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Name _____ ()

Class: _____



**CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS /
5 NORMAL (ACADEMIC)**

**MATHEMATICS
PAPER 1**

**4048/01
2 hours**

Classes: 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, hand in **separately**:

1. Section A with exam cover sheet
2. Section B

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
Total marks	/80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2}$$

Name _____ ()

Class: _____

Answer **all** the questions.

Section A

1 Calculate $\frac{11.27^{\frac{1}{4}}}{30.67 - 5.23}$.

Write your answer correct to 3 significant figures.

Answer [1]

2 Without using a calculator, show that $7^{103} - 7^{101}$ is a multiple of 2.

Answer

[1]

3 The following stem-and-leaf diagram shows the masses of 10 parcels that arrive at the post office.

0	1	2	6
1	3	7	
2	2	5	
3	1	4	9

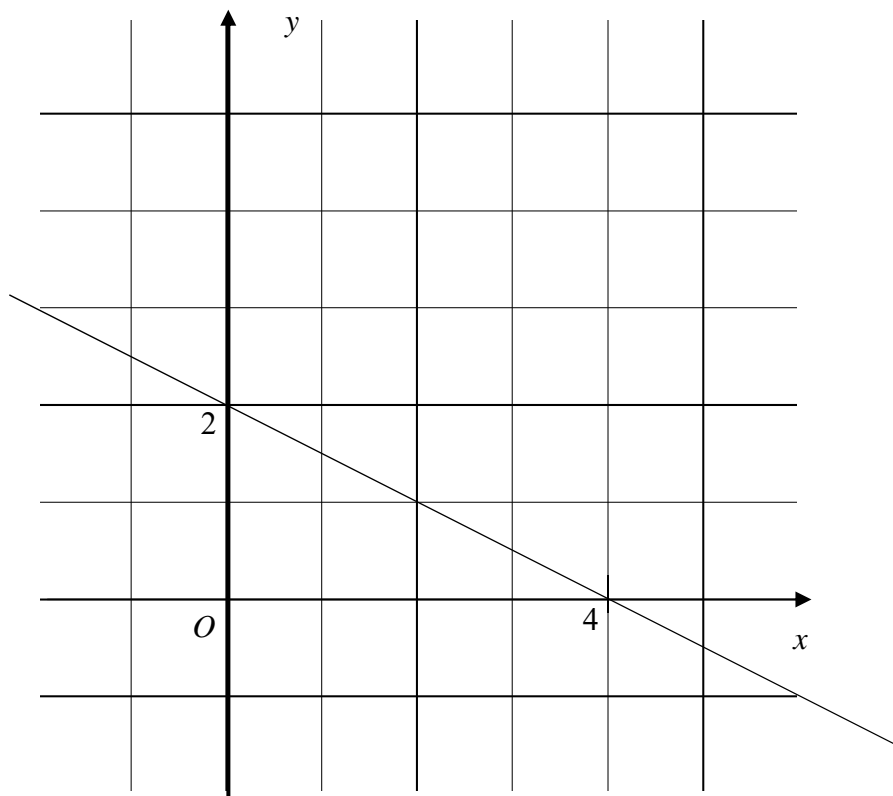
Key: 1 | 3 represents 13 kg

A parcel is chosen at random.

Find, as a fraction in its simplest form, the probability that the parcel has a mass between 12 kg and 32 kg.

Answer [1]

4 The diagram shows the line $y = -\frac{1}{2}x + 2$.



The line $y = -\frac{1}{2}x + 2$ undergoes a translation represented by the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Draw the line after translation, on the diagram above.

[1]

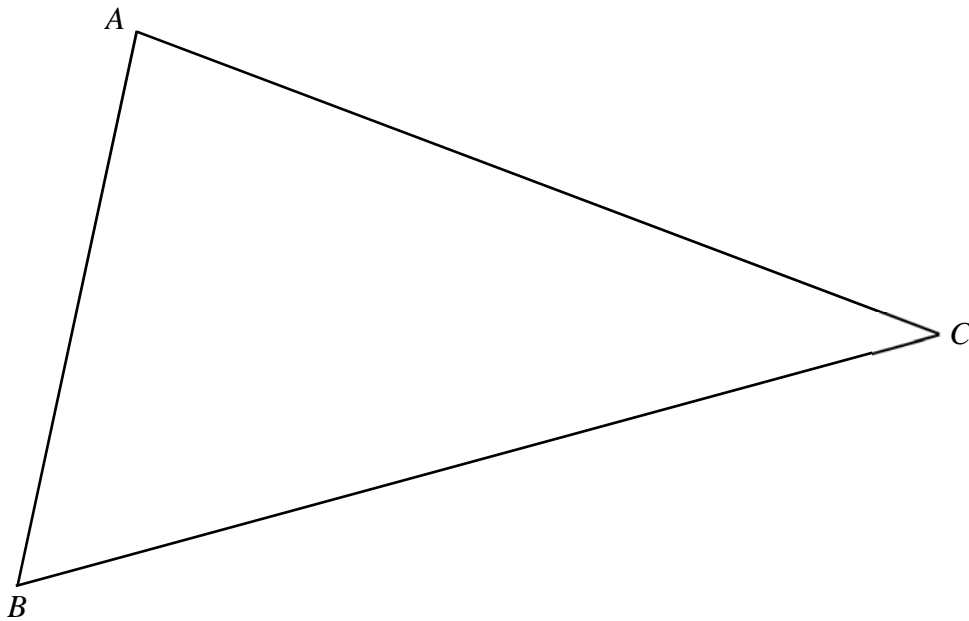
5 Use prime factorisation to explain why 72 is **not** a perfect cube.

Answer

..... [2]

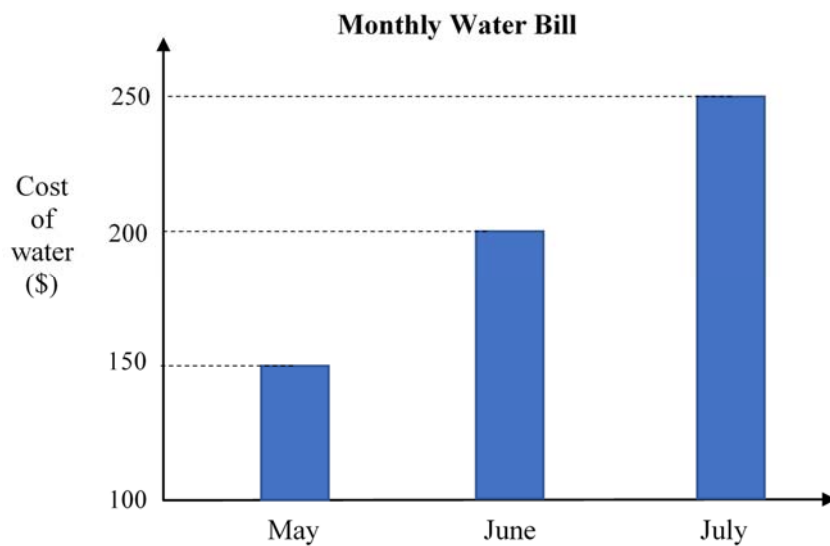
6 The diagram shows a triangle ABC .

Label the point O that is equidistant from B and C , and also equidistant from AB and BC .



[2]

7 Wally draws this graph to show his monthly water bill for each of the last three months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

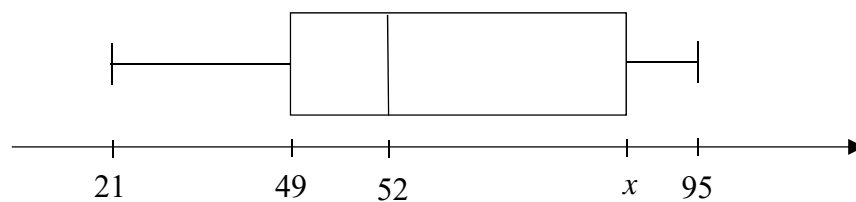
.....

..... [2]

8 The table shows the scores of 10 students in a Mathematics test.

Test score	Frequency
21	2
49	3
55	1
65	1
80	1
95	2

The test scores are also represented in the box-and-whisker plot below.



(a) Find the value of x .

Answer $x = \dots\dots\dots$ [1]

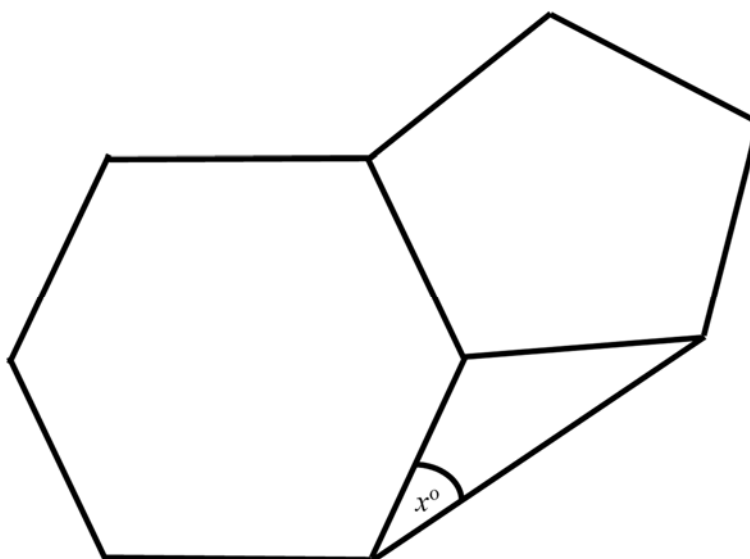
(b) Calculate the standard deviation of the test scores.

Answer $\dots\dots\dots$ [1]

9 Solve $3x^2 - 2x - 11 = 0$.

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

10 The diagram shows a regular hexagon and a regular pentagon.



Find x .

Answer $x = \dots\dots\dots$ [3]

11 (a) Simplify $2(3x + 5) - 2(1 - 2x)$.

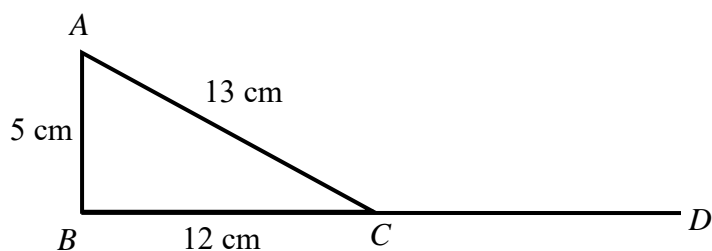
Answer [1]

(b) Factorise completely $18 - 24x + 8x^2$.

Answer [2]

12 ABC is a triangle.

$AB = 5$ cm, $BC = 12$ cm and $AC = 13$ cm.



(a) Show that ABC is a right-angled triangle.

Answer

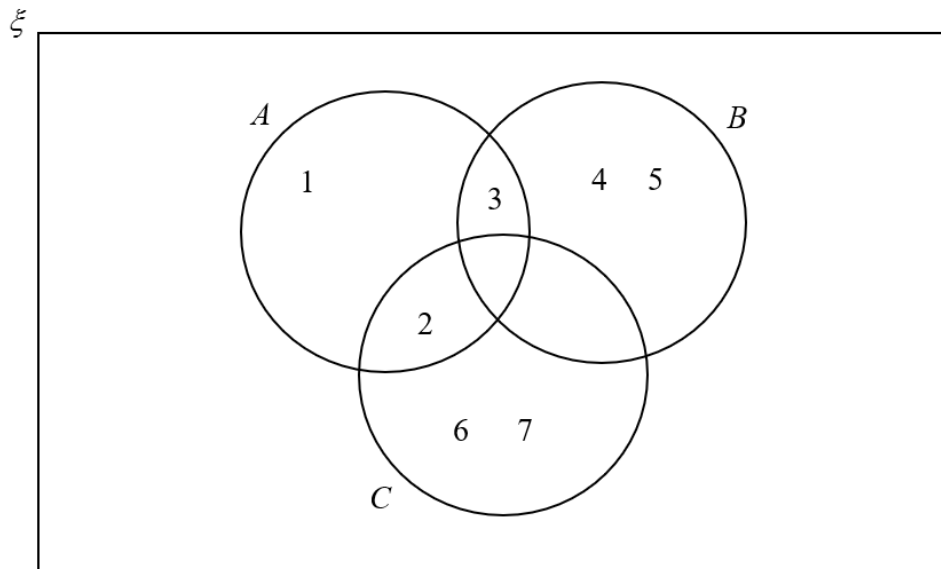
[2]

(b) Find $\sin \widehat{ACD}$.

Answer [1]

13 $\xi = \{\text{integers } x : 1 \leq x \leq 7\}$

The Venn diagram shows the elements of ξ and three sets A , B and C .



Use one of the symbols below to complete each statement.

$\emptyset \quad \subset \quad \not\subset \quad \notin \quad \in \quad \xi$

- (a) $\{4,5\}$ B [1]
- (b) 2 $A \cup B$ [1]
- (c) $B \cap C =$ [1]

14 The body mass index (BMI) of a person is defined as

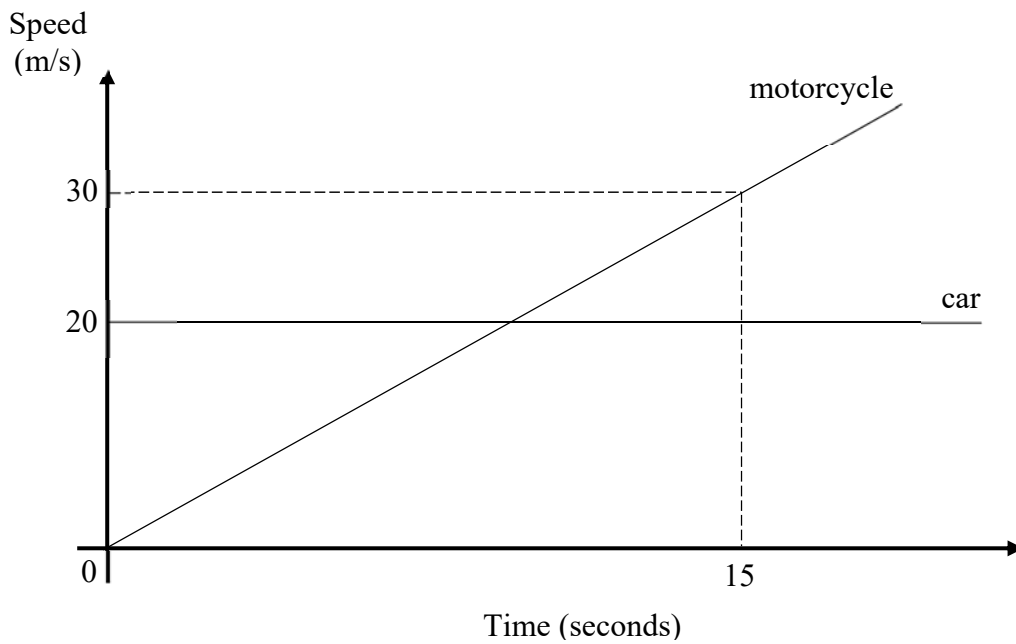
$$\text{BMI} = \frac{\text{mass}}{\text{height}^2}.$$

Sam's mass was 62 kg. One year later, Sam's mass increased by 10%, while his BMI increased by 2.01 kg/m². Sam's height remained the same.

Find Sam's height.

Answer m [3]

- 15 The diagram shows the speed–time graphs for a car and a motorcycle travelling along a straight road.



- (a) Calculate the acceleration of the motorcycle.

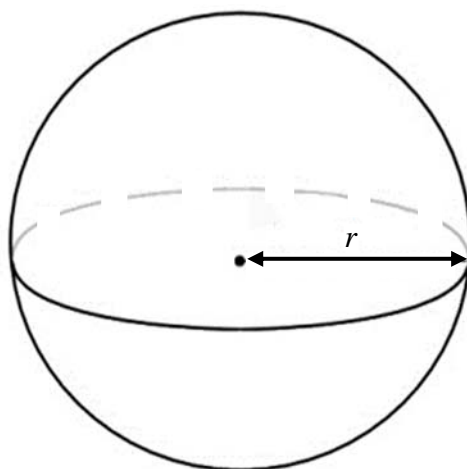
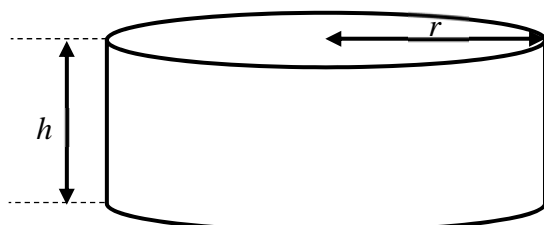
Answer m/s² [1]

- (b) Both the motorcycle and the car were beside each other at the start.
At t seconds, the motorcycle overtook the car.

Find the value of t .

Answer $t =$ [2]

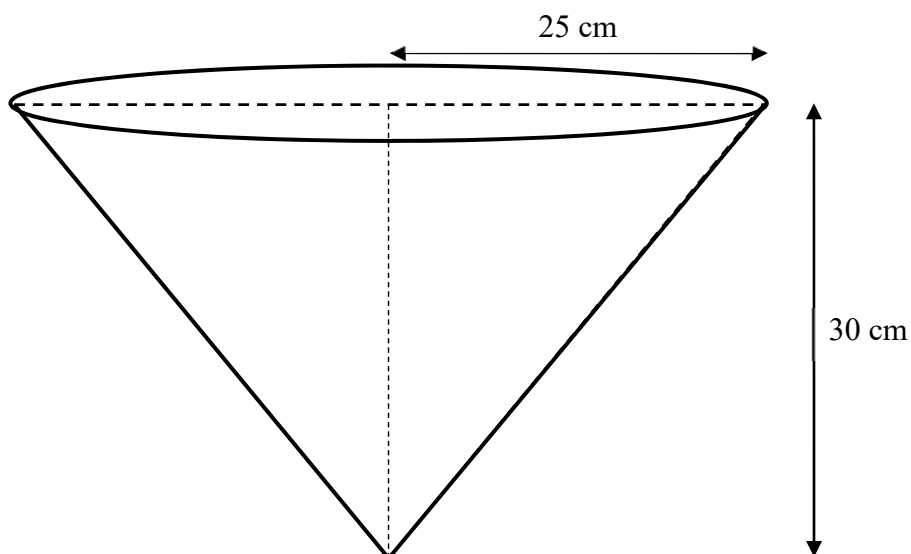
- 16** A solid cylinder has radius r cm and height h cm.
 A solid sphere has radius r cm.
 The total surface areas of the solid cylinder and the sphere are equal.



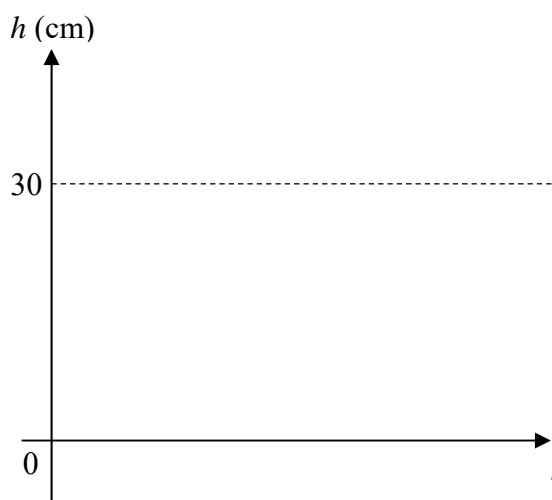
Work out, in terms of r , the total volume of the cylinder.

Answer [3]

17 Water is being poured into the cone below at a constant rate. The cone is initially empty.



(a) Sketch a graph on the axes below to show how the height, h of the water level in the cone increases with time, t .



[1]

(b) Calculate the volume of water in the cone when $h = 10$ cm.

Answer cm^3 [3]

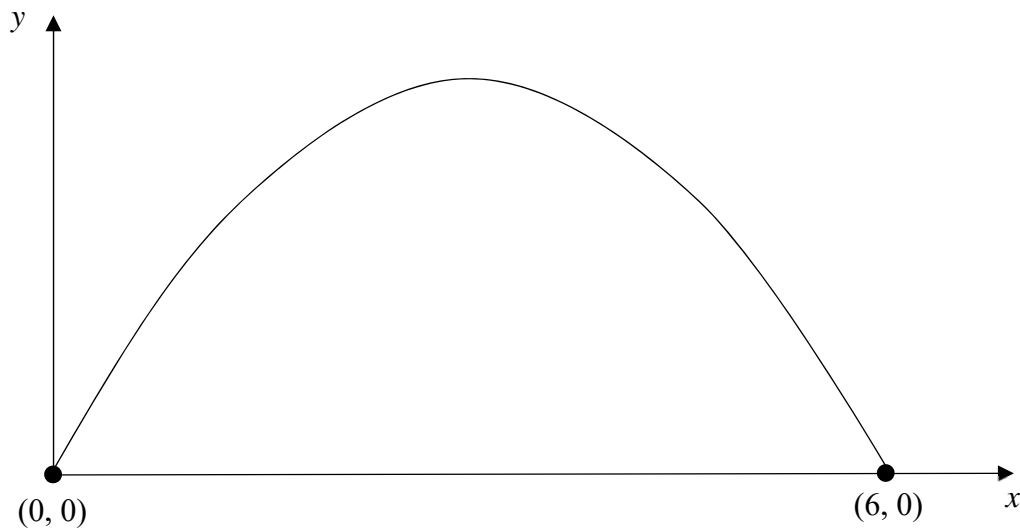
Name _____ ()

Class: _____

Answer **all** the questions.

Section B

- 18** The sketch shows the graph of $y = -x^2 + ax + b$.
The points $(0, 0)$ and $(6, 0)$ lie on the graph.



- (a)** Show that $a = 6$ and $b = 0$.

Answer

[2]

- (b)** Find the coordinates of the maximum point of the graph.

Answer (..... ,) [2]

19 A map of Singapore has a scale of 1: 80 000.

(a) The actual length of the Singapore River is 3.2 km.

Calculate the length, in centimetres, of the river on the map.

Answer cm [2]

(b) The actual area of the Bishan-Ang Mo Kio Park is 620 000 m².

Calculate the area, in square centimetres, of the park on the map.

Answer cm² [2]

20 The point L is (1, 2) and the point M is (16, 16).

(a) Find $|\overline{LM}|$.

Answer $|\overline{LM}| =$ units [2]

20 (b) The point N is such that $\overrightarrow{LN} = \frac{1}{3}\overrightarrow{NM}$.

Find the position vector \overrightarrow{ON} .

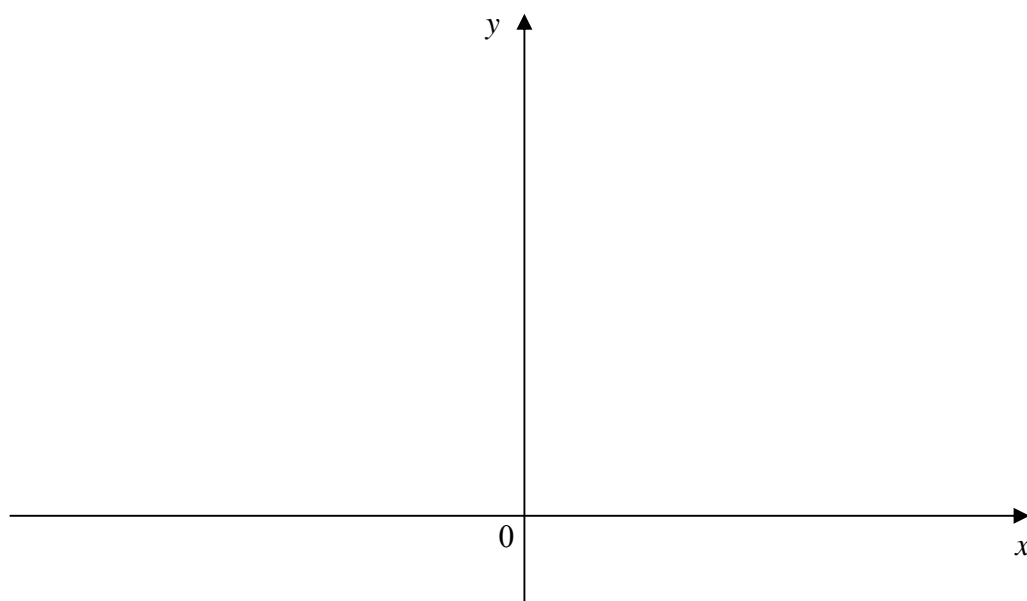
Answer $\overrightarrow{ON} = \dots\dots\dots$ [2]

21 The point $(-2, 1)$ lies on the graph $y = \frac{a}{x^2}$.

(a) Find the value of a .

Answer $a = \dots\dots\dots$ [1]

(b) Hence, sketch the graph of $y = \frac{a}{x^2}$ on the axes below.



[1]

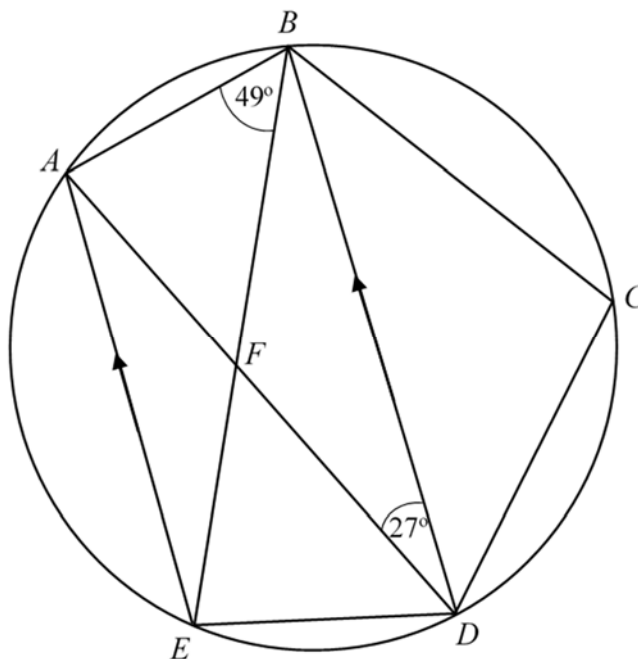
- 21 (c)** Explain how you can tell from the graph, the number of solutions to the equation $\frac{a}{x^2} = k$ for positive values of k .

Answer

.....

..... [2]

- 22** The diagram shows a circle that passes through A, B, C, D and E .
 The lines AE and BD are parallel.
 Angle $ADB = 27^\circ$ and angle $ABE = 49^\circ$.



- (a)** Find the angle AFE .
 Show your working and give reasons.

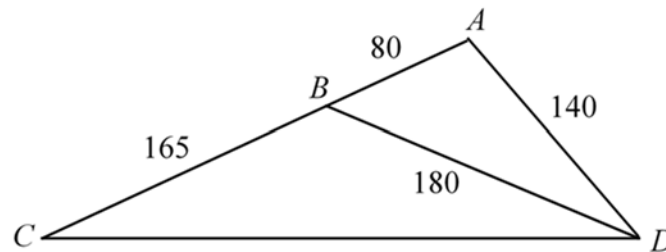
Answer $^\circ$ [3]

22 (b) Find angle BCD .

Show your working and give reasons.

Answer° [2]

23 ABD is a triangle where $AB = 80$ cm, $AD = 140$ cm, and $BD = 180$ cm.
 AB is produced to C and $BC = 165$ cm.



(a) Show that triangle ACD is similar to triangle ADB .

Answer

[2]

(b) Calculate the length CD .

Answer cm [1]

23 (c) Calculate the perpendicular distance from A to BD .

Answer cm [3]

24 Singapore adopts a progressive tax structure.
The table shows the tax rates for various annual income brackets.

Assessable annual Income	Chargeable income	Income tax rate (%)	Gross tax payable
≤ \$20 000	0	0	0
\$20 001 - \$30 000	First \$20 000	0	0
	Next \$10 000	2	\$200
\$30 001 - \$40 000	First \$30 000	–	\$200
	Next \$10 000	3.50	\$350
\$40 001 - \$80 000	First \$40 000	–	\$550
	Next \$40 000	7	\$2800

(Adapted from www.iras.gov.sg.)

Note: For “–” under Income tax rate, refer to Gross tax payable for the amount of tax.

(a) Calculate the amount of tax a manager has to pay if his assessable annual income is \$55 000.

Answer \$ [2]

- 24 (b)** To reduce the amount of tax payable, the manager makes use of the Supplementary Retirement Scheme (SRS). Each dollar deposited into the SRS reduces the assessable annual income by a dollar.

Calculate the amount of tax savings the manager enjoys if he deposits \$15 300 into the SRS.

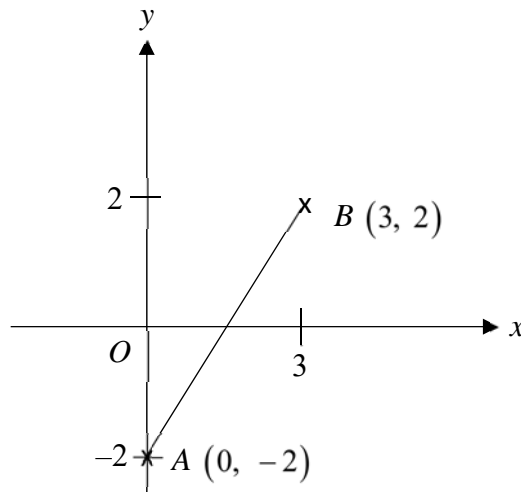
Answer \$ [2]

- (c)** Suppose the manager further invests the \$15 300 he had deposited in the SRS in a savings bond which provides a compound interest of 2.63% per year for 10 years.

Calculate the amount of money he has in the SRS after 10 years.
Give your answer correct to the nearest dollar.

Answer \$ [2]

25 Point A has coordinates $(0, -2)$. Point B has coordinates $(3, 2)$.



(a) Find the equation of the line AB .

Answer [2]

(b) Find the length of the line AB .

Answer units [2]

(c) The point D is $(0, 4)$.

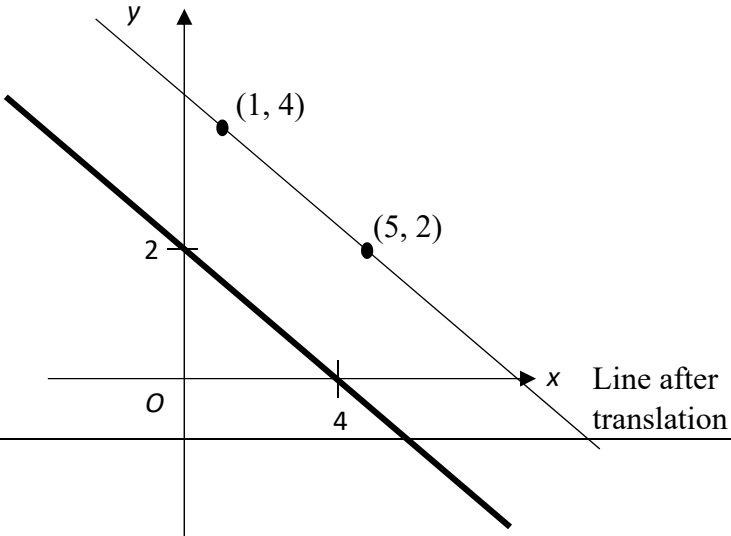
Write down the coordinates of the point C such that $ABCD$ is a parallelogram.

Answer (.....,) [1]

(d) Find the area of the parallelogram $ABCD$.

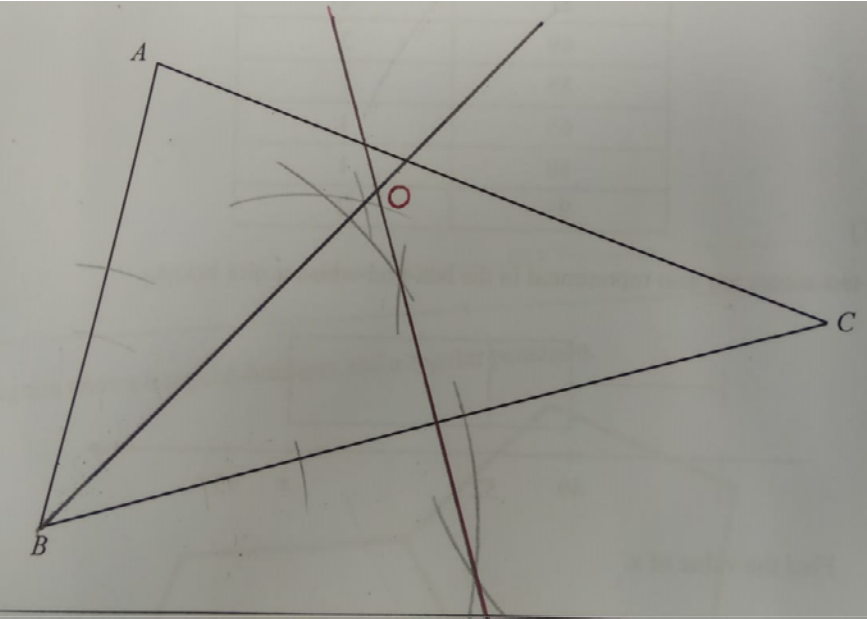
Answer units² [2]

CHIJ KC
2018 S4E/5N E Math Preliminary Examination
Paper 1 Solutions

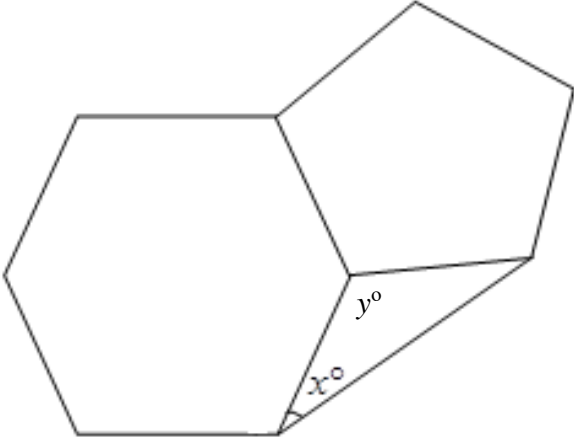
Question No.	Solution
1	0.0720
2	$7^{103} - 7^{101}$ $= 7^2 (7^{101}) - 7^{101}$ $= 7^{101} (7^2 - 1)$ $= 7^{101} (48)$ $= 7^{101} (2^4 \times 3)$ <p><i>Alternative Solution:</i></p> <p>7 raised to any index is odd. Hence 7^{103} and 7^{101} are both odd. The difference of two odd numbers is an even number.</p>
3	No. of parcels between 12 kg and 32: 5 parcels $P = \frac{5}{10} = \frac{1}{2}$
4	The point (0, 2) after translation will become (1, 4). The point (4, 0) after translation will become (5, 2)
Issues with vector translation NOT moving to coordinate (1, 2)	<p>Award <u>1 mark</u> for a straight line // to original line and passing through the above points.</p> <p>Do not penalise if students do not label points (1, 4) and/ or (5, 2)</p> 

- Common factor is 701^{101}
- Not the same as $7^{103-101} = 7^{103} - 7^{101}$
- 49 or 7 are NOT multiples of 2
- $7^2 \neq 14$

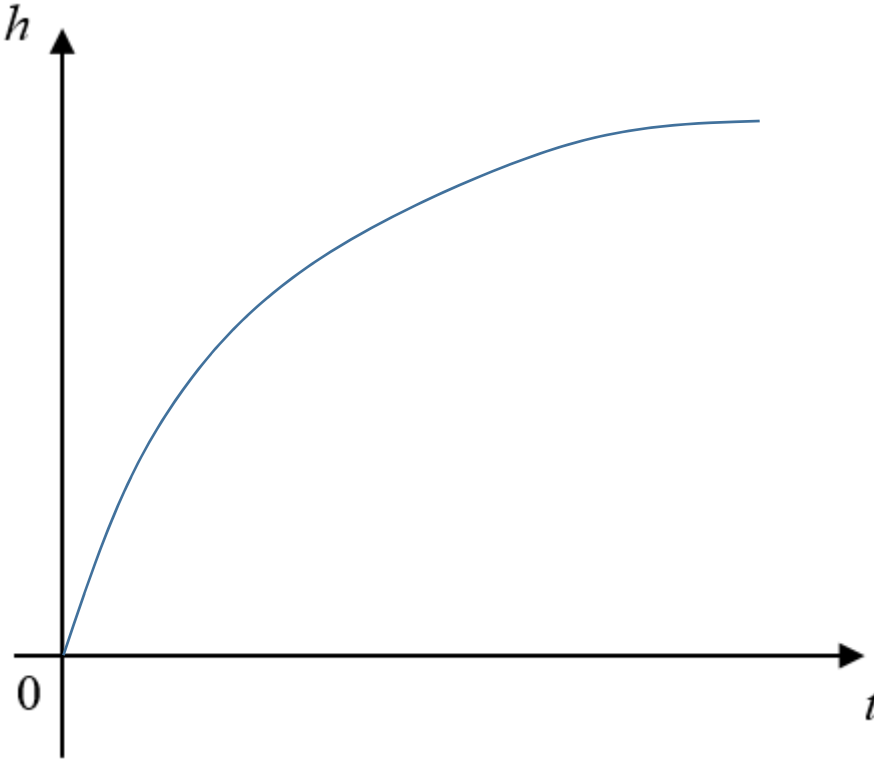
- Given answers:**
- $\frac{5}{10} = \frac{1}{5}$
 - $\frac{5}{10} = \frac{2}{5}$

<p>5</p>	$72 = 2^3 \times 3^2$ $\sqrt[3]{72} = \sqrt[3]{2^3 \times 3^2}$ $= 2 \times 3^{\frac{2}{3}}$ <p>Since $3^{\frac{2}{3}}$ is not an integer, 72 is not a perfect cube.</p> <p>(A) Index of factor 3 is not a multiple of 3. / 3^2 is not a perfect cube</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Perfect cube – cube of an integer (any index in <u>multiples of 3</u>)</p> <ul style="list-style-type: none"> - “3^2 is not a multiple of 3” o $= 3 \times 3$ which is a multiple of 3 <p>VS</p> <ul style="list-style-type: none"> - Index of factor 3 (means differently) </div>
<p>6</p>	<p>1 mark for angle bisector of angle ABC AND perpendicular bisector of BC; 1 mark for marking out the intersection of the two bisectors as O.</p> 
<p>7</p>	<p>The bar charts <u>do not start from \$0</u>.</p> <p>A student may <u>infer the water bill directly from the height of the bar charts</u> without looking at the axes and conclude that the <u>water bill in June is twice that of May, and the water bill for July was thrice that of May</u>.</p> <p>(A) maybe a base amount of \$100 even with no usage Perceive water bill to be <u>lower</u> than true value (lower height of graph)</p>

	<ul style="list-style-type: none"> - Question is not about reading of values (already indicated on y-axis) - Many unable to state the misinterpretation (usually gave very vague responses)
<p>8</p>	<p>(a) $x = 80$ i.e. the upper quartile, position no. 8. [A1]</p> <p>(b)</p> $\text{S.D.} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= \sqrt{\frac{39785}{10} - \left(\frac{579}{10}\right)^2}$ $= \sqrt{\frac{39785}{10} - \left(\frac{579}{10}\right)^2}$ $= 25.0$ <div style="border: 1px solid black; padding: 5px; margin-left: 200px; width: fit-content;"> <p>Many error arises from manual calculation of $\sum fx^2$, etc</p> </div> <p>Do not penalise if student directly cites value for S.D. from the calculator.</p>
<p>9</p>	$3x^2 - 2x - 11 = 0$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-11)}}{6}$ $= \frac{2 \pm 11.66190379}{6}$ $= 2.276983 \text{ or } -1.610317298$ $x = 2.28 \text{ or } x = -1.61$

<p>10</p>	<p>Interior angle of hexagon = $\frac{(6-2)180^\circ}{6} = 120^\circ$</p> <p>Interior angle of pentagon = $\frac{(5-2)180^\circ}{5} = 108^\circ$</p> <p>angle $y = 360^\circ - 120^\circ - 108^\circ = 132^\circ$</p> <p>$x = \frac{180^\circ - 132^\circ}{2}$ (isoceles triangle)</p> <p>$= 24^\circ$</p> 
<p>11(a)</p>	<p>$2(3x + 5) - 2(1 - 2x)$</p> <p>$= 6x + 10 - 2 + 4x$</p> <p>$= 10x + 8$</p> <p>$= 2(5x + 4)$</p>
<p>11(b)</p>	<p>$18 - 24x + 8x^2$</p> <p>$= 2(4x^2 - 12x + 9)$</p> <p>$= 2(2x - 3)^2$</p> <p>Accept: $2(3-2x)^2$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p style="color: red; margin: 0;">Reject:</p> <p style="margin: 0;">$2(3 - 2x)(3 - 2x)$</p> </div>
<p>12(a)</p>	<p>$AC^2 = 13^2 = 169$</p> <p>$AB^2 + BC^2 = 12^2 + 5^2 = 169$</p> <p>Since $AC^2 = AB^2 + BC^2$, by the converse of Phythagoras theorem, ABC is a right-angle triangle.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p style="color: red; margin: 0;">Many did not conclude. Just apply PT.</p> </div>

12(b)	Using $\sin(180 - \theta) = \sin \theta$ $\sin \widehat{ACD} = \sin \widehat{ACB}$ $= \frac{5}{13}$ Accept: 0.385 B1
13	<p>(a) $\{4, 5\} \subset B$</p> <p>(b) $2 \in A \cup B$</p> <p>(c) $B \cap C = \emptyset$</p>
14	$\frac{62 \times 1.1}{h^2} - \frac{62}{h^2} = 2.01$ $\frac{68.2}{h^2} - \frac{62}{h^2} = 2.01$ $68.2 - 62 = 2.01h^2$ $h^2 = 3.084577$ $h = 1.756$ $= 1.76 \text{ m, negative answer rejected}$ Alternative Method: Since the mass increased by 10%, the BMI must have also increased by 10%. This is because BMI and mass are directly proportional. 10% of old BMI = 2.01 Old BMI = 20.1 $\frac{62}{h^2} = 20.1$ $h = 1.756$ $= 1.76 \text{ m}$
15(a)	$a = \frac{30 - 0}{15 - 0}$ $= 2 \text{ m/s}^2$

<p>15(b)</p>	<p>let the time at which the motorcycle overtakes the car be t :</p> <p style="text-align: center;">Distance travelled by car = $20t$</p> <p style="text-align: center;">Distance travelled by motorcycle = $\frac{1}{2}(t)(2t)$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p style="color: red;">- Overtaking is NOT intersection of two lines (D-T graph)</p> </div> <div style="text-align: right; margin-top: 10px;"> $20t = \frac{1}{2}(t)(2t)$ $20t = t^2$ $t^2 - 20t = 0$ $t(t - 20) = 0$ $t = 0 \text{ or } t = \underline{20}$ </div>
<p>16</p>	<p>S.A. of cylinder = S.A. of sphere</p> $2\pi r^2 + 2\pi rh = 4\pi r^2$ $2\pi rh = 2\pi r^2$ $r = h$ <p>Vol. of cylinder = $\pi r^2 h$</p> $= \pi r^3$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto; color: red;"> <p>Accept: $3.14 r^3$</p> </div>
<p>17(a)</p>	

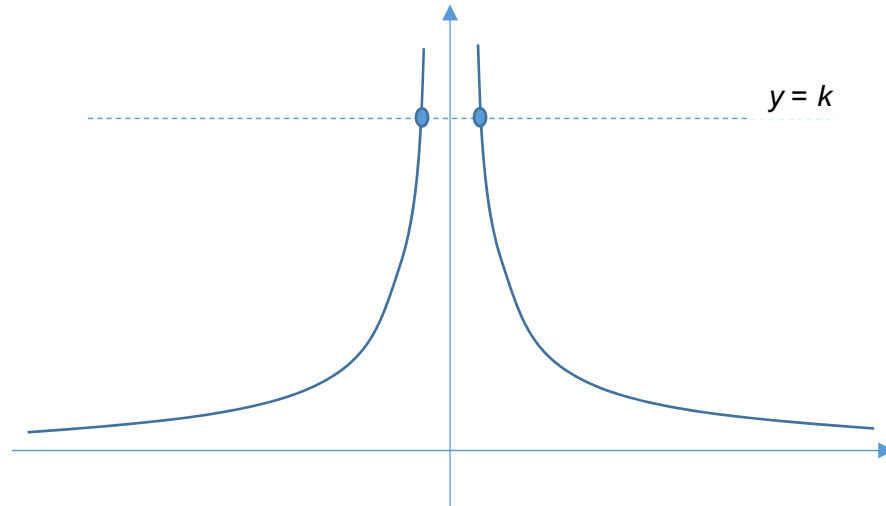
<p>17(b)</p>	$\frac{V_{10 \text{ cm}}}{V_{30 \text{ cm}}} = \left(\frac{h_{10 \text{ cm}}}{h_{30 \text{ cm}}} \right)^3$ $\frac{V_{10 \text{ cm}}}{\frac{1}{3}\pi(25)^2(30)} = \left(\frac{1}{3} \right)^3$ $V_{10 \text{ cm}} = \left(\frac{1}{27} \right) \frac{1}{3}\pi(25)^2(30)$ $= 727.22$ $= 727 \text{ cm}^3$	<p>When height changes, radius will change.</p> <p>- Need to find radius at</p>
<p>18</p>	<p>(a)</p> $y = -(x-0)(x-6)$ $= -x(x-6)$ $= -x^2 + 6x + 0$ $\therefore a = 6, b = 0$ <p>Alternatively solution:</p> <p>Subst in (0, 0)</p> $0 = b$ <p>Subst in (6, 0)</p> $0 = -36 + 6a$ $a = 6$ <p>(b)</p> $y = -x^2 + 6x$ $= -(x^2 - 6x)$ $= -[(x-3)^2 - 9]$ $= -(x-3)^2 + 9$ <p>\therefore the max. point is (3, 9)</p> <p>Alternative method:</p> <p>The quadratic curve is symmetrical about $x = 3$</p> <p>The max point is thus at $x = 3$</p> $y = -(3)^2 + 6(3) = \underline{9}$	

	<p>Common Errors:</p> <ul style="list-style-type: none"> Students substituted $a = 6$, $b = 0$ immediately, and then showed the provided coordinates were correct. Since the question asked students to prove $a = 6$, $b = 0$, students cannot substitute the values for a and b immediately. Students should substitute the provided coordinates to show $a = 6$, $b = 0$. Some students had the misconception that $b = y$-intercept. This is not true, this is not a straight-line graph.
<p>19</p>	<p>(a)</p> $\text{Length of Singapore River on Map} = \frac{3.2 \times 1000 \times 100}{80000}$ $= 4.0 \text{ cm}$ <p>(b)</p> <p>Actual size of Bishan-Ang Mo Kio Park = $620\,000 \text{ m}^2$</p> $= 620\,000 (10^2 \text{ cm})^2$ $= 620\,000 \times 10^4 \text{ cm}^2$ $= 6.2 \times 10^9 \text{ cm}^2$ <p>map:Actual</p> <p>1 cm : 80 000 cm</p> $1 \text{ cm}^2 : 6.4 \times 10^9 \text{ cm}^2$ $\text{Size of park on Map} = \frac{6.2 \times 10^9}{6.4 \times 10^9}$ $= 0.96875 \text{ cm}^2$ <p>Common Errors:</p> <ul style="list-style-type: none"> 1 cm on map : 800 m on actual ground. Students commonly wrote 1 cm : 80 m, or 1 cm : 80 km. 1 cm² on map : 800² m² on actual ground i.e. 1 cm² on map : 640 000 m². Students commonly wrote 1 cm : 800 m², or 1 cm : 6400 m². Students also could not convert m² to cm² e.g. students commonly wrote $620\,000 \text{ m}^2 = 6.2 \times 10^7 \text{ cm}^2$ when it should be $6.2 \times 10^9 \text{ cm}^2$.
<p>20(a)</p>	$ \overline{LM} = \sqrt{(16-2)^2 + (16-1)^2}$ $= 20.518$ $= 20.5 \text{ units}$

	<p>Common Errors:</p> <ul style="list-style-type: none"> Some students determined the vector \overline{LM} incorrectly. Of these students, a handful incorrectly determined \overline{LM} as $\overline{OL} - \overline{OM}$ which is \overline{ML} instead. Students who got this question wrong often could not recall the correct formula for the length of a vector.
20(b)	$\overline{LN} = \frac{1}{3} \overline{NM}$ $\overline{ON} - \overline{OL} = \frac{1}{3} (\overline{OM} - \overline{ON})$ $\frac{4}{3} \overline{ON} = \overline{OL} + \frac{1}{3} \overline{OM}$ $\overline{ON} = \frac{3}{4} \overline{OL} + \frac{1}{4} \overline{OM}$ $= \frac{3}{4} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 16 \\ 16 \end{pmatrix}$ $= \begin{pmatrix} 4.75 \\ 5.5 \end{pmatrix}$ <p>Accept: $\begin{pmatrix} 4\frac{3}{4} \\ 5\frac{1}{2} \end{pmatrix}$</p> <p>Marker's Comments M1 awarded only if student has expressed the vectors in the position vectors, \overline{LO}, \overline{NO}, must be converted to \overline{OL} and \overline{ON}.</p>
21	<p>(a)</p> $y = \frac{a}{x^2}$ <p>Subst (-2, 1) into the equation</p> $1 = \frac{a}{4}$ $a = 4$ <p>Common Errors</p> <ul style="list-style-type: none"> a = - 4. Careless mistake arose because students did not put brackets – poor book keeping i.e. if students had put brackets a = (-2)², they would be less likely to make the careless mistake.

(b)

1 mark for correct shape, and curve drawn in the 1st and 2nd quadrant.



Common Errors

Some graphs resembled the solution provided above but was not given credit because:

- (1) the curve deflected backwards, or
- (2) the curve intersected the y or x axis, or
- (3) the curve was skewed/ unsymmetrical, or
- (4) only one side of the graph was provided.

(c)

There will be two solutions.

Any **horizontal** straight line above $y = 0$ will intersect the graph twice i.e. once in the 2nd quadrant, and another time in the first quadrant.

22

(a)

$$\begin{aligned}\angle AEB &= \angle ADB \quad (\text{angles in the same segment}) \\ &= 27^\circ\end{aligned}$$

$$\begin{aligned}\angle EAD &= \angle ADB \quad (\text{alternate angles since } AE \parallel BD) \\ &= 27^\circ\end{aligned}$$

$$\begin{aligned}\angle AFE &= 180^\circ - \angle AEB - \angle EAD \\ &= 126^\circ\end{aligned}$$

(b)
 $\angle AFE = \angle DAB + \angle ABE$ (Exterior angle = Sum of Interior Opposite angles)

$$126^\circ = \angle DAB + 49^\circ$$

$$\begin{aligned}\angle DAB &= 126^\circ - 49^\circ \\ &= 77^\circ\end{aligned}$$

$$\begin{aligned}\angle BCD &= 180^\circ - \angle DAB \quad (\text{Opposite angles of a cyclic quadrilateral add up to } 180^\circ) \\ &= 180^\circ - 77^\circ \\ &= 103^\circ\end{aligned}$$

Common Mistakes

Students (incorrectly) assume F is the centre of the circle, or triangle AEB is a right angle triangle, or triangles AFE and BFD are isosceles triangles.

23

(a)
The two triangles share $\angle CAD$

$$\frac{AB}{AD} = \frac{80}{140} = \frac{4}{7}$$

$$\frac{AD}{AC} = \frac{140}{80+165} = \frac{4}{7}$$

$$\therefore \frac{AB}{AD} = \frac{AD}{AC} = \frac{4}{7} \quad [\text{must show the two fractions} = \frac{4}{7}]$$

\therefore triangle ACD is similar to triangle ADB because the corresponding sides are of the same proportion, and the included angle is the same

Accept: If students suggest triangle ACD is similar to triangle ADB because of SAS.

(b)

$$\frac{AD}{AC} = \frac{DB}{CD}$$

$$\frac{140}{245} = \frac{180}{CD}$$

$$CD = 315 \text{ cm}$$

(c)

$$\cos(\angle ABD) = \frac{80^2 + 180^2 - 140^2}{2(80)(180)}$$

$$\angle BAD = 48.18968^\circ$$

$$\text{Area of triangle } ABD = \frac{1}{2}(BD)(\text{perpendicular distance}) = \frac{1}{2}(AB)(BD)\sin(\angle ABD)$$

$$\text{perpendicular distance} = (80)\sin(48.18968^\circ)$$

$$\text{P. distance} = 59.6284$$

$$= 59.6 \text{ cm}$$

Common Errors

For part (a), many students wrote the following, which resulted in a penalty for bad presentation. Of course $4/7$ is equal to $4/7$.

$$\begin{aligned}\frac{AB}{AD} &= \frac{AD}{AC} \\ \frac{80}{140} &= \frac{140}{80+165} \\ \frac{4}{7} &= \frac{4}{7}\end{aligned}$$

Part (C) was poorly done. For e.c.f., there will be no A1 marks.

24

(a)

First \$40 000, tax = \$550

$$\begin{aligned}\text{Next } \$15\,000, \text{ tax} &= \frac{7}{100} \times 15\,000 \\ &= \$1050\end{aligned}$$

$$\begin{aligned}\text{Total Tax} &= \$550 + \$1050 \\ &= \$1600\end{aligned}$$

(b)

If manager deposits \$15 300 into SRS,
his assessable annual income becomes \$39 700

First \$30 000, tax = \$200

Next \$9700, tax = \$339.50

Total tax = \$539.50

$$\text{Amount saved} = \$1600 - \$539.50 = \underline{\$1060.50}$$

(c)

$$\begin{aligned}A &= P \left(1 + \frac{R}{100} \right)^n \\ &= 15300 \left(1 + \frac{2.63}{100} \right)^{10} \\ &= \$19835.11 \\ &= \$19835\end{aligned}$$

Common Errors

- For part (a), most students knew that they had to refer to the last row of the table. Those who got this wrong usually calculated total tax = \$550 + 2800 or \$550 + \$2800 + 7% of \$15 000. The answer is just \$550 + 7% of the remainder i.e. \$15 000.
- Students who were unable to do part (a) usually could not do part (b) as well.
- For part (c), many students did not heed the question's instruction to leave the answer to the nearest dollar, resulting in the loss of 1 mark.

25

(a)

$$\text{gradient, } m = \frac{y_1 - y_2}{x_1 - x_1}$$

$$= \frac{2 - (-2)}{3 - 0}$$

$$= \frac{4}{3}$$

$$y = \frac{4}{3}x - 2$$

(b)

$$\text{length } AB = \sqrt{4^2 + 3^2}$$
$$= 5$$

Accept if student uses $\sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$

(c)

$C : (3, 8)$ i.e. 6 units above $(3, 2)$

(d)

Area of parallelogram = base \times height

$$= BC \times (\text{distance from } AD \text{ to } BC)$$

$$= 6 \times 3$$

$$= 18 \text{ units}^2$$

Common Errors

- For part (c), a common mistake was $(-3, 0)$. Although students would get a parallelogram, the letters would not be in running order i.e. ABDC.
- When calculating the parallelogram's area, a handful of students took the product of the length of the sides which is incorrect. It should be base \times perpendicular height.

Name: _____ ()

Class: _____



**CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS /
5 NORMAL (ACADEMIC)**

**MATHEMATICS
PAPER 2**

**4048/02
2 hours 30 minutes**

Classes: 401, 402, 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, hand in **separately**:

1. Section A with cover page
2. Section B

The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

Section A

1 (a) Simplify $\left(\frac{ab}{c}\right)^3 \times ab^{-3}c^2$. [2]

(b) Express as a single fraction in its simplest form $\frac{x}{(1-5x)^2} + \frac{3}{1-5x}$. [2]

(c) Factorise completely $3rx - 3y + x - 9ry$. [2]

(d) Solve the inequality $-2 \leq \frac{4x-10}{5} < 2$. [2]

(e) It is given that $\frac{x}{4-hy} = \frac{1}{4y+h}$.

Express y in terms of x and h . [3]

2 Every Thursday, Sana jogs a distance of j km and then walks a distance of w km. She jogged at 9 km/h and walked at 5 km/h.

(a) Write down an expression, in terms of j , for the length of time that Sana jogged. [1]

(b) Sana travelled a total distance of 8 km.
She jogged half an hour more than she walked.

Write down two simultaneous equations in j and w to represent this information. [2]

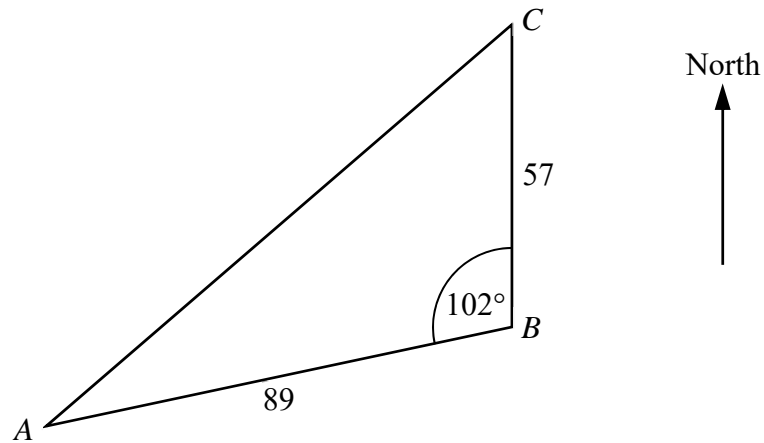
(c) Solve your simultaneous equations to find j and w . [3]

(d) Find Sana's average speed for the total distance. [2]

(e) One Thursday, Sana increases her speed by 120% for the distance of w km.

Find the percentage decrease in the time that she takes to travel the w km. [2]

3



ABC is a triangular plot of land.
 $AB = 89$ m, $BC = 57$ m and angle $ABC = 102^\circ$.
 B is due south of C .

- (a) Calculate AC . [2]
- (b) Calculate the bearing of C from A . [2]
- (c) P is a point vertically above B .
 The height BP is 14 m.
 - (i) Calculate the angle of elevation of P from A . [2]
 - (ii) $PABC$ is a pyramid with vertex P and base ABC .
 Calculate the volume of the pyramid $PABC$.
 Give your answer correct to the nearest 10 m^3 . [3]

4 The first four terms in a sequence of numbers are given below.

$$T_1 = 6 + (1 - 2)^2 - 2 = 5$$

$$T_2 = 6 + (2 - 2)^2 - 4 = 2$$

$$T_3 = 6 + (3 - 2)^2 - 6 = 1$$

$$T_4 = 6 + (4 - 2)^2 - 8 = 2$$

(a) Find T_5 . [1]

(b) Show that the n th term of the sequence, T_n , is given by $n^2 - 6n + 10$. [2]

(c) T_k and T_{3k} are terms in the sequence.

It is given that $\frac{T_{3k}}{T_k} = 17$.

Show that this equation simplifies to

$$2k^2 - 21k + 40 = 0. \quad [3]$$

(d) Solve the equation $2k^2 - 21k + 40 = 0$. [3]

(e) Explain why one of the solutions in part (d) must be rejected as the position of T_k in the sequence. [1]

Section B

Start Section B on a new sheet of writing paper.

- 5** A café serves cappuccinos (C) and lattes (L).
 Each cup of cappuccino contains 60 ml of espresso, 60 ml of milk and 60 ml of foam.
 Each cup of latte contains 60 ml of espresso, 300 ml of milk and no foam.

- (a) This information can be represented in the 3×2 matrix, \mathbf{V} .

$$\mathbf{V} = \begin{pmatrix} & \text{C} & \text{L} \\ & & \\ & & \\ & & \end{pmatrix} \begin{array}{l} \text{Espresso} \\ \text{Milk} \\ \text{Foam} \end{array}$$

Copy and complete the matrix \mathbf{V} . [1]

- (b) In one day, the café sold 26 cups of cappuccino and 45 cups of latte.

Evaluate the matrix $\mathbf{S} = \mathbf{V} \begin{pmatrix} 26 \\ 45 \end{pmatrix}$. [2]

- (c) Explain what each element in matrix \mathbf{S} represents. [1]

- (d) 60 ml of espresso costs 30 cents.

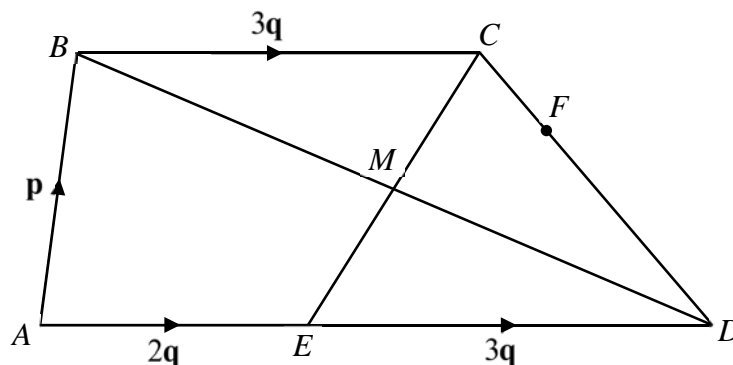
The elements of the matrix \mathbf{E} , where $\mathbf{E} = \mathbf{UV}$, represent the costs, in cents, of the espresso contained in each cup of cappuccino and each cup of latte.

Write down the matrix \mathbf{U} . [1]

- (e) Foam is made from milk.
 100 ml of milk makes 350 ml of foam.

Calculate the largest number of cups of cappuccino that 2 litres of milk can make. [3]

6



$ABCD$ is a quadrilateral and E is a point on AD .

M is the point of intersection of BD and CE .

$\vec{AB} = \mathbf{p}$, $\vec{AE} = 2\mathbf{q}$ and $\vec{BC} = \vec{ED} = 3\mathbf{q}$.

(a) Show that triangles BMC and DME are congruent.
Give a reason for each statement you make. [3]

(b) Express, as simply as possible, in terms of \mathbf{p} and \mathbf{q} ,

(i) \vec{AC} , [1]

(ii) \vec{BD} , [1]

(iii) \vec{AM} . [1]

(c) F is the point on CD such that $CF : FD = 2 : 5$.

(i) Explain why A , M and F lie in a straight line. [3]

(ii) Find the ratio area of triangle AME : area of triangle FMD . [1]

7 Answer the whole of this question on a sheet of graph paper.

A ball is thrown upwards.

The height of the ball, y metres, t seconds after it is thrown is given by the formula

$$y = t^3 - 10t^2 + (22.6)t .$$

The table shows some corresponding values of t and y , correct to 1 decimal place.

t	0	0.5	1	1.5	2	2.5	3	3.25
y	0.0	8.9	13.6	14.8	13.2	9.6	4.8	p

(a) Find the value of p . [1]

(b) Using a scale of 4 cm to represent 1 second, draw a horizontal t -axis for $0 \leq t \leq 4$.
Using a scale of 1 cm to represent 1 metre, draw a vertical y -axis for $0 \leq y \leq 16$.

On your axes, draw a graph to show the height of the ball for $0 \leq t \leq 3.25$. [3]

(c) Use your graph to find the height of the ball 0.8 seconds after it is thrown. [1]

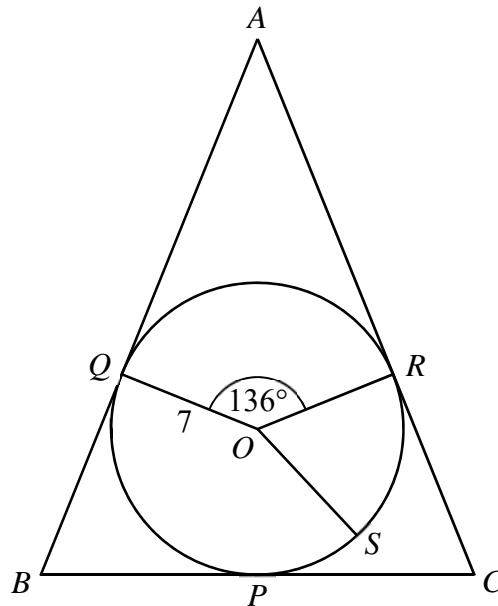
(d) By drawing a tangent, find the gradient of the curve at (2, 13.2).
State the units of your answer. [3]

(e) When the ball is thrown, a feather is dropped at a height of 9 metres.
The feather falls vertically downwards at a constant speed.
4 seconds after the ball is thrown, the feather is at a height of 5 metres.

(i) On the same axes, draw a line to show the height of the feather for $0 \leq t \leq 4$. [1]

(ii) Use your line to find when the ball first falls below the feather. [1]

8



The diagram shows a circle with centre O and radius 7 cm.
 P , Q , R and S are points on the circle.
 The tangents to the circle at P , Q and R form the triangle ABC .
 Triangle ABC is isosceles with $AB = AC$.
 Angle $QOR = 136^\circ$.

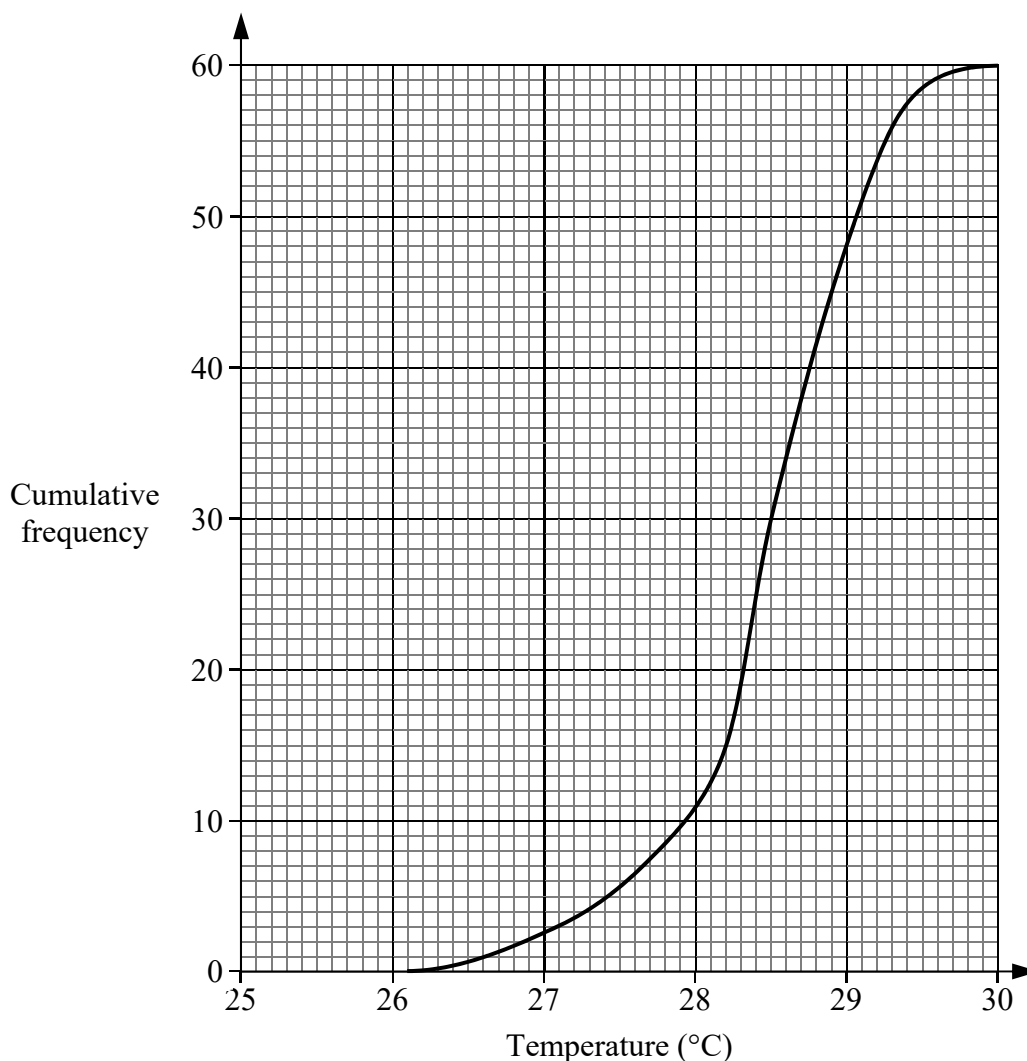
- (a) Show that angle $OAR = 22^\circ$.
 Give a reason for each step of your working. [3]

- (b) Calculate the area of the triangle ABC . [4]

- (c) Angle $ROS = \theta$ radians.
 The perimeter of the sector ORS is $2(\theta + 10)$ cm.

 Calculate the length of the arc RS . [3]

- 9 (a) The temperature at Simei was recorded every day for 60 days.
The cumulative frequency curve below shows the distribution of the temperatures.



- (i) Use the curve to estimate
- (a) the median temperature, [1]
 - (b) the interquartile range of the temperatures. [2]
- (ii) Estimate the number of days that had temperatures above 29°C. [1]

The temperature at Jurong was recorded every day for the same period.
The interquartile range of the temperatures at Jurong is 1.5°C.

- (iii) Make a comment comparing the temperatures at Simei and at Jurong. [1]
- (iv) The temperatures at Jurong are converted to degrees Fahrenheit (°F) using the formula

$$\text{temperature in } ^\circ\text{F} = 1.8 \times (\text{temperature in } ^\circ\text{C}) + 32.$$

- Find the interquartile range, in °F, of the converted temperatures. [1]

- (b)** A drawer contains 2 blue socks and 6 white socks.
Two socks are taken from the drawer at random without replacement.
If the two socks are different colours, then a third sock is taken from the drawer.
Otherwise, no third sock is taken.
- (i)** Draw a tree diagram to show the probabilities of the possible outcomes. [3]
- (ii)** Find, as a fraction in its simplest form, the probability that
- (a)** the first two socks taken are white, [1]
- (b)** a third sock is taken and it is the same colour as the first sock. [2]
-

10 Meg would like to buy an air conditioner.

(a) Meg writes down how long she would use the air conditioner in the following table.

Monday to Thursday	6 hours each day
Friday	7 hours 15 minutes
Saturday and Sunday	8 hours each day

Find the mean length of time that she would use the air conditioner each day. [2]

Meg is deciding between two models of air conditioner.

The next page shows information that she needs, including the electricity consumptions of the two models.

(b) Based on her usage, Meg estimates that the electricity consumptions in 1 year will be 1755 kWh for Model S and 1066.5 kWh for Model E.

Explain how she found these estimates. [1]

(c) The total cost of an air conditioner includes its price, the cost of the electricity it consumes and the cost of servicing it.

Electricity costs 25.3 cents per kWh, including GST.

Meg would like the air conditioner to be serviced once every 4 months.

Based on her usage, which model will have a lower total cost after 7 years of use?

Justify your decision with calculations. [7]

(You should assume that the costs of electricity and servicing remain the same.)

Residential Air Conditioners

	Model S (Standard)	Model E (Energy efficient)
Price of air conditioner	\$650	\$1300
Electricity consumption in one year	2080 kWh	1264 kWh

Notes:

- Prices include GST
- Electricity consumptions are based on 8 hours of use each day

Service Contracts

Frequency	Price per service before 7% GST
1 service every 2 months	\$25
1 service every 3 months	\$30
1 service every 4 months	\$35

Offer: 40% discount on service contract with purchase of Model S

End of paper

Penalties:
Presentation / Rounding off [P/Ro] – 1m
Units [U] – 1m

Solution

$$\begin{aligned} 1 \quad (\mathbf{a}) \quad & \left(\frac{ab}{c}\right)^3 \times ab^{-3}c^2 \\ & = \frac{(ab)^3}{c^3} \times ab^{-3}c^2 \\ & = \frac{a^3b^3}{c^3} \times \frac{ac^2}{b^3} \\ & = \frac{a^4}{c} \end{aligned}$$

$$\begin{aligned} 1 \quad (\mathbf{b}) \quad & \frac{x}{(1-5x)^2} + \frac{3}{1-5x} \\ & = \frac{x+3(1-5x)}{(1-5x)^2} \\ & = \frac{3-14x}{(1-5x)^2} \end{aligned}$$

$$\begin{aligned} 1 \quad (\mathbf{c}) \quad & 3rx - 3y + x - 9ry \\ & = 3rx - 9ry - 3y + x \\ & = 3r(x-3y) + (x-3y) \\ & = (3r+1)(x-3y) \end{aligned}$$

$$\begin{aligned} 1 \quad (\mathbf{d}) \quad & -2 \leq \frac{4x-10}{5} \text{ and } \frac{4x-10}{5} < 2 \\ & -10 \leq 4x-10 \text{ and } 4x-10 < 10 \\ & 0 \leq 4x \text{ and } 4x < 20 \\ & 0 \leq x \text{ and } x < 5 \\ & 0 \leq x < 5 \end{aligned}$$

1 (e) $\frac{x}{4-hy} = \frac{1}{4y+h}$
 $x(4y+h) = 4-hy$
 $4xy+hx = 4-hy$
 $4xy+hy = 4-hx$
 $(4x+h)y = 4-hx$
 $y = \frac{4-hx}{4x+h}$

2 (a) $\frac{j}{9}$ hours

2 (b) $j+w=8$
 $\frac{j}{9} = \frac{w}{5} + \frac{1}{2}$

2 (c) $j=8-w$

Substituting into $\frac{j}{9} = \frac{w}{5} + \frac{1}{2}$,

$$\frac{8-w}{9} = \frac{w}{5} + \frac{1}{2}$$

$$5(8-w) = 9w + \frac{45}{2}$$

$$40-5w = 9w + \frac{45}{2}$$

$$14w = 17.5$$

$$w = 1.25$$

Substituting into $j=8-w$,

$$j = 8 - 1.25$$

$$= 6.75$$

$$\therefore j = 6.75, w = 1.25$$

2 (d) Total distance = 8

$$\begin{aligned}\text{Total time} &= \frac{6.75}{9} + \frac{1.25}{5} \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Average speed} &= \frac{8}{1} \\ &= 8 \text{ km/h}\end{aligned}$$

2 (e) New speed = $5 \times 220\%$
 $= 11$

$$\text{New time} = \frac{1.25}{11}$$

Percentage decrease

$$\begin{aligned}& \frac{1.25}{5} - \frac{1.25}{11} \\ &= \frac{5 - 11}{\left(\frac{1.25}{5}\right)} \times 100\% \\ &= 54.5\% \text{ (3 s.f.)}\end{aligned}$$

Alternative method:

$$\begin{aligned}\text{Ratio of old speed : new speed} \\ &= 100 : (100 + 120) \\ &= 5 : 11\end{aligned}$$

Length of time is inversely proportional to speed,
so the ratio of old time to new time is 11 : 5 .

Percentage decrease

$$\begin{aligned}&= \frac{11 - 5}{11} \times 100\% \\ &= 54.5\% \text{ (3 s.f.)}\end{aligned}$$

3 (a) $AC^2 = 89^2 + 57^2 - 2(89)(57)\cos 102^\circ$
 $AC = 115.24$
 $= 115 \text{ m (3 s.f.)}$

3 (b) $\frac{\sin \angle ACB}{89} = \frac{\sin 102^\circ}{115.24}$
 $\angle ACB = \sin^{-1}\left(\frac{\sin 102^\circ}{115.24} \times 89\right)$
 $= 49.062^\circ$
Bearing of C from $A = 049.1^\circ$ (1 d.p.)

3 (c) (i) Angle of elevation of P from A
 $= \tan^{-1}\left(\frac{14}{89}\right)$
 $= 8.9^\circ$ (1 d.p.)

3 (c) (ii) Area of triangle ABC
 $= \frac{1}{2}(89)(57)\sin 102^\circ$
 $= 2481.1$

Volume of pyramid $PABC$

$$= \frac{1}{3}(2481.1)(14)$$
$$= 11578$$
$$= 11580 \text{ m}^3 \text{ (to nearest } 10 \text{ m}^3\text{)}$$

4 (a) $T_5 = 6 + (5 - 2)^2 - 10$
 $= 5$

4 (b) $T_n = 6 + (n - 2)^2 - 2n$
 $= 6 + n^2 - 4n + 4 - 2n$
 $= n^2 - 6n + 10$

$$4 \quad (c) \quad \frac{(3k)^2 - 6(3k) + 10}{k^2 - 6k + 10} = 17$$

$$\frac{9k^2 - 18k + 10}{k^2 - 6k + 10} = 17$$

$$9k^2 - 18k + 10 = 17k^2 - 102k + 170$$

$$8k^2 - 84k + 160 = 0$$

$$2k^2 - 21k + 40 = 0 \text{ (shown)}$$

$$4 \quad (d) \quad (2k - 5)(k - 8) = 0$$

$$2k - 5 = 0 \text{ or } k - 8 = 0$$

$$k = 2.5 \text{ or } k = 8$$

4 (e) $k = 2.5$ must be rejected
because it is not a positive integer

$$5 \quad (a) \quad \mathbf{V} = \begin{pmatrix} 60 & 60 \\ 60 & 300 \\ 60 & 0 \end{pmatrix}$$

$$5 \quad (b) \quad \mathbf{S} = \begin{pmatrix} 60 & 60 \\ 60 & 300 \\ 60 & 0 \end{pmatrix} \begin{pmatrix} 26 \\ 45 \end{pmatrix}$$

$$= \begin{pmatrix} 60 \times 26 + 60 \times 45 \\ 60 \times 26 + 300 \times 45 \\ 60 \times 26 + 0 \times 45 \end{pmatrix}$$

$$= \begin{pmatrix} 4260 \\ 15060 \\ 1560 \end{pmatrix}$$

5 (c) The elements in \mathbf{S} represent the total volumes, in ml,
of espresso, milk and foam in the drinks sold.

Alternative answer:

In the drinks sold, there was a total of
4260 ml of espresso,
15060 ml of milk, and
1560 ml of foam.
These are the elements in \mathbf{S} .

5 (d) From $\mathbf{E} = \begin{pmatrix} 30 & 30 \end{pmatrix}$,

$$\mathbf{E} = \mathbf{U} \begin{pmatrix} 60 & 60 \\ 60 & 300 \\ 60 & 0 \end{pmatrix}$$

$$\begin{aligned} \therefore \mathbf{U} &= \begin{pmatrix} 30 & 0 & 0 \\ 60 & & \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix} \end{aligned}$$

5 (e) Volume of milk to make 60 ml of foam

$$\begin{aligned} &= 60 \times \frac{100}{350} \\ &= 17.143 \end{aligned}$$

Volume of milk to make one cup of cappuccino

$$\begin{aligned} &= 60 + 17.143 \\ &= 77.143 \end{aligned}$$

Largest number of cups of cappuccino

$$\begin{aligned} &= \frac{2 \times 1000}{77.143} \\ &= 25.926 \\ &= 25 \text{ (rounded down to nearest integer)} \end{aligned}$$

6 (a) $\overline{BC} = \overline{ED}$, so BC and ED are parallel.

$$\angle BCM = \angle DEM \quad (\text{alternate angles, } BC \parallel ED)$$

$$BC = DE \quad (\text{since } \overline{BC} = \overline{ED})$$

$$\angle CBM = \angle EDM \quad (\text{alternate angles, } BC \parallel ED)$$

Therefore, triangles BMC and DME are congruent (ASA).

Alternative method:

$\overline{BC} = \overline{ED}$, so BC and ED are parallel.

$\angle BMC = \angle DME$ (vertically opposite angles)

$\angle BCM = \angle DEM$ (alternate angles, $BC \parallel ED$)

$BC = DE$ (since $\overline{BC} = \overline{ED}$)

Therefore, triangles BMC and DME are congruent (ASA).

6 (b) (i) From triangle ABC ,

$$\overline{AC} = \mathbf{p + 3q}$$

6 (b) (ii) From triangle ABD ,

$$\overline{BD} = \mathbf{5q - p}$$

6 (b) (iii) Since triangles BMC and DME are congruent,

$$BM = DM$$

$$\overline{BM} = \frac{1}{2} \overline{BD}$$

$$\overline{AM} = \overline{AB} + \overline{BM}$$

$$= \mathbf{p} + \frac{1}{2} \overline{BD}$$

$$= \mathbf{p} + \frac{1}{2} (5q - p)$$

$$= \frac{1}{2} \mathbf{p} + \frac{5}{2} \mathbf{q}$$

$$\begin{aligned}
 \text{6 (c) (i) } \overrightarrow{CD} &= \overrightarrow{AD} - \overrightarrow{AC} \\
 &= 5\mathbf{q} - (\mathbf{p} + 3\mathbf{q}) \\
 &= 2\mathbf{q} - \mathbf{p}
 \end{aligned}$$

$$\begin{aligned}
 \overrightarrow{CF} &= \frac{2}{7}\overrightarrow{CD} \\
 &= \frac{2}{7}(2\mathbf{q} - \mathbf{p})
 \end{aligned}$$

$$\begin{aligned}
 \overrightarrow{AF} &= \overrightarrow{AC} + \overrightarrow{CF} \\
 &= \mathbf{p} + 3\mathbf{q} + \frac{2}{7}(2\mathbf{q} - \mathbf{p}) \\
 &= \frac{5}{7}\mathbf{p} + \frac{25}{7}\mathbf{q} \\
 &= \frac{10}{7}\left(\frac{1}{2}\mathbf{p} + \frac{5}{2}\mathbf{q}\right) \\
 &= \frac{10}{7}\overrightarrow{AM}
 \end{aligned}$$

This shows that \overrightarrow{AF} and \overrightarrow{AM} are parallel.

Also, \overrightarrow{AF} and \overrightarrow{AM} have the common point A .

Therefore, the points A , M and F lie on a straight line.

$$\begin{aligned}
 \text{6 (c) (ii) } \text{area of triangle } AME &: \text{area of triangle } EMD = 2 : 3 \\
 \text{area of triangle } EMD &: \text{area of triangle } CMD = 1 : 1 \\
 \text{area of triangle } CMD &: \text{area of triangle } FMD = 7 : 5
 \end{aligned}$$

$$\text{area of triangle } AME : \text{area of triangle } FMD = 14 : 15$$

$$\text{7 (a) } p = 2.153125$$

- 7 (b) Horizontal axis drawn covering $0 \leq t \leq 4$ with correct scale
 Vertical axis drawn covering $0 \leq y \leq 16$ with correct scale
 All 8 points plotted
 Smooth curve drawn through plotted points



7 (c) 12.2 m (or from your graph)

7 (d) Tangent drawn at $t = 2$ and estimated (change in y)/(change in x)
Gradient = -4.8 to -6.1 (exact answer: -5.4)
Units are m/s

7 (e) (i) Line drawn from $(0, 9)$ to $(4, 5)$

7 (e) (ii) 2.85s, 2.875s, 2.9s (or from your graph)

8 (a) $\angle ARO = 90^\circ$ (tangent perpendicular to radius)

$$\angle AOR = \frac{136^\circ}{2} \quad (\text{tangents from external point})$$

$$= 68^\circ$$

$$\angle OAR = 180^\circ - 90^\circ - 68^\circ \quad (\text{angles in a triangle})$$

$$= 22^\circ \text{ (shown)}$$

Alternative method:

$$\angle AQO = \angle ARO = 90^\circ \quad (\text{tangent perpendicular to radius})$$

$$\angle QAR = 360^\circ - 136^\circ - 90^\circ - 90^\circ \quad (\text{angles in a quadrilateral})$$

$$= 44^\circ$$

$$\angle OAR = \frac{44^\circ}{2} \quad (\text{tangents from external point})$$

$$= 22^\circ \text{ (shown)}$$

$$8 \quad (b) \quad \sin 22^\circ = \frac{7}{AO}$$

$$AO = \frac{7}{\sin 22^\circ}$$

$$= 18.686$$

$$\angle ACB = \frac{180^\circ - 44^\circ}{2}$$

$$= 68^\circ$$

$$\angle OCP = \frac{68^\circ}{2}$$

$$= 34^\circ$$

$$\tan 34^\circ = \frac{7}{PC}$$

$$PC = \frac{7}{\tan 34^\circ}$$

$$= 10.378$$

Area of triangle ABC

$$= \frac{1}{2}(BC)(AP)$$

$$= \frac{1}{2}(2 \times PC)(AO + 7)$$

$$= \frac{1}{2}(2 \times 10.378)(18.686 + 7)$$

$$= 267 \text{ cm}^2 \text{ (3 s.f.)}$$

Alternative methods:

- Find AR and RC , then area is $\frac{1}{2}(AC)^2 \sin \angle BAC$
- Find AR and RC , then area is $2 \times \frac{1}{2}(AR)(7) + 4 \times \frac{1}{2}(RC)(7)$

8 (c) Perimeter of sector ORS

$$= 7 + 7 + r\theta$$

$$= 14 + 7\theta$$

$$14 + 7\theta = 2(\theta + 10)$$

$$5\theta = 6$$

$$\theta = 1.2$$

Length of arc RS

$$= 7\theta$$

$$= 7(1.2)$$

$$= 8.4 \text{ cm}$$

9 (a) (i) (a) Median = 28.5°C

9 (a) (i) (b) Interquartile range = $28.9 - 28.2$
 $= 0.7^\circ\text{C}$

9 (a) (ii) $60 - 48 = 12$ days

9 (a) (iii) The temperatures at Jurong have a larger spread than the temperatures at Simei.

Alternative answer:

The temperatures at Jurong were less consistent than the temperatures at Simei.

9 (a) (iv) After every temperature is multiplied by 1.8,

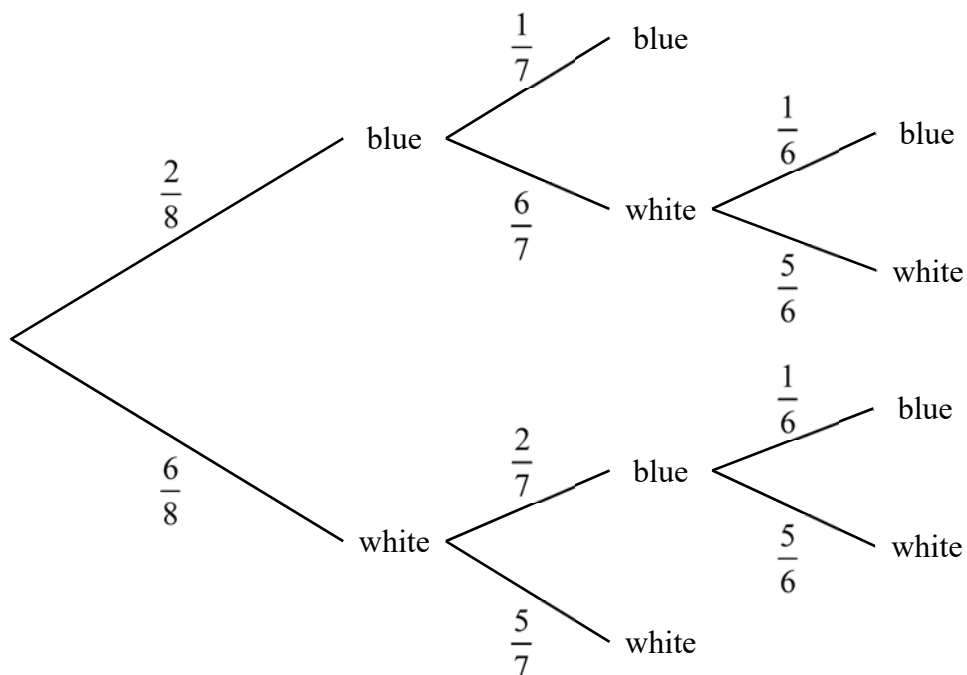
$$\text{Interquartile range} = 1.8 \times 1.5$$

$$= 2.7$$

After 32 is added to every temperature,

$$\text{Interquartile range} = 2.7^\circ\text{F}$$

9 (b) (i)



9 (b) (ii) (a) $\frac{6}{8} \times \frac{5}{7} = \frac{15}{28}$

9 (b) (ii) (b) $\left(\frac{2}{8} \times \frac{6}{7} \times \frac{1}{6}\right) + \left(\frac{6}{8} \times \frac{2}{7} \times \frac{5}{6}\right)$
 $= \frac{3}{14}$

10 (a) $\frac{6 \times 4 + 7.25 + 8 \times 2}{7}$
 $= 6.75$ hours

10 (b) Meg multiplied the given annual electricity consumptions by $\frac{6.75}{8}$.

10 (c) Model S:

$$\begin{aligned} &\text{Cost of electricity per year} \\ &= 25.3 \times 1755 \\ &= 44401.5 \text{ cents} \\ &= \$444.02 \end{aligned}$$

$$\begin{aligned} &\text{Cost of servicing per year before discount} \\ &= \$35 \times \frac{12}{4} \times \frac{107}{100} \\ &= \$112.35 \end{aligned}$$

$$\begin{aligned} &\text{Total cost of servicing per year} \\ &= \$112.35 \times \frac{100 - 40}{100} \\ &= \$67.41 \end{aligned}$$

$$\begin{aligned} &\text{Total cost of Model S} \\ &= \$650 + 7 \times (\$444.02 + \$67.41) \\ &= \$4230.01 \end{aligned}$$

Model E:

$$\begin{aligned} &\text{Cost of electricity per year} \\ &= 25.3 \times 1066.5 \\ &= 26982.45 \text{ cents} \\ &= \$269.82 \end{aligned}$$

$$\begin{aligned} &\text{Total cost of Model E} \\ &= \$1300 + 7 \times (\$269.82 + \$112.35) \\ &= \$3975.19 \end{aligned}$$

Since \$3975.19 is less than \$4230.01, Model E has a lower total cost.