Name ______________________________( )   Class:______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2017
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4048/01
PAPER 1

Duration: 2 hours

Classes: 401, 402, 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, hand in separately:

1. **Section A**
2. **Section B**
3. **Section C**

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
Total marks	**/80**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Name: ______________________ ( ) Class: ______

Answer all the questions.

Section A [22 marks]

1 **(a)** Simplify $\frac{x+1}{x^2-9}-\frac{2}{3-x}$.

Answer .... [4]

(b) Simplify $\frac{\left(abc^{-2}\right)^3}{\left(a^{-4}b^{-1}\right)^{-1}}\times\frac{a^{-6}b^{-7}}{\left(bc^{2}\right)^{-4}}$, leave your answer in positive indices.

Answer .... [3]

2 Given that $\frac{k}{3}=\sqrt{\frac{A-3b^2}{cA}}$, express A in terms of b, c and k.

Answer $k=$ .... [3]

3 Factorise the following completely.

(a) $18x^2y+27xy-9xy^3$

Answer .... [1]

(b) $27a^2-12b^2$

Answer .... [1]

(c) $3rs-3s-r+1$

Answer .... [1]

4 Given that $-5 \le x \le 2$ and $-6 \le y \le -1$, find

(a) the largest possible value of $x-y$,

Answer .... [1]

(b) the smallest possible value of y^2-x^2,

Answer .... [1]

(c) the smallest possible value of $(x-y)^2$.

Answer .... [1]

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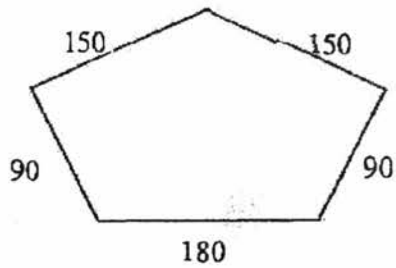
5 A small bus interchange has 2 feeder buses. Bus number 801 leaves the interchange at 15-minute intervals while number 802 at 25-minutes intervals. If both buses leave together on a particular day, how many times will they leave together in the next 5 hours?

Answer .... times [3]

6 A pond with the shape of a pentagon is shown below (measurements are given in metres and not drawn to scale).



Lamp posts are to be constructed around the pond with the following requirements:

(I) The lamp posts are to be equally spaced from each other.
(II) One lamp post must be constructed at each vertex of the pentagon.
(III) Minimum number of lamp posts are to be constructed to save cost.

Find

(a) the distance between any two lamp posts.

Answer .... [1]

(b) the number of lamp posts to be constructed.

Answer .... [2]

Section B [18 marks]

7 When written as the product of their prime factors,

$A = 2^{m+2} \times 3^{n}$

$B = 2^{m} \times 3^{n+1} \times 5$, where m and n are positive constants.

Find the lowest common multiple of A and B, giving your answer as a product of its prime factors.

Answer .... [2]

8 Solve the simultaneous equations.

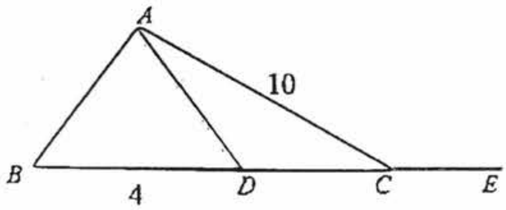
$$\frac{1}{2}x + y = 1,$$
$$\frac{1}{4}x - 3y = 11$$

Answer $x =$ ....

$y =$ .... [3]

Name: ______________________ (   ) Class: ______

9 In the diagram, $BDCE$ is a straight line, $BD = 4$ cm, $AC = 10$ cm and $AB = AD$.
Given that the area of triangle ABD is 16 cm^2, calculate



(a) the vertical height of triangle ABD. [2]

(b) the value of $\sin \angle ACD$. [1]

Answer vertical height = . ….. cm [2]
$\sin \angle ACD$ = …………… ° [1]

(c) the value of $\cos \angle ACE$.

Answer $\cos \angle ACE$ = ……………… ° [2]

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10 During their quest to reach the South Pole on the first day of the new millennium, the Singapore Antarctica 2000 Expedition team experienced temperatures ranging from -35°C to -5°C while their family members in Singapore experienced temperatures ranging from a°C to b°C, where $a < b$.
Find, in terms of a and/or b,

(a) the greatest difference in temperatures between the South Pole and Singapore.

Answer ………………… °C [1]

(b) the smallest difference in temperatures between the South Pole and Singapore.

Answer ………………… °C [1]

11 Two maps of a new town are drawn. On the first map, a school is represented by an area of 3 cm^2.

The school is represented by an area of 12 cm^2 on the second map.

Given that the scale of the first map is 1 : 80000, find the scale of the second map in the form of 1: n.

Answer 1 : .... [4]

12 Mrs Ang invested $36 000 in a bank that pays compound interest of 3.2 % per annum, payable every 3 months.

Calculate the amount that Mrs Ang has in the bank after 6 years.

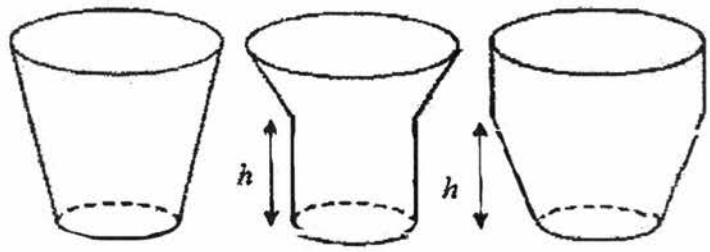
Answer $....................... [2]

Name:_________________________ (   ) Class: ______

Section C [40 marks]

13 Liquid X is poured into three different tanks at a constant rate.

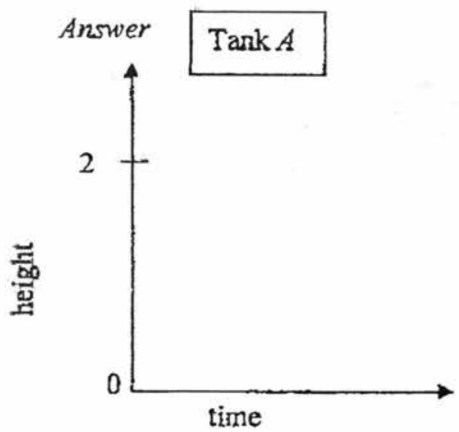
The height of each tank is 2 metres.

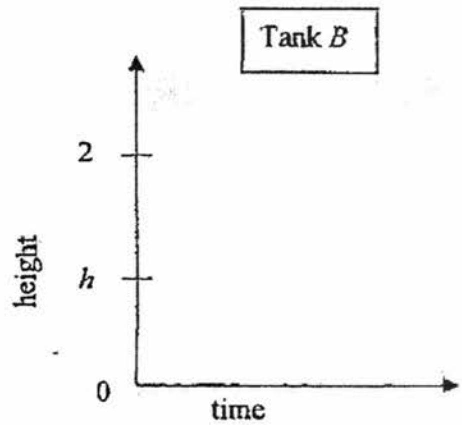


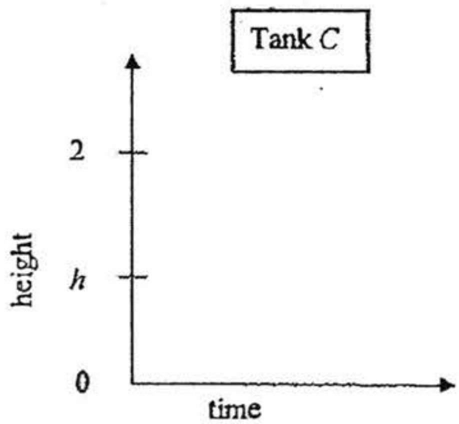
Tank A Tank B Tank C

On each of the grids below, sketch the graphs to show how the height of the water changes with time for each tank.

Answer

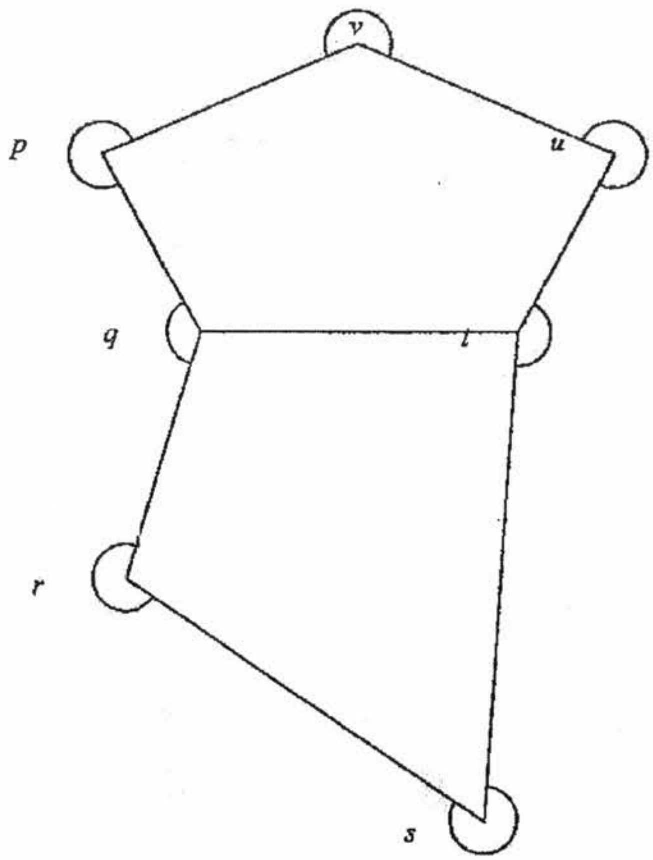






[3]

14 **(a)** Calculate the sum of the angles p, q, r, s, t, u and v shown in the diagram.



Answer ....° [2]

(b) A regular polygon has n sides.

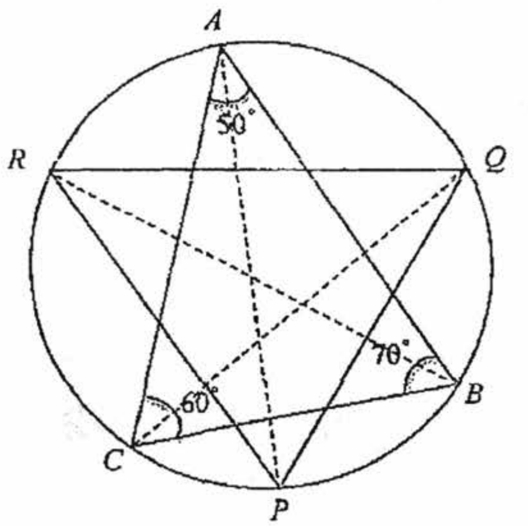
Each exterior angle is $\frac{n}{40}$ degrees.

Find the size of each interior angle in this polygon.

Answer ....° [2]

Name: ______________________ ( ) Class: ______

15 In the figure, the vertices of triangle ABC and triangle PQR touch the circumference of the circle.
Given that angle $CAB = 50^{\circ}$, angle $ABC = 70^{\circ}$ and angle $BCA = 60^{\circ}$ and AP, BR and CQ are angle bisectors of angle CAB, angle ABC and angle BCA respectively, find the values of angles RPQ, PQR and PRQ.



Answer angle RPQ = ...° [2]

angle PQR = ...° [1]

angle PRQ = ...° [1]

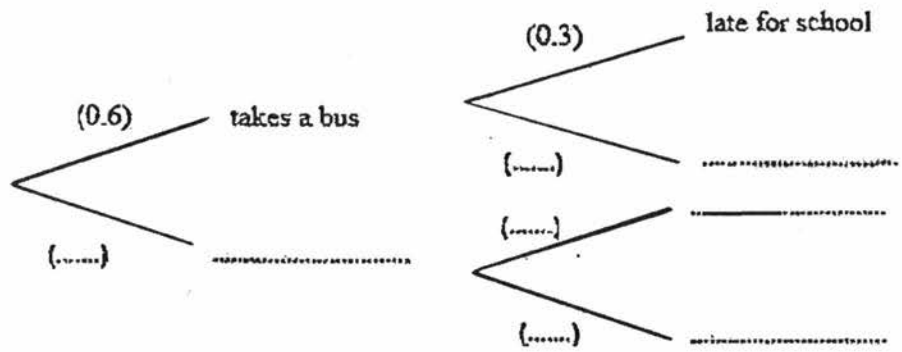
16 The probability that Katie takes a bus is 0.6.

If she takes a bus, the probability that she is late for school is 0.3.

If she does not take a bus, the probability that she is late for school is 0.2.

(a) Complete the probability tree given below

Answer



[3]

(b) Calculate the probability that Katie is not late to school.

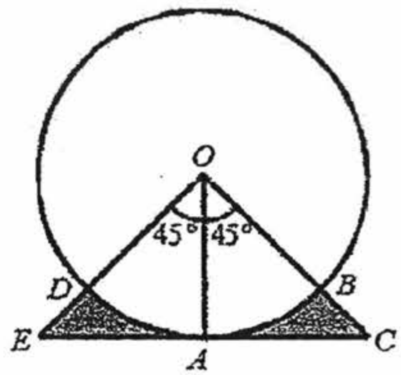
Answer .... [2]

Name: ______________________ (   ) Class: ______

17 In the diagram, the circle, centre O, passes through D, A and B.

The tangent at A meets OB produced at C and OD produced at E.

The radius of the circle is 4 cm and angle AOB = angle $AOE = 45°$.



(a) The area of the shaded region can be expressed as $(a - b\pi)$ cm^2, where a and b are constants.

Find the values of a and b.

Answer a = ... [2]

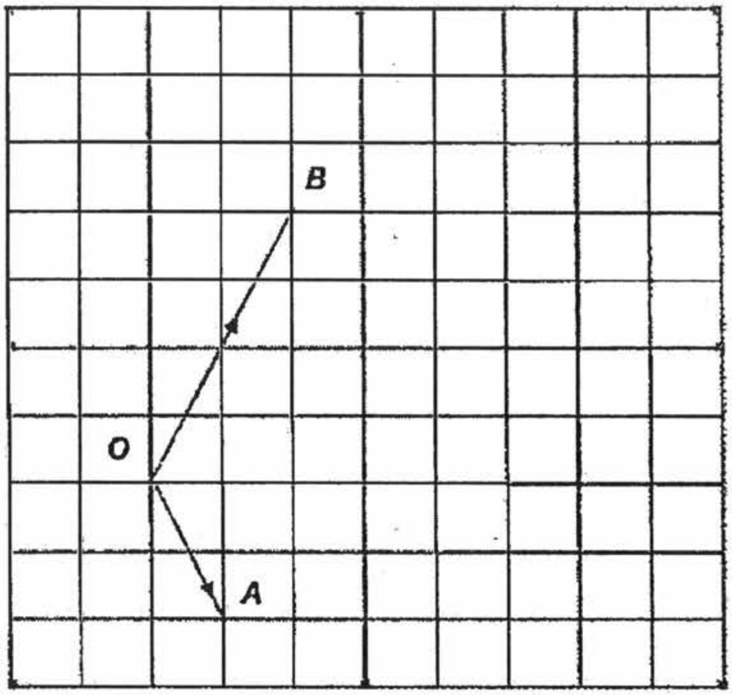
b = ... [2]

(b) The perimeter of the shaded region can be expressed as $\left(p\pi + 2\sqrt{q}\right)$ cm .
Find the values of p and q.

Answer p = ... [2]

q = ... [2]

18 Vectors $\overrightarrow{OB}$ and $\overrightarrow{OA}$ are drawn below.



Given that $\overrightarrow{OP} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

(a) **(i)** locate point P on the grid, mark it with a cross X and label it, [1]

(ii) express $\overrightarrow{OP}$ in terms of $\overrightarrow{OB}$ and/or $\overrightarrow{OA}$.

Answer $\overrightarrow{OP}$ = ... [1]

(b) $OBQA$ is a parallelogram.

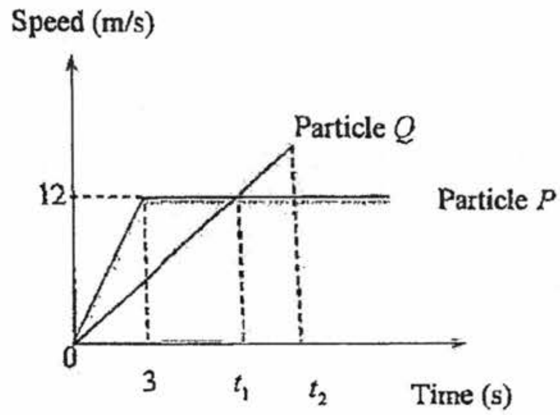
(i) locate point Q on the grid, mark it with a cross X and label it, [1]

(ii) find the column vector representing $\overrightarrow{OQ}$.

Answer $\overrightarrow{OQ}$ = ... [1]

19 The diagram shows the speed-time graphs of two particles P and Q. Both particles

P and *Q* start from rest. *P* accelerates uniformly for 3 seconds until it reaches a speed of 12 m/s. It then continues to travel at this constant speed. *Q* starts from the same point as *P* but accelerates from rest at a constant rate of 3 m/s ^{2}.



(a) Write down the value of t_1, where the speeds *P* and *Q* are the same.

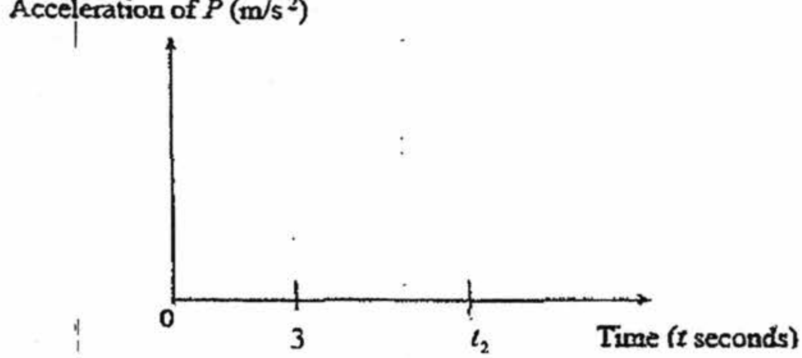
Answer t_1 = ... [1]

(b) Given that *Q* overtakes *P* t_2 seconds after the start of the motion, find the value of t_2

Answer t_2 = ... [3]

(c) In the answer space below, sketch the acceleration-time graph of *P* for $0 \le t \le t_2$.

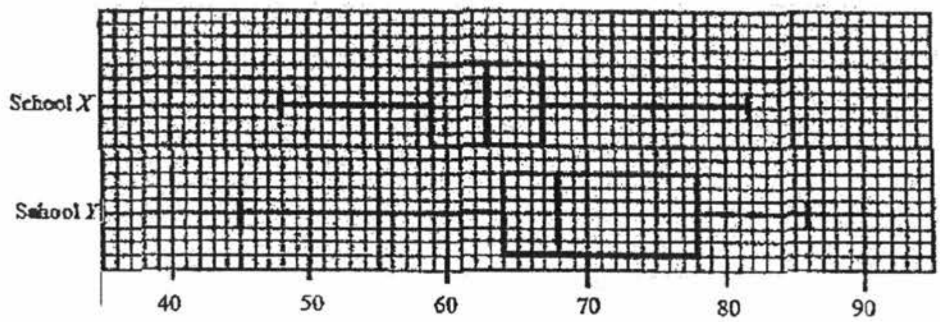
Answer



[1]

20 All the students from 2 schools *X* and *Y* took the same examination paper.

The box-and -whisker diagram below shows the results for the two schools.



(a) State, with a reason, which school achieved a better result.

Answer ....

....

.... [1]

(b) State, with a reason, which school has a more uniformly-distributed mark.

Answer ....

....

.... [1]

21 The numbers in the Number Triangle are consecutive even numbers.

Name: ______________________ (   ) Class: ______

Row	Number Triangle	Sum of row (R)	No. of even numbers (E)	Average of Row (A)
1	2	2	1	2
2	4 6	10	2	5
3	8 10 12	30	3	10
4	14 16 18 20	68	4	p
5	22 24 26 28 30	130	5	26
6	32 34 36 38 40 42	q	6	37

(a) Find the values of p and q.

Answer p = ……, q =……… [2]

(b) Write down a formula connecting A and E.

Answer ……………………… [1]

(c) Write down a formula connecting R and E.

Answer ……………………… [1]

(d) Justify, with reason why the number 6400 could not appear in the column A.

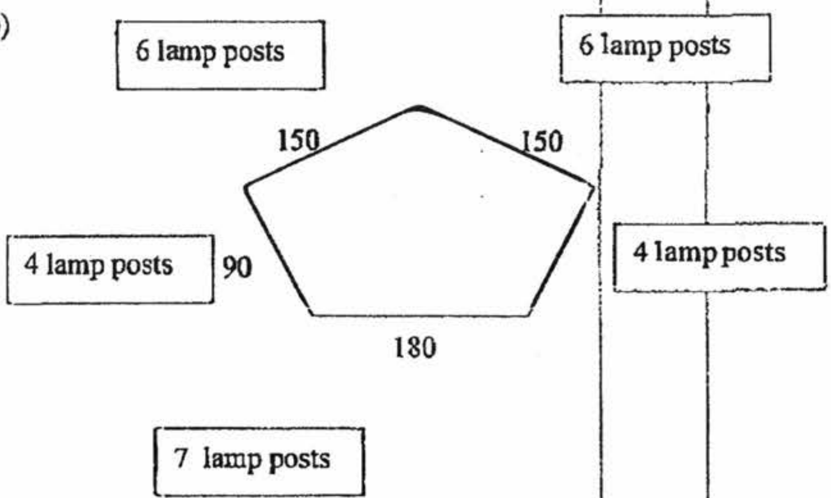
Answer ……………………………………………………………………

……………………………………………………………………

…………………………………………………………………… [1]

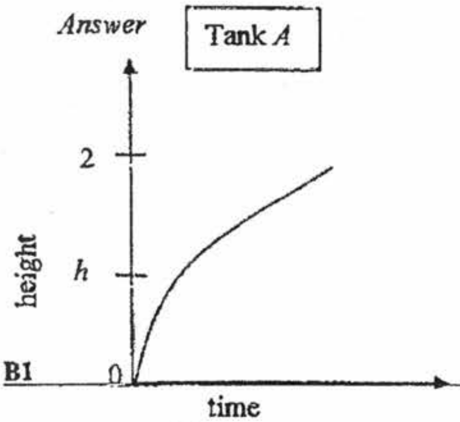
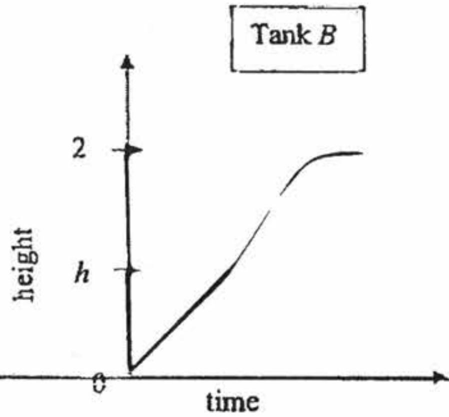
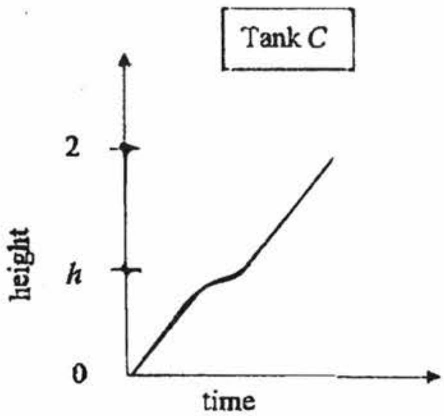
End of Paper

Qn	Solution		
	Section A		
1a	$\frac{x+1}{x^2-9}-\frac{2}{3-x}=\frac{x+1}{(x-3)(x+3)}+\frac{2}{x-3}$ $=\frac{x+1+2(x+3)}{(x-3)(x+3)}$ $=\frac{3x+7}{(x+3)(x-3)}$		
1b	$\frac{(abc^{-2})^3}{(a^{-4}b^{-1})^{-1}}\times\frac{a^{-6}b^{-7}}{(bc^2)^{-4}}=\frac{a^3b^3c^{-6}}{a^4b^1}\times\frac{a^{-6}b^{-7}}{b^{-4}c^{-8}}$ $=\frac{a^{-3}b^{-4}c^{-6}}{a^4b^{-3}c^{-8}}$ $=a^{-7}b^{-1}c^2$ $=\frac{c^2}{a^7b}$		
2	$\frac{k}{3}=\sqrt{\frac{A-3b^2}{cA}}$ $\frac{k^2}{9}=\frac{A-3b^2}{cA}$ $k^2cA=9A-27b^2$ $A(k^2c-9)=-27b^2$ $A=\frac{27b^2}{9-k^2c}$ OR $A=\frac{-27b^2}{(ck^2-9)}$		
3	(a) $9xy(2x+3-y^2)$ (b) $3(3a-2b)(3a+2b)$ (c) $(r-1)(3s-1)$		

Qn	Solution		
4	(a) 8 (b) -24 (c) 0		
5	Bus 801 Bus 802 3 | 15 25 5 | 5 25 5 | 1 5 1 1 LCM is 75 5 hours = 300 mins $\frac{300}{75}$ = 4 *times*		
6	(a) HCF of 150, 90, 180 is 30m (b)  6 + 6 + 4 + 4 + 7 = 27 lamp posts Double counting answer 27 – 5 = 22 lamp posts		
	SECTION B [18m]		

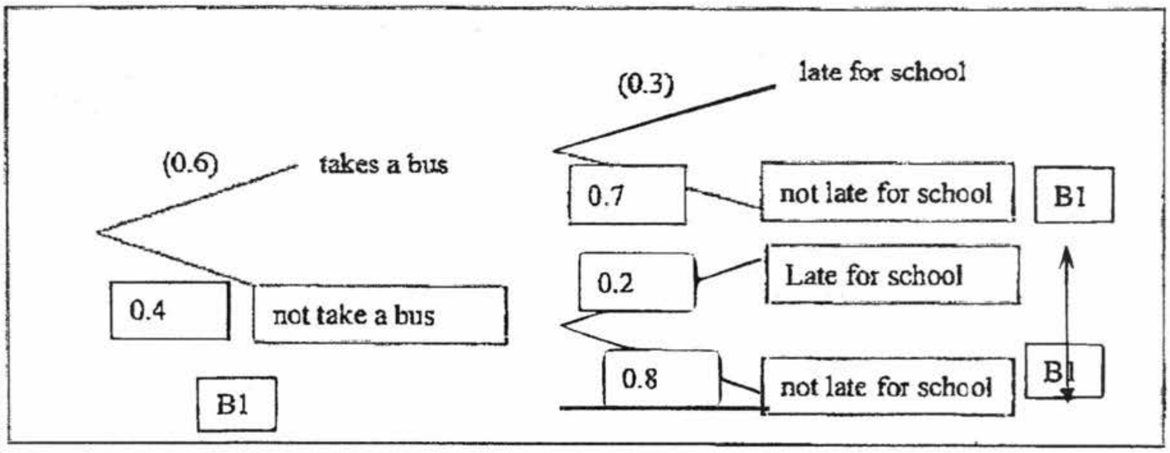
2017 4E/5N P1 E Mathematics Prelim Marking Scheme

Qn	Solution		
7	$A = 2^m \times 2^2 \times 3^n$ $B = 2^m \times 3^n \times 3 \times 5$ $\underline{LCM = 2^{m+2}} \times 3^{n+1} \times 5$		
8	$x = 2 - 2y$ $\frac{1}{4}(2-2y) = 11 + 3y$ $y = -3$ $x = 8$		
9	(a) $\frac{1}{2} \times 4 \times h = 16$ $h = 8$ *(b)* $\sin \angle ACD = \frac{8}{10} = \frac{4}{5}$		
	(c) $XC = \sqrt{10^2 - 8^2} = 6$ $\cos \angle ACE = -\frac{6}{10} = -\frac{3}{5}$		
10	(a) $35 + b$ (b) $5 + a$		
11	$1 \text{ cm}^2 : 64 \times 10^8 \text{ cm}^2$ Map 1 $3 \text{ cm}^2 : 192 \times 10^8 \text{ cm}^2$ Map 2 $12 \text{ cm}^2 : 192 \times 10^8 \text{ cm}^2$ $1 \text{ cm}^2 : 16 \times 10^8 \text{ cm}^2$ 1: 40000		
12	Amount = $\$36000\left(1 + \frac{\frac{3.2}{4}}{100}\right)^{24} = \$43586.83$		

Qn	Solution		
13	**Section C [40m]**  B1  B1  B1		
14	(a) Total angles in the 2 polygons $= 540^0 + 360^0$ $= 900^0$ sum of all required angles $= 7 \times 360^0 - 900^0$ $= 1620^0$		
	$\frac{360^0}{n} = \frac{n}{40}$ (b) $n^2 = 14400$ $n = 120$		
15	angle *RPQ* = angle *RPA* + angle *APQ* = angle RPA + angle ACQ (angles in the same segment) $= 35^0 + 30^0 = 65^0$ angle PQR = angle PQC + angle CQR = angle PAC + angle CBR = 25 + 35 = 60° angle PRQ = 25 + 30 = 55°		

Qn | **Solution**

16 a



16b $0.6 \times 0.7 + 0.4 \times 0.8 = 0.74$
M1 **A1**

(multiplication of probability from the tree)

17 (a)

Angle $OAE = 90^0$

$OA = AE = AC$

Area of shaded region $= \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 4^2 \times \pi/2$

$= 16 - 4\pi$

$x = 16$

$y = 4$

17 (b)

$OE = \sqrt{4^2 + 4^2} = \sqrt{32}$

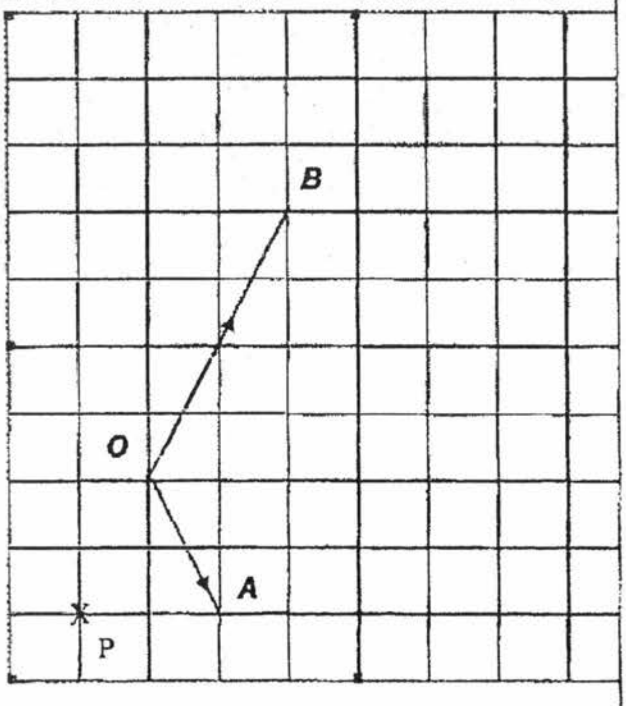
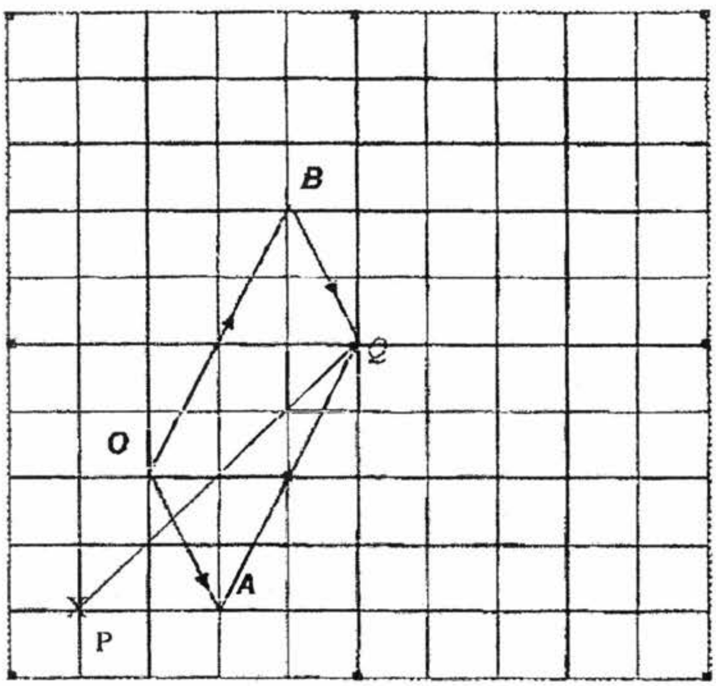
$4\left(\frac{\pi}{2}\right) + 8 + 2\left(\sqrt{32} - 4\right)$

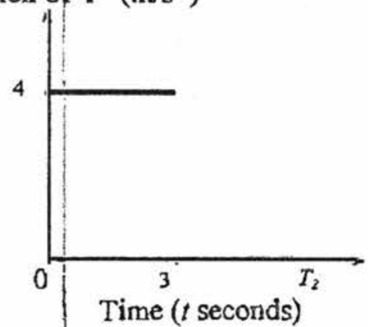
$= 2\pi + 8 + 2\sqrt{32} - 8$

$= 2\pi + 2\sqrt{32}$

$p = 2$

$q = 32$

Qn	Solution		
18 ai			A1
18 aii	$\overrightarrow{OP} = -\frac{1}{2}\overrightarrow{OB}$		A1
18 b			A1

	18bii $\overrightarrow{PQ}=\begin{pmatrix}4\\4\end{pmatrix}$ A1		
19	(a) 4s (b) $\frac{1}{2}\times T_2\times 3T_2=\frac{1}{2}\times 3\times 12+12(T_2-3)$ $T_2{}^2-8T_2+12=0$ $(T_2-6)(T_2-2)=0$ $T_2=6$ (c) Acceleration of P (m/s ^{2}) 		
20	(a) Sch $\underline{Y}$ achieved better results because it has a higher median of 68 as compared to 63 for X. (b) Sch $\underline{X}$ is more uniform because of a smaller interquartile range of 8 as compared to 14 for Y.		
21	a. p = 17 q = 222 b. $A=E^2+1$ c. $R=E^3+E$ d. 6400= 80^2, a perfect square number, but the number in column A are not perfect square numbers.		

Answer all the questions.

Section A [30 marks]

1 (a) Expand and simplify $(4x-1)^2-(8x+1)(2x-1)$. [2]

(b) Express $\frac{4x^2-9}{x^2+x-20}\div\frac{4x^2-6x}{16-x^2}$ as a fraction in its lowest term. [3]

(c) Solve the equation $\frac{x}{3}-\frac{2x-1}{x-3}=-2$, leaving your answer correct to 3 decimal places. [3]

(d) y is directly proportional to x^2.
It is known that $y = 144$ for a particular value of x. [3]
Find the percentage change in y when the value of x decreases by 25%.

---

2 During a school's sports day, the number of first, second and third positions won by the different houses are given in the table below.
The number of points won for individual and group events are also given in the table.

Houses	Individual events			Group events		
	First	Second	Third	First	Second	Third
Blue	7	5	4	3	2	0
Green	5	4	6	1	2	1
Red	4	5	5	1	2	2
Yellow	4	6	5	1	0	3
Points	5	3	1	10	6	2

(a) It is given that $\mathbf{A}=\begin{pmatrix}7&5&4\\5&4&6\\4&5&5\\4&6&5\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}5\\3\\1\end{pmatrix}$, evaluate the matrix $\mathbf{P}=\mathbf{AB}$. [2]

(b) Given matrix $\mathbf{C}=\begin{pmatrix}3&2&0\\1&2&1\\1&2&2\\1&0&3\end{pmatrix}$.

(i) Represent the group event scoring system in a 3×1 matrix $\mathbf{D}$. [1]

(ii) Evaluate the matrix $\mathbf{Q}=\mathbf{CD}$ and explain what do the elements of $\mathbf{Q}$ represent. [2]

(c) The scores of individual events and group events are added for each house. Using matrix manipulation, determine which house won the overall championship. [2]

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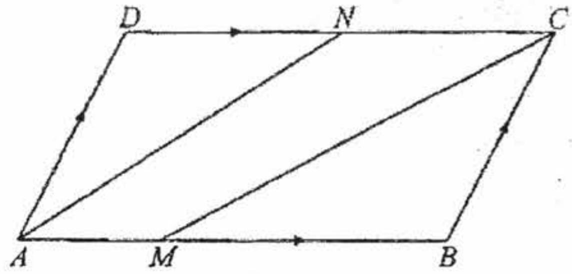
3 $ABCD$ is a parallelogram.
N is the midpoint of DC and M is the point on AB such that $2AM = MB$.



Given that $\overrightarrow{AB} = 6\mathbf{a}$ and $\overrightarrow{AD} = 4\mathbf{b}$,

(a) Express as simply as possible, in terms of **a** and/or **b**.

(i) $\overrightarrow{AM}$ [1]

(ii) $\overrightarrow{MC}$ [1]

(iii) $\overrightarrow{AN}$ [1]

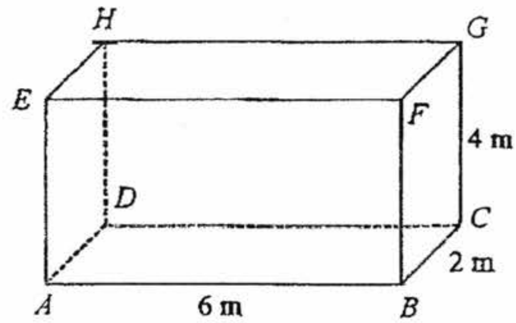
(b) Find the numerical value of

(i) $\dfrac{\text{area of triangle } ADN}{\text{area of parallelogram } ABCD}$, [1]

(ii) $\dfrac{\text{area of triangle } ADN}{\text{area of triangle } AMN}$. [2]

---

4 The diagram shows a rectangular cuboid $ABCDEFGH$.
$AB = 6$ m, $BC = 2$ m and $CG = 4$ m.



(a) Show that angle $HBD = 32.3°$, correct to 1 decimal place. [2]

(b) Calculate angle AFC. [3]

(c) Calculate the greatest angle of elevation of the point H when viewed from the line AB. [1]

Section B [70 marks]
Please begin Question 5 on a NEW sheet of paper

5 (a) Chloe has a total of 126 marks in x tests.
In the next two tests, she scored 9 marks and 8 marks respectively.

Find, in terms of x, her mean mark for the

(i) first x tests, [1]

(ii) $(x + 2)$ tests. [1]

Her mean mark for the first x tests was one greater than her mean mark for the $(x + 2)$ tests.

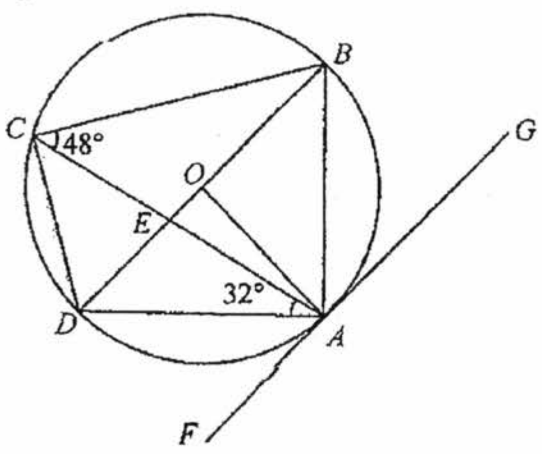
(iii) write an equation in x to represent this information and show that it reduces to $x^2 + 19x - 252 = 0$. [3]

(iv) Solve the equation to find the number of tests Chloe took initially. [3]

(b) Amanda has a mean of 13.5 marks for the first $(x + 1)$ tests, but her mark on the last test gave her a mean of 14 marks for the $(x + 2)$ tests.

Calculate the number of marks Amanda scored in the last test. [2]

6 In the diagram, O is the centre of the circle through A, B, C and D.
FG is the tangent to the circle at A.
AC intersects BD at E.
Angle $ACB = 48°$ and angle $CAD = 32°$.



(a) Calculate the following angles, stating your reasons clearly.

(i) Angle ABO [2]

(ii) Angle CDA [2]

(iii) Angle GAB [2]

(b) Explain why BD is not parallel to GF. [2]

7 (a) The frequency table shows the weekly expenditure on food of n students from School X.

Weekly expenditure (\$$x$)	Frequency
$30 < x \le 40$	8
$40 < x \le 50$	17
$50 < x \le 60$	34
$60 < x \le 70$	p
$70 < x \le 80$	3

(i) If $\frac{5}{16}$ of the n students have a weekly expenditure of at most \$50, show that the value of p is 18. [2]

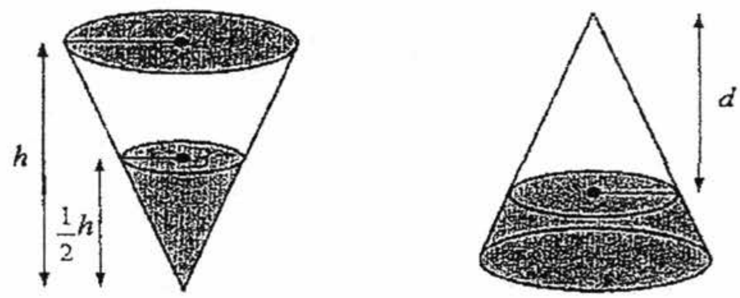
(ii) Calculate an estimate of

(a) the mean weekly expenditure on food, [1]

(b) the standard deviation. [1]

(iii) The standard deviation of the weekly expenditure on food of students from School Y was \$5.62.
Using this information, comment on one difference between the two distributions. [1]

(b) The diagram shows an inverted cone of height h and radius r.
It contains water to a depth of $\frac{1}{2}h$.

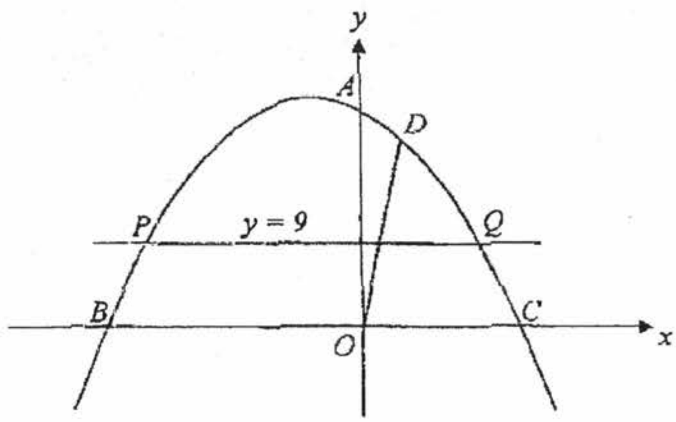


(i) Find the ratio of area of surface B to area of surface A. [1]

(ii) Find the volume of the water if the cone can hold 480 cm^3 of water when full. [2]

(iii) The cone is now inverted such that the liquid rests on the flat circular base of the cone, as shown in the diagram on the right.
Find, in terms of h, an expression for d, the vertical distance of the liquid surface from the tip of the cone. [3]

8 The diagram shows the curve $y = (4 - x)(x + k)$, where k is a constant.
The curve cuts the y-axis at the point $A(0, 24)$, and the x-axis at B and C.



(a) Show that the value of k is 6. [1]

(b) Write down the coordinates of B and C. [2]

(c) Find the coordinates of the maximum point on the curve. [2]

(d) $D(1, m)$ is a point on the given curve.
Find the value of m and the equation of the line OD. [3]

(e) The line $y = 9$ intersects the curve at P and Q. Find the coordinates of P and Q. [3]

9 A student needed to make a circular face mask for a school performing arts event. She took a circular sheet of radius 10 cm and removed two circles, each of radius 2.5 cm for two eyes and an isosceles triangle of base 2 cm and equal sides of 3 cm each for a nose, as shown in **Diagram I**.

The mouth is shown in the **Diagram II**.
It is formed by an arc, *AXB*, of a circle, centre *O* and radius 3 cm.
AYB is the arc of another circle with diameter, *AB*, 3 cm.

She painted the remaining area.

Diagram I

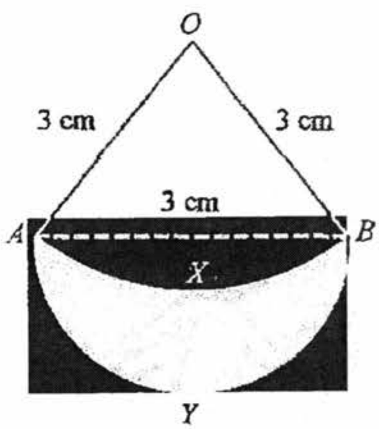


Diagram II

(a) Calculate the area removed. [7]

(b) Calculate the area of mask that was painted. [2]

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 5 - \frac{x^2}{10} - \frac{4}{x}$. Some corresponding values are given in the following table.

x	0.5	0.7	1	2	3	4	5	6	7	8
y	−3.0	−0.8	0.9	2.6	2.8	k	1.7	0.7	−0.5	−1.9

(a) Calculate the value of k. [1]

(b) Taking 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 8$ and a vertical y-axis for $-3 \le y \le 3$, draw the graph of $y = 5 - \frac{x^2}{10} - \frac{4}{x}$ for the values of x in the range $0.5 \le x \le 8$. [3]

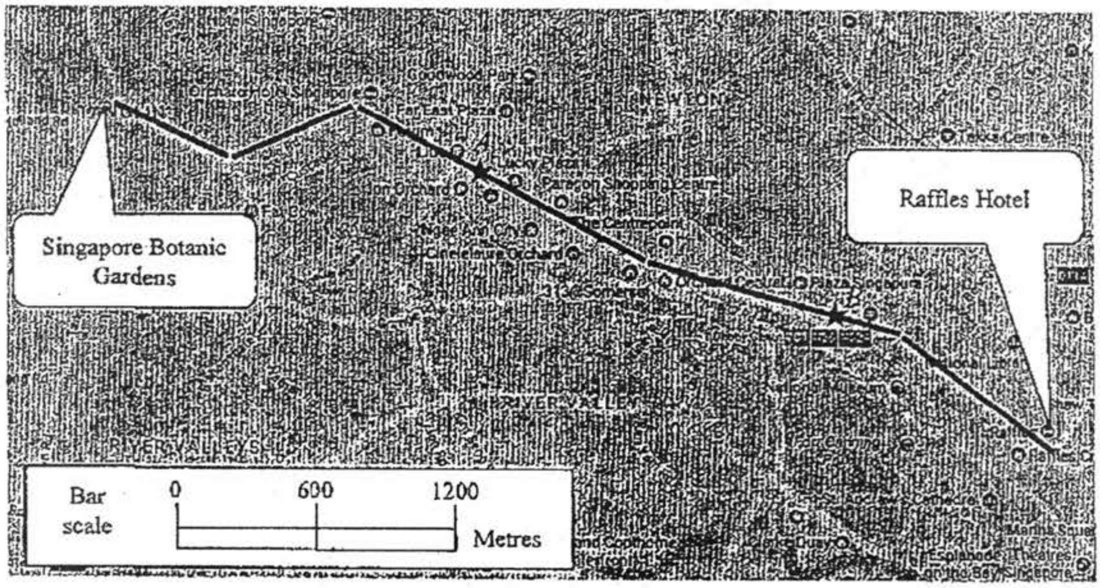
(c) Use your graph to find the greatest value of $5 - \frac{x^2}{10} - \frac{4}{x}$ in the interval $0.5 \le x \le 8$. [1]

(d) By drawing a tangent, find the gradient of the graph at the point where $x = 2$. [2]

(e) Use your graph to solve $5 - \frac{x^2}{10} - \frac{4}{x} = 2$ in the range $0.5 \le x \le 8$. [3]

(f) By drawing a suitable straight line, find the range of values of x in the interval $0.5 \le x \le 8$ for which $5 - \frac{x^2}{10} - \frac{4}{x} \ge x$. [2]

11 Cheryl works at the Singapore Botanic Gardens.
She needs to rush down to meet a client at Raffles Hotel.
The quickest route from Cheryl's location to Raffles Hotel is indicated on the map with black solid lines.
The bar scale on the lower left corner of the map provides the corresponding actual ground distance.



(a) Calculate the actual travelling distance, in kilometres, between Cheryl's location and Raffles Hotel, giving your answer correct to 2 significant figures. [2]

(b) At 6.14 pm, Cheryl decided to call for a ride from Singapore Botanic Gardens to Raffles Hotel.

Information about FastDel Cab and Aber services and other travelling details are on the opposite page.
Along the way, there are two ERP gantries, indicated by A and B with a star each on the map.

Determine which service Cheryl should choose.
Justify your answer with relevant working. [7]

Travelling time

From	To	Duration
Singapore Botanic Gardens	Orchard ERP (*A*)	6 minutes
Orchard ERP	Handy Road ERP (*B*)	5 minutes
Handy Road ERP	Raffles Hotel	4 minutes

ERP Charges

Orchard (*A*)		**Handy Road Gantry (*B*)**	
12.00 pm – 5.29 pm	$0.50	12.00 pm – 12.04 pm	$0.50
5.30 pm – 5.34 pm	$1.00	12.05 pm – 1.59 pm	$1.00
5.35 pm – 5.59 pm	$1.50	2.00 pm – 2.04 pm	$1.50
6.00 pm – 6.54 pm	$2.00	2.05 pm – 2.54 pm	$2.00
6.55 pm – 6.59 pm	$1.50	2.55 pm – 2.59 pm	$1.50
7.00 pm – 7.59 pm	$1.00	3.00 pm – 5.29 pm	$1.00
		5.30 pm – 5.59 pm	$0.50
		6.00 pm – 7.54 pm	$1.00
		7.55 pm – 7.59 pm	$0.50

FastDel Cab Service

The first 1 km or less	$3.20
Every 400 m thereafter or less up to 10 km	$0.22
Every 350 m thereafter or less after 10 km	$0.22

Current Booking
Peak Period ($3.30)
Monday to Friday (Except Public Holidays): 6.00 am – 9.29 am
Monday to Sunday & Public Holidays: 6.00 pm – 11.59 pm

Peak Period Surcharge (25% of metered fare)
Monday to Friday (Except Public Holidays): 6.00 am – 9.29 am
Monday to Sunday & Public Holidays: 6.00 pm – 11.59 pm

ERP Charge
Passengers are required to bear the ERP charge shown on the upper display of the In-vehicle Unit. The ERP charge is deducted each time the taxi passes under the ERP gantry, payable on top of metered fare

Aber Service

Base Fare	$3.00
Travelling time per minute	$0.20
Travelling distance per km	$0.45
6 pm to 8 pm peak period surge	2.5× of normal fare

End of Paper

4E5N Mathematics Preliminary Exam 2017 (Paper 2)

Section A

1(a)	$(4x-1)^2-(8x+1)(2x-1)$ $=16x^2-8x+1-(16x^2-6x-1)$ $=16x^2-8x+1-16x^2+6x+1$ $=-2x+2$	
1(b)	$\frac{(4x^2-9)}{(x^2+x-20)}\div\frac{(4x^2-6x)}{(16-x^2)}$ $=\frac{(2x-3)(2x+3)}{(x+5)(x-4)}\div\frac{2x(2x-3)}{-(x-4)(x+4)}$ $=\frac{(2x-3)(2x+3)}{(x+5)(x-4)}\times\frac{-(x-4)(x+4)}{2x(2x-3)}$ $=\frac{-(2x+3)(x+4)}{2x(x+5)}$	
1(c)	$\frac{x}{3}-\frac{2x-1}{x-3}=-2$ $\frac{x(x-3)-3(2x-1)}{3(x-3)}=-2$ $x^2-3x-6x+3=-6(x-3)$ $x^2-9x+3=-6x+18$ $x^2-3x-15=0$ $x=\frac{-(3)\pm\sqrt{(-3)^2-4(1)(-15)}}{2(1)}$ $=5.653$ or -2.653	
1(d)	$y=kx^2$ $144=kx^2$ Original value: x New value: $0.75x$ $Y=kX^2$ $Y=k(0.75x)^2$ $=0.5625kx^2$ $=0.5625(144)$ $=81$ Percentage change $=\frac{81-144}{144}\times100$ $=-43.75\%$	

2(a)	$P = \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ 4 & 5 & 5 \\ 4 & 6 & 5 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 54 \\ 43 \\ 40 \\ 43 \end{pmatrix}$	
2(bi)	$D = \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$	
2(bii)	$Q = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix}\begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 42 \\ 24 \\ 26 \\ 16 \end{pmatrix}$ The elements of **Q** represent the **total score from group events for each house respectively**.	
2(c)	Total score $= \begin{pmatrix} 54 \\ 43 \\ 40 \\ 43 \end{pmatrix} + \begin{pmatrix} 42 \\ 24 \\ 26 \\ 16 \end{pmatrix}$ $= \begin{pmatrix} 96 \\ 67 \\ 66 \\ 59 \end{pmatrix}$ **Blue** house won overall championship.	

3(ai)	$2AM = MB$ $\frac{AM}{MB} = \frac{1}{2}$ $\overline{AM} = \frac{1}{3}\overline{AB}$ $= \frac{1}{3}(6\mathbf{a})$ $= 2\mathbf{a}$	
3(aii)	$\overline{MC} = \overline{MB} + \overline{BC}$ $= \frac{2}{3}(6\mathbf{a}) + 4\mathbf{b}$ $= 4\mathbf{a} + 4\mathbf{b}$	
3(aiii)	$\overline{DN} = \frac{1}{2}\overline{DC}$ $= 3\mathbf{a}$ $\overline{AN} = \overline{AD} + \overline{DN}$ $= 3\mathbf{a} + 4\mathbf{b}$	
3(bi)	$\frac{\text{area of triangle } ADN}{\text{area o parallelogram } ABCD} = \frac{\frac{1}{2}(h)(DN)}{(h)(DC)}$ $= \frac{\frac{1}{2}(DN)}{(DC)}$ $= \frac{1}{2} \times \frac{1}{2}$ $= \frac{1}{4}$	
3(bii)	$\frac{\text{area of triangle } ADN}{\text{area of triangle } AMN} = \frac{DN}{AM}$ $= \frac{\frac{1}{2}(DC)}{\frac{1}{3}(DC)}$ $= \frac{3}{2}$	

4(a)	$DB^2 = 6^2 + 2^2$ $= 40$ $DB = \sqrt{40}$ $= 6.3245$ $\tan \angle HBD = \frac{4}{\sqrt{40}}$ $\angle HBD = \tan^{-1}\left(\frac{4}{\sqrt{40}}\right)$ $= 32.311°$ $= 32.3°$ (1 d.p.)	
4(b)	$AF^2 = 6^2 + 4^2$ $\quad FC^2 = 2^2 + 4^2$ $= 52$ $\quad = 20$ $AF = \sqrt{52}$ $\quad FC = \sqrt{20}$ $= 7.2111$ $\quad = 4.4721$ $AC = DB$ $= \sqrt{40}$ $= 6.3245$ $AC^2 = AF^2 + FC^2 - 2(AF)(FC)\cos\angle AFC$ $\cos\angle AFC = \frac{AF^2 + FC^2 - AC^2}{2(AF)(FC)}$ $= \frac{52 + 20 - 40}{2(\sqrt{52})(\sqrt{20})}$ $\angle AFC = \cos^{-1}\left(\frac{32}{2(\sqrt{52})(\sqrt{20})}\right)$ $= 60.255°$ $= 60.3°$ (1 d.p.)	
4(c)	$\tan \angle HAD = \frac{4}{2}$ $\angle HAD = \tan^{-1}(2)$ $= 63.434°$ $= 63.4°$ (1 d.p.) $\therefore$ greatest angle of elevation is 63.4°	

Section B

5(a)	Mean mark for first x tests = $\frac{126}{x}$	
5(b)	Mean mark for first (x+2) tests = $\frac{126+9+8}{x+2}$ $= \frac{143}{x+2}$	
5(c)	$\frac{126}{x}-\frac{143}{x+2}=1$ $\frac{126(x+2)-143x}{x(x+2)}=1$ $126x+252-143x=x^2+2x$ $252-17x=x^2+2x$ $x^2+19x-252=0$ (shown)	
5(d)	$x^2+19x-252=0$ $(x-9)(x+28)=0$ $x=9$ or -28 (reject) ∴ Chloe took 9 tests initially.	
5(e)	Number of marks Amanda scored in the last test $= 14(x+2)-13.5(x+1)$ $= 14(11)-13.5(10)$ $= 19$	
6(ai)	$\angle BDA = 48°$ (angles in the same segment) $\angle ABO = 90° - 48°$ (right angle triangle in semicircle) $= 42°$ OR $\angle DCE = 90° - 48°$ (right angle triangle in semicircle) $= 42°$ $\angle ABO = 42°$ (angles in the same segment) OR $\angle AOB = 48° \times 2$ $= 96°$ (angle at centre is twice angle at circumference) [illegible]2° (isosceles triangle AOB)	

6(aii)	$\angle DCE = 42^\circ$ (angles in the same segment) $\angle CDA = 180^\circ - 42^\circ - 32^\circ$ (sum of angles in triangle) $= 106^\circ$ OR $\angle CBD = 32^\circ$ (angles in the same segment) (angles in opposite segement are supplementary) $\angle CDA = 180^\circ - 32^\circ - 42^\circ$ $= 106^\circ$	
6(aiii)	$\angle OAB = 42^\circ$ (base angles of isosceles triangle) $\angle OAG = 90^\circ$ (tangent perpendicular to radius) $\angle GAB = 90^\circ - 42^\circ$ $= 48^\circ$ OR $\angle GAB = 48^\circ$ (alternate segment theorem)	
6(b)	Since $\angle OBA \neq \angle GAB$, it does not satisfy the property of alternate angles with a set of parallel line. Hence, BD is not parallel to GF OR If *BD* is parallel to *GF*, $\angle OBA = \angle GAB$, based on alternate angles. Since $\angle OBA \neq \angle GAB$, *BD* is not parallel to *GF*.	
7(ai)	$\frac{5}{16}$ -------- $8 + 17 = 25$ students $\therefore\ 8+17+34+p+3 = \frac{25}{5} \times 16$ $62 + p = 80$ $p = 18$ (shown)	
7(aiia)	Mean $= \frac{\sum fx}{\sum f}$ $= \$53.875$ $= \$53.88$ (2 d.p.)	
7(aiib)	Standard deviation $= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= 9.8734$ $= 9.87$ (3 s.f.)	

7(aiii)	The weekly expenditure on food for School X has a **wider spread** (**less consistent**) than that for School Y as the standard deviation for School X is greater than that of School Y.	
7(bi)	$\dfrac{\text{area of surface B}}{\text{area of surface A}} = \left(\dfrac{\frac{1}{2}h}{h}\right)^2$ $= \dfrac{1}{4}$	
7(bii)	$\dfrac{\text{Volume of water}}{\text{Volume of full cone}} = \left(\dfrac{1}{2}\right)^3$ $\dfrac{\text{Volume of water}}{480} = \dfrac{1}{8}$ $\text{Volume of water} = \dfrac{1}{8} \times 480$ $= 60 \text{ cm}^3$	
7(biii)	$\text{Remainder volume} = 480 - 60 = 420 \text{ cm}^3$ $\dfrac{\text{Volume of empty part}}{\text{Volume of full cone}} = \left(\dfrac{d}{h}\right)^3$ $\dfrac{420}{480} = \left(\dfrac{d}{h}\right)^3$ $\dfrac{d}{h} = \sqrt[3]{\dfrac{7}{8}}$ $d = 0.95647h$ $= 0.956h$ (3 s.f.)	
8(a)	At $A(0, 24)$, $24 = (4-0)(0+k)$ $24 = 4k$ $k = 6$	
8(b)	$B(-6, 0)$ $C(4, 0)$	
8(c)	Line of symmetry: $x = \dfrac{-6+4}{2} = -1$ At $x = -1$, $-1+6)$ $\therefore$ Coordinate of maximum point $= (-1, 25)$	

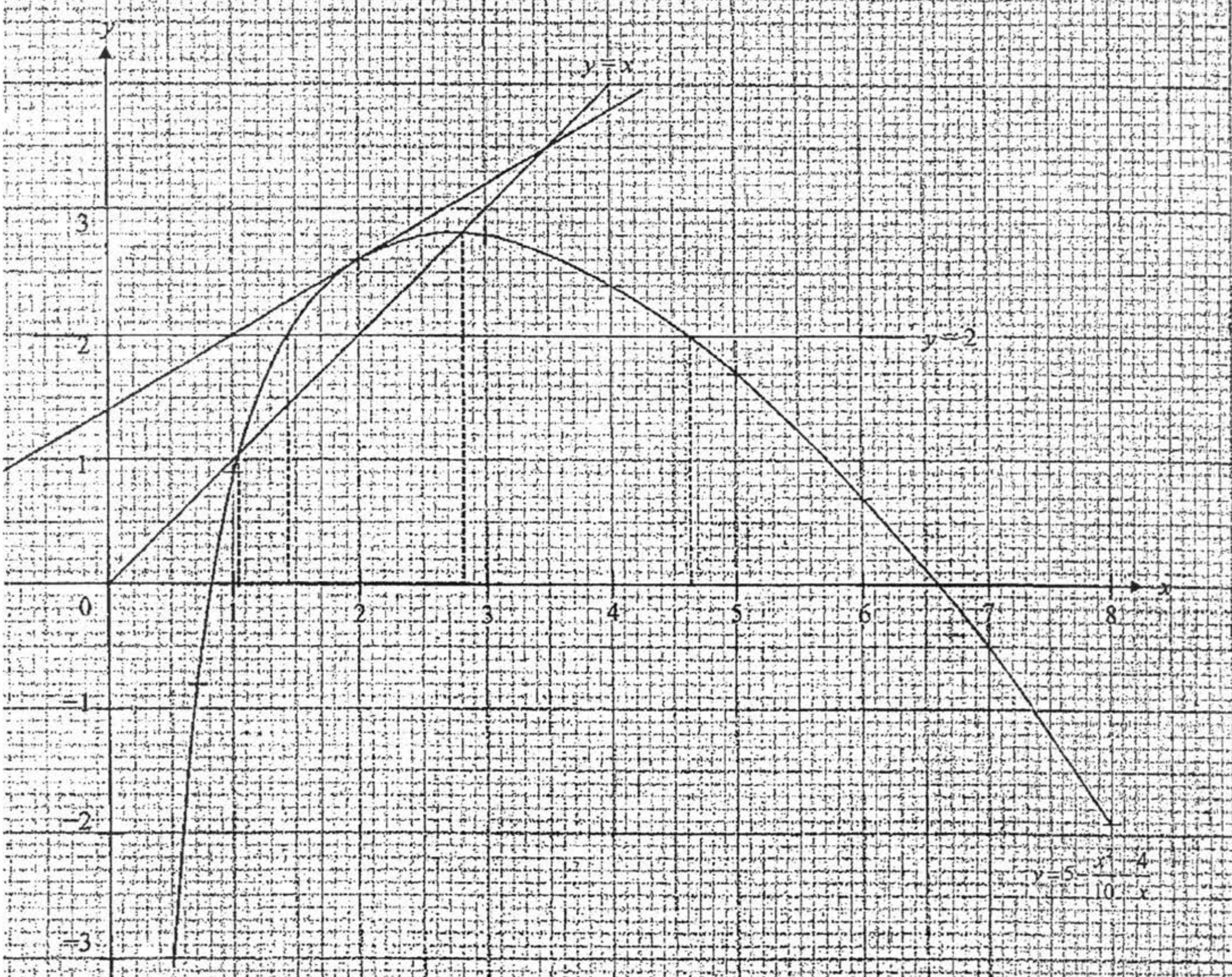
8(d)	At $x = 1$, $m = (4-1)(1+6)$ $= 21$ $\text{gradient} = \frac{21}{1}$ $= 21$ $\therefore$ Equation of line: $y = 21x$	
8(e)	Sub. y = 9 into equation of graph, $9 = (4-x)(x+6)$ $9 = -x^2 - 2x + 24$ $x^2 + 2x - 15 = 0$ $(x-3)(x+5) = 0$ $x = 3 \quad \text{or} \quad -5$ P(−5, 9) Q(3, 9)	
9(a)	Area of eyes $= 2 \times \pi r^2$ $= 2 \times (2.5)^2 \pi$ $= 12.5\pi \quad \text{cm}^2$ For isosceles triangle, $\cos\alpha = \frac{3^2 + 3^2 - 2^2}{2(3)(3)}$ $= \frac{14}{18}$ $\alpha = \cos^{-1}\left(\frac{14}{18}\right)$ $= 38.942°$ Area of nose $= \frac{1}{2}(3)(3)\sin 38.942°$ $= 2.8284 \quad \text{cm}^2$ OR $h = \sqrt{3^2 - 1^2} = \sqrt{8}$ ıgle $= \frac{1}{2} \times 2 \times \sqrt{8}$ $= 2.8284 \text{ cm}^2$	

	For mouth, $\beta = 60°$ Area of semicircle $= \frac{1}{2}\pi(1.5)^2$ $= \frac{9}{8}\pi \quad cm^2$ Area of sector $= \frac{60}{360}\pi(3)^2$ $= \frac{3}{2}\pi \quad cm^2$ Area of triangle $= \frac{1}{2}(3)(3)\sin 60°$ $= 3.89711 \quad cm^2$ OR $h = \sqrt{3^2 - 1.5^2} = \sqrt{\frac{27}{4}}$ Area of triangle $= \frac{1}{2} \times 3 \times \sqrt{\frac{27}{4}}$ $= 3.89711 \ cm^2$ Area of mouth $= \frac{9}{8}\pi - \left(\frac{3}{2}\pi - 3.89711\right)$ $= 2.71901 \quad cm^2$ Total area removed $= 12.5\pi + 2.8284 + 2.71901$ $= 44.8173$ $= 44.8 \ cm^2 \quad (3\text{ s.f.})$	
9(b)	Area of whole mask $= \pi r^2$ $= 100\pi \quad cm^2$ Area of mask painted $= 100\pi - 44.8173$ $= 269.341$ $= 269 \quad cm^2 \quad (3\text{ s.f.})$	

11(a)	Total distance on map $= 1.8 + 1.9 + 4.7 + 3.8 + 2.8$ $= 15$ cm Actual distance $= \frac{15}{2} \times 600$ $= 4500$ m $= 4.5$ km	
11(b)	**FastDel service** Base fare = \$3.20 400m thereafter or less: $\frac{3500\text{ m}}{400\text{ m}} = 8.75 \approx 9$ Normal fare = \$3.20 + 9 × \$0.22 = \$5.18] **A** Normal fare + peak surcharge = \$5.18 × 1.25 = \$6.475 **B** Total metered fare = \$6.475 + booking + ERP = \$6.475 + \$3.30 + \$3.00 = \$12.775 = \$12.78 (2 d.p.) **Aber service** Base fare = \$3.00 Travelling time fare = \$0.20 × 15 = \$3.00 Distance fare = \$0.45 × 4.5 = \$2.025 Normal fare = \$3 + \$3 + \$2.025 = \$8.025 Total fare = \$8.025 × 2.5 = \$20.0625 = \$20.06 (2 d.p.) Cheryl should choose **FastDel** service .	

Name ________________ Index No. ________

Subject ________________ Class ________ Date ________



10(a) k = 2.4 [B1]

10(b)
Axes [B1]
Plotting [B1]
Graph [B1]

10(c)
greatest value
(2.70,

10(d)
Tangent [B1]
Gradient = 0.6 [B1] (0.4 – 0.8)

10(e)
Line $y = 2$ [B1]
$x = 1.45$ or 4.65 [B1, B1] (± 0.1)

10(f)
Line $y = x$ [B1]
$1.05 \le x \le 2.8$ [B1] (± 0.1)

) cm x 24 cm