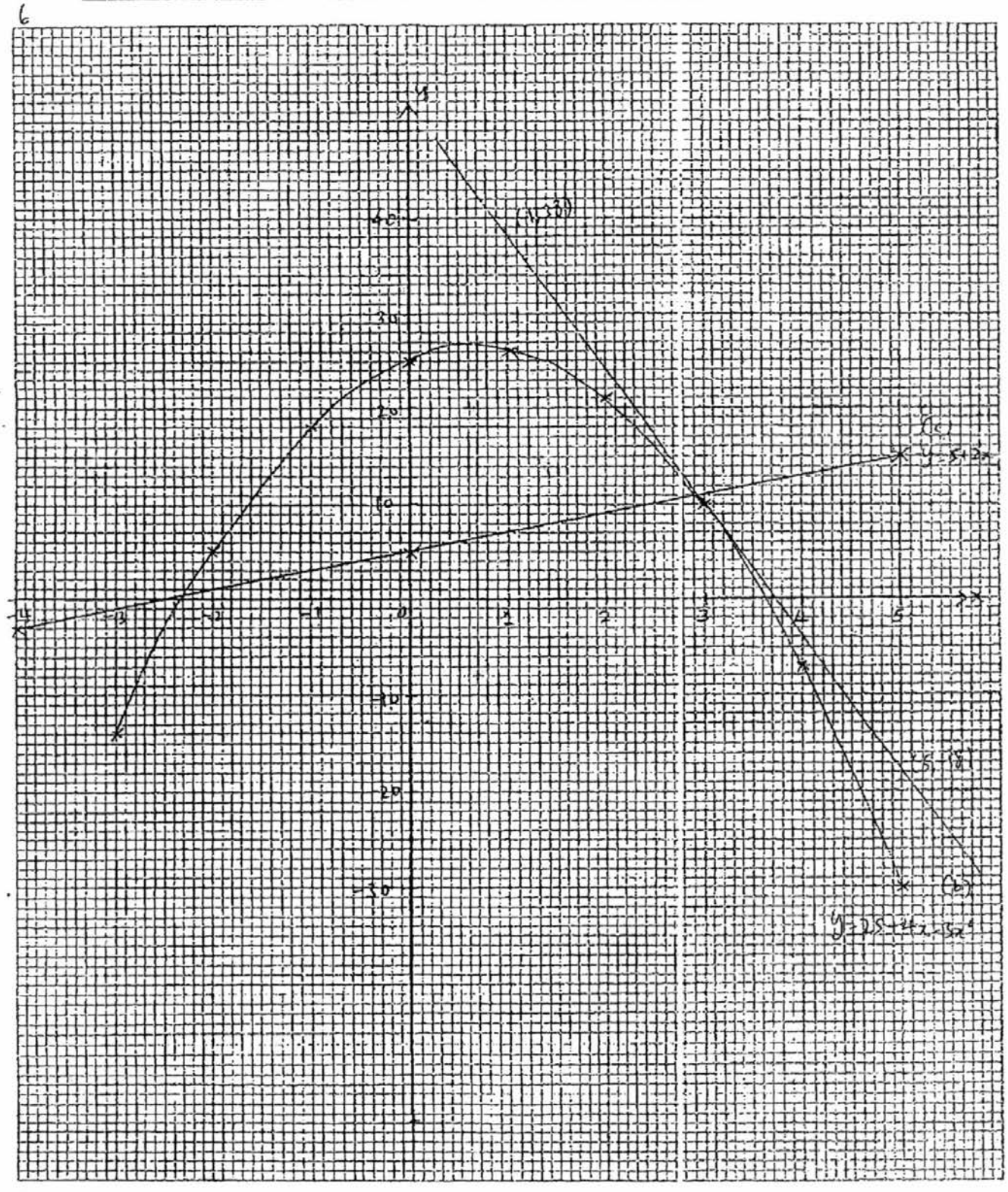
From graph, $-2.3 \leq x \leq 3.6$

(-2.4 to -2.2) **(3.5 to 3.7)**

Name _____ Index No _____

Subject EM P2 2016 Class _____ Date _____

6



20 cm × 24 cm

7) i) $\overrightarrow{BA} = 3\underline{a} + \underline{b}$

ii) $\overrightarrow{BD} = 3\underline{a} + 3\underline{a} + \underline{b}$

$= 6\underline{a} + \underline{b}$

$\overrightarrow{BF} = \frac{1}{4}(6\underline{a} + \underline{b})$ OR $\frac{3}{2}\underline{a} + \frac{1}{4}\underline{b}$

OR $\frac{1}{2}(3\underline{a} + \frac{1}{2}\underline{b})$

iii) $\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

$= -(3\underline{a} + \underline{b}) + \frac{3}{2}\underline{a} + \frac{1}{4}\underline{b}$

$= -\frac{3}{2}\underline{a} - \frac{3}{4}\underline{b}$

$= -\frac{3}{4}(2\underline{a} + \underline{b})$

b) $\overrightarrow{AG} = \overrightarrow{AB} + \overrightarrow{BG}$

$= -3\underline{a} - \underline{b} + \underline{a}$

$= -2\underline{a} - \underline{b}$

$= -(2\underline{a} + \underline{b})$

OR $\overrightarrow{FG} = \overrightarrow{FB} + \overrightarrow{BG}$

$= -\frac{3}{2}\underline{a} - \frac{1}{4}\underline{b} + \underline{a}$

$= -\frac{1}{2}\underline{a} - \frac{1}{4}\underline{b}$

$= -\frac{1}{4}(2\underline{a} + \underline{b})$

A, G and F lie on a straight line because $\overrightarrow{AF} = k\overrightarrow{AG}$, k is a constant.

And $\overrightarrow{AF} = \frac{3}{4}\overrightarrow{AG}$ $(|\overrightarrow{AF}| = \frac{3}{4}|\overrightarrow{AG}|)$

or $\overrightarrow{FG} = \frac{1}{3}\overrightarrow{AF}$

or $\overrightarrow{FG} = \frac{1}{4}\overrightarrow{AG}$

7(c)(i) $\dfrac{\text{Area of } \Delta ABF}{\text{Area of } \Delta ABD} = \dfrac{\frac{1}{2} \times BF \times h}{\frac{1}{2} \times BD \times h}$

$= \dfrac{BF}{BD}$

$= \dfrac{1}{4}$

(c)(ii) $\dfrac{\text{Area of } \Delta BFG}{\text{Area of } \Delta AFD} = \left(\dfrac{1}{3}\right)^2$

$= \dfrac{1}{9}$

$\dfrac{\text{Area of } \Delta ABF}{\text{area of } \Delta AFD} = \dfrac{1}{3}$

$= \dfrac{3}{9}$

OR

$\dfrac{\text{area of } \Delta AFD}{\text{area of } \Delta ABD} = \dfrac{\frac{1}{2} \times FP \times h}{\frac{1}{2} \times BD \times h}$

$= \dfrac{FD}{BD}$

$= \dfrac{3}{4}$

$= \dfrac{9}{12}$

$\dfrac{\text{Area of } \Delta BFG}{\text{area of } \parallel\text{gram } ABCD} = \dfrac{1}{24}$

8 a) i) The bearing of D from C $= 215° - 180°$
$= 035°$

ii) $BC^2 = 5.7^2 + 3.9^2 - 2(5.7)(3.9)\cos 35°$

$BC = 4.711559$ (7sf)

$BC = 4.71$ km (3 sf)

iii) $\frac{\sin A\hat{D}B}{4.8} = \frac{\sin 35°}{5.7}$

$\sin A\hat{D}B = \frac{\sin 35°}{5.7} \times 4.8$

$A\hat{D}B = 43.615°$ (3dp)

$\therefore A\hat{B}D = 180° - 43.615° - 35°$
$= 81.385°$ (3dp)
$= 81.4°$ (1dp)

iv) let shortest dist. from D to BC be y

$\frac{1}{2} \times 5.7 \times 3.9 \times \sin 35° = \frac{1}{2} \times 4.711559 \times y$

$\therefore y = 3.86491 = 3.86$ km
(6sf) (3sf)

8b) $\tan 0.2^\circ = \frac{\text{ht of tower}}{6.9}$

ht of tower $= 0.024086$ km (6 dp)

$= 24$ m

(nearest m)

8c) $\frac{5.7}{50} + \frac{35}{60} + \frac{3.9}{40}$

$= \frac{4769}{6000}$

$= 0.794833$ hr (6 dp)

$= 47.69$ mins

$= 47.7$ min (to nearest tenth)

9 ai) Median mark = 48 marks (Accept 47.5 to 48.5)

ii) Interquartile Range = 78 − 50

= 28 marks

iii) (0.85 × 80 = 68 students)

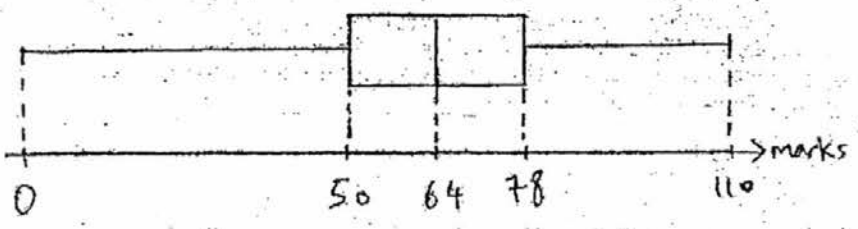
Mark for Grade A in Mathematics = 67 marks

(Accept 66 to 68 marks)

b) The median for Maths is 48 marks lower than the median for Science which is 64 marks

So, Maths is the more difficult paper.

c)



q d)

$1 + p + 6 + q = 15$

$p + q = 8$ ①

$$\frac{1.5 + 2.5p + 3.5(6) + 4.5q}{15} = 3.5$$

$2.5p + 4.5q = 52.5 - 1.5 - 21$

$2.5p + 4.5q = 30$ — ②

From ① $q = 8 - p$ sub into ②

$2.5p + 4.5(8 - p) = 30$

$2.5p + 36 - 4.5p = 30$

$-2p = -6$

$p = 3$

$\therefore q = 5$

d ii) S.D = 0.894 kg (3sf)

d iii) The durians in Shop B are heavier (as mean bigger) the masses of durians in Shop B are more consistent or has smaller spread. (as SD is lower).

10 a) $\cos\angle ABC = \dfrac{4.8^2 + 4.8^2 - 7^2}{2(4.8)(4.8)}$

$= 93.635^\circ$ (3dp)

$\angle ABC = 93.6^\circ$ (1dp)

b) Shaded Area $= \frac{1}{2}(4.8)(4.8)\sin 93.635 - \frac{1}{2}\pi(2)^2$

$= 5.213664\ cm^2$ (7sf)

$= 5.21\ cm^2$ (3sf)

c) Vol of block $= 5.213664 \times 10$

$= 52.13664\ cm^3$ (7sf)

$= 52.1\ cm^3$ (3sf)

$\frac{M}{V} = D$

d) Mass of block $= 19.3 \times 52.13664$

$= 1006.237\ g$ (7sf)

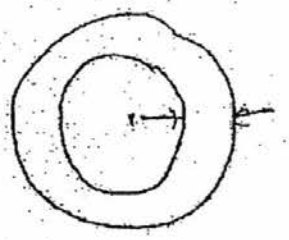
∴ Price of block $= \$56\ 349.26$ (2dp)

10 (cont')

Circumference of wrist = 15 cm

$2\pi R = 15$

$R = \frac{15}{2\pi}$ cm



Cross sectional area

$= \pi\left(\frac{15}{2\pi} + 0.5\right)^2 - \pi\left(\frac{15}{2\pi}\right)^2$

$= \pi[8.33664 - 5.199317)$

$= 8.285396 \text{ cm}^2$.

Vol $= 8.285396 \times 1$

$= 8.285396 \text{ cm}^3$

Since volume of bangle < volume of metal block,

it is possible.

END